Can cosmic neutrinos be detected by bremsstrahlung from a metal?

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We examine the proposal of Loeb and Starkman to detect cosmic-background neutrinos by coherent bremsstrahlung from electrons in a neutral metal. We show that the positive ions in the metal exert a restoring force which suppresses the radiation by $\sim 10^{-20}$.

I. INTRODUCTION

Loeb and Starkman have recently proposed a method for detecting cosmic-background neutrinos.[1] The method relies on the coherent interaction of neutrinos with bulk matter to get an enhanced cross section for neutrinos bouncing off a slab of metal. The neutrinos accelerate conduction electrons which then radiate low-energy photons. Loeb and Starkman calculate the photon production rate assuming free electrons and find a signal which is small but in principle detectable. In Sec. II we present a simple picture of the soft-photon production as the classical bremsstrahlung of a lump of charge which has been given some momentum perpendicular to the slab by the reflection of a neutrino. In Sec. III we show that this agrees with the soft-photon limit of the quantum calculation. In Sec. IV we show that the positive-ion background suppresses the motion of electrons, reducing the production rate of photons by $(\omega/\omega_p)^4 \sim 10^{-20}$, where ω is the photon energy and ω_p is the plasma frequency of the metal.

The presence of large restoring forces in the metal suggests that detecting the cosmic-background neutrinos through bremsstrahlung from a neutral object is not possible, independent of the particular geometry or material used. It may, however, be possible to detect the neutrinos from the bremsstrahlung of a charged object.

II. CLASSICAL PICTURE FOR SOFT PHOTONS

A neutrino scattering off a charged particle accelerates it and causes it to emit radiation. The radiation at low frequency depends only on the momentum transfer to the particle, $\Delta\mathbf{p}$, and not on the details of the interaction:[2]

$$\frac{d^2N}{d\Omega_\gamma d\omega} = \frac{1}{4\pi^2\omega}\frac{Q^2}{m^2}(\epsilon\cdot\Delta\mathbf{p})^2 , \tag{2.1}$$

where ω is the photon energy, ϵ is its polarization vector, Q is the particle's charge, and m its mass. (The units used in this paper are $\hbar = c = k_B = 1$, $e^2 \approx \frac{1}{137}$.) Since the formula depends only on the charge-to-mass ratio, a given momentum transfer will generate the same photon spectrum whether the scattering is off one particle or off many particles acting as a lump. In the soft-photon limit, the probability distribution for radiation is obtained by multiplying the classical radiation distribution (2.1) by the probability that the neutrino will scatter.

The probability for a neutrino to scatter off an extended object, a slab in this case, is found by summing the scattering amplitudes off individual charges times the appropriate phase shift for each scatterer:

$$\frac{d\sigma(\Delta\mathbf{p})}{d\Omega_\nu} = \left| \int d^3\mathbf{r}\, n(\mathbf{r}) e^{i\Delta\mathbf{p}\cdot\mathbf{r}} \right|^2 \frac{d\sigma^0}{d\Omega_\nu}$$
$$= |\tilde{n}(\Delta\mathbf{p})|^2 \frac{d\sigma^0}{d\Omega_\nu} , \tag{2.2}$$

where σ^0 is the cross section for scattering off a single particle, $\tilde{n}(\Delta\mathbf{p})$ is the Fourier transform of the number density $n(\mathbf{r})$, and $\Delta\mathbf{p}$, the momentum given to the slab by the neutrino, is an implicit function of the outgoing neutrino solid angle Ω_ν. The total cross section for a neutrino incident at angle θ_ν to scatter off a large, thin, uniform slab is[3]

$$\sigma(\theta_\nu) = \int \frac{d\sigma(\Delta\mathbf{p})}{d\Omega_\nu} d\Omega_\nu$$
$$= \tfrac{1}{2}(n g_V G_F)^2 [2E_\nu^2 - k\cdot k'] A \cos\theta_\nu \frac{\sin^2(a\Delta p/2)}{(\Delta p/2)^4} , \tag{2.3}$$

where $k = (E_\nu, \mathbf{k})$ is the initial neutrino four-momentum, k' is the final neutrino momentum, $\Delta\mathbf{p} = \mathbf{k}' - \mathbf{k}$, A is the slab area, a is the slab thickness, and n is the number density of particles, each with weak vector charge g_V. [The weak charge appears as

$$\mathcal{L} = \cdots + \frac{G_F}{\sqrt{2}} \bar{\psi}_\nu \gamma_\mu (g_V - g_A\gamma_5)\psi_\nu \bar{\psi}_e \gamma^\mu (1-\gamma_5)\psi_e .$$

For the coupling of neutrinos to bulk spinless matter, g_A may be set to 0.] For coherent scattering, the translation invariance requires $\Delta\mathbf{p}_\parallel = 0$, so the neutrino is reflected at the angle θ_ν. The term in square brackets is m_ν^2 for nonrelativistic neutrinos and $2E_\nu^2 \sin^2\theta_\nu$ for relativistic neutrinos.

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What are the scatterers in a thin metal slab that contribute to the photon production? Loeb and Starkman argue that they are the conduction electrons. The bound electrons are coupled to the ions with energies ~ 1 eV so that at the low frequencies characteristic of the cosmic-background neutrinos they do not oscillate freely and their contribution to the bremsstrahlung radiation is strongly suppressed. Only the free electrons have any hope of decoupling their motion from that of the slab. Restricting the calculation to these electrons implies that $n = n_e$, the conduction-electron density, and that $g_V^i = (4\sin^2\theta_W - 1)/2 + \delta_e^i$ where θ_W is the weak angle and δ_e^i takes account of charged-current interactions in the case of electron neutrinos.

We can treat these conduction electrons as a lump initially at rest, even though they have huge Fermi momenta, since they are exchanging momentum among themselves on a much faster time scale than they are with the incident neutrino $[(1\ \mathrm{eV})^{-1} \ll \Delta p^{-1}]$. This is the same reason that nuclei may be treated as single-point particles when they interact with sufficiently low-frequency neutrinos. The slab, however, cannot be treated as a point particle since it is larger than a neutrino wavelength; its cross section must include the Fourier-transform factor in (2.2).

For the bremsstrahlung radiation we may also treat the electrons as a lump. The radiation formula (2.1) is valid as long as the photon wavelength is larger than the size of the system which is accelerated. For this problem, the size of the electron lump which receives momentum from the neutrino can be seen from Eq. (2.2) to be of order

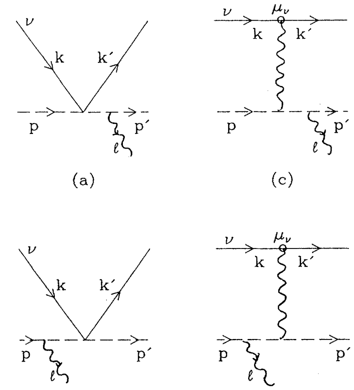

FIG. 1. Feynman diagrams (following Ref. 1) for bremsstrahlung in scattering of neutrinos off an "electron lump" for both weak-interaction (a and b) and magnetic-moment (c and d) processes. The electron lump is spinless and is therefore shown as a scalar. The neutrino four-momentum transfer is $q \equiv k' - k$ and the electron-lump three-momentum transfer is $\Delta\mathbf{p} \equiv \mathbf{p}' - \mathbf{p}$. The photon momentum is $l = (\omega, \mathbf{l})$.

$|\mathbf{r}| \approx \Delta p^{-1}$. By momentum conservation, $\omega \lesssim \Delta p$, so the electrons radiate as a point lump.

The cross section for soft bremsstrahlung is therefore given by the product of Eqs. (2.1) and (2.3) with $Q/m = e/m_e$ and $n = n_e$:

$$\frac{d\sigma_{\mathrm{brem}}(\theta_\nu)}{d\ln\omega\, d\Omega_\gamma} = \frac{Aa^2}{8\pi^2}\left[n_e g_V G_F \frac{e}{m_e}\right]^2 \frac{[2E_\nu^2 - k\cdot k'](\epsilon\cdot\Delta\mathbf{p})^2}{|\mathbf{k}|(\Delta p/2)} \frac{\sin^2(a\Delta p/2)}{(a\Delta p/2)^2} . \tag{2.4}$$

Summing over photon angles and polarizations and making the assumption that the slab is thin compared to a neutrino wavelength, we find

$$\frac{d\sigma_{\mathrm{brem}}(\theta_\nu)}{d\ln\omega} = \frac{4}{3\pi} Aa^2 \left[n_e g_V G_F \frac{e}{m_e}\right]^2 E_\nu^2 \cos\theta_\nu \times \begin{cases} 1 , \\ 2\sin^2\theta_\nu , \end{cases} \tag{2.5}$$

where the upper and lower forms refer to the limits of nonrelativistic and relativistic neutrinos, respectively.

III. QUANTUM-MECHANICAL CALCULATION

For the quantum calculation we consider the two diagrams Figs. 1(a) and 1(b). The incoming and outgoing charged "particle" is a lump of N electrons with electric charge Ne, weak charge Ng_V, mass Nm_e, and spin zero. The arbitrary size N disappears from the final result. Squaring the matrix element $\mathcal{M}_{1a} + \mathcal{M}_{1b}$, averaging over initial and summing over final neutrino helicities, and including the phase-space factors gives, for a point particle,

$$\frac{d\sigma^0_{\mathrm{brem}}}{d\Omega_{\Delta p}\, d\ln\omega\, d\Omega_\gamma} = \frac{1}{32\pi^4}\left[Ng_V G_F \frac{e}{m_e}\right]^2 \frac{[2(B\cdot k)(B\cdot k') - (k\cdot k')B^2](\Delta p)^3}{|\mathbf{k}||\mathbf{k}'\cdot\Delta\mathbf{p}|} , \tag{3.1}$$

where we have kept only the nonrelativistic electron-lump terms. Here $\Delta\mathbf{p}$ is the difference between the lump's final and initial momenta, k and k' are the initial and final neutrino momenta, and B is a timelike vector associated with the neutrino momentum transfer, $q \equiv k' - k$,

$$B^0 \equiv -\epsilon\cdot\mathbf{q} = \epsilon\cdot\Delta\mathbf{p}, \quad \mathbf{B} = 0 . \tag{3.2}$$

[We introduce the vector B because we will later refer to Eq. (3.1) with $B^i \neq 0$.] The slab is not a point but a continuous uniform distribution of electrons so we multiply by the square of the Fourier transform of the slab, as was done in the previous section. Inserting the lump density $n = n_e/N$ and integrating over $\Omega_{\Delta\mathbf{p}}$, we find

$$\frac{d\sigma_{\text{brem}}(\theta_\nu)}{d\ln\omega\, d\Omega_\gamma} = \frac{Aa^2}{8\pi^2}\left[n_e g_V G_F \frac{e}{m_e}\right]^2 \frac{[2E_\nu E'_\nu - k\cdot k'](\epsilon\cdot\Delta\mathbf{p})^2}{|\mathbf{k}||\mathbf{k}'\cdot\hat{\mathbf{z}}|}\frac{\sin^2(a\Delta p/2)}{(a\Delta p/2)^2}, \tag{3.3}$$

where now $\Delta\mathbf{p} = \Delta p\hat{\mathbf{z}}$. In the soft-photon limit, $E'_\nu \to E_\nu$, $|\mathbf{k}'\cdot\hat{\mathbf{z}}| \to \Delta p/2$, and Eq. (3.3) agrees exactly with the classical bremsstrahlung expression (2.4).

Loeb and Starkman consider the same diagrams and perform a similar summation. (This is not obvious because they have chosen different integration variables and have left their result as a numerical integral.) However, they let the in-coming charged particles be the individual conduction electrons and do an *incoherent* average over initial spins. This is inconsistent with the electrons' *coherent* interaction. Following Loeb and Starkman's prescription yields Eq. (3.1), but with $B^i = \omega\epsilon^i$. In the soft-photon limit, the components B^i become negligible compared to B^0 and the two results coincide.

IV. ELECTROSTATIC FORCES IN THE SLAB

In this section we consider the electrostatic forces inside the slab arising from the motion of the electron lump. As a result of the reflection of the neutrino, the electron lump, initially at rest, gets pushed straight down into the slab. When the electrons have moved a distance Δz, there is a residual surface density of ions on the top, $\sigma = \Delta z n_e e$, and an oppositely charged surface density on the bottom of the slab which generate an electric field $4\pi\sigma$. Since the charge-to-mass ratio is e/m_e, the force per unit mass is just $\omega_p^2\Delta z$, where $\omega_p^2 = 4\pi n_e e^2/m_e$ is the plasma frequency. The relative motion of the electrons perpendicular to the slab is therefore constrained by a harmonic-oscillator potential whose characteristic energy, ω_p, is 5 orders of magnitude higher than that of the driving force. The amplitude of the induced electric dipole is reduced by a factor $(\omega/\omega_p)^2$ relative to the free case. The radiated power is therefore suppressed by a factor $(\omega/\omega_p)^4 \sim 10^{-20}$. A rigorous analysis using Maxwell's equations with a driving force shows that the problem is truly electrostatic and therefore yields the same result.[4]

Although a coherent interaction requires that the final momentum of the electrons be perpendicular to the slab, the electrons in their intermediate state can move transversely. The transverse momentum is equal to that of the out-going photon, $l_\parallel$, and the power radiated due to electrons free to move only in this direction is given by Eq. (3.3), with $(\epsilon\cdot\Delta\mathbf{p}) \to (\epsilon\cdot l_\parallel)$.

The transverse motion of the electrons generating the radiation is not free, however. Roughly speaking, the oscillation of a lump of electrons inside the metal causes charges to pile up, generating a restoring force. An emitted photon of energy ω and momentum l is generated by motions perpendicular to l on a scale $\sim\omega^{-1}$. Charges pile up in the transverse direction on a distance scale $\sim l_z^{-1}$. A lump of electrons with volume $l_z^{-2}a$ moving a distance Δx generates a charge $Q = n_e e l_z^{-1} a\Delta x$ at each of its two sides. Up to factors of order unity, the potential energy is $Q^2 l_z$ and the force per unit mass is $\omega_p^2 l_z a\Delta x$. This estimate gives, just as for the perpendicular motion, a harmonic-oscillator restoring force and a corresponding suppression in the emission rate of $\omega^2/(\omega_p^4 a^2\cos^2\theta_\gamma)$. A rigorous analysis using Maxwell's equations yields exactly this result[4] although, in the Maxwell picture, the electric fields are induced rather than electrostatic.

Multiplying the bremsstrahlung cross section (3.3) by these suppression factors gives

$$\sigma_{\text{brem}} \sim A\left[\frac{g_V}{e}G_F\right]^2\frac{|\mathbf{k}|^6}{E_\nu^2}, \tag{4.1}$$

independent of the slab thickness and the number density and mass of the electrons. Very little bremsstrahlung is produced because the neutrino probes the slab on a scale on which the slab is neutral.

Loeb and Starkman also considered the case where the neutrino interacts with the slab through a magnetic moment, Figs. 1(c) and 1(d). The motion of electrons in the slab is suppressed, independent of the details of the driving force, so the decrease in the photon production rate is the same as for the weakly interacting case discussed above.

In this paper we considered only perfect conductors. It is easy to see by considering the forces on an oscillating dipole that imperfect conductors cause an additional suppression $[1+(\omega\gamma/\omega_{\text{rest}}^2)^2]^{-1}$, where γ is the damping constant and ω_{rest} is the frequency of the restoring force.

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