Neutrino–two-photon vertex

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We consider the flavor-diagonal neutrino–two-photon vertex in the standard model. Whereas for massless neutrinos this vertex is of order $G_F \alpha p_\nu / m_W^2$, for massive neutrinos a contribution to this amplitude of order $G_F \alpha m_\nu / m_e^2$ is identified and calculated. This contribution can be thought of as arising from a triangle graph containing electrons. The neutrino–two-photon vertex also gets a significant contribution from a second-order effect if the electromagnetic moments are anomalously large, as recently proposed to solve the solar-neutrino problem. The effects of the neutrino–two-photon vertex on neutrino-photon scattering and on photon pair annihilation into neutrinos are calculated for situations that may be of astrophysical or cosmological interest. The long-range force between a neutrino and a charge arising from two-photon exchange is also discussed.

I. INTRODUCTION

The study of neutrino interactions with photons has a long history. Early work on this topic was done in the context of what was believed in the 1960s, namely, that neutrinos had zero mass and that their interactions were invariant under the operation

$$\nu \to -\gamma_5 \nu \,, \tag{1.1}$$

where ν is any neutrino field. This has the well-known consequence that matrix elements of the electromagnetic current operator j_μ satisfy

$$\langle \nu | j_\mu | \nu' \rangle = F \bar{u} \gamma_\mu (1-\gamma_5) u' \,. \tag{1.2}$$

Here the form factor F is a function of momentum transfer q^2 which vanishes when $q^2=0$. In particular, this implies that matrix elements of the magnetic moment operator, both diagonal and off diagonal in neutrino flavor, vanish.

A similar analysis can be given for matrix elements of two powers of the electromagnetic current, which govern such processes as neutrino-photon scattering. It was shown by Gell-Mann[1] that for theories in which massless neutrinos interact locally with charged leptons, this matrix element vanishes when contracted with the wave function of physical photons. Later, calculations were carried out in which the local interaction assumption is relaxed but the neutrinos are kept massless.[2] In such theories, including the standard model in which the neutrinos interact directly with gauge bosons, there is a neutrino–two-photon matrix element, but this is second order in the Fermi constant. Furthermore, the invariance of (1.1) implies that the matrix element is proportional to an extra power of the neutrino momentum. The result of these two effects is that any massless neutrino–two-photon interaction is extremely small.

The situation for neutrino-photon interactions changes dramatically if the neutrino has a small mass. It is well known that in such theories neutrinos can have magnetic moments proportional to their mass.[3] It is less well known that the neutrino–two-photon vertex also is very different when the neutrino has a small mass. In particular, this vertex is nonzero even in the local theory, and no longer proportional to an extra power of neutrino momentum. Notice was made of this fact by Crewther, Finjord, and Minkowski,[4] who calculated the amplitude for the massive neutrino–two-photon vertex. We have independently calculated this amplitude and verified their result.

All this work has been done within the context of the standard model, the only deviation, if one considers it a deviation, being the introduction of a mass term for neutrinos. Recently, to provide an exotic solution to the solar-neutrino problem, there has been a great deal of interest in the possibility that neutrinos have electromagnetic moments much larger than predicted by the standard model.[5] One previously unnoticed consequence of a large neutrino electromagnetic moment is the enhancement of the neutrino–two-photon vertex via a second-order effect in the moment. This occurs because the magnetic-moment interaction, just as a mass term, violates the condition given in Eq. (1.1). Here we will calculate the neutrino–two-photon vertex due to the electromagnetic moment and show that it can be larger than the standard-model contribution for a wide range of neutrino masses and moments.

The expressions for the neutrino–two-photon vertex can be used to calculate several physical processes. The simplest is the Compton scattering of photons by neutrinos, and the crossing-related annihilation rate for photons into neutrino pairs. The latter may be of astrophysical interest in connection with the possible cooling of stars by neutrino pair emission. These processes were considered long ago[2] in the context of higher-order matrix elements studied earlier for massless neutrinos and found to be extremely small. Furthermore, in those theories the neutrino pairs are emitted with opposite helicities, and there is no change in helicity when a neutrino scatters a photon. On the other hand, we will see that the two-photon amplitude due to the neutrino mass is such that in the high-energy limit, the pairs are emitted with the same helicities, and neutrino-photon scattering always involves a change in neutrino helicity.

The neutrino Compton amplitude can also be used to calculate a long-range interaction between neutrinos and charged particles such as electrons. This interaction was estimated long ago also,[6] based on the Compton amplitudes derived in Ref. 2. The form of the potential becomes much simpler if the Compton amplitude due to neutrino mass or moment is used, and its magnitude is much larger, but it is unclear whether it is large enough to observe.

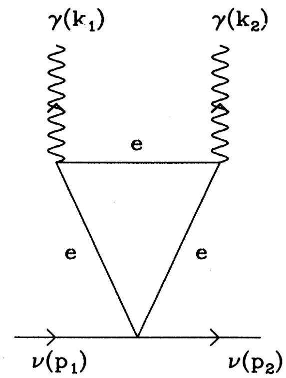

FIG. 1. To leading order in G_F, the Feynman graphs contributing to the neutrino–two-photon vertex can be collapsed to this triangle graph.

II. THE NEUTRINO–TWO-PHOTON VERTEX

The neutrino–two-photon vertex gets a contribution proportional to a single power of G_F if the neutrino has nonzero mass. This contribution can be depicted by the Feynman diagram in Fig. 1, where the intermediate (W,Z) have been contracted to a point. As pointed out by Rosenberg[7] in the present context and later by Adler[8] and by Bell and Jackiw[9] in the context of the axial-vector anomaly, care must be taken to ensure that the amplitude calculated from the triangle graph is gauge invariant. Using the results of Rosenberg, we find

$$\mathcal{M}^{\text{mass}}=\frac{\sqrt{2}e^2G_Fc_a m_\nu}{(2\pi)^4}\times\bar{u}_\nu(p_2)\gamma_5 u_\nu(p_1)\epsilon^{\sigma\rho\alpha\beta}k_{1\alpha}k_{2\beta}\epsilon_\sigma^{(1)}\epsilon_\rho^{(2)}A_3 \tag{2.1}$$

which agrees with Ref. 4. The superscript on $\mathcal{M}^{\text{mass}}$ denotes that this is the contribution to the amplitude arising from nonzero mass, to be distinguished from the contribution arising from nonzero magnetic moment discussed below. In Eq. (2.1), c_a depends on neutrino flavor, with $|c_a|^2=\frac{1}{4}$ for all three species; the momentum is routed as in Fig. 1, and $\epsilon^{(1)},\epsilon^{(2)}$ are the polarization vectors of the two photons. We note that because of the γ_5 and the $\epsilon^{\sigma\rho\alpha\beta}$, $\mathcal{M}^{\text{mass}}$ is a scalar, which if it were written in coordinate space would be proportional to the product $\mathbf{E}\cdot\mathbf{B}$. In Eq. (2.1), A_3 is a function of the momentum transfer $t\equiv(p_1-p_2)^2$. We will need A_3 in two limits, where $|t|\ll m_e^2$ and where $m_W^2\gg|t|\gg m_e^2$. In the first case we find

$$\lim A_3\rightarrow\frac{2\pi^2}{3m_e^2}\quad\text{for } |t|\ll m_e^2\ , \tag{2.2}$$

whereas for the second case

$$\lim A_3\rightarrow\frac{-8\pi^2}{t}\quad\text{for } m_W^2\gg|t|\gg m_e^2\ . \tag{2.3}$$

Note that in either case $\mathcal{M}^{\text{mass}}$ contains only a single power of G_F. The terms we have neglected by contracting the (W,Z) lines to a point in Fig. 1 and by ignoring graphs in which photons attach to W lines will contain two powers of G_F. Provided that the neutrino mass is not extremely small, these corrections will be negligible in the energy range of interest to us. The relevant criterion for the neutrino mass is approximately that

$$m_\nu/E_\nu\gg m_e^2/m_W^2\ . \tag{2.4}$$

For neutrino energies E_ν on the order m_e, this condition will be satisfied if the neutrino mass is greater than 10^{-4} eV.

For the case $|t|\ll m_e^2$, we can think of $\mathcal{M}^{\text{mass}}$ as representing a mixed electric-magnetic polarizability of the neutrino. Analogous terms have been identified for other particles.[10]

It is worth noting the reason why a neutrino-photon interaction through a term such as $\mathcal{M}^{\text{mass}}$ can arise for massive neutrinos even in a local theory. The original argument of Gell-Mann can be paraphrased as follows. In a local theory, the neutrinos enter the matrix element for $\gamma+\gamma\rightarrow\nu+\bar{\nu}$ only as a product of spinors with no additional factors of neutrino momentum; that is, the interaction is an s-wave interaction. Conservation of angular momentum implies that the total neutrino spin must equal the total photon spin along the same direction. But in the center-of-mass system, the total photon spin must equal zero or two units. Thus the total spin of the spin-$\frac{1}{2}$ neutrino and antineutrino must be 0. On the other hand, for massless chiral particles a (left-handed) neutrino and a (right-handed) antineutrino must carry opposite helicity, and so sum to one unit of spin in the center-of-mass system. Therefore, the process $\gamma+\gamma\rightarrow\nu+\bar{\nu}$ cannot occur. However, if the neutrino has mass, then the helicities of ν and $\bar{\nu}$ can be the same, and the total spin can be zero units, allowing the process to occur. This is what happens for the matrix element $\mathcal{M}^{\text{mass}}$.

Finally we note that, unlike the magnetic moment operator, the amplitude for neutrinos to scatter off photons is nonzero for Majorana neutrinos as well as for Dirac neutrinos.

Now we turn to the neutrino–two-photon vertex when the neutrino has an anomalously large electromagnetic moment. Such an interaction can be written as

$$\mathcal{L}_{\rm int}=\tfrac{1}{2}F_{\mu\nu}\bar{\nu}_i\sigma^{\mu\nu}(A_{ij}+B_{ij}\gamma_5)\nu_j\ , \tag{2.5}$$

where i,j label the different neutrino generations, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, and the matrices A and B have dimensions of e/m. Hermiticity of the interaction requires A to be Hermitian and B anti-Hermitian; CP invariance requires the matrices to be real. So assuming CP invariance, A is a real, symmetric matrix and B is a real antisymmetric matrix. This interaction can generate a neutrino–two-photon vertex through the Feynman graph in Fig. 2. In this figure, the indices on the neutrinos label the generation. The off-diagonal amplitude with $i\neq j$ gives a contribution to the decay of a heavy neutrino into a lighter one and two photons, a process examined in detail in Ref. 11. It is straightforward to check that, for allowed values of A and B and reasonable mixing angles, this contribution is smaller than the one studied in Ref. 11 in the relevant mass regime, so we will focus on the diagonal case, $i=j$. In this case, assuming CP invariance, we find

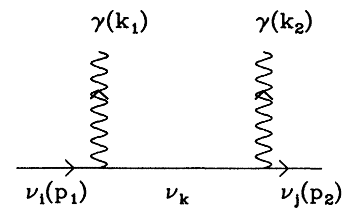

FIG. 2. The contribution to the neutrino–two-photon vertex arising from a nonzero electromagnetic moment. The graph with k_1 and k_2 interchanged must also be considered.

$$\begin{aligned}\mathcal{M}^{\rm moment}=i&\frac{\epsilon^{(1)\alpha}k_1^{\beta}\epsilon^{(2)\mu}k_2^{\nu}}{m_i^2-m_k^2-2p_1\cdot k_1}\bar{u}_i(p_2)\\ &\times\{(A_{ik}^2+B_{ik}^2)(p_1-k_1)^{\lambda}[(g_{\mu\alpha}g_{\nu\beta}-g_{\mu\beta}g_{\nu\alpha}+i\epsilon_{\mu\nu\alpha\beta}\gamma_5)\gamma_\lambda+iT_{\mu\nu\alpha\beta\lambda}]\\ &\quad+2A_{ik}B_{ki}(p_1-k_1)^{\lambda}[(g_{\mu\beta}g_{\nu\alpha}-g_{\mu\alpha}g_{\nu\beta})\gamma_5\gamma_\lambda-i\epsilon_{\mu\nu\alpha\beta}\gamma_\lambda+iT_{\mu\nu\alpha\beta\lambda}\gamma_5]\\ &\quad+(A_{ik}^2-B_{ik}^2)m_k(g_{\mu\alpha}g_{\nu\beta}-g_{\mu\beta}g_{\nu\alpha}+i\epsilon_{\mu\nu\alpha\beta}\gamma_5-iS_{\mu\nu\alpha\beta})\}u_i(p_1)\end{aligned} \tag{2.6}$$

plus a set of terms with k_1 and k_2 (along with $\epsilon^{(1)}$ and $\epsilon^{(2)}$) interchanged. Here the intermediate neutrino species k must be summed over. The tensors S and T are defined as

$$S_{\mu\nu\alpha\beta}\equiv g_{\mu\alpha}\sigma_{\nu\beta}+g_{\nu\beta}\sigma_{\mu\alpha}-g_{\nu\alpha}\sigma_{\mu\beta}-g_{\mu\beta}\sigma_{\nu\alpha} \tag{2.7}$$

and

$$T_{\mu\nu\alpha\beta\lambda}\equiv 2\sigma_{\mu\nu}(g_{\lambda\alpha}\gamma_\beta-g_{\lambda\beta}\gamma_\alpha)-S_{\mu\nu\alpha\beta}\gamma_\lambda\ . \tag{2.8}$$

One interesting limit of this amplitude is the nonrelativistic limit, wherein there exists a neutrino species with a mass m_k larger than any other mass or energy under consideration. Then the last line of (2.6) gives the dominant contribution, and after including the contribution from the $k_1\leftrightarrow k_2$ piece, the nonrelativistic amplitude is simply

$$\begin{aligned}\mathcal{M}_{\rm NR}^{\rm moment}=&\frac{-2i(A_{ik}^2-B_{ik}^2)}{m_k}\epsilon^{(1)\alpha}k_1^{\beta}\epsilon^{(2)\mu}k_2^{\nu}\\ &\times\bar{u}_i(p_2)(g_{\mu\alpha}g_{\nu\beta}-g_{\mu\beta}g_{\nu\alpha}\\ &\qquad+i\epsilon_{\mu\nu\alpha\beta}\gamma_5)u_i(p_1)\ .\end{aligned} \tag{2.9}$$

The γ_5 piece here couples the neutrino to $F\tilde{F}=\mathbf{E}\cdot\mathbf{B}$, so this part adds to the amplitude due to the mass in Eq. (2.1). The relative magnitude of the contribution from the moment compared with that from the mass is

$$\frac{\mathcal{M}^{\rm mass}}{\mathcal{M}_{\rm NR}^{\rm moment}}\sim 10^6\left[\frac{10^{-10}\mu_B}{A}\right]^2\left[\frac{m_\nu m_k}{m_e^2}\right]\ , \tag{2.10}$$

where m_ν is the mass of the neutrino of interest and m_k is the mass of the heaviest internal neutrino in Fig. 2. We have normalized A by 10^{-10} Bohr magnetons, because that is roughly the upper limit arising from experiments. We see, therefore, that if the off-diagonal magnetic moment is sufficiently large, there is a wide range of neutrino masses for which the moment-induced contribution to the two-photon vertex is the dominant one.

The $g_{\mu\alpha}g_{\nu\beta}-g_{\mu\beta}g_{\nu\alpha}$ term in (2.9) has no analogue in $\mathcal{M}^{\rm mass}$; it couples $\bar{u}u$ to $F^2=E^2-B^2$. The consequences of such a coupling for the long-range force on neutrinos will be discussed in Sec. IV.

III. COMPTON SCATTERING AND PAIR PRODUCTION

Armed with the amplitude of (2.1), we can now calculate the cross sections for various physical processes involving photons and neutrinos. Two of possible importance are Compton scattering $-\gamma+\nu\to\gamma+\nu-$ and pair production $-\gamma+\gamma\to\nu\bar{\nu}$.

For the cross sections we need to square the amplitude in (2.1), sum over final spins and polarizations; and average over initial spins and polarizations. A short calculation shows that

$$\frac{1}{4}\sum_{\text{spins, polarizations}} |\mathcal{M}_c|^2 = \frac{\alpha^2 G_F^2 c_a^2 m_\nu^2}{32\pi^6}(-t)^3 A_3^2 \ , \qquad (3.1)$$

where again the four-momentum transfer is defined as $t \equiv (p_1 - p_2)^2$. The subscript c identifies this as the amplitude for Compton scattering: $\nu\gamma \to \nu\gamma$. The pair-production amplitude squared is related to this by letting $-t \to s$ where s is the center-of-mass energy squared. Therefore,

$$\frac{1}{4}\sum_{\text{spins, polarizations}} |\mathcal{M}_p|^2 = \frac{\alpha^2 G_F^2 c_a^2 m_\nu^2}{32\pi^6} s^3 A_3^2 \ . \qquad (3.2)$$

The cross section for the reverse process $\nu\bar{\nu} \to \gamma\gamma$ considered in Ref. 4 is more subtle; it is necessary to specify the helicity of the incoming neutrinos. In Ref. 4 all neutrino states are summed over, which somewhat obscures which states are contributing. In order to see this better, let us first consider the ultrarelativistic limit in which a negative (positive) helicity state is a purely left (right) chiral neutrino. Since the two-photon–neutrino vertex produces (or annihilates) neutrinos of the same chirality, the cross section for a negative-helicity neutrino and negative-helicity antineutrino to annihilate and produce two photons can simply be obtained by integrating Eq. (3.2) over the photon phase space. However, a much more common situation is one in which only negative-helicity neutrinos and positive-helicity antineutrinos exist. Then, since the incoming ν's have opposite chiralities, the annihilation cross section vanishes. If we move out of the ultrarelativistic regime and include the effects of the mass, then a negative-helicity neutrino, for example, has a right chiral component suppressed by a factor of m/E, so the cross section for $\nu(-)\bar{\nu}(+) \to \gamma\gamma$ is suppressed relative to the reverse process by a factor of order $(m/E)^2$.

The cross section for neutrino Compton scattering can be obtained from Eq. (3.1) by integrating over phase space. We will evaluate it in the high-energy limit: $E(p_1) \simeq p_1 \gg m_e^2$. In this limit we can take the high-$|t|$ limit of the Feynman integral, (2.3). Even at high energies, $|t|$ can be very small (corresponding to forward angle scattering), so at first it may not appear valid to use this limit. However the amplitude squared is already proportional to t^3, so it vanishes at small momentum transfer. As a result, the small $|t|$ part of A_3 does not contribute at high energies, and we are justified in taking the limit above. With this, the cross section is

$$\sigma(\nu\gamma \to \nu\gamma) = \frac{1}{2p_1 2k_1}\int \frac{d^3p_2}{(2\pi)^3 2p_2}\int \frac{d^3k_2}{(2\pi)^3 2k_2}(2\pi)^4\delta^4(p_1+k_1-p_2-k_2)\tfrac{1}{4}\sum |\mathcal{M}_c|^2$$
$$= \frac{\alpha^2 G_F^2 c_a^2 m_\nu^2}{16\pi^3}(1-\cos\theta) = 3.65\times 10^{-52}\ \text{cm}^2 \left[\frac{m_\nu}{m_e}\right]^2 (1-\cos\theta) \ , \qquad (3.3)$$

where θ is the angle between $\mathbf{p}_1$ and $\mathbf{k}_1$; i.e., $\cos\theta = -1$ in the center-of-mass frame. This expression is generally valid since $c_a^2 = \frac{1}{4}$ for all three neutrino species. An important feature of this cross section is that, unlike the ordinary weak cross section, it is constant at high energies. This is particularly relevant for cosmological considerations. Since this reaction flips the helicity of the neutrinos, Compton scattering in the early Universe is one way of producing positive-helicity neutrinos. Our present understanding of primordial nucleosynthesis is compatible with observations only if positive-helicity neutrinos are not abundant at the time of light element production. Since this cross section is small, though, and does not increase with increasing energies, very few positive-helicity neutrinos are produced via this process even at the high temperatures expected in the early Universe.[12]

Parenthetically we point out that there are several other processes that might be effective at producing positive-helicity neutrinos in the early Universe. These include electron-neutrino scattering via photon exchange if the neutrino dipole moment is large enough;[13] plasmon decays $\gamma \to \nu\bar{\nu}$ again for a large neutrino dipole moment, electron-neutrino scattering via W and Z exchange due to the small positive-helicity component of massive left-handed neutrinos,[14] and decays of mesons into neutrino-antineutrino pairs. All of the amplitudes for these processes are proportional to G_F (since they are weak pro-

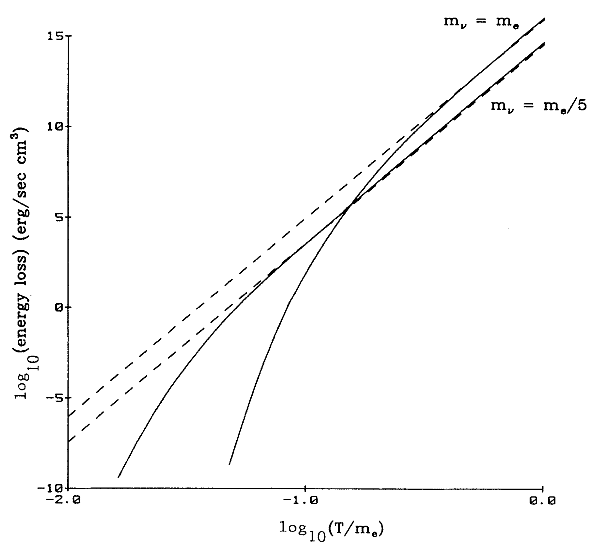

FIG. 3. Energy-loss rate of a star at temperature T due to photon pair production of neutrinos. The solid lines give the exact rate for two different values of the neutrino mass. The dashed lines are the small m_ν approximation of Eq. (3.6) wherein the rate falls off simply as m_ν^2.

cesses) times m_ν (since this is the only way a right-handed neutrino enters in the standard model). Dimensionally, then, it is apparent that the cross sections for all these processes go simply as $(G_F m_\nu)^2$ at high energies; they are typically not dangerous if the neutrino mass is not too large. This is true in the standard model, where the right-handed neutrino appears in the Lagrangian only via its mass term (coupling to a Higgs boson). However, extensions of the standard model wherein the right-handed neutrino has other couplings may be severely constrained by primordial nucleosynthesis.

The cross section for pair production is obtained in a similar fashion. This process is of potential importance as an energy-loss mechanism in stars, where temperatures are typically beneath the electron mass. Therefore, the energies involved are smaller than m_e, and we can safely take the small t limit of A_3, (2.2). Then using (3.2) and integrating over phase space gives

$$\begin{aligned}\sigma(\gamma\gamma\to\nu\bar{\nu}) &= \frac{1}{2k_1 2k_2}\int\frac{d^3p_1}{(2\pi)^3 2E(p_1)}\int\frac{d^3p_2}{(2\pi)^3 2E(p_2)}(2\pi)^4\delta^4(p_1+p_2-k_1-k_2)\tfrac{1}{4}\sum|\mathcal{M}_p|^2 \\ &= \frac{\alpha^2 G_F^2 c_a^2 m_\nu^2}{1152\pi^3}\left[\frac{s^3}{2k_1k_2m_e^4}\right]\left[1-\frac{4m_\nu^2}{s}\right]^{1/2} \\ &= 5.08\times10^{-54}\ \text{cm}^2\left[\frac{m_\nu}{m_e}\right]^2\left[\frac{s^3}{2k_1k_2m_e^4}\right]\left[1-\frac{4m_\nu^2}{s}\right]^{1/2}. \end{aligned} \tag{3.4}$$

As an example of when such a process might be relevant, let us consider a star of temperature T. Photons exist in the star with the Planck distribution $f(k)=2(e^{k/T}-1)^{-1}$. These photons are typically trapped for long times because their mean free paths are so short. Neutrinos, on the other hand, escape freely from stars. The production of neutrinos by photons is therefore an energy-loss mechanism for the star. The rate at which energy is lost can be obtained by integrating $f(k_1)$ and $f(k_2)$ times the cross section times the energy loss over all momenta:

$$\begin{aligned}\frac{\text{energy loss}}{\text{time volume}} &= \frac{1}{2}\int\frac{d^3k_1}{(2\pi)^3}\frac{2}{e^{k_1/T}-1}\int\frac{d^3k_2}{(2\pi)^3}\frac{2}{e^{k_2/T}-1}(k_1+k_2)\sigma(\gamma\gamma\to\nu\bar{\nu}) \\ &= \frac{\alpha^2G_F^2c_a^2m_\nu^2}{18\,432\pi^7m_e^4}\int_0^\infty\frac{dk_1}{e^{k_1/T}-1}\int_{m_\nu^2/k_1}^\infty\frac{dk_2}{e^{k_2/T}-1}(k_1+k_2)\int_{4m_\nu^2}^{4k_1k_2}ds\,s^3\left[1-\frac{4m_\nu^2}{s}\right]^{1/2}. \end{aligned} \tag{3.5}$$

The factor of $\frac{1}{2}$ in front of the integrals on the first line avoids double photon counting. This expression for the energy-loss rate simplifies when the neutrino mass is very small. Then all the integrations can be performed and

$$\begin{aligned}\lim_{\text{small } m_\nu}\frac{\text{energy loss}}{\text{time volume}} &= \frac{20\zeta(5)\zeta(6)}{\pi^7}\frac{\alpha^2G_F^2c_a^2m_\nu^2T^{11}}{m_e^4} \\ &= 1.0\times10^{16}\frac{\text{erg}}{\text{sec cm}^3}\left[\frac{m_\nu}{m_e}\right]^2\left[\frac{T}{m_e}\right]^{11}. \end{aligned} \tag{3.6}$$

This can be compared with the energy-loss rate calculated by Rosenberg for the density dependent process $\gamma N\to N\nu\bar{\nu}$ (where N is a nucleus). He found that at a density of 5 cm^{-3}, the energy-loss rate is $2.3\times10^9(T/m_e)^{10}$ erg sec^{-1} cm^{-3}. So pair production of neutrinos causes greater energy loss than the process $\gamma N\to N\nu\bar{\nu}$ for large enough temperatures and neutrino masses. At low temperatures the rate of Eq. (3.6) falls off very rapidly, while at high temperatures of the order of the electron mass, ordinary weak processes are much more effective than photon pair production in producing neutrinos. There may be an intermediary temperature regime, however, in which pair production by photons is the dominant energy-loss mechanism. All the other processes which lead to neutrino emission become ineffective at very low densities, while photon pair production is independent of density. Therefore, the most likely stars to be dominantly affected by this process are those with intermediate temperatures and very low densities.

For such a process to be effective, a large neutrino mass is needed. Therefore, the limit $m_\nu\to0$ cannot be taken in evaluating the above integrals. Instead, the s integral in (3.5) can be calculated analytically. The remaining double integration has been performed numerically and the energy-loss rate is shown as a function of temperature in Fig. 3 for various values of the neutrino mass.

So far we have worked in the context of the standard model. As mentioned in Sec. II, extensions of the standard model could have much larger neutrino mixed polarizabilities. We can place a model-independent limit on the neutrino mixed polarizability by rewriting some of the above formulas. First, we define the neutrino mixed polarizability α_ν as the coefficient in front of the amplitude

$$\mathcal{M}=\alpha_\nu\bar{u}_\nu(p_2)\gamma_5u_\nu(p_1)\epsilon^{\sigma\rho\alpha\beta}k_{1\alpha}k_{2\beta}\epsilon_\sigma^{(1)}\epsilon_\rho^{(2)}. \tag{3.7}$$

According to (2.1) the standard-model value of the polarizability is

$$\alpha_\nu^{\rm SM}=\sqrt{2}\alpha G_F c_a m_\nu A_3/(4\pi^3)\ . \tag{3.8}$$

In general, the polarizability depends on the momenta; in the standard model, for example, A_3 depends on s for the pair-production process. However, we may assume that at energies lower than the electron mass, it is constant (as in the standard model). If we make this assumption, then the energy-loss rate due to pair production in general can simply be read off from (3.6); we need only multiply by a factor of $(\alpha_\nu/\alpha_\nu^{\rm SM})^2$. As an example of a limit which emerges from such considerations, let us consider red giants. Sutherland *et al.*[15] restrict the energy loss due to neutrinos escaping the core to be less than 5×10^8 erg cm^{-3}sec^{-1}. In the core the temperature is approximately $T\simeq1.6\times10^8$ K$=.027m_e$. Therefore, using (3.6) we find

$$\alpha_\nu<7\times10^4\alpha_\nu^{\rm SM}(m_e/m_\nu)=4\times10^{-10}\ {\rm MeV}^{-3}\ . \tag{3.9}$$

Finally we note that the neutrino–two-photon vertex induced by a large (but within experimental limits) electromagnetic moment does not violate the limit (3.9). Indeed, it does not significantly affect any of the rates we have calculated in this section. For example, using the nonrelativistic limit (2.9), we get a contribution to the cross section for pair production of order

$$\sigma^{\rm moment}(\gamma\gamma\to\nu\bar\nu)\sim\frac{(A^2-B^2)^2s^2}{m_k^2} \tag{3.10}$$

which is equal to 10^{-64} cm^2—much smaller than that due to the mass-induced amplitude—for an intermediate neutrino mass and center-of-mass energy of order m_e even if the moments A and B are taken to be as large as 10^{-10} Bohr magnetons. The cross section for Compton scattering of neutrinos off photons induced by a large magnetic moment is similarly small.

IV. LONG-RANGE FORCES ON NEUTRINOS

Because of the neutrino Compton matrix element $\mathcal{M}^{\rm mass}$, neutrinos will experience a long-range force arising from two-photon exchange in the presence of a charged spin-$\frac{1}{2}$ particle, analogous to the familiar dispersion forces of atomic physics. A general formalism for calculating such forces from the Compton amplitude has recently been given,[10] and that formalism could be applied to the present case. However, for the purpose of obtaining the leading term in an expansion of the force in inverse powers of the distance, it is easier to use a "classical" method discussed long ago.[16]

To do this we first infer from $\mathcal{M}^{\rm mass}$ an effective Hamiltonian for the interaction of a neutrino with external electromagnetic fields.

$$H_{\rm eff}^{\rm mass}=\lambda\bar u_\nu\gamma_5 u_\nu \mathbf{E}\cdot\mathbf{B}\ , \tag{4.1}$$

where

$$\lambda\equiv i\frac{-\alpha m_\nu G_F c_a\sqrt{2}}{12\pi m_e^2}\ . \tag{4.2}$$

The additional factor of i in $H_{\rm eff}^{\rm mass}$ is required to convert from a matrix element to an effective Hamiltonian for a spin-$\frac{1}{2}$ particle.

To get the leading term in the long-range potential acting between a charge and a neutrino, we substitute into $H_{\rm eff}^{\rm mass}$ the electric and magnetic fields produced by the charge. For a negative spin-$\frac{1}{2}$ charge of mass M and no anomalous magnetic moment, these are given by

$$\mathbf{E}=\frac{-e\mathbf{r}}{4\pi r^3},\quad \mathbf{B}=\frac{3\boldsymbol{\mu}\cdot\mathbf{r}\mathbf{r}-\boldsymbol{\mu}r^2}{4\pi r^5}\ . \tag{4.3}$$

Here $\boldsymbol{\mu}$ is the magnetic moment vector of the charge, given by $\boldsymbol{\mu}=-e\mathbf{S}_c/M$, where $\mathbf{S}_c$ is the spin operator of the charge. Note that in these units $e^2/4\pi=\alpha$, the fine-structure constant.

Substituting these fields into (4.1) we obtain a potential acting on the neutrino:

$$V_{2\gamma}^{\rm mass}=\frac{\lambda}{4\pi}\bar u_\nu\gamma_5 u_\nu\frac{2\alpha\mathbf{S}_c\cdot\mathbf{r}}{Mr^6}\ . \tag{4.4}$$

So, for large r, this contribution to the two-photon-exchange potential acting between a neutrino and charge is proportional to the spin of the charge.

Because of the γ_5 in V, that potential also depends on the neutrino spin. In the nonrelativistic limit for the neutrino,

$$\bar u_\nu\gamma_5 u_\nu\to\mathbf{S}_\nu\cdot\mathbf{Q}/m_\nu\ , \tag{4.5}$$

where $\mathbf{Q}$ is the momentum transfer. Thus in the nonrelativistic limit the potential acting between a neutrino and a charge is

$$V_{2\gamma}^{\rm mass}=\frac{2\alpha\lambda}{iMm_\nu}\frac{\mathbf{S}_\nu\cdot\mathbf{S}_c r^2-6\mathbf{S}_\nu\cdot\mathbf{r}\mathbf{S}_c\cdot\mathbf{r}}{4\pi r^8}\ . \tag{4.6}$$

We note that the factor of m_ν disappears from the final answer for $V_{2\gamma}^{\rm mass}$ in this limit.

The potential (4.4) may be compared with other long-range potentials that act on neutrinos. For massless neutrinos, there are two such potentials, both analyzed in Ref. 6 (in addition to gravity which we do not consider here). One arises from the neutrino Compton amplitude that exists even for massless neutrinos, which is quadratic in G_F. This potential is noncentral and is proportional to

$$V_{2\gamma}^{m_\nu=0}\sim\frac{\alpha^2 G_F E_\nu}{m_W^2 r^4}\ . \tag{4.7}$$

This is much smaller than $V_{2\gamma}^{\rm mass}$ at atomic distances but eventually gets larger than $V_{2\gamma}^{\rm mass}$.

The other long-range interaction that occurs for massless neutrinos arises from the exchange of neutrino pairs themselves.[6] This is a central potential, proportional to G_F^2/r^5. The two-neutrino-exchange potential between a neutrino and electron is smaller than $V_{2\gamma}^{\rm mass}$ at distances smaller than about 10^{-3} cm. Of course, at distances of order m_ν^{-1} the interaction due to neutrino pair exchange damps out exponentially, while the two-photon interaction remains.

Finally, if the neutrino has a mass and is a Dirac, rather than a Majorana particle, it can have a magnetic dipole moment proportional to the mass. This will also lead to a long-range dipole-dipole interaction with a spin-$\frac{1}{2}$ charge. That interaction is proportional to the tensor operator

$$T=(\mathbf{S}_\nu\cdot\mathbf{S}_c r^2-3\mathbf{S}_\nu\cdot\mathbf{r}\mathbf{S}_c\cdot\mathbf{r})/r^2$$

as well as to the product $\mu_\nu\mu_c/r^3$, where μ is the magnitude of the dipole moment.

In the standard model, for a Dirac neutrino, μ_ν has been calculated,[3] with the result $\mu_\nu\sim em_\nu G_F$. The magnitude of the corresponding long-range potential then goes as

$$\alpha G_F m_\nu/Mr^3 .$$

This potential is smaller than $V_{2\gamma}^{\text{mass}}$ for separations less than $\lambda_c(\alpha m_c/m_\nu)^{1/3}$, where λ_c is the Compton wavelength of the charge. If the charge is an electron and for a neutrino of mass 10^{-2} eV, this separation is about 10^{-8} cm.

The neutrino–two-photon vertex Eq. (2.6) arising from neutrino magnetic moments also generates long-range potentials that can be even larger than that in Eq. (4.6). Again the form of this interaction can be inferred directly from an effective Hamiltonian corresponding to the Compton amplitude in Eq. (2.9). This effective Hamiltonian may be written as

$$H_{\text{eff}}^{\text{moment}}=-ik\bar{u}_\nu u_\nu(E^2-B^2)+k\bar{u}_\nu\gamma_5 u_\nu\mathbf{E}\cdot\mathbf{B} . \quad (4.8)$$

Here k is given by

$$k=2i\frac{A_{ik}^2-B_{ik}^2}{m_k} . \quad (4.9)$$

It is of interest to note that if the interaction (2.6) is dominantly of the magnetic type, that is if $A>B$, then in the effective Hamiltonian (4.8) the E^2 term appears with a positive coefficient. This implies that the corresponding electric polarizability of the neutrino is *negative.* This would be an example of the phenomenon of "diaelectricity," which is unknown in atomic physics, and not definitely known to exist in any situation.

The final term in $H_{\text{eff}}^{\text{moment}}$ is of the same form as $H_{\text{eff}}^{\text{mass}}$ of Eq. (4.1), and so the corresponding long-range potential will differ only in magnitude from Eq. (4.6). However, the potential arising from the first term will be quite different. That potential can again be found by substituting into $H_{\text{eff}}^{\text{moment}}$ the fields $\mathbf{E}$ and $\mathbf{B}$ given by Eq. (4.3). The result is

$$V_{2\gamma}^{\text{moment}}=\frac{k}{16\pi^2}\left[\frac{e^2}{r^4}-\frac{3e^2}{2M^2r^6}\right] \quad (4.10)$$

for a spin-$\frac{1}{2}$ charge. Since $e^2=4\pi\alpha$ and k is also proportional to e^2 when A,B are expressed in Bohr magnetons, the factor of $16\pi^2$ disappears from the answer written in terms of α.

We note that this potential is spin independent and falls off as r^{-4} for large r. Therefore, so long as k is not zero, $V_{2\gamma}^{\text{moment}}$ will be larger than $V_{2\gamma}^{\text{mass}}$ for sufficiently large separations. For a neutrino magnetic moment of 10^{-10} Bohr magnetons and an intermediate neutrino mass of 1 MeV, the minimum separation for this to occur is about 10^{-7} cm.

We can also compare $V_{2\gamma}^{\text{moment}}$ with the potential $V_{2\gamma}^{m_\nu=0}$, quadratic in G_F, arising from two-photon exchange even for massless neutrinos. These two potentials fall off as the same power of the separation, but depend differently on neutrino energy. Their ratio is given approximately by

$$V_{2\gamma}^{\text{moment}}/V_{2\gamma}^{m_\nu=0}\sim 10^{21}\left[\frac{A}{\mu_B}\right]^2(m_e/m_k)(m_e/E) . \quad (4.11)$$

Therefore, if the intermediate neutrino mass m_k is relatively small and the neutrino energy is small, then $V_{2\gamma}^{\text{moment}}$ can be larger.

Finally, if there is an off-diagonal magnetic moment of the type needed to generate $V_{2\gamma}^{\text{moment}}$, then neutrinos may also have diagonal magnetic moments of comparable magnitude, in which case the dipole-dipole interaction between this moment and that of a charge will exist. Unlike $V_{2\gamma}^{\text{moment}}$, the latter will be proportional to the spin of the charge, and so vanish for unpolarized charges. Note further that if the source of $V_{2\gamma}^{\text{moment}}$ is an off-diagonal electric dipole moment, there can be no diagonal counterpart so long as time reversal invariance is a good approximation. In that case the potential $V_{2\gamma}^{\text{moment}}$ can be the dominant long-range potential (other than gravity) that acts on neutrinos.

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