PHYSICAL REVIEW D VOLUME 43, NUMBER 3 1 FEBRUARY 1991

Nucleon-nucleon Hamiltonian from Skyrmions

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(Received 11 October 1989)

We derive a Hamiltonian for two interacting nucleons starting directly from the Skyrme Lagrangian. Apart from the order-N_c (number of colors) "adiabatic" potential, we find a term of order $\sqrt{N_c}$ due to the fact that the static two-soliton configuration is not a solution to Hamilton's equations. Using a double expansion in N_c^{-1} and the range of the interaction, we show that this term leads to attraction at intermediate range in the scalar channel. The expansion provides a systematic framework for discussing the nucleon-nucleon interaction using solitons. A comparison is made between the soliton approach and the conventional approach based on multipion exchange.

I. INTRODUCTION

In the past few years the Skyrme model[1–3] has provided a number of insights to the nucleon problem using solely mesonic degrees of freedom. The relevance of the model for hadronic physics at low energy is supported by arguments based on large-N_c QCD (N_c=number of colors).[4]

The Skyrme model is rooted in the nonlinear σ model where the underlying dynamics is that of pions. Baryons emerge as stable classical solitons with bulk properties that are in fair agreement with experiment. It is now known however that the model in its canonical form displays pathological features when applied to πN scattering.[5,6] Fortunately this problem is not present in the vector-stabilized versions.[7] They will be used below. A notable feature of the Skyrme model is that, in addition to the properties of individual baryons, one may also analyze their interactions, such as the nuclear force. This is a distinct advantage over other models of hadrons, such as the nonrelativistic quark model or the bag model.

Originally, Skyrme[8,9] proposed to use the asymptotic form of the two-soliton solution to derive the asymptotic potential between two Skyrmions as a function of their relative separation and orientation. Following on this suggestion Jackson, Jackson, and Pasquier[10] have extracted the nucleon-nucleon potential by projecting the orientation-dependent potentials onto nucleon wave functions. This approach has been further investigated by many others.[11,12] Their results compare favorably with the empirically motivated potentials at short and long distances but fail to reproduce the intermediate-range attraction in the scalar channel which is at the origin of nuclear binding.

The result has urged some to argue that nuclear matter is not bound in the large-N_c limit. Others have argued that the attraction can be achieved by introducing higher-derivative terms.[13] This suggestion has proved to be limited by the classical stability criterion. Even worse, it would amplify the pathology for πN scattering. Inspired by boson-exchange models, others have argued that quantum effects are important for accounting for the missing medium-range attraction.[14,13,15]

In addition to the lack of nuclear binding, another unresolved problem is that of ansatz dependence. So far, most of the approaches to the two-nucleon problem have focused on extracting the nucleon-nucleon interaction using the product ansatz. However, since the product ansatz is not a solution to the equation of motion, its use requires justification. Indeed recently, it has been argued that one should adopt another ansatz on the grounds of consistency.[16]

The problem of ansatz dependence could in principle be avoided if the full classical solutions were used. However, most of the numerical attempts to date[17,18] to solve the scattering problem involve asymptotic Skyrmions and do not relate simply to nucleon-nucleon scattering. Overall, the numerical simulations appear to be relatively unstable due to the fact that the scattering configuration is a saddle point rather than a true minimum. The problem does not arise for static multi-Skyrmion solutions,[19] but these do not bear directly on the scattering problem.

In this paper, we address both the issues of nuclear binding and ansatz dependence. We will show how one can systematically analyze the two-nucleon problem starting from an asymptotic ansatz. Our description emphasizes the role of the pion field in describing unambiguously the nucleon-nucleon interaction using a double expansion in the coupling constant ($1/\sqrt{N_c}$) and the range of the interaction ($e^{-m_\pi r}$). We find that, apart from the conventional terms involving the collective variables to order N_c, there are additional terms in the Hamiltonian to order $\sqrt{N_c}$, due to the fact that the starting *static* ansatz for the two-soliton problem is not a solution of Hamilton's (Euler-Lagrange) equations. The conventional part is repulsive but ansatz dependent in the intermediate range. The ansatz dependence is largely canceled by an iteration of the $\sqrt{N_c}$ term. This new term is also attractive, and provides a potential source of nuclear binding in the intermediate range to order N_c. Our analysis is reminiscent of the Born-Oppenheimer approach in atomic physics where the atomic potential (here nuclear potential) involves not only the Coulombic interaction (direct part) but the self-consistently induced electron potential (pion potential).

In Sec. II, we discuss the general approach for quantizing two solitons in $1+1$ dimensions. We show in particular that a systematic expansion scheme yields attraction at intermediate range which also restores ansatz independence. The result is general and extends naturally to higher dimensions. In Sec. III, we discuss the quantization of the Skyrme model in the two-nucleon sector. To avoid the pathology of the canonical Skyrme model we use the ω-stabilized Skyrmion. In Sec. IV, we give the explicit form of the two-nucleon Hamiltonian to order N_c^0. For completeness we include also the collective term to order N_c^{-1}. In Sec. V, we show how to extract the two-nucleon potential. The limitations behind the concept of a static potential are also discussed. The pion attraction induced in the spin-isospin zero channel is explicitly displayed. In Sec. VI, we analyze the consistency of the present construction with large-N_c counting. It is argued that the scattering amplitudes obtained from the soliton approach cannot be reproduced by conventional boson exchange. Our conclusion and prospects are summarized in Sec. VII.

II. MODELS IN 1+1 DIMENSIONS

Let us consider a model Lagrangian in $1+1$ dimensions:

$$L=\int dx\,[\tfrac{1}{2}(\dot{\varphi})^2-\tfrac{1}{2}(\varphi')^2-V(\varphi(x))] \tag{1}$$

where $\dot{\varphi}$ stands for the partial time derivative of φ and φ' the partial space derivative of φ. We will assume that the potential V has degenerate minima $\varphi_\pm$ and supports a static soliton solution $\varphi=\varphi_s(x)$ with

$$\varphi_s(x)=\varphi_\pm+O(e^{\mp mx}),\quad x\to\pm\infty\ . \tag{2}$$

We will also assume that there is a small parameter g (typically the meson-meson coupling) such that $\varphi_s=O(g^{-1})$ and $V(\varphi_s)=O(g^{-2})$. By the virial theorem, the soliton mass is given by

$$M_s=\int dx\,(\varphi_s')^2=2\int dx\,V(\varphi_s(x))\ , \tag{3}$$

which is of order g^{-2} and hence large. This justifies the use of a semiclassical description to leading order.

Quantum perturbation theory around such soliton solutions is well understood.[20] In addition to the soliton, there exist ordinary meson states with mass m of order g^0. The meson-meson scattering amplitude is of order g^2 as in the vacuum sector, whereas the meson-soliton scattering amplitude is of order g^0. Special care is required to maintain translational invariance. This is usually achieved by introducing collective coordinates, although other methods are possible.

Let us assume that V has more than two degenerate minima, so that a classical two-soliton solution exists. In general, the solution will no longer be static, leading to considerably more complexity in quantizing the theory. This is more so for realistic models where an exact classical solution is very unlikely. One must therefore be satisfied with an approximate solution and construct the quantum theory upon it.

An approximation to the exact two-soliton solution can be constructed as follows. We first choose a suitable ansatz $\varphi_{\rm cl}(x;r)$ which obeys the correct boundary condition in x, and approaches two single solitons asymptotically ($r\to\infty$). For example, if the minima of V are equally spaced, a possible choice would be

$$\varphi_{\rm cl}(x-X;r)=\varphi_s\left[x-X+\frac{r}{2}\right]+\varphi_s\left[x-X-\frac{r}{2}\right]\ . \tag{4}$$

The collective variables X and r are time dependent. At large separations r corresponds to the distance between the two solitons and X is the center-of-mass coordinate. Clearly these identifications are only meaningful asymptotically. At shorter distances, the solitons interact strongly and it is meaningless to separate them.

Substituting $\varphi(t,x)=\varphi_{\rm cl}(x-X(t);r(t))$, the starting Lagrangian truncates to an effective Lagrangian of the form

$$L_{\rm trun}=\tfrac{1}{2}M_{11}(r)\dot{X}^2+M_{12}(r)\dot{X}\dot{r}+\tfrac{1}{2}M_{22}(r)\dot{r}^2-U(r)\ , \tag{5}$$

where M is a 2×2 inertial matrix

$$M_{ij}=\int dx\,\phi^{(i)}(x)\phi^{(j)}(x) \tag{6}$$

with $\phi^{(1)}=-\varphi_{\rm cl}'$ and $\phi^{(2)}=\partial_r\varphi_{\rm cl}$, and U is the conventional (static) potential of the form

$$U=\tfrac{1}{2}M_{11}+\int dx\,V(\varphi_{\rm cl}(x))\ . \tag{7}$$

The variational principle applied to the corresponding action $\int dt\,L_{\rm trun}$ fixes the time evolution of the collective variables $X(t)$ and $r(t)$. This can also be recast in Hamiltonian form using

$$H_{\rm trun}=\tfrac{1}{2}(M^{-1})_{11}(r)P^2+(M^{-1})_{12}(r)Pp+\tfrac{1}{2}(M^{-1})_{22}(r)p^2+U(r)\ , \tag{8}$$

where P and p are conjugate to X and r, respectively. For the ansatz (4), the off-diagonal element M_{12} vanishes, and (8) simplifies to

$$H=\frac{P^2}{2M_{11}(r)}+\frac{p^2}{2M_{22}(r)}+U(r)\ . \tag{9}$$

For far-separated solitons ($r\to\infty$),

$$H\to\frac{P^2}{4M_s}+\frac{p^2}{M_s}+2M_s \tag{10}$$

where M_s is the single soliton mass (3). In the asymptotic region, (10) correctly describes two free solitons with total momentum P and relative momentum p. Clearly the distinction between masses and potential is only meaningful asymptotically. For any finite separation the masses are r dependent. As a result, the "kinetic term" actually contains a potential which is velocity dependent. However, since the solitons are heavy, this effect is small for fixed momentum transfer.

The potential U is of order g^{-2} and can be regarded as the "adiabatic potential" between two solitons. However, comparison with atomic physics shows that U is different from the potential that would be obtained from an actual application of the adiabatic approximation. Indeed, in atomic physics one solves the Schrödinger equation for the light degrees of freedom (electrons) with fixed positions for the heavy degrees of freedom (nuclei). The resulting energy is then added to the Coulomb energy to obtain the potential as a function of the internuclear distance. In our case the light degrees of freedom are the mesons and the heavy degrees of freedom the solitons. It is clear that the "adiabatic potential" U corresponds only to the Coulomb energy since no Schrödinger equation was solved and the truncated Hamiltonian is purely classical.

More importantly, the above construction is ansatz dependent. Indeed, if we vary the starting ansatz the "adiabatic potential" varies so that

$$\delta U=\int dx\, S(x)\delta\varphi_{\rm cl}(x)\ , \tag{11}$$

where

$$S(x)=-\varphi_{\rm cl}''(x)+V^{(1)}(\varphi_{\rm cl}(x)) \tag{12}$$

and $V^{(n)}(\varphi)=d^nV(\varphi)/d\varphi^n$. For large r, $\delta\varphi_{\rm cl}$, and S may both be taken to be of order $g^{-1}e^{-mr}$ from (2). Hence the potential is well defined in the far region $r>\ln g^{-1}/m$, but not in the intermediate region $r<\ln g^{-1}/m$.

Not surprisingly, the two difficulties are related. To see this, we need to quantize the theory including the light (meson) degrees of freedom. We write

$$\varphi(t,x)=\varphi_{\rm cl}(x-X(t);r(t))+\eta(t,x-X(t))\ , \tag{13}$$

where η is the quantum meson field. We can use (13) to rewrite the Lagrangian in terms of η, X, r, and their velocities (background-field expansion). In principle, a calculation to all orders should yield results for physical observables that are independent of the form of $\varphi_{\rm cl}$, for otherwise the original Lagrangian would not fix the theory within a given topological sector. In practice of course, we cannot compute to all orders. However, since the dependence on the ansatz appears for small r, one may hope for an expansion such that the quantities which are sensitive only to the long and intermediate ranges of r are insensitive to the choice of the starting ansatz. This will be shown below.

In quantizing the theory, the fact that the Lagrangian depends only on φ and not individually on $\varphi_{\rm cl}$ and η leads to some complications. This follows from the fact that (13) contains redundant degrees of freedom as in a gauge theory. Indeed, under the infinitesimal (local) transformations

$$X\to X+\epsilon,\quad \eta\to\eta+\epsilon\varphi_{\rm cl}'+\epsilon\eta'\ , \tag{14}$$

or

$$r\to r+\epsilon,\quad \eta\to\eta-\epsilon\partial_r\varphi_{\rm cl}\ , \tag{15}$$

φ and hence the Lagrangian are left invariant. In the Hamiltonian formalism, the redundancy manifests itself as the presence of constraints among the momenta. Indeed, the momenta read

$$P=\frac{\partial L}{\partial\dot X}=+\int dx\,(\varphi')^2\dot X-\int dx\,(\varphi'\partial_r\varphi)\dot r-\int dx\,\varphi'\dot\eta\ ,$$
$$p=\frac{\partial L}{\partial\dot r}=-\int dx\,(\varphi'\partial_r\varphi)\dot X+\int dx\,(\partial_r\varphi)^2\dot r+\int dx\,\partial_r\varphi\dot\eta\ , \tag{16}$$

$$\Pi(x)=\frac{\delta L}{\delta\dot\eta}=-[\varphi'(x)]\dot X+[\partial_r\varphi(x)]\dot r+\dot\eta(x)\ ,$$

which are subject to the first-class constraints

$$\chi_1\equiv P+\int dx\,\varphi'(x)\Pi(x)=0\ ,$$
$$\chi_2\equiv p-\int dx\,\partial_r\varphi_{\rm cl}(x)\Pi(x)=0\ . \tag{17}$$

In fact χ_1 and χ_2 are the generators of the infinitesimal transformations (14) and (15). The Hamiltonian is

$$\begin{aligned}H&=P\dot X+p\dot r+\int dx\,\Pi(x)\dot\eta(x)-L\\&=\tfrac12\int dx\,\Pi^2(x)+U(r)+\int dx\,S(x)\eta(x)\\&\quad+\tfrac12\int dx\,\eta(x)[-\partial_x^2+V^{(2)}(\varphi_{\rm cl}(x))]\eta(x)\\&\quad+\sum_{n=3}^{\infty}\frac{1}{n!}\int dx\,V^{(n)}(\varphi_{\rm cl}(x))[\eta(x)]^n+\dot X\chi_1+\dot r\chi_2\ .\end{aligned} \tag{18}$$

The presence of a term linear in η is an immediate consequence of the fact that the ansatz does not obey the equation of motion. The terms proportional to the constraints in (18) reflect the redundancy in the degrees of freedom. To fix this we must choose a gauge. According to (14) and (15), the terms proportional to $\varphi_{\rm cl}'$ and $\partial_r\varphi_{\rm cl}$ can be absorbed into a change in X and r to first order. They are also quasizero modes in the sense that ($x\to\pm\infty$)

$$[-\partial_x^2+V^{(2)}(\varphi_{\rm cl}(x))]\varphi_{\rm cl}'(x)=S'(x)\to 0\ ,$$
$$[-\partial_x^2+V^{(2)}(\varphi_{\rm cl}(x))]\partial_r\varphi(x)=\partial_r S(x)\to 0 \tag{19}$$

and hence nonpropagating in time. It is therefore natural to adopt the conditions (gauge choice)

$$\psi_1\equiv-\int dx\,\varphi_{\rm cl}'(x)\eta(x)=0\ ,$$
$$\psi_2\equiv\int dx\,\partial_r\varphi_{\rm cl}(x)\eta(x)=0\ . \tag{20}$$

Since $[\chi_i,\psi_j]=iM_{ij}(1+O(g))$ has a nonvanishing determinant, the first-class constraints (17) are now turned into second-class constraints. As a result the commutators are turned to Dirac commutators[21] given by

$$[A,B]_D=[A,B]-[A,\psi_i]c_{ij}[\chi_j,B]+[A,\chi_i]c_{ji}[\psi_j,B]\ , \tag{21}$$

with $c_{ij}[\chi_j,\psi_k]=\delta_{ik}$ and the constraints (17) and (20) hold at the operator level. As a result the Hamiltonian is uniquely determined (up to ordering problems and renormalization).

To leading order,

$$[\Pi(x),\eta(y)]_D = -iP_T(x,y)+O(g) ,$$
$$[\Pi(x),X]_D = i\varphi'_{cl}(x)(M^{-1})_{11} - i\partial_r\varphi_{cl}(M^{-1})_{21}+O(g^2) , \tag{22}$$
$$[\Pi(x),r]_D = i\varphi'_{cl}(x)(M^{-1})_{12} - i\partial_r\varphi_{cl}(M^{-1})_{22}+O(g^2) ,$$

where P_T is the projector off the quasizero mode states:

$$P_T(x,y)=\delta(x-y)-\phi^{(i)}(x)M_{ij}^{-1}\phi^{(j)}(y) . \tag{23}$$

It follows that $\Pi=\Pi_L+\Pi_T$ where $\Pi_T=P_T\Pi P_T$ is conjugate to the meson degrees of freedom and Π_L refers to the soliton (collective) degrees of freedom. The corresponding Hamiltonian reads

$$H=H_0+H_1+H_2+H_3 \tag{24}$$

with

$$H_0=\tfrac{1}{2}\int dx\, \Pi_L^2(x)+U(r) ,$$
$$H_1=\int dx\, S(x)\eta(x) ,$$
$$H_2=\tfrac{1}{2}\int dx\, [\Pi_T(x)]^2 + \tfrac{1}{2}\int dx\, \eta(x)[-\partial_x^2+V^{(2)}(\varphi_{cl}(x))]\eta(x) , \tag{25}$$
$$H_3=\sum_{n=3}^{\infty}\frac{1}{n!}V^{(n)}(\varphi_{cl}(x))[\eta(x)]^n .$$

H_0 and H_2 commute to order g. Using the constraint, we can solve for Π_L and rewrite H_0 in the form

$$H_0=\tfrac{1}{2}\bar{P}(M^{-1})_{11}\bar{P}+\bar{P}(M^{-1})_{12}p+\tfrac{1}{2}p(M^{-1})_{22}p+U(r) , \tag{26}$$

$$\bar{P}=P-\int dx\, \Pi(x)\eta'(x) ,$$

which is similar to the classical (truncated Hamiltonian discussed above (8). In particular, the potential part is identical. The replacement $P\rightarrow\bar{P}$ may also be understood by noting that P is a constant of motion and should be identified with the total momentum of the system. Hence to obtain the momentum of the solitons, one must subtract the momentum in the meson field.

Now, we are ready to supply the missing part of the potential in the adiabatic approximation. The terms involving the meson degrees of freedom are H_1, H_2, H_3, which are of order g^{-1}, g^0, g, respectively. If we ignore H_3, it is easy to solve the Schrödinger equation $(H_1+H_2)|\rangle=E(r)|\rangle$ to obtain the extra potential

$$E(r)=-\tfrac{1}{2}\int dx\int dy\, S(x)G_T(x,y)S(y)+O(g^0) , \tag{27}$$

where $G_T(x,y)$ is the static transverse Green's function given by

$$P_T[-\partial^2+V^{(2)}(\varphi_{cl})]P_TG_T=P_T1 . \tag{28}$$

We note that the unprojected static (inverse) meson propagator G^{-1} has the form

$$G^{-1}=-\partial^2+\mathcal{V}_1^{(2)}+\mathcal{V}_2^{(2)}+O(e^{-mr}) , \tag{29}$$

where the potentials $\mathcal{V}_{1,2}^{(2)}$ are evaluated around solitons 1 and 2, respectively. The finite-range corrections to (29) are ansatz dependent and operative only in the three-meson range and down in the potential, as discussed below. Hence, to zeroth order in the meson range, the spectrum of (29) follows from the single-particle spectra 1 and 2 which are positive nondefinite. Indeed, (29) is characterized by quasizero modes that are split evenly about zero (due to hopping between the exact zero modes of 1 and 2), and scattering states that are doubly degenerate for each value of ω_k^2 (Possible bound states below threshold in the single-soliton sector, as for ϕ^4 models for instance, would translate to split bound states in the two-soliton case under consideration, much like in the molecular system.) In the neighborhood of 1 (2) the scattering wave functions are proportional to those of the single-soliton wave functions 1 (2). Since $G_T=P_TGP_T$, the quasi-zero-mode states are excluded from G_T. As a result, the spectrum of G_T is definite positive. Corrections to the spectrum due to the overlapping part of the potentials are down exponentially.

Equation (27) is the basic result we need. First, notice that the meson contribution to the potential is *attractive* and of order g^{-2}. As in the Born-Oppenheimer approximation it is the self-consistent part to be added to the Coulomb potential U. The $O(g^0)$ piece in (27) comes from the zero-point energy, and from the fact that the shifted field $\bar{\eta}=\eta+G_TJ$ has a modified commutation relation with Π_T.

Second, notice that a change in the starting ansatz yields a change in (27) of the form

$$\delta E(r)=-\int dx\int dy\, S(x)G_T(x,y)\times[-\partial_y^2+V^{(2)}(\varphi_{cl}(y))]\delta\varphi_{cl}(y) -\tfrac{1}{2}\int dx\int dy\, S(x)\delta G_T(x,y)S(y) . \tag{30}$$

For variations orthogonal to the quasizero modes, the first term cancels against the variation of U as given by (11), whereas the second term is of order $g^{-2}e^{-3mr}$ and should be balanced by higher-order corrections. Hence a proper application of the adiabatic approximation indeed eliminates most of the ansatz dependence in the intermediate region $g^{-2}e^{-2mr}$. Higher-order corrections such as $g^{-3}e^{-3mr}$, $g^{-4}e^{-4mr}, \ldots$ (with $mr>\ln g^{-1}$) are expected. They are of shorter range. Also one should *not* eliminate ansatz dependence for variations proportional to quasizero modes, since otherwise the potential would be independent of r.

Results similar to (27) have been obtained by Bogomolny[22] in the context of instanton–anti-instanton interactions, and by Saito[23] in the context of two-nucleon interactions, although the issue of ansatz dependence was not discussed in the latter. On the other hand, Amadò *et al.*[16] have proposed to fix the classical ansatz so that

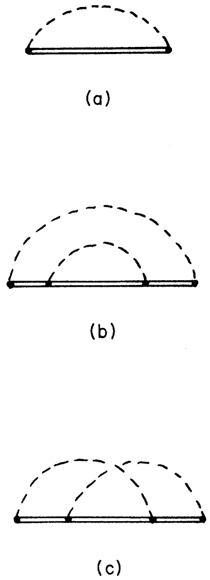

FIG. 1. (a) The contribution of the iterated pion-two-nucleon vertex to the nucleon-nucleon potential at intermediate range, i.e., $N_c e^{-2m_\pi r}$. (b), (c) Higher order corrections to the potential at shorter range $N_c^2 e^{-4m_\pi r}$.

the static source term S is proportional to the quasizero mode,

$$S(x;r)=a(r)\partial_r \varphi_{\rm cl}(x;r)\ , \tag{31}$$

and H_1 vanishes through the gauge condition (20). The authors have further argued that the vanishing of H_1 is necessary for a consistent interpretation, and hence that (31) singles out a unique ansatz. As we have seen, however, the "classical" Yukawa interaction" (27) arising from H_1 actually serves to maintain consistency in the intermediate regime. Therefore, (30) is certainly a natural choice, but by no means a mandatory one.

Our analysis is yet incomplete on two accounts. First, the shift in η induces extra terms in H_3 which are also of order g^{-2} (but of shorter range), which again underlines the fact that there is no static two-soliton solution. Second, for scattering states the adiabatic approximation does not work as well as for bound states. Therefore we must look at the full Lippmann-Schwinger equation or the T matrix.

If we go over into the asymptotic domain, it is clear that H_0 and H_2 remain nonvanishing, and they must be kept to define scattering states (distorted waves). On the other hand, H_1 vanishes asymptotically, so it may be treated as an interaction together with H_3. Since the background field $V^{(2)}(\varphi_{\rm cl})$ is static, we may define a unique vacuum state $a_k|0\rangle=0$ by diagonalizing the meson Hamiltonian H_2. More specifically, we expand as

$$\begin{aligned}\eta(x)&=\frac{1}{2\pi}\int\frac{dk}{\sqrt{2\omega_k}}[a_k\xi_k(x)+a_k^\dagger\xi_k^*(x)]\ ,\\ P_T[-\partial_x^2+V^{(2)}(\varphi_{\rm cl}(x))]P_T\xi_k&=\omega_k^2\xi_k\ .\end{aligned} \tag{32}$$

Since H_0 and H_2 commute to order g, we may take the scattering states to be $|\Psi\rangle=|ss\rangle\otimes|0\rangle+O(g)$, where $|ss\rangle$ is a two-soliton eigenstate of H_0.

If we confine ourselves to the leading behavior in g, it is easy to rearrange the distorted-wave Born series for the T matrix in terms of proper diagrams (see Fig. 1):

$$\begin{aligned}{}_f\langle\Psi|T|\Psi\rangle_i&={}_f\langle\Psi|H_1+H_3|\Psi\rangle_i+{}_f\langle\Psi|(H_1+H_3)\frac{1}{E-H_0-H_2+i\epsilon}(H_1+H_3)|\Psi\rangle_i+\cdots\\ &={}_f\langle ss|\Sigma|ss\rangle_i+{}_f\left\langle ss\left|\Sigma\frac{1}{E-H_0-H_2+i\epsilon}\Sigma\right|ss\right\rangle_i+\cdots\ .\end{aligned} \tag{33}$$

It follows that the effective two-soliton Hamiltonian with the meson degrees of freedom integrated out is given by $H_{ss}=H_0+H_2+\Sigma$, where, to lowest order [Fig. 1(a)],

$$\Sigma^{(2)}=\frac{1}{2\pi}\int\frac{dk}{2\omega_k}\int dx\int dy\ S(x)\xi_k(x)\frac{1}{E-H_0-\omega_k+i\epsilon}\xi_k^*(y)S(y)\ . \tag{34}$$

It is seen that (34) is attractive below threshold $E<2M_s+m$. If we further take $E\sim H_0$ and use closure, (34) reduces to the induced potential $E(r)$ discussed above (27). In principle, one may also compute higher-order terms. It is not difficult to see that they are stronger, but of shorter range. For example, Figs. 1(b) and 1(c) would be of order $g^{-4}e^{-4mr}$. Therefore, if g is small, but not too small, $e^{-mr}<g<1$, it makes sense to retain only (27) or (33).

There is one final point. The soliton-soliton interaction energy is generally known to be of strength g^{-2}, and our expansion scheme seems to violate this result. However, this is only apparent. The order g^{-2} result is obtained when φ is expanded around the exact classical solution $\hat{\varphi}$, and the resulting meson field $\hat{\eta}$ is counted as order g^0. If we reexpand around a classical ansatz, and treat η as order g^0, then $\hat{\eta}-\eta=\varphi_{\rm cl}-\hat{\varphi}_{\rm cl}$ is of order $g^{-1}e^{-mr}$, and we recover the $(g^{-1}e^{-mr})^n$ terms. We recall that the validity of this expansion holds for ranges such that $mr>\ln g^{-1}$.

III. QUANTIZATION OF SKYRMIONS

We now analyze the (3+1)-dimensional case, with nucleons as solitons. To avoid the pathologies of the original Skyrme model we will use the ω-stabilized version. (For a previous analysis of the soliton-soliton interaction in this model, see Ref. 12.) For that consider the nonlinear σ model minimally coupled to a massive ω meson

$$\mathcal{L}=-\frac{f_\pi^2}{4}\mathrm{Tr}(L_\mu L^\mu)+\frac{m_\pi^2 f_\pi^2}{4}\mathrm{Tr}(U+U^\dagger-2)$$
$$-\tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu}+\tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu+g_\omega\omega_\mu B^\mu\ , \qquad (35)$$

where $U=\exp(i\pi^a\tau^a/f_\pi)$ is the chiral field, $L_\mu=U^\dagger\partial_\mu U$, $\omega_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu$, and

$$B^\mu=-\frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2}\mathrm{Tr}L_\nu L_\alpha L_\beta \qquad (36)$$

the topological (baryon) current. In the language of large-N_c QCD, f_π and g_ω are of order $\sqrt{N_c}$, whereas the masses m_π, m_ω are of order N_c^0. The expansion parameter is $1\sqrt{N_c}$.

Since $\dot\omega_0$ does not appear in (35), ω_0 can be eliminated from the Lagrangian using its equation of motion

$$(-\nabla^2+m_\omega^2)\omega_0=\partial_0\partial_i\omega^i-g_\omega B_0\ . \qquad (37)$$

For the pion, it is convenient to work with the left and right isospin currents $V_0^a\pm A_0^a$ which are related to the conventional canonical momentum as

$$\frac{\partial\mathcal{L}}{\partial\dot\pi^a}=\frac{i}{2}\mathrm{Tr}\left[\frac{\partial U}{\partial\pi^a}U^\dagger\tau^b(V_0^b+A_0^b)\right]$$
$$=-\frac{i}{2}\mathrm{Tr}\left[U^\dagger\frac{\partial U}{\partial\pi^a}\tau^b(V_0^b-A_0^b)\right] \qquad (38)$$

and to each other through

$$V_0^a+A_0^a=-\mathbf{R}^{ab}(U)(V_0^b-A_0^b)\ , \qquad (39)$$

where

$$\mathbf{R}^{ab}(U)=\tfrac{1}{2}\mathrm{Tr}(\tau^a U\tau^b U^\dagger) \qquad (40)$$

is the rotation matrix associated with U. It may be checked from (38) that the currents generate the expected left and right isospin transformations

$$[(V_0^a-A_0^a)(\mathbf{x},t),U(\mathbf{y},t)]=U(\mathbf{x},t)\tau^a\delta^3(x-y)\ , \qquad (41)$$
$$[(V_0^a+A_0^a)(\mathbf{x},t),U(\mathbf{y},t)]=-\tau^a U(\mathbf{x},t)\delta^3(x-y) \qquad (42)$$

and satisfy the Gell-Mann algebra

$$[(V_0^a\pm A_0^a)(\mathbf{x},t),(V_0^b\pm A_0^b)(\mathbf{y},t)]$$
$$=2i\epsilon^{abc}(V_0^c\pm A_0^c)(\mathbf{x},t)\delta^3(x-y)\ . \qquad (43)$$

For the ω-stabilized model, the left and right isospin currents are given by

$$V_0^a-A_0^a=+f_\pi^2 L_0^a+\frac{g_\omega}{4\pi^2}\epsilon^{abc}\epsilon^{ijk}\omega_i L_j^b L_k^d\ ,$$
$$V_0^a+A_0^a=-f_\pi^2 R_0^a-\frac{g_\omega}{4\pi^2}\epsilon^{abc}\epsilon^{ijk}\omega_i R_j^b R_k^d\ . \qquad (44)$$

With this in mind the Hamiltonian density associated with (35) reads

$$H=+\frac{1}{2f_\pi^2}\left[V_0^a-A_0^a+\frac{ig_\omega}{8\pi^2}\omega^i\epsilon_{ijk}\mathrm{Tr}(L_jL_k\tau^a)\right]^2$$
$$-\frac{f_\pi^2}{4}\mathrm{Tr}(L_i^2)-\frac{f_\pi^2 m_\pi^2}{4}\mathrm{Tr}(U+U^\dagger-2)$$
$$+\tfrac{1}{2}\omega_{0i}^2+\tfrac{1}{4}\omega_{ij}^2+\tfrac{1}{2}m_\omega^2\omega_i^2+\frac{1}{2m_\omega^2}(\partial_i\omega_{0i}+g_\omega B_0)^2\ , \qquad (45)$$

which is positive definite. We note that ω_{0i} is the electric field induced by the ω meson, which obeys the usual commutation relations

$$[\omega_{0i}(\mathbf{x},t),\omega^j(\mathbf{y},t)]=i\delta_i^j\delta^3(x-y)\ . \qquad (46)$$

Because of the nonlinear character of the model Hamilton's equations admit static single soliton solutions of the type

$$\hat U(\mathbf{x})=\exp[i\boldsymbol{\tau}\cdot\hat{\mathbf{x}}F(r)]\ \text{ and }\ \boldsymbol{\omega}=\mathbf{0} \qquad (47)$$

with $F(r)$ subject to the boundary conditions $F(0)=\pi$ and $F(\infty)=0$ so that one has unit baryon number

$$\int B_0=-\frac{1}{\pi}[F(r)-\tfrac{1}{2}\sin 2F(r)]_{r=0}^{r=\infty}=1\ . \qquad (48)$$

Owing to isospin invariance, an equally good solution is obtained as $A\hat U(\mathbf{x})A^\dagger$ where A is an SU(2) matrix. The same solution is obtained by a spatial rotation, corresponding to the fact that (47) is invariant under simultaneous rotation in isospin and ordinary space.

When quantizing the system in the single soliton sector, one must promote A to a collective variable, together with the center-of-mass coordinate $\mathbf{X}$. There is some freedom in defining the remaining pion field since we may write either

$$U=A\hat U(\mathbf{x}-\mathbf{X})A^\dagger U_\pi(\mathbf{x}-\mathbf{X},t)$$

or (49)

$$U=U_\pi(\mathbf{x}-\mathbf{X},t)A\hat U(\mathbf{x}-\mathbf{X})A^\dagger\ .$$

In either case, the pion field transforms simply under chiral SU(2)×SU(2):

$$U_\pi\to g_R U_\pi g_L^\dagger,\quad A\to g_L A$$

or (50)

$$U_\pi\to g_R U_\pi g_L^\dagger,\quad A\to g_R A\ .$$

This would not be the case for other choices such as $U=\sqrt{U_\pi}A\hat U A^\dagger\sqrt{U_\pi}$ advocated in Ref. 24. For definiteness, we use the first choice.

For the single soliton case, a comparison of (50) with (38) yields

$$(V_0^a-A_0^a)(\mathbf{x})=\Pi^a(\mathbf{x}-\mathbf{X})\ ,$$
$$\int(V_0^a+A_0^a)(\mathbf{x})=-\int\mathbf{R}^{ab}(U_\pi)\Pi^b(\mathbf{x}-\mathbf{X})+2I^a\ , \qquad (51)$$

where Π and I are the generators analogous to $V_0 \pm A_0$ for U_π and A, with the rotation matrix $\mathbf{R}^{ab}(U_\pi)$ defined as in (40) More specifically

$$[\Pi^a(\mathbf{x},t), U_\pi(\mathbf{y},t)] = U_\pi(\mathbf{x},t)\tau^a\delta^3(x-y) , \quad (52)$$

$$[\Pi^a(\mathbf{x},t), \Pi^b(\mathbf{y},t)] = 2i\epsilon^{abc}\Pi^c(\mathbf{x},t)\delta^3(x-y) , \quad (53)$$

and also

$$[I^a, A] = -\frac{\tau^a}{2}A, \quad [I^a, I^b] = i\epsilon^{abc}I^c . \quad (54)$$

We may also introduce $J^a = -I^b\mathbf{R}^{ba}(A)$ which obeys

$$[J^a, A] = A\frac{\tau^a}{2}, \quad [J^a, J^b] = i\epsilon^{abc}J^c . \quad (55)$$

Relations similar to (38) hold for Π, I, J. It may be checked from (54) and (55) that I (J) generates isospin (spin) transformations on the soliton part $A\hat{U}A^\dagger$.

The decomposition (49) into a soliton part and a pion field however is not unique, since the total field U is invariant under the infinitesimal transformations

$$\delta A = iA\epsilon^a\tau^a, \quad \delta U_\pi = -i\epsilon^a\hat{U}^\dagger[\tau^a,\hat{U}]A^\dagger U_\pi \quad (56)$$

as well as

$$\delta\mathbf{X} = \boldsymbol{\epsilon}, \quad \delta U_\pi = A\boldsymbol{\epsilon}\cdot\hat{U}^\dagger\nabla\hat{U}A^\dagger U_\pi + \boldsymbol{\epsilon}\cdot\nabla U_\pi . \quad (57)$$

The pionic modes in Eqs. (56) and (57) are just the six isospin and translational zero modes. It follows that there must be an isospin constraint

$$\chi^a = J^a - \tfrac{1}{4}\int \mathrm{Tr}(A\hat{U}^\dagger[\tau^a,\hat{U}]A^\dagger\tau^b)\mathbf{R}^{bc}(U_\pi)\Pi^c = 0 \quad (58)$$

as well as a momentum constraint

$$\chi^i = P^i - \frac{i}{2}\int \mathrm{Tr}(A\hat{U}^\dagger\partial_i\hat{U}A^\dagger\tau^a)\mathbf{R}^{ab}(U_\pi)\Pi^b - \frac{i}{2}\int \mathrm{Tr}(U_\pi^\dagger\partial_i U_\pi\tau^a)\Pi^a = 0 . \quad (59)$$

The constraints χ are the generators of the gauge transformations (56) and (57) as before. It can be checked that the isospin-zero modes appearing in (58) is orthogonal to the translation zero modes appearing in (59):

$$\int \mathrm{Tr}(\hat{U}^\dagger[\tau^a,\hat{U}]\tau^b)\mathrm{Tr}(\tau^b\hat{U}^\dagger\partial_i\hat{U}) = 0 . \quad (60)$$

(In the presence of a mass term for the pion, the zero modes are normalizable modes below threshold.) Further details about the quantization in the single soliton sector may be found in Ref. 25, so we proceed to the two-soliton case.

As in 1+1 dimensions, an approximate two-soliton configuration can be constructed using a starting ansatz that satisfies the proper boundary condition in space, and approaches two single solitons asymptotically. We choose the product ansatz

$$U(\mathbf{x},t) = A_1\hat{U}\left[\mathbf{x}-\mathbf{X}-\frac{\mathbf{r}}{2}\right]A_1^\dagger U_\pi(\mathbf{x}-\mathbf{X},t) \times A_2\hat{U}\left[\mathbf{x}-\mathbf{X}+\frac{\mathbf{r}}{2}\right]A_2^\dagger , \quad (61)$$

where all the collective variables A_1, A_2, $\mathbf{X}$, $\mathbf{r}$ are time dependent. Here $\mathbf{X}$ plays the role of the center-of-mass coordinate and $\mathbf{r}$ the role of the relative separation, whereas A_1 and A_2 characterize the spin and isospin of the two nucleons. It should be stressed however that this identification is only meaningful asymptotically where the solitons are far separated. Under chiral transformations,

$$A_1 \to g_R A_1, \quad U_\pi \to g_R U_\pi g_L^\dagger, \quad A_2 \to g_L A_2 , \quad (62)$$

which leads to the identifications

$$\int (V_0^a - A_0^a)(\mathbf{x}) = \int \Pi^a(\mathbf{x}-\mathbf{X}) + 2I_2^a ,$$
$$\int (V_0^a + A_0^a)(\mathbf{x}) = -\int \mathbf{R}^{ab}(U_\pi)\Pi^b(\mathbf{x}-\mathbf{X}) + 2I_1^a . \quad (63)$$

The analysis of the constraints is also similar to the case of a single soliton. The result is

$$J_1^a - \tfrac{1}{4}\int \mathrm{Tr}(A_1\hat{U}_1^\dagger[\tau^a,\hat{U}_1]A_1^\dagger\tau^b)\mathbf{R}^{bc}(U_\pi)\Pi^c = 0 , \quad (64)$$

$$J_2^a - \tfrac{1}{4}\int \mathrm{Tr}(A_2\hat{U}_2^\dagger[\tau^a,\hat{U}_2]A_2^\dagger\tau^b)\Pi^b = 0 , \quad (65)$$

$$p_1^i - \int \frac{i}{2}\mathrm{Tr}(A_1\hat{U}_1^\dagger\partial_i\hat{U}_1A_1^\dagger\tau^a)\mathbf{R}^{ab}(U_\pi)\Pi^b - \int \frac{i}{4}\mathrm{Tr}(U_\pi^\dagger\partial_i U_\pi\tau^a)\Pi^a = 0 , \quad (66)$$

$$p_2^i - \int \frac{i}{2}\mathrm{Tr}(A_2\partial_i\hat{U}_2\hat{U}_2^\dagger A_2^\dagger\tau^a)\Pi^a - \int \frac{i}{4}\mathrm{Tr}(U_\pi^\dagger\partial_i U_\pi\tau^a)\Pi^a = 0 , \quad (67)$$

where $\mathbf{p}_1$ and $\mathbf{p}_2$ are, respectively, the conjugate to $\mathbf{X}-\mathbf{r}/2$ and $\mathbf{X}+\mathbf{r}/2$, and we are using the compact notation

$$\hat{U}_{1,2} = \hat{U}(\mathbf{x} \mp \mathbf{r}/2) .$$

Notice that, for large separations, the constraint equations reduce to the expected constraints for individual solitons involving the single-soliton zero modes.

If we denote by $J^\alpha = (J_1^a, p_1^i, J_2^a, p_2^i)$ the row vector for the generalized collective momenta and by Φ^{ab} the corresponding matrix containing the asymptotic zero modes, then the set of first-class constraints simplifies to

$$\chi^\alpha = J^\alpha + \int \Phi^{ab}\Pi^b = 0 . \quad (68)$$

The χ's are the infinitesimal generators of the gauge symmetry discussed above. To uniquely define the physical Hamiltonian, we need to fix the gauge. We choose the gauge conditions so that the quantum pion field is orthogonal to the zero modes asymptotically:

$$\psi^\alpha = \tfrac{1}{2}\int \hat{\Phi}^{ab}\mathrm{Tr}(\tau^a U_\pi) = 0 . \quad (69)$$

Here $\hat{\Phi}$ follows from Φ by setting $U_\pi = 1$.

To leading order, $[\chi^\alpha, \psi^\beta] = M^{\alpha\beta}[1 + O(N_c^{-1/2})]$, where M is a 12×12 inertial matrix with entries proportional to the overlap between the soft modes

$$M^{\alpha\beta} = \int \hat{\Phi}^{\alpha c}\hat{\Phi}^{\beta c} . \quad (70)$$

In particular, the diagonal elements are the norm of the single-soliton zero modes. They give the mass and the (pionic) moment of inertia for a single Skyrmion. (The ω contribution to the moment of inertia using a Hamiltonian formalism can be found in Ref. 25.) The off-diagonal elements depend explicitly on the relative separation r, and vanish as $r \to \infty$.

Since M is positive definite, the constraints are now second class and may be turned to strong operators identities. Correspondingly, the commutators are changed to Dirac commutators as before. In order to disentangle the collective degree of freedom from the fluctuations, we further choose to decompose the left-handed current $\Pi = \Pi_L + \Pi_T$ such that Π_L is proportional to the zero mode matrix $\hat{\Phi}$. More explicitly

$$\int \hat{\Phi}^{ab} \Pi_T^b = 0 \ . \tag{71}$$

Solving for the longitudinal part in (68) yields

$$\Pi_L^a = -\hat{\Phi}^{\alpha a} (M^{-1})^{\alpha\beta} \bar{J}^{\beta} \ , \quad \bar{J}^{\alpha} = J^{\alpha} + \int (\Phi - \hat{\Phi})^{\alpha a} \Pi^a \ . \tag{72}$$

We have ignored the ordering problem since we will be mostly interested in the Hamiltonian to order N_c^{-1}, where the problem does not appear. For higher-order calculations, the ordering problem can be addressed in principle following the discussion of Christ and Lee.[26]

IV. TWO-NUCLEON HAMILTONIAN

Having fixed the gauge, we may expand the Hamiltonian in powers of $1/\sqrt{N_c}$, after substituting (61) into (45). The Hamiltonian density to order N_c is given by

$$H_0 = +\frac{\Pi_L^2}{2} - \frac{f_\pi^2}{4} \mathrm{Tr}(L_{1i}^2 + R_{2i}^2 + [L_{1i}, R_{2i}]_+) + \frac{g_\omega^2}{2m_\omega^2} \left[\frac{-1}{24\pi^2} \epsilon_{ijk} \mathrm{Tr}(L_{1i}+R_{2i})(L_{1j}+R_{2j})(L_{1k}+R_{2k}) \right]^2 - \frac{f_\pi^2 m_\pi^2}{4} \mathrm{Tr}(\hat{U}_1 \hat{U}_2 + \hat{U}_2^\dagger \hat{U}_1^\dagger - 2) \tag{73}$$

with the definitions ($A = A_1^\dagger A_2$)

$$L_{1i} = \hat{U}_1^\dagger \partial_i \hat{U}_1 \quad \text{and} \quad R_{2i} = A \partial_i \hat{U}_2^\dagger \hat{U}_2 A^\dagger \ . \tag{74}$$

In deriving (73) we have used the fact that the ω mass is large so that $-\nabla^2/m_\omega^2 \sim 0$. (This approximation is good for the soliton profiles. It affects somehow the rms radius and charge distributions.) The term proportional to the ω coupling constant is just the classical baryon current $\hat{B}_0$. It is the sum of the classical contribution due to the individual baryon currents plus an overlapping contribution of zero net winding number

$$\hat{B}_0 = B_0^{(1)} + B_0^{(2)} + \frac{1}{8\pi^2} \epsilon_{ijk} \partial_i \mathrm{Tr} L_{1j} R_{2k} \ ,$$
$$B_0^{(1)} = -\frac{1}{24\pi^2} \epsilon_{ijk} \mathrm{Tr} L_{1i} L_{1j} L_{1k} \ , \tag{75}$$
$$B_0^{(2)} = -\frac{1}{24\pi^2} \epsilon_{ijk} \mathrm{Tr} R_{2i} R_{2j} R_{2k} \ .$$

As in the (1+1)-dimensional discussion above, we have included in (73) the order N_c^{-1} due to the collective motion. In general, the inertial parameters through the mass matrix M depend on the relative separation r and orientations A of the nucleon. They give a velocity-dependent contribution to the classical potential to order N_c^{-1}. Asymptotically, this term reduces to the free Hamiltonian for two quantum-mechanical particles

$$\int \frac{\Pi_L^2}{2} = \frac{1}{2} \bar{J}^{\alpha} (M^{-1})^{\alpha\beta} \bar{J}^{\beta} \to \frac{\bar{J}_1^2}{2\Lambda_\pi} + \frac{\bar{J}_2^2}{2\Lambda_\pi} + \frac{\bar{P}_1^2}{2M_\pi} + \frac{\bar{P}_2^2}{2M_\pi} \ , \tag{76}$$

where the soliton moment of inertia and mass (pion contribution) are given by

$$\Lambda_\pi = \frac{f_\pi^2}{16} \int \mathrm{Tr}(\hat{U}^\dagger [\tau^a, \hat{U}] \tau^b) \mathrm{Tr}(\hat{U}^\dagger [\tau^a, \hat{U}] \tau^b) \ , \quad M_\pi = -\frac{f_\pi^2}{4} \int \mathrm{Tr}(\hat{U}^\dagger \partial_i \hat{U} \tau^a) \mathrm{Tr}(\hat{U}^\dagger \partial_i \hat{U} \tau^a) \ . \tag{77}$$

Since the starting ansatz (61) is not a solution to Hamilton's equations, there are terms in the Hamiltonian linear in the quantum pion field to order $\sqrt{N_c}$. Indeed, using $U_\pi = 1 + i\tau\cdot\pi/f_\pi + \cdots$ we have, for the Hamiltonian density,

$$H_1 = \frac{if_\pi}{2} \mathrm{Tr}(\tau\cdot\pi \{ \partial_i (L_{1i} + R_{2i}) + [L_{1i}, R_{2i}]_+ \}) + \frac{-ig_\omega^2}{8\pi^2 f_\pi m_\omega^2} \hat{B}_0 \epsilon_{ijk} \partial_k \mathrm{Tr}[\tau\cdot\pi (L_{1i}+R_{2i})(L_{1j}+R_{2j})] + \frac{-if_\pi m_\pi^2}{4} \mathrm{Tr}[\tau\cdot\pi (\hat{U}_2 \hat{U}_1 - \hat{U}_1^\dagger \hat{U}_2^\dagger)] \ . \tag{78}$$

Finally the order N_c^0 contribution to the Hamiltonian density reads

$$H_2=\frac{1}{2f_\pi^2}\left[\mathbf{R}^{ab}(\hat{U}_2)(\tilde{V}_0^b-\tilde{A}_0^b)+\frac{ig_\omega}{8\pi^2}\omega^i\epsilon_{ijk}\mathrm{Tr}[\tau^a(L_{1j}+R_{2j})(L_{1k}+R_{2k})]\right]^2+\tfrac{1}{2}(\partial_i\pi)^2+\frac{m_\pi^2}{8}\mathrm{Tr}(\hat{U}_1\hat{U}_2+\hat{U}_2^\dagger\hat{U}_1^\dagger)\pi^2$$
$$+\frac{i}{2}\mathrm{Tr}[\tau\cdot(\pi\times\partial_i\pi)(L_{1i}-R_{2i})]+\tfrac{1}{4}\mathrm{Tr}([L_{1i},\tau\cdot\pi][R_{2i},\tau\cdot\pi])+\tfrac{1}{2}\tilde{E}^2+\tfrac{1}{2}\omega_{ij}\omega_{ij}+\frac{m_\omega^2}{2}\omega_i^2+\frac{1}{2m_\omega^2}(\partial_i\tilde{E}_i+g_\omega\tilde{B}_0)^2$$
$$-\left[\frac{g_\omega}{4\pi f_\pi m_\omega}\right]^2\hat{B}_0\epsilon_{ijk}\partial_i\mathrm{Tr}\{2\tau\cdot(\pi\times\partial_j\pi)(L_{1k}+R_{2k})+[L_{1j},\tau\cdot\pi][R_{2k},\tau\cdot\pi]\}\ , \tag{79}$$

where $(\tilde{V}_0-\tilde{A}_0)$ is the order $\sqrt{N_c}$ part of the left-handed isospin current operator, $\tilde{E}_i=\omega_{0i}+\partial_i\hat{\omega}_0$, and

$$\tilde{B}_0=-\frac{i}{8\pi^2 f_\pi}\epsilon_{ijk}\partial_k\mathrm{Tr}[(L_1-R_2)_i(L_1-R_2)_j\tau\cdot\pi]\ , \tag{80}$$

which is of order $1/\sqrt{N_c}$. The quantities with carets involve only the classical configurations. Notice that since $g_\omega\sim\sqrt{N_c}$ the ω couples to order $1/\sqrt{N_c}$ to the two nucleons. This coupling is velocity dependent and yields contributions of order N_c to the inertial parameters.[25]

In the field of the two Skyrmions the (unprojected) static pion propagator G is given by

$$(G^{-1})^{ab}=\left[-\nabla^2+\frac{m_\pi^2}{4}\mathrm{Tr}(U_1U_2+U_2^\dagger U_1^\dagger)\right]\delta^{ab}+\frac{i}{2}\epsilon^{abc}\mathrm{Tr}[\tau^c(L_1-R_2)_i]\vec{\nabla}_i+\tfrac{1}{4}\mathrm{Tr}([L_{1i},\tau^a][R_{2i},\tau^b])$$
$$-\frac{1}{2}\left[\frac{g_\omega}{8\pi^2 f_\pi m_\omega}\right]^2\epsilon_{ijk}\epsilon_{lmn}\mathrm{Tr}\tau^a(L_1-R_2)_i(L_1-R_2)_j\overleftarrow{\nabla}_k\vec{\nabla}_n\mathrm{Tr}\tau^b(L_1-R_2)_l(L_1-R_2)_m$$
$$-\left[\frac{g_\omega}{4\pi f_\pi m_\omega}\right]^2\hat{B}_0\epsilon_{ijk}\mathrm{Tr}\{2\epsilon^{abc}\tau^c\vec{\nabla}_i(L_1+R_2)_k\vec{\nabla}_j+[L_{1j},\tau^a][R_{2k},\tau^b]\}\ . \tag{81}$$

To leading order in the pion range, we have

$$(G^{-1})^{ab}=(-\nabla^2+m_\pi^2)\delta^{ab}+\mathcal{V}_1^{ab}+\mathcal{V}_2^{ab}+O(e^{-m_\pi r})\ , \tag{82}$$

where $\mathcal{V}_{1,2}$ are the potentials created by the individual Skyrmions. As discussed in Sec. II, the spectrum of G_T following from (82) by projection is definite positive.

V. TWO-NUCLEON POTENTIAL

Given the Hamiltonian, Eqs. (73), (78), and (79) of the previous section, we may proceed to extract the two-nucleon potential V as in the (1+1)-dimensional case. To order $N_c e^{-2m_\pi r}$ we have a contribution to the potential V from the conventional term H_0 and from the pion term Σ.

The conventional contribution to the potential reads

$$V_0=f_\pi^2\int\left[\hat{L}_{1j}^{\,b}\hat{R}_{2j}^{\,c}+\frac{\beta}{2}(\hat{B}_0^{(1)}+\hat{B}_0^{(2)})\epsilon_{lmn}\partial_l\hat{L}_m^{\,b}\hat{R}_n^{\,c}\right]\mathbf{R}^{bc}$$
$$+\frac{1}{2}\left[\frac{g_\omega}{8\pi^2 m_\omega}\right]^2\int(\epsilon_{ijk}\partial_i\hat{L}_j^{\,b}\hat{R}_k^{\,c})$$
$$\times(\epsilon_{lmn}\partial_l\hat{L}_m^{\,e}\hat{R}_n^{\,f})\mathbf{R}^{bc}\mathbf{R}^{ef}$$
$$-f_\pi^2 m_\pi^2\int(\phi_1\phi_2-\phi_1^b\phi_2^c\mathbf{R}^{bc}-1)\ , \tag{83}$$

where we have defined

$$\hat{L}_i^{\,a}=-\frac{i}{2}\mathrm{Tr}(\tau^a\hat{U}_1^\dagger\partial_i\hat{U}_1),\quad \hat{R}_i^{\,a}=-\frac{i}{2}\mathrm{Tr}(\tau^a\partial_i\hat{U}_2\hat{U}_2^\dagger)\ ,$$
$$\hat{U}_i=\phi_i+i\tau^a\phi_i^a,\quad \beta=\left[\frac{g_\omega}{2\pi f_\pi m_\omega}\right]^2. \tag{84}$$

$\hat{B}_0^{(i)}$ is the classical baryon current for a single soliton defined previously. The rotation matrix $\mathbf{R}$ is short for $\mathbf{R}(A=A_1^\dagger A_2)$.

The contribution from the pion term is

$$\Sigma=-\tfrac{1}{2}\int d\mathbf{x}\,d\mathbf{y}\,\hat{S}^a(\mathbf{x})G_T^{ab}(\mathbf{x},\mathbf{y})\hat{S}^b(\mathbf{y})\ , \tag{85}$$

where $\hat{S}$ is the classical pion source and G_T is the projected static Green's function in the background of two fixed nucleons. The explicit form of $\hat{S}$ can be read off from (78):

$$\hat{S}^a=S_0^a+S_1^{abc}\mathbf{R}^{bc}+S_2^{bcd}\mathbf{R}^{ad}\mathbf{R}^{bc}\ ,$$
$$S_0^a=f_\pi m_\pi^2(\phi_2-1)\phi_1^a\ ,$$
$$S_1^{abc}=2f_\pi(\delta_{ij}+\beta\epsilon_{ijk}\partial_k(\hat{B}_0^{(1)}+\hat{B}_0^{(2)}))\epsilon^{adb}\hat{L}_i^{\,d}\hat{R}_j^{\,c}$$
$$-\frac{f_\pi\beta}{4\pi^2}\epsilon_{ijk}\partial_i\hat{L}_j^{\,a}\partial_k(\epsilon_{lmn}\partial_l\hat{L}_m^{\,b}\hat{R}_n^{\,c})$$
$$+f_\pi m_\pi^2((\phi_1-1)\phi_2^c\delta^{ab}+\epsilon^{adb}\phi_1^d\phi_2^c)\ ,$$
$$S_2^{abc}=\frac{f_\pi\beta}{4\pi^2}\epsilon_{ijk}\partial_i\hat{R}_j^{\,a}\partial_k(\epsilon_{lmn}\partial_l\hat{L}_m^{\,b}\hat{R}_n^{\,c})\ . \tag{86}$$

The contribution Σ to the two-nucleon potential is *attractive*, so it is a potential source of nuclear binding. For simplicity, in the equations below, we will ignore the spin-isospin distortions generated by the two nucleons on the propagating pion, i.e., $G_T \sim (-\nabla^2+m_\pi^2)^{-1}$, so that the pion contribution becomes

$$\Sigma \sim -\tfrac{1}{2}\int \hat{S}^a(-\nabla^2+m_\pi^2)^{-1}\hat{S}^a \ . \tag{87}$$

We note that this approximation affects systematically the character of the potential in all channels. As a first step, it should provide us with the qualitative insights that are needed at this stage. In a more comprehensive analysis of the potential the distortions in the pion propagator should be included. Notice that for the one- and two-pion range, the distorted pion propagator follows from (82). Also, notice that even though (82) depends on the various spin-isospin orientations, both (85) and (87) lead attraction in the central channel. Indeed, the singlet component follows from

$$\Sigma_1 = \int_{SU(2)} d\mu(A)\Sigma \ . \tag{88}$$

It is negative definite since the Haar measure is positive or null over SU(2).

To sort out the various spin-isospin channels we first note that the rotation matrix $\mathbf{R}^{ab}$ belongs to the (3,3) representation of SU(2)×SU(2) generated by I_1 and I_2. The symmetric product yields

$$\mathbf{R}^{bc}\mathbf{R}^{ef} = \tfrac{1}{3}\delta^{be}\delta^{cf} + \tfrac{1}{2}\epsilon^{bei}\epsilon^{cfj}\mathbf{R}^{ij} + \tfrac{1}{2}(\mathbf{R}^{bc}\mathbf{R}^{ef} + \mathbf{R}^{ec}\mathbf{R}^{bf} - \tfrac{2}{3}\delta^{be}\delta^{cf}) \tag{89}$$

corresponding to

$$[(3,3)\otimes(3,3)]_S = (1,1)\oplus(3,3)\oplus(5,5) \ . \tag{90}$$

The last term operates only between $N\Delta$ and $\Delta\Delta$ states. We will not need it below. For completeness we also quote the result for the product of three rotation matrices:

$$\begin{aligned}\mathbf{R}^{ab}\mathbf{R}^{cd}\mathbf{R}^{ef} = {} & \tfrac{1}{6}\epsilon^{ace}\epsilon^{bdf} + \tfrac{2}{5}(\delta^{ec}\delta^{df}\mathbf{R}^{ab} + \delta^{ae}\delta^{bf}\mathbf{R}^{cd} + \delta^{ac}\delta^{bd}\mathbf{R}^{ef}) \\ & - \tfrac{1}{10}(\delta^{ec}\delta^{bf}\mathbf{R}^{ad} + \delta^{ec}\delta^{bd}\mathbf{R}^{af} + \delta^{ea}\delta^{fd}\mathbf{R}^{cb} + \delta^{ac}\delta^{df}\mathbf{R}^{eb} + \delta^{ae}\delta^{bd}\mathbf{R}^{cf} + \delta^{ac}\delta^{bf}\mathbf{R}^{ed}) \\ & \oplus(5,5)\oplus(7,7)\end{aligned} \tag{91}$$

corresponding to

$$[(3,3)\otimes(3,3)\otimes(3,3)]_S = (1,1)\oplus(3,3)\oplus(5,5)\oplus(7,7) \ . \tag{92}$$

The last two terms belong to higher-spin representations and do not operate in the nucleon-nucleon channel. Their explicit form is not necessary. We may further decompose the (3,3) piece as

$$\mathbf{R}^{ab} = \tfrac{1}{3}(\mathbf{R}_T^{ab} + \delta^{ab}\mathbf{R}_S) \ , \tag{93}$$

where $\mathbf{R}_T$ (tensor) and $\mathbf{R}_S$ (spin-spin) transform separately under the diagonal subgroup of SU(2)×SU(2)

$$\mathbf{R}_T^{ab} = 3\mathbf{R}^{ab} - \delta^{ab}\mathbf{R}^{cc} \quad \text{and} \quad \mathbf{R}_S = \mathbf{R}^{cc} \ . \tag{94}$$

For two nucleon states, the matrix element of the rotation matrix is normalized as

$$\langle S_1S_2T_1T_2|\mathbf{R}^{ab}(A_1^\dagger A_2)|S_1S_2T_1T_2\rangle = \tfrac{1}{9}S_1^aS_2^b(\mathbf{T}_1\cdot\mathbf{T}_2) \ , \tag{95}$$

where $\mathbf{S}$ and $\mathbf{T}$ are the spin and isospin of the single nucleons, respectively.

Using the above relations it is straightforward to decompose the potential derived above in the scalar, spin-spin, and tensor parts. Indeed, in the scalar channel we have (to order $N_c e^{-2m_\pi r}$)

$$\begin{aligned}V_1 = {} & +\frac{1}{6}\left[\frac{g_\omega}{4\pi^2 m_\omega}\right]^2 \int [\epsilon_{ijk}\partial_i(\hat{L}_j^{\,b}\hat{R}_k^{\,c})][\epsilon_{lmn}\partial_l(\hat{L}_m^{\,b}\hat{R}_n^{\,c})] \\ & -\tfrac{1}{2}\int (S_0^a\Delta S_0^a + \tfrac{1}{3}S_1^{abc}\Delta S_1^{abc} + \tfrac{1}{3}S_2^{abc}\Delta S_2^{abc} \\ & + \tfrac{2}{3}S_0^a\Delta S_2^{abb} + \epsilon^{bae}\epsilon^{cgf}\tfrac{1}{3}S_1^{abc}\Delta S_2^{efg}) \\ & - f_\pi^2 m_\pi^2 \int (1-\phi_1)(1-\phi_2) \ ,\end{aligned} \tag{96}$$

where we have defined $\Delta = (-\nabla^2+m_\pi^2)^{-1}$. The source terms $S_{0,1,2}$ are given in (86). Notice the relative minus sign between the conventional contribution due to ω exchange and the pionic contribution.

In the spin-spin channel the potential is given by

$$\begin{aligned}V_S = {} & \frac{f_\pi^2}{3}\int \{\hat{L}_{1j}^{\,b}\hat{R}_{2j}^{\,b} - \beta(\hat{B}_0^{(1)} + \hat{B}_0^{(2)})\epsilon_{lmn}\partial_l(\hat{L}_m^{\,b}\hat{R}_n^{\,b}) + m_\pi^2\phi_1^b\phi_2^b\} \\ & + \frac{1}{12}\left[\frac{g_\omega}{4\pi^2 m_\omega}\right]^2 \int \{[\epsilon_{ijk}\partial_i(\hat{L}_j^{\,b}\hat{R}_k^{\,b})^2] - [\epsilon_{ijk}\partial_i(\hat{L}_j^{\,b}\hat{R}_k^{\,c})][\epsilon_{lmn}\partial_l(\hat{L}_m^{\,b}\hat{R}_n^{\,c})]\} - \tfrac{1}{6}\int S_0^a\Delta S_2^{bba} + 2S_0^a\Delta S_1^{abb} - S_0^a\Delta S_1^{bab} \\ & - \tfrac{1}{12}\int (S_1^{abb}\Delta S_1^{acc} - S_1^{abc}\Delta S_1^{acb} + S_2^{abb}\Delta S_2^{acc} - S_2^{abc}\Delta S_2^{acb}) - \tfrac{2}{15}\int (S_1^{aab}\Delta S_2^{ccb} + S_1^{abc}\Delta S_2^{bca} + S_1^{abb}\Delta S_2^{acc}) \\ & + \tfrac{1}{30}\int (S_1^{aba}\Delta S_2^{bcc} + S_1^{aab}\Delta S_2^{bcc} + S_1^{aab}\Delta S_2^{cbc} + S_1^{abc}\Delta S_2^{acb} + S_1^{abc}\Delta S_2^{bac} + S_1^{abc}\Delta S_2^{abc}) \ .\end{aligned} \tag{97}$$

The first two terms give the conventional contribution to the spin-spin interaction. The last terms are the pion contributions to the same order. Finally, the contribution to the tensor part reads

$$V_{\mathbf{T}}^{bc}=\frac{f_\pi^2}{3}\int[\hat{L}\,{}^b_{1j}\hat{R}\,{}^c_{2j}-\beta(\hat{B}\,{}^{(1)}_0+\hat{B}\,{}^{(2)}_0)\epsilon_{lmn}\partial_l(\hat{L}\,{}^b_m\hat{R}\,{}^c_n)+m_\pi^2\phi_1^b\phi_2^c]-\frac{1}{6}\left[\frac{g_\omega}{4\pi^2 m_\omega}\right]^2\int[\epsilon_{ijk}\partial_i(\hat{L}\,{}^c_j\hat{R}\,{}^b_k)][\epsilon_{lmn}\partial_l(\hat{L}\,{}^d_m\hat{R}\,{}^d_n)]$$
$$+\frac{1}{6}\left[\frac{g_\omega}{4\pi^2 m_\omega}\right]^2\int[\epsilon_{ijk}\partial_i(\hat{L}\,{}^d_j\hat{R}\,{}^b_k)][\epsilon_{lmn}\partial_l(\hat{L}\,{}^c_m\hat{R}\,{}^d_n)]-\tfrac{1}{6}\int S_0^c\Delta S_2^{aba}+S_0^a\Delta S_2^{cab}-S_0^c\Delta S_2^{aab}-S_0^a\Delta S_2^{cba}+2S_0^a\Delta S_1^{abc}$$
$$-\tfrac{1}{6}\int(S_1^{adb}\Delta S_1^{acb}-S_1^{acb}\Delta S_1^{add}+S_2^{adb}\Delta S_2^{acb}-S_1^{acb}\Delta S_2^{add})-\tfrac{2}{15}\int(S_1^{aad}\Delta S_2^{bcd}+S_1^{bad}\Delta S_2^{adc}+S_1^{abc}\Delta S_2^{add})$$
$$+\tfrac{1}{30}\int(S_1^{bac}\Delta S_2^{add}+S_1^{aac}\Delta S_2^{bdd}+S_1^{aad}\Delta S_2^{bdc}+S_1^{abd}\Delta S_2^{adc}+S_1^{bad}\Delta S_2^{acd}+S_1^{abd}\Delta S_2^{acd})\ . \tag{98}$$

The first three terms are the conventional contributions to the tensor force. The last terms stem from the pionic contribution. To order $N_c e^{-2m_\pi r}$ the total potential reads

$$V=V_1\mathbf{1}+V_{\mathbf{S}}\mathbf{R}_{\mathbf{S}}+V_{\mathbf{T}}^{bc}\mathbf{R}_T^{bc}\ . \tag{99}$$

Its detailed numerical analysis will be given elsewhere.[27]

VI. COUNTING N_c

Assuming that the central potential we have derived does lead to nuclear binding, it is natural to ask whether it may be identified with the two-pion exchange force of boson-exchange models.[28] Or in a broader context, one might ask how does a picture based on meson exchange compare with QCD counting arguments.[4] As we have noted in the Introduction, the literature is quite discordant on this point, so a general discussion seems worthwhile. To avoid ambiguities, we assume that the N_c counting is done with respect to the exact classical solutions.

In the model we have used, the pion decay constant scales like $f_\pi\sim\sqrt{N_c}$ while $g_\omega\sim\sqrt{N_c}$. As a result, the nucleon mass scales like $M_N\sim N_c$. This scaling is consistent with the nonrelativistic quark-model result. General soliton theory requires that the meson-nucleon coupling constant is of order $N_c^{1/2}$ and the nucleon-nucleon potential of order N_c. In general, an $O(N_c^0)$ attraction cannot overcome an $O(N_c)$ repulsion to produce binding. Therefore if the $1/N_c$ (semiclassical) expansion is of any use, nuclear matter should bind in the large-N_c limit. Furthermore, the effect should be sought at the classical (order-N_c) level and not at the quantum (order-N_c^0) level. Our results are compatible with this counting.

This conclusion should be taken with some care, since nuclear binding is rather a delicate effect. On the other hand, so is chemical binding, but then the Born-Oppenheimer expansion is certainly successful there, and perhaps so is the expansion we employed here. It has also been argued[29] that the large-N_c expansion can break down for nuclear matter. However, the argument applies only in the low-density limit, where the interaction energy $V\sim N_c e^{-mr}$ is smaller than the kinetic energy $T\sim N_c^{-1}$, quite opposite to the regime we are interested in.

Let us now consider nucleon-nucleon scattering in the soliton- and boson-exchange pictures. In the soliton model, it is natural to work at fixed velocity and scattering angle, since the classical equations of motion are independent of the soliton mass (N_c). This implies that the momentum transfer is of order N_c, and the scattering amplitude is of order N_c^0. This is not a region where simple boson exchange is expected to work. Nevertheless, if we try to fit with one-boson exchange we find a pseudovector coupling $f_{\pi NN}$ of order N_c^0 or less, in contrast with $N_c^{1/2}$ from the soliton model.

Comparison at fixed momentum transfer does not help either. The low-energy pion-nucleon coupling that follows from chiral symmetry is[30]

$$\mathcal{L}=-M_N\bar{\psi}\left[\frac{1+\gamma_5}{2}U+\frac{1-\gamma_5}{2}U^\dagger\right]\psi \tag{100}$$

with $U=(\sigma+i\boldsymbol{\tau}\cdot\boldsymbol{\pi})/f_\pi$. The scalar and pseudoscalar coupling are equal and of order $g_{\sigma NN}=g_{\pi NN}\sim\sqrt{N_c}$, while the axial-vector coupling is of order $g_A\sim N_c^0$. By a chiral rotation, one may trade the pseudoscalar for a pseudovector coupling of order $f_{\pi NN}\sim g_{\pi NN}/M_N\sim 1/\sqrt{N_c}$.

At the tree level (100) generates a one-pion-exchange potential that scales like $V_\pi\sim N_c^{-1}$. Similarly, the potential resulting from two-pion exchange scales like $V_{2\pi}\sim N_c^{-2}$, that from three-pion exchange scales like $V_{3\pi}\sim N_c^{-3}$, and so on. (We note that the nucleon propagator scales as N_c^0, not N_c^{-1}.) This pattern is not supported by the soliton description.

In general, the discrepancy cannot be eliminated by adopting a Lagrangian other than (100). To reproduce an order-N_c potential within one-pion-exchange range, we need a pseudovector coupling of order $\sqrt{N_c}$ or a pseudoscalar coupling of order $N_c^{3/2}$. However, that would lead to a two-pion-exchange potential $V_{2\pi}$ of order N_c^2 with pseudovector coupling, or worse with pseudoscalar coupling. Clearly, the more pions the worse. (This has also been noted by Banerjee and Cohen.[31])

One may attempt to avoid this situation by invoking strong-coupling cancellation.[32,33] However, this would require a cutoff of order N_c^0 for the pions, since the cancellation only works near mass shell. If the cutoff is introduced by hand, effects of the cutoff appear for momentum transfer of order N_c^0, and if the cutoff is introduced through a form factor, the one-boson-exchange diagram is already affected. In either case the boson-exchange picture breaks down. Finally, in practical terms, the Bonn potential, for example, uses a cutoff of the order of the nucleon mass, and it is certainly hard to interpet this as $O(N_c^0)$. Therefore, we conclude that the conventional boson-exchange approach to the nucleon-nucleon potential seems to have difficulties with the description of the nucleon as a soliton in the large-N_c limit.

Actually, the difference between the soliton approach and the conventional chiral Lagrangian approach (100) goes much deeper. In soliton models the axial-vector current scales with N_c, e.g.,

$$A_\beta^a = \frac{if_\pi^2}{4}\mathrm{Tr}\tau^a(U^\dagger\partial_\beta U - U\partial_\beta U^\dagger) - \frac{ig_\omega}{16\pi^2}\epsilon_{\mu\nu\alpha\beta}\omega^\mu\partial^\nu \mathrm{Tr}\tau^a(U^\dagger\partial^\alpha U + U\partial^\alpha U^\dagger) \quad (101)$$

and the nucleon axial-vector coupling is large, $g_A \sim N_c$, in contact with $g_A \sim N_c^0$ for the chiral Lagrangian (100). (This discrepancy was also noted in Ref. 34.) The scaling behavior for the soliton case, however, is incompatible with the Adler-Weisberger relation[35]

$$g_A^2 = 1 + \frac{2f_\pi^2}{\pi}\int_{m_\pi}^{\infty}\frac{d\nu}{\nu^2}\sqrt{\nu^2 - m_\pi^2}\,(\sigma_+ - \sigma_-) \quad (102)$$

since the cross section $\sigma_\pm$ for meson-soliton scattering is order N_c^0, and the integral is at most of order $\ln N_c$.

The discrepancy also appears in pion-nucleon scattering, where the isospin-$\frac{1}{2}$ and -$\frac{3}{2}$ S-wave scattering lengths in the Skyrme model do not satisfy the Tomozawa-Weinberg relation[36] to leading order. In general, although soliton models obey the Gell-Mann algebra and realize chiral symmetry in the Nambu-Goldstone mode, they do not obey the conventional results of current algebra, since the limits $m_\pi^2 \to 0$ and $N_c \to \infty$ do not commute. For a detailed analysis of this point as well as comparison with previous literature we refer to Ref. 37.

Another interesting question is whether the soliton nature of the nucleon can show up directly in the analyticity properties of scattering amplitudes, independently of Lagrangians. Analysis of two-dimensional models and monopoles in four dimensions suggest that it does. In addition to its theoretical interest, the issue is of direct relevance for the nuclear force problem, since the Mandelstam representation constitutes the true basis for boson-exchange models, and in particular the analysis of the nucleon-nucleon interaction in the intermediate range. We hope to discuss these issues in the near future.

VII. CONCLUSION

We have outlined a general construction for deriving the nucleon-nucleon Hamiltonian that is free from the ambiguities associated with the arbitrariness in the starting ansatz. Our method is general and applies to any model with solitons. Using ω stabilized Skyrmions to avoid the problems found in the canonical Skyrme model, we have explicitly extracted the potential in the intermediate range and have shown that it is attractive in the spin-isospin 0 channel. The intermediate-range attraction is important for nuclear binding. This way, our approach may lead to the resolution of a long-standing issue in soliton models of the nucleon, starting from first principles.

Our analysis can be extended naturally to the SU(3) case, to investigate the interaction between octet and decuplet baryons, where the Wess-Zumino term is expected to play an important role. It will also be interesting to see how the pion contribution affects the exchange currents in the medium range region as shown in Fig. 2. These effects are particularly relevant for the description of the isoscalar form factor in the case of the deuteron for instance. Some of these issues will be taken up next.

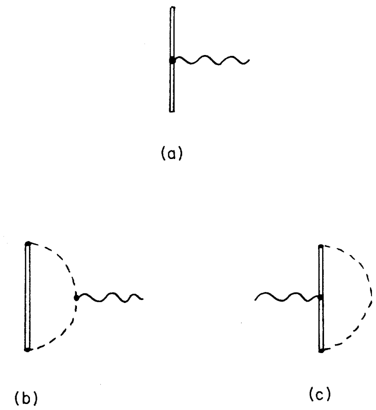

FIG. 2. (a) The contribution to the two-nucleon form factor from the direct terms. (b), (c) Contributions arising from pion-exchange at intermediate range.

ACKNOWLEDGMENTS

We would like to thank Veisteinn Thorsson for correcting a mistake in our early version of the source term, and a careful reading of the manuscript. One of us (H.Y.) would like to thank Professor M. Oka for interesting him in the subject and for correspondence.

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