$(D, D_s^+) \to VV$ decays in two models: An SU(3)-symmetry model and a factorization model, with final-state interactions

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We have studied all decays of the kind $(D,D_s^+)\to VV$ in two models: an SU(3)-symmetry model with nonet symmetry and a factorization model, and the inclusion of final-state-interaction phases. We show that the factorization model makes predictions in agreement with data, with fewer free parameters. Detailed predictions for all decay modes are made and the two models contrasted.

I. INTRODUCTION

In this paper we have studied the decays of the D mesons (D^0, D^+, D_s^+) into two vector mesons in two different schemes: (i) an SU(3)-symmetry scheme and (ii) a facorization model, with final-state-interaction (FSI) phases, for Cabibbo-angle-favored, -suppressed, and -doubly-suppressed decay modes. There is a paucity of data on $(D,D_s^+)\to VV$ decays but we have utilized whatever data are available at this moment to determine, and restrict, the parameters of the models. It is hoped that this study will prove to be a useful guide to future experiments, particularly high-statistics experiments with future machines such as τ-charm or B factories.

In Sec. II we have investigated an SU(3)-symmetry scheme and made detailed predictions for all the Cabibbo-angle-favored, -suppressed, and -doubly-suppressed modes in nonet symmetry. Given the state of experimental data, we did not consider it worthwhile to discuss nonet-symmetry breaking. We have also discussed the pitfalls of this model.

In Sec. III we have discussed $(D,D_s^+)\to VV$ decays in a factorization scheme with FSI phases. To keep the description simple, we have ignored the annihilation amplitude. In any case, data at present do not warrant the inclusion of an annihilation amplitude.

In Sec. IV we have discussed the merits and demerits of the two descriptions. In our opinion, the factorization model fares much better, in particular, in describing the (ϕ-ω) sector. The reason, as we discuss in detail in Sec. IV, being that dynamical symmetries such as the conserved-vector-current (CVC) hypothesis are easily implemented in the factorization scheme, where one deals with matrix elements of currents. In the symmetry model, as also in diagrammatic approaches, implementation of such symmetries is not natural and, at best, implemented in an *ad hoc* manner.

II. $(D,D_s^+)\to VV$ IN SU(3) SYMMETRY WITH FSI

Because of the identity of the particles in the final state, the SU(3) structure of the decay amplitude parallels that of $(D,D_s^+)\to PP$ (P=pseudoscalar meson) decays discussed by us in Refs. 1 and 2. Only the symmetric $\mathbf{8}_s$ and $\mathbf{27}$ representations of $V\otimes V$ are allowed. In nonet symmetry, the decay amplitude is written as[2] [see also Refs. 3–5 for SU(3) analysis in the charmed-meson sector]

$$\begin{aligned}A(D\to VV,\ \text{nonet sym}) = {} & c\,(V_a^m V_m^c P^b)H^a_{[bc]} \\ & + d\,(V_a^m V_m^c P^b)H^a_{\{bc\}} \\ & + e\,(P^m V_m^c V_a^b)H^a_{\{bc\}} \\ & - \frac{e}{5}(V_m^m V_a^b P^c)H^a_{\{bc\}}\ . \quad (1)\end{aligned}$$

There are three independent parameters, c, d, and e, in the model. In nonet symmetry where V_b^a represents a nonet of vector mesons, the trace of V_b^a does not vanish; hence the last term, which arises from the $\mathbf{27}$ representation of $V\otimes V$. $H^a_{[bc]}$ is the weak spurion belonging to the $\mathbf{6^*}$ representation of SU(3) and $H^a_{\{bc\}}$ to the $\mathbf{15}$ representation. For Cabibbo-angle-favored decays, $H^a_{bc}=H^2_{13}$, for Cabibbo-angle-suppressed decays, $H^a_{bc}=H^2_{12}-H^3_{13}$ and for doubly-Cabibbo-angle-suppressed decays, $H^a_{bc}=H^3_{12}$.

Nonet-symmetry breaking could be introduced by writing[6]

$$\begin{aligned}A(D\to VV,\ &\text{broken nonet sym}) \\ &= A(D\to V_8V_8) \\ &\quad + (V_1 V_k^l P^m)(aH^k_{[lm]} + bH^k_{\{lm\}})\ , \quad (2)\end{aligned}$$

where $A(D\to V_8V_8)$ represents the decay amplitude to vector *octets* only and is obtained from (1) by using V_b^a as traceless tensors, so that, in particular, the last term in (1) does not contribute. The amplitude for the decays involving an SU(3) singlet V_1 arises from the last term of (2). In a broken-nonet-symmetry model there are five parameters (a,b,c,d,e). The relation between (a,b) and (c,d,e) in *nonet* symmetry is

$$a=-\frac{2}{\sqrt{3}}c\ ,$$
$$b=\frac{2}{\sqrt{3}}\left[d+\frac{e}{5}\right]\ . \tag{3}$$

In Tables I–III we have listed the decay amplitudes for Cabibbo-angle-favored, -suppressed, and -doubly-suppressed decays of D^0, D_s^+, and D in nonet symmetry. We have also shown the isospin split of the decay amplitudes and introduced FSI phases in column 3 of these tables. Nonet symmetry could be broken by allowing a and b to take on arbitrary values. Throughout our discussion we have used ideally a mixed singlet and octet, such that

$$\phi=V_8\cos\theta_V-V_0\sin\theta_V\ ,$$
$$\omega=V_8\sin\theta_V+V_0\cos\theta_V\ , \tag{4}$$

with

$$\tan\theta_V=\frac{1}{\sqrt{2}},\quad \theta_V=35.3^\circ\ .$$

The following analysis is presented in nonet symmetry only.

First we notice from Tables I–III that only certain combinations of parameters are determined by data. For example, $D\rightarrow\rho\bar{K}^*$ decay amplitudes are determined by the parameters $A\equiv c+d+e/3$, $r\equiv 4e/3A$ and

$$\delta^{\rho\bar{K}^*}\equiv\delta_{1/2}^{\rho\bar{K}^*}-\delta_{3/2}^{\rho\bar{K}^*}$$

($\delta_{1/2}$ and $\delta_{3/2}$ being isospin $\frac{1}{2}$ and $\frac{3}{2}$ FSI phases. To determine these parameters we use Mark III Collaboration[7] data:

$$B(D^0\rightarrow K^{*-}\rho^+)=6.2\pm2.3\pm2.0\%\ ,$$
$$B(D^0\rightarrow\bar{K}^{*0}\rho^0)=1.9\pm0.3\pm0.7\%\ , \tag{5}$$
$$B(D^+\rightarrow\bar{K}^{*0}\rho^+)=4.8\pm1.2\pm1.4\%\ .$$

The Mark III analysis[7] employs the definition of isospin amplitudes [primes are used here to distinguish these amplitudes from the ones introduced in Eq. (8) to follow]

$$A(D^0\rightarrow K^{*-}\rho^+)=\frac{1}{\sqrt{3}}[A'_{3/2}\exp(i\delta_{3/2}^{\rho\bar{K}^*})+\sqrt{2}A'_{1/2}\exp(i\delta_{1/2}^{\rho\bar{K}^*})]\ ,$$
$$A(D^0\rightarrow\bar{K}^{*0}\rho^0)=\frac{1}{\sqrt{3}}[\sqrt{2}A'_{3/2}\exp(i\delta_{3/2}^{\rho\bar{K}^*})-A'_{1/2}\exp(i\delta_{1/2}^{\rho\bar{K}^*})]\ , \tag{6}$$
$$A(D^+\rightarrow\bar{K}^{*0}\rho^+)=\sqrt{3}A'_{3/2}\exp(i\delta_{3/2}^{\rho\bar{K}^*})$$

and derives[7]

$$r'\equiv A'_{3/2}/A'_{1/2}=0.29\pm0.077\ ,$$
$$37^\circ\leq(\delta_{1/2}^{\rho\bar{K}^*}-\delta_{3/2}^{\rho\bar{K}^*})\leq 90^\circ \tag{7}$$

Our tabulation of $D\rightarrow\rho\bar{K}^*$ decay amplitudes in Table I uses a definition

$$A(D^0\rightarrow K^{*-}\rho^+)=\frac{1}{3}[A_{3/2}\exp(i\delta_{3/2}^{\rho\bar{K}^*})+2(A_{1/2}\exp(i\delta_{1/2}^{\rho\bar{K}^*})]\ ,$$
$$A(D^0\rightarrow\bar{K}^{*0}\rho^0)=\frac{\sqrt{2}}{3}[A_{3/2}\exp(i\delta_{3/2}^{\rho\bar{K}^*})-A_{1/2}\exp(i\delta_{1/2}^{\rho k\bar{K}^*})]\ , \tag{8}$$
$$A(D^+\rightarrow\bar{K}^{*0}\rho^+)=A_{3/2}\exp(i\delta_{3/2}^{\rho\bar{K}^*})$$

so that

$$A_{1/2}=(3/2)^{1/2}A'_{1/2}\ ,$$
$$A_{3/2}=\sqrt{3}A'_{3/2}\ , \tag{9}$$

and

$$r\equiv A_{3/2}/A_{1/2}=\sqrt{2}r'=0.41\pm0.11\ .$$

We use $B(D^+\rightarrow\rho^+\bar{K}^{*0})$ in (5) to determine e and then we use (9) to determine $A\equiv(c+d+e/3)$ from r. Our solution is (c, d, and e are expressed in units of $10^{11/2}$ $\text{MeV}^{1/2}\text{s}^{-1/2}$)

$$|e|=78\pm14\ ,$$
$$|A|\equiv c+d+e/3=250\pm80\ . \tag{10}$$

TABLE I. Cabibbo-angle-favored $(D,D_s^+)\rightarrow VV$ decay amplitudes in the nonet-symmetry model. Symbols: $A\equiv c+d+e/3$, $B\equiv c-d+e/3$, $r=4e/3A$, $p=4e/3B$, $\delta^{\rho\bar{K}^*}\equiv\delta_{1/2}^{\rho\bar{K}^*}-\delta_{3/2}^{\rho\bar{K}^*}$, $\delta^{\rho\rho}\equiv\delta_0^{\rho\rho}-\delta_2^{\rho\rho}$, $\delta^{K^*\bar{K}^*}\equiv\delta_0^{K^*\bar{K}^*}-\delta_1^{K^*\bar{K}^*}$. FSI=final-state interaction. A factor of $\cos^2\theta_C$ is omitted; ω and ϕ are ideally mixed.

Mode	Amplitude in nonet symmetry	Amplitude with FSI phases
$D^0\rightarrow K^{*-}\rho^+$	$A(1+r/2)$	$A\exp(i\delta_{1/2}^{\rho\bar{K}^*})[1+r/2\exp(-i\delta^{\rho\bar{K}^*})]$
$\rightarrow\bar{K}^{*0}\rho^0$	$-(1/\sqrt{2})A(1-r)$	$-(1/\sqrt{2})A\exp(i\delta_{1/2}^{\rho\bar{K}^*})[1-r\exp(-i\delta^{\rho\bar{K}^*})]$
$\rightarrow\omega\bar{K}^{*0}$	$(1/\sqrt{2})A(1+r/5)$	$(1/\sqrt{2})A\exp(i\delta_{1/2}^{\omega\bar{K}^*})(1+r/5)$
$D^+\rightarrow\bar{K}^{*0}\rho^+$	$2e$	$2e\exp(i\delta_{3/2}^{\rho\bar{K}^*})$
$D_s^+\rightarrow\rho^+\rho^0$	Forbidden	
$\rightarrow K^{*+}\bar{K}^{*0}$	$-B(1-p)$	$-B\exp(i\delta_1^{K^*\bar{K}^*})(1-p)$
$\rightarrow\phi\rho^+$	$-4e/5$	$-4e/5\exp(i\delta_1^{\rho\phi})$
$\rightarrow\omega\rho^+$	$-\sqrt{2}B(1-p/10)$	$-\sqrt{2}B\exp(i\delta_1^{\rho\omega})(1-p/10)$

TABLE II. Cabibbo-angle-suppressed $(D,D_s^+)\to VV$ decay amplitudes in the nonet-symmetry model. Symbols as in Table I. A factor of $\sin\theta_C\cos\theta_C$ is omitted.

Mode	Amplitude in nonet symmetry	Amplitude with FSI phases
$D^0\to\rho^-\rho^+$	$A(1+r/2)$	$A\exp(i\delta_0^{\rho\rho})[1+(r/2)\exp(-i\delta^{\rho\rho})]$
$\to\rho^0\rho^0$	$A(1-r)$	$A\exp(i\delta_0^{\rho\rho})[1-r\exp(-i\delta^{\rho\rho})]$
$\to K^{*+}K^{*-}$	$-A(1+r/2)$	$-(1/2)A(1+r/2)\exp(i\delta_0^{K^*\bar{K}^*})[1+\exp(-i\delta^{K^*\bar{K}^*})]$
$\to K^{*0}\bar{K}^{*0}$	0	$-(1/2)A(1+r/2)\exp(i\delta_0^{K^*\bar{K}^*})[1-\exp(-i\delta^{K^*\bar{K}^*})]$
$\to\phi\rho^0$	$(4e)/(5\sqrt{2})$	$(4e)/(5\sqrt{2})\exp(i\delta_1^{\rho\phi})$
$\to\omega\rho^0$	$-A(1-2r/5)$	$-A\exp(i\delta_1^{\rho\omega})(1-2r/5)$
$\to\omega\phi$	$2\sqrt{2}e/5$	$2\sqrt{2}e/5\exp(i\delta_0^{\omega\phi})$
$\to\omega\omega$	$A(1+r/5)$	$A(1+r/5)\exp(i\delta_0^{\omega\omega})$
$D^+\to\rho^+\rho^0$	$-\sqrt{2}e$	$-\sqrt{2}e\exp(i\delta_1^{\rho\rho})$
$\to K^{*+}\bar{K}^{*0}$	$-B(1+p/2)$	$-B\exp(i\delta_1^{K^*\bar{K}^*})(1+p/2)$
$\to\phi\rho^+$	$6e/5$	$6e/5\exp(i\delta_1^{\rho\phi})$
$\to\omega\rho^+$	$-\sqrt{2}B(1-17p/20)$	$-\sqrt{2}(B-17p/20)\exp(i\delta_1^{\rho\omega})$
$D_s^+\to\rho^0K^{*+}$	$(1/\sqrt{2})B(1-p)$	$(1/\sqrt{2})B\exp(i\delta_{1/2}^{\rho K^*})[1-p\exp(-i\delta^{\rho K^*})]$
$\to\rho^+K^{*0}$	$B(1+p/2)$	$B\exp(i\delta_{1/2}^{\rho K^*})[1+p/2\exp(-i\delta^{\rho K^*})]$
$\to\phi K^{*+}$	$-B(1-8p/5)$	$-B(1-8p/5)\exp(i\delta_{1/2}^{\phi K^*})$
$\to\omega K^{*+}$	$(1/\sqrt{2})B(1+4p/5)$	$(1/\sqrt{2})B(1+4p/5)\exp(i\delta_{1/2}^{\omega K^*})$

The relative signs of e and A are not determined by data. If we assume the relative signs of e and A to be even (i.e., $r>0$) then $\delta^{\rho K^*}$ is determined to be in the first quadrant and given by (7). However, if we assume the relative sign between e and A to be odd ($r<0$) then the phase angle $\delta^{\rho\bar{K}^*}$ is in the second quadrant: $90°\le\delta^{\rho\bar{K}^*}\le 143°$. We shall make predictions with both signs of r.

The parameter $B\equiv c-d+e/3$ is poorly determined. We have used the following ratio from the Amsterdam-Bristol-CERN-Cracow-Munich-Rutherford (ACCMOR) Collaboration[8] to determine B:

$$B(D_s^+\to K^{*+}\bar{K}^{*0})/B(D_s^+\to\phi\pi^+)=2.4\pm1.6\ . \qquad (11)$$

With e from (10) and $B(D_s^+\to\phi\pi^+)$ *assumed* to be[9] 3%, we obtain (in units of $10^{11/2}\ \mathrm{MeV}^{1/2}\,\mathrm{s}^{-1/2}$)

$$B=(430^{+100}_{-140}),\ \text{or}\ (220^{+100}_{-140})\ . \qquad (12)$$

The first solution uses $r=0.4$ and the second $r=-0.4$. Henceforth we will assume the convention $A>0$ and $B>0$. Sextet dominance, $c>(d,e)$, ensures that A and B have the same sign. The errors in (12) are due largely to the data in (11).

The parameter $p\equiv 4e/3B$ which enters the description of several amplitudes in Table I is poorly determined. However, sextet dominance $[c>(d,e)]$ ensures that $r\approx p$.

Finally the parameters $C\equiv c-d-e/3$ and $q\equiv 4e/3C$ only enter the description of doubly-Cabibbo-angle-suppressed rates (see Table III) and are not determined by any data. In principle, C can be derived from the knowledge of B and e. However, the large uncertainties in B make this exercise fruitless. *Our predictions in the double-Cabibbo-angle-suppressed sector are made in the approximation.* $C\approx A$ and $q\approx r$, which would be a good approximation if c were to be much larger than d and e. Tables IV–VI summarize the predicted branching ratios in nonet symmetry.

We comment below on some of the salient features of these tables.

(1) $B(D_s^+\to\omega\rho^+)$ is almost certainly predicted to be too large, even though no measurements exist. We point out, and we later return to this point in Secs. III and IV,

TABLE III. Double-Cabibbo-angle-suppressed $(D,D_s^+)\to VV$ decay amplitudes in the nonet-symmetry model. Symbols: $C=c-d-e/3$, $q=4e/3C$. The rest as in Table I. A factor of $\sin^2\theta_C$ is omitted.

Mode	Amplitude in nonet symmetry	Amplitude with FSI phases
$D^0\to\rho^-K^{*+}$	$A(1+r/2)$	$A\exp(i\delta_{1/2}^{\rho K^*})[1+r/2\exp(-i\delta^{\rho K^*})]$
$\to\rho^0K^{*0}$	$-(1/\sqrt{2})A(1-r)$	$-(1/\sqrt{2})A\exp(i\delta_{1/2}^{\rho K^*})[1-r\exp(-i\delta^{\rho K^*})]$
$\to\omega K^{*0}$	$(1/\sqrt{2})A(1+r/5)$	$(1/\sqrt{2})A(1+r/5)\exp(i\delta_{1/2}^{\omega K^*})$
$D^+\to\rho^+K^{*0}$	$-C(1-q/2)$	$-C\exp(i\delta_{1/2}^{\rho K^*})[1-q/2\exp(-i\delta^{\rho K^*})]$
$\to\rho^0K^{*+}$	$-(1/\sqrt{2})C(1+q)$	$-(1/\sqrt{2})C\exp(i\delta_{1/2}^{\rho K^*})[1+q\exp(-i\delta^{\rho K^*})]$
$\to\omega K^{*+}$	$-(1/\sqrt{2})C(1-q/5)$	$-(1/\sqrt{2})C(1-q/5)\exp(i\delta_{1/2}^{\omega K^*})$
$D_s^+\to K^{*+}K^{*0}$	$2e$	$2e\exp(i\delta_1^{KK})$

TABLE IV. Branching ratios (in %) for Cabibbo-angle favored $(D,D_s^+)\rightarrow VV$ decays in the nonet-symmetry model. In units of $10^{11/2}$ $\text{MeV}^{1/2}\text{s}^{-1}$: $A=250\pm80$, $B=430^{+100}_{-140}$ with $r=0.4$ and $\delta^{\rho\bar{K}^*}=60°$ or 220^{+100}_{-140} with $r=-0.4$ and $\delta^{\rho\bar{K}^*}=120°$.

Model	Branching ratio (%) experiment	Branching ratio (%) theory
$D^0\rightarrow K^{*-}\rho^+$	$6.2\pm2.3\pm2.0$ [a]	Fitted ($A,\delta^{\rho\bar{K}^*}$)
$\rightarrow\bar{K}^{*0}\rho^0$	$1.9\pm0.3\pm0.7$ [a]	Fitted ($A,\delta^{\rho\bar{K}^*}$)
$\rightarrow\omega\bar{K}^{*0}$		2.8 ± 1.7 [c]
		2.0 ± 1.2 [d]
$D^+\rightarrow\bar{K}^{*0}\rho^+$	$4.8\pm1.2\pm1.4$ [a]	Fitted (e)
$D_s^+\rightarrow K^{*+}\bar{K}^{*0}$	$\dfrac{B(D_s^+\rightarrow K^{*+}\bar{K}^{*0})}{B(D_s^+\rightarrow\phi\pi^+)}=2.4\pm1.6$ [b]	Fitted (B)
$\rightarrow\phi\rho^+$	$\dfrac{B(D_s^+\rightarrow\phi\rho^+)}{B(D_s^+\rightarrow\phi\pi^+)}<1.8$ b	0.26 ± 0.10
$\rightarrow\omega\rho^+$		62^{+30}_{-40} [e]
		16^{+18}_{-14} [f]

[a]Mark III Collaboration, Ref. 7.
[b]ACCMOR Collaboration, Ref. 8.
[c]Uses $r=0.4$, $A=250\pm80$.
[d]Uses $r=-0.4$, $A=250\pm80$.
[e]Uses $r=0.4$, $B=430^{+100}_{-140}$.
[f]Uses $r=-0.4$, $B=220^{+100}_{-140}$.

that SU(3)-symmetry treatment[2] or diagrammatic approaches[10] applied to $D\rightarrow VP$ decays, generate rather a large value for $B(D_s^+\rightarrow\omega\pi^+)$ also. The problem with a large value for $B(D_s^+\rightarrow\omega\rho^+)$ in SU(3) symmetry has the same origin, as we will discuss in Secs. III and IV.

(2) Among the Cabibbo-angle-suppressed rates, $B(D^0\rightarrow\phi\rho^0)$ and $B(D^0\rightarrow\phi\omega)$ appear to be strongly suppressed. Nonet symmetry predicts $B(D^0\rightarrow\phi\rho^0)=(0.38\pm0.14)\times10^{-2}\%$ while the ACCMOR Collaboration[8] gives $B(D^0\rightarrow\phi\rho^0)/B(D^0\rightarrow K^-\pi^+)=0.009^{+0.006}_{-0.004}$. Using[9] $B(D^0\rightarrow K^-\pi^+)\approx4\%$, one gets $B(D^0\rightarrow\phi\rho^0)\simeq(3.6^{+2.4}_{-1.6})\times10^{-2}\%$. Thus the theoretical central value is an order of magnitude lower than the experimental central value.

TABLE V. Branching ratios (in %) for Cabibbo-angle-suppressed $(D,D_s^+)\rightarrow VV$ decays in the nonet-symmetry model. Parameters as in Table IV. Also used $r=p$, $\delta^{\rho\bar{K}^*}=\delta^{\rho\bar{K}^*}_{1/2}-\delta^{\rho\bar{K}^*}_{3/2}=60°$, $\delta^{\rho\rho}=\delta^{\rho\rho}_0-\delta^{\rho\rho}_2=60°$, $\delta^{K^*\bar{K}^*}=\delta^{K^*\bar{K}^*}_0-\delta^{K^*\bar{K}^*}_1=60°$. The first entry uses $r=p=0.4$ (and $B=430^{+100}_{-140}$ if it enters the amplitude); the second entry uses $r=p=-0.4$ (and $B=220^{+100}_{-140}$ if it enters the amplitude); single entries are independent of the sign of r.

Mode	Branching ratio (%) theory	Mode	Branching ratio (%) theory
$D^0\rightarrow\rho^+\rho^-$	0.54 ± 0.32	$D^+\rightarrow\rho^+\rho^0$	0.21 ± 0.08
	0.37 ± 0.22		
$\rightarrow\rho^0\rho^0$	0.17 ± 0.10	$\rightarrow K^{*+}\bar{K}^{*0}$ [c]	0.76 ± 0.48
	0.34 ± 0.20		$0.15^{+0.23}_{-0.15}$
$\rightarrow K^{*+}K^{*-}$	0.15 ± 0.08	$\rightarrow\phi\rho^+$	0.044 ± 0.016
	$0.07^{+0.06}_{-0.04}$		
$\rightarrow K^{*0}\bar{K}^{*0}$ [a]	0.05 ± 0.03	$\rightarrow\omega\rho^+$	$1.85^{+1.88}_{-1.53}$
	$0.02^{+0.02}_{-0.01}$		$3.15^{+2.35}_{-2.20}$
$\rightarrow\phi\rho^0$ [b]	$(0.38\pm0.14)\times10^{-2}$	$D_s^+\rightarrow\rho^0K^{*+}$	0.42 ± 0.23
			$0.14^{+0.16}_{-0.13}$
$\rightarrow\omega\rho^0$	0.30 ± 0.18	$\rightarrow\rho^+K^{*0}$ [d]	0.69 ± 0.37
	0.56 ± 0.34		$0.23^{+0.26}_{-0.21}$
$\rightarrow\omega\phi$	$(0.35\pm0.13)\times10^{-2}$	$\rightarrow\phi K^{*+}$	0.04 ± 0.02
			$0.21^{+0.24}_{-0.18}$
$\rightarrow\omega\omega$	0.23 ± 0.14	$\rightarrow\omega K^{*+}$	0.90 ± 0.47
	0.17 ± 0.10		$0.063^{+0.070}_{-0.055}$

[a]ACCMOR Collaboration, Ref. 8, $B(D^0\rightarrow K^{*0}\bar{K}^{*0})/B(D^0\rightarrow K^-\pi^+)=0.03\pm0.02$.
[b]ACCMOR Collaboration, Ref. 8, $B(D^0\rightarrow\phi\rho^0)/B(D^0\rightarrow K^-\pi^+)=0.009^{+0.006}_{-0.004}$.
[c]ACCMOR Collaboration, Ref. 8, $B(D^+\rightarrow K^{*+}\bar{K}^{*0})/B(D^+\rightarrow\phi\pi^+)=1.7\pm1.6$.
[d]ACCMOR Collaboration, Ref. 8, $B(D_s^+\rightarrow K^{*0}\rho^+)/B(D_s^+\rightarrow\phi\pi^+)<1.3$.

TABLE VI. Branching ratios (in %) for double-Cabibbo-angle-suppressed $(D,D_s^+)\rightarrow VV$ decays in the nonet-symmetry model. For lack of information we have used $C=A$ and $q=r$. The first entry uses $r=0.4$ and the second $r=-0.4$. Single entries are independent of the sign of r.

Mode	To get branching ratio in % multiply by 10^{-2}	Mode	To get branching ratio in % multiply by 10^{-2}
$D^0\rightarrow\rho^-K^{*+}$	1.74 ± 1.0	$D^+\rightarrow\rho^+K^{*0}$	2.9 ± 1.7
$\rightarrow\rho^0K^{*0}$	0.53 ± 0.32	$\rightarrow\rho^0K^{*+}$	2.7 ± 1.7
$\rightarrow\omega K^{*0}$	0.77 ± 0.46	$\rightarrow\omega K^{*+}$	1.38 ± 0.83
	0.56 ± 0.33		1.90 ± 1.14
		$D_s^+\rightarrow K^{*+}K^{*0}$	0.46 ± 0.17

(3) In SU(3) symmetry, for arbitrary values of c, d, e, and the phase $\delta^{\rho\bar{K}^*}$, one finds,[11] from Tables I and III,

$$B(D^0\rightarrow\rho^-K^{*+})/B(D^0\rightarrow\rho^+K^{*-})=\tan^4\theta_C\ ,$$
$$B(D^0\rightarrow\rho^0K^{*0})/B(D^0\rightarrow\rho^0\bar{K}^{*0})=\tan^4\theta_C\ , \tag{13}$$
$$|A(D_s^+\rightarrow K^{*+}K^{*0})|=\tan^2\theta_C|A(D^+\rightarrow\rho^+\bar{K}^{*0})\ ,$$
$$|A(D^0\rightarrow\omega K^{*0})|=\tan^2\theta_C|A(D^0\rightarrow\omega\bar{K}^{*0})|\ .$$

(4) Since the parameter $C=c-d-e/3$ is different from $A\equiv c+d+e/3$ and $r\equiv4e/3A$ different from $q\equiv4e/3C$, no simple relations can be derived among $B(D^+\rightarrow\rho^0K^{*+})$, $B(D^+\rightarrow\rho^+K^{*0})$, and $B(D^+\rightarrow\rho^+\bar{K}^{*0})$. *However, in the approximation $A=C,q=r$ and with $\delta^{\rho\bar{K}^*}=60°$*, we get

$$B(D^+\rightarrow\rho^0K^{*+})/B(D^+\rightarrow\rho^+\bar{K}^{*0})=2.2\tan^2\theta_C\ ,$$
$$B(D^+\rightarrow\rho^+K^{*0})/B(D^+\rightarrow\rho^+\bar{K}^{*0})=2.33\tan^4\theta_C\ . \tag{14}$$

III. $(D,D_s^+)\rightarrow VV$ IN A FACTORIZATION MODEL

The factorization model has been discussed in Refs. 12–14. The effective weak Hamiltonian for charm→hadronic decays is as follows [we are using the notation introduced by Bauer, Stech, and Wirbel[13] (BSW)].

Cabibbo-angle-favored decays:

$$H_\omega(\Delta C=\Delta S=-1)=\frac{G_F\cos^2\theta_C}{\sqrt{2}}[a_1(\bar{u}d)_H(\bar{s}c)_H+a_2(\bar{u}c)_H(\bar{s}d)_H]\ , \tag{15}$$

where

$$a_1=\tfrac{1}{2}(C_++C_-)+\frac{\xi}{2}(C_+-C_-)=\tfrac{1}{3}(2C_++C_-)\ \text{with}\ \xi=\tfrac{1}{3}\ , \tag{16}$$

and

$$a_2=\tfrac{1}{2}(C_+-C_-)+\frac{\xi}{2}(C_++C_-)=\tfrac{1}{3}(2C_+-C_-)\ \text{with}\ \xi=\tfrac{1}{3}\ . \tag{17}$$

$C_\pm$ are the QCD factors[12–15] which reduce to unity in absence of QCD corrections.

Cabibbo-angle-suppressed decays:

$$H_w(\Delta C=-1,\Delta S=0)=\frac{G_F}{\sqrt{2}}\sin\theta_C\cos\theta_C\{a_1[(\bar{u}s)_H(\bar{s}c)_H-(\bar{u}d)_H(\bar{d}c)_H]+a_2[(\bar{u}c)_H(\bar{s}s)_H-(\bar{u}c)_H(\bar{d}d)_H]\}\ . \tag{18}$$

Double-Cabibbo-angle-suppressed decays:

$$H_w(\Delta C=-\Delta S=-1)=-\frac{G_F}{\sqrt{2}}\sin^2\theta_C[a_1(\bar{u}s)_H(\bar{d}c)_H+a_2(\bar{u}c)_H(\bar{d}s)_H]\ . \tag{19}$$

We also use the following normalizations as in BSW:[13]

$$\langle P(K)|A_\mu|0\rangle=-if_Pk^\mu\ . \tag{20}$$

P stands for a 0^- meson and A_μ the axial-vector current:

$$\langle V(k)|V_\mu|0\rangle=\epsilon_\mu^*m_Vf_V\ . \tag{21}$$

V stands for a 1^- meson and V_μ the vector current. The normalization of f_P is such that $f_\pi=133$ MeV and $F_V=0.221$ GeV for ρ and K^* mesons in BSW.[13] The matrix element of the currents between the hadron states is given in terms of the form factors as[16,17]

$$\langle V(k)|V_\mu(0)|D(P)\rangle=\frac{2}{m_D+m_V}\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}P^\rho k^\sigma V(q^2)\ , \tag{22}$$

$$\langle V(k)|A_\mu(0)|D(P)\rangle = i\left[\epsilon_\mu^*(m_D+m_V)A_1(q^2) - \frac{\epsilon^*\cdot(P-k)}{(m_D+m_V)}(P+k)_\mu A_2(q^2) - \frac{\epsilon^*\cdot(P-k)}{q^2}(2m_V)(P-k)_\mu A_3(q^2) + \frac{\epsilon^*\cdot(P-k)}{q^2}(2m_V)(P-k)_\mu A_0(q^2)\right] \tag{23}$$

with $A_0(0)=A_3(0)$. $A_3(q^2)$ is related to $A_1(q^2)$ and $A_2(q^2)$ by

$$(2m_V)A_3(q^2)=(m_D+m_V)A_1(q^2)-(m_D-M_V)A_2(q^2)\ . \tag{24}$$

Because of (24) the divergence of (23) is given by $A_0(q^2)$ alone.

In the factorization model the spectator processes arise from matrix elements of the currents in the generic form

$$A(D\to V_1V_2)=\langle V_2|V_\mu|0\rangle\langle V_1|J^\mu|D\rangle\ , \tag{25}$$

where J^μ stands for the V-A combination of currents. On using the Hamiltonians in Eqs. (15), (18), and (19) together with the definitions in Eqs. (22), (23), and (24) in (25), one gets a generic form for the decay amplitude:

$$A(D\to V_1V_2)=\left[\frac{2m_{V_2}f_{V_2}}{m_D+m_{V_1}}\epsilon_{\mu\nu\rho\sigma}\epsilon_1^{*\mu}\epsilon_2^{*\nu}P_D^\rho P_{V_1}^\rho V(q^2)+im_{V_2}f_{V_2}\left[\epsilon_1^*\cdot\epsilon_2^*(m_D+m_{V_1})A_1(q^2) - \epsilon_1^*\cdot(P_D-P_{V_1})\epsilon_2^*\cdot\frac{P_D+P_{V_1}}{m_D+m_{V_1}}A_2(q^2)\right]\right] \times(\text{nonkinematic factors})\ . \tag{26}$$

The (nonkinematic factors) is a product of $[G_F/\sqrt{2})\cos^2\theta_C]$ for the Cabibbo-angle-favored decays and the appropriate QCD factors a_1 and a_2. Note also that only $V(q^2)$, $A_1(q^2)$, and $A_2(q^2)$ are relevant to describe $D\to VV$ decays. The rate $\Gamma(D\to V_1V_2)$ has a generic form

$$\Gamma(D\to V_1V_2)=(\text{nonkinematic factors})^2\left[\frac{p_c}{8\pi m_D^2}\right](m_{V_2}f_{V_2})^2(m_D+m_{V_1})^2 \times\left\{\frac{8m_D^2p_c^2}{(m_D+m_{V_1})^4}|V(q^2)|^2+\left[2+\left[\frac{m_D^2-m_{V_1}^2-m_{V_2}^2}{2m_{V_1}m_{V_2}}\right]^2\right]|A_1(q^2)|^2 + \frac{4m_D^4p_c^4}{m_{V_1}^2m_{V_2}^2(m_D+m_{V_1})^4}|A_2(q^2)|^2+\frac{2(m_D^2-m_{V_1}^2-m_{V_2}^2)}{(m_D+m_{V_1})^2}\frac{m_D^2p_c^2}{m_{V_1}^2m_{V_2}^2}\text{Re}[A_1^*(q^2)A_2(q^2)]\right\}, \tag{27}$$

where p_c, the center-of-mass momentum of the two vector mesons, is given by

$$p_c^2=\frac{1}{4m_D^2}[m_D^2-(m_{V_1}-m_{V_2})^2][m_D^2-(m_{V_1}+m_{V_2})^2]\ . \tag{28}$$

For a typical decay, such as $D\to\bar{K}^*\rho$, the coefficient of $|A_1(q^2)|^2$ is larger than that of $|V(q^2)|^2$ by a factor ≈ 50, larger than that of $|A_2(q^2)|^2$ by a factor ≈ 75, and larger than the coefficient of the interference term by a factor of ≈ 6. The terms proportional to $A_1(q^2)$, $V(q^2)$, and $A_2(q^2)$ represent S, P, and D waves in the final state. Thus S-D interference could be significant if both $A_1(q^2)$ and $A_2(q^2)$ would be large at $q^2=m_{V_2}^2$. However, if $A_2(q^2)$ is small, as appears to be the case[18] in semileptonic $D^+\to K^{*0}e^+\nu_e$, then the retention of $A_1(q^2)$ only (or S waves only) would appear to be an excellent approximation. *We have adopted this approximation in our calculations.* Further, in order to keep our description as simple as possible, we have ignored the annihilation contribution. There is good evidence[19] from the analysis of $D\to VP$ decays that the annihilation term is small compared to the spectator term. With these assumptions we have listed the decay amplitudes without FSI phases for Cabibbo-angle-favored, -suppressed, and -doubly-suppressed decays in Tables VII–IX.

In the following we describe how FSI phases are built in. We will describe in detail the isospin split of $D\to\rho\bar{K}^*$ decay amplitude and the incorporation of FSI

TABLE VII. Cabibbo-angle-favored $(D,D_s^+)\to VV$ decay amplitudes in the factorization model *without* FSI phases. A factor of $(G_F/\sqrt{2})\cos^2\theta_C A_1(q^2)$ is omitted.

Mode	Amplitude without FSI phases	Mode	Amplitude without FSI phases
$D^0\to K^{*-}\rho^+$	$a_1(m_D+m_{K^*})m_\rho f_\rho$	$D_s^+\to\rho^+\rho^0$	Forbidden
$\to\bar{K}^{*0}\rho^0$	$\frac{a_2}{\sqrt{2}}(m_D+m_\rho)m_{K^*}f_{K^*}$	$\to K^{*+}\bar{K}^{*0}$	$a_2(m_{D_s}+m_{K^*})m_{K^*}f_{K^*}$
$\to\omega\bar{K}^{*0}$	$\frac{a_2}{\sqrt{2}}(m_D+m_\omega)m_{K^*}f_{K^*}$	$\to\phi\rho^+$	$a_1(m_{D_s}+m_\phi)m_\rho f_\rho$
$D^+\to\bar{K}^{*0}\rho^+$	$a_1(m_D+m_{K^*})m_\rho f_\rho$ $+a_2(m_D+m_\rho)m_{K^*}f_{K^*}$	$\to\omega\rho^+$	0

phases and simply write down the final result for the other amplitudes that involve two isospins in the final state. We have also adopted the attitude that FSI's simply rotate the amplitudes. A multichannel FSI could change the magnitude of the amplitude also. In this paper we do not consider such effects.

Consider the Cabibbo-angle-favored $D\to\rho\bar{K}^*$ decays. We define the decay amplitude in terms of the isospin amplitudes with their phases as in Eq. (8). By switching off the FSI phases and comparing the amplitudes in (8) with those in Table VII we get

$$A_{1/2}^{\rho\bar{K}^*}=\left[a_1(m_D+m_{K^*})m_\rho f_\rho -\frac{a_2}{2}(m_D+m_\rho)m_{K^*}f_{K^*}\right]\frac{G_F}{\sqrt{2}}\cos^2\theta_C A_1(q^2)\,,$$
$$A_{3/2}^{\rho\bar{K}^*}=[a_1(m_D+m_{K^*})m_\rho f_\rho +a_2(m_D+m_\rho)m_{K^*}f_{K^*}]\frac{G_F}{\sqrt{2}}\cos^2\theta_C A_1(q^2)\,. \tag{29}$$

Using the BSW tabulation[13] of f_V's and $a_1=1.2$, $a_2=-0.5$, we get

$$r^{\rho\bar{K}^*}=A_{3/2}^{\rho\bar{K}^*}/A_{1/2}^{\rho\bar{K}^*}=0.44 \tag{30}$$

which agrees remarkably well [see (9)] with the central value of r determined by the Mark III Collaboration.[7] Having obtained $A_{1/2}^{\rho\bar{K}^*}$ and $A_{3/2}^{\rho\bar{K}^*}$, we finally cast $D\to\rho\bar{K}^*$ decay amplitudes in the form given in Table I by writing

$$A(D^0\to\rho^+K^{*-})=A^{\rho\bar{K}^*}e^{i\delta_{1/2}^{\rho\bar{K}^*}}(1+\tfrac{1}{2}r^{\rho\bar{K}^*}e^{-i\delta^{\rho\bar{K}^*}})\,,$$
$$A(D^0\to\rho^-\bar{K}^{*0})=-\frac{A^{\rho\bar{K}^*}}{\sqrt{2}}e^{i\delta_{1/2}^{\rho\bar{K}^*}}(1-r^{\rho\bar{K}^*}e^{-i\delta^{\rho\bar{K}^*}})\,,$$
$$A(D^+\to\rho^+\bar{K}^{*0})=A_{3/2}^{\rho\bar{K}^*}e^{i\delta_{3/2}^{\rho\bar{K}^*}}\,, \tag{31}$$

where

TABLE VIII. Cabibbo-angle-suppressed $(D,D_s^+)\to VV$ decay amplitudes in the factorization model *without* FSI phases. A factor of $-(G_F/\sqrt{2})\sin\theta_C\cos\theta_C A_1(q^2)$ is omitted.

Mode	Amplitude without FSI phases	Mode	Amplitude without FSI phases
$D^0\to\rho^-\rho^+$	$a_1(m_D+m_\rho)m_\rho f_\rho$	$D^+\to\rho^+\rho^0$	$-\frac{1}{\sqrt{2}}(a_1+a_2)(m_D+m_\rho)m_\rho f_\rho$
$\to\rho^0\rho^0$	$-a_2(m_D+m_\rho)m_\rho f_\rho$	$\to K^{*+}\bar{K}^{*0}$	$-a_1(m_D+m_{K^*})m_{K^*}f_{K^*}$
$\to K^{*+}K^{*-}$	$-a_1(m_D+m_{K^*})m_{K^*}f_{K^*}$	$\to\phi\rho^+$	$a_2(m_D+m_\rho)m_\phi f_\phi$
$\to K^{*0}\bar{K}^{*0}$	0	$\to\omega\rho^+$	$\frac{a_1}{\sqrt{2}}(m_D+m_\omega)m_\rho f_\rho$ $+\frac{a_2}{\sqrt{2}}(m_D+m_\rho)m_\omega f_\omega$
$\to\phi\rho^0$	$\frac{a_2}{\sqrt{2}}(m_D+m_\rho)m_\phi f_\phi$	$D_s^+\to\rho^0K^{*+}$	$-\frac{a_2}{\sqrt{2}}(m_{D_s}+m_{K^*})m_\rho f_\rho$
$\to\omega\rho^0$	$\frac{a_2}{2}[(m_D+m_\rho)m_\omega f_\omega$ $-(m_D+m_\omega)m_\rho f_\rho]$	$\to\rho^+K^{*0}$	$a_1(m_{D_s}+m_{K^*})m_\rho f_\rho$
$\to\omega\phi$	$\frac{a_2}{\sqrt{2}}(m_D+m_\omega)m_\phi m_\phi$	$\to\phi K^{*+}$	$a_1(m_{D_s}+m_\phi)m_{K^*}f_{K^*}$ $+a_2(m_{D_s}+m_{K^*})m_\phi f_\phi$
$\to\omega\omega$	$a_2(m_D+m_\omega)m_\omega f_\omega$	$\to\omega K^{*+}$	$\frac{a_2}{\sqrt{2}}(m_{D_s}+m_{K^*})m_\omega f_\omega$

TABLE IX. Double-Cabibbo-angle-suppressed $(D,D_s^+)\to VV$ decay amplitudes in the factorization model *without* FSI phases. A factor of $-(G_F/\sqrt{2})\sin^2\theta_C A_1(q^2)$ is omitted.

Mode	Amplitude without FSI phases	Mode	Amplitude without FSI phases
$D^0\to\rho^-K^{*+}$	$a_1(m_D+m_\rho)m_{K^*}f_{K^*}$	$D^+\to\rho^+K^{*0}$	$a_2(m_D+m_\rho)m_{K^*}f_{K^*}$
$\to\rho^0K^{*0}$	$\frac{a_2}{\sqrt{2}}(m_D+m_\rho)m_{K^*}f_{K^*}$	$\to\rho^0K^{*+}$	$-\frac{a_1}{\sqrt{2}}(m_D+m_\rho)m_{K^*}f_{K^*}$
$\to\omega K^{*0}$	$\frac{a_2}{\sqrt{2}}(m_D+m_\omega)m_{K^*}f_{K^*}$	$\to\omega K^{*+}$	$\frac{a_1}{\sqrt{2}}(m_D+m_\omega)m_{K^*}f_{K^*}$
		$D_s^+\to K^{*+}K^{*0}$	$(a_1+a_2)(m_{D_s}+m_{K^*})m_{K^*}f_{K^*}$

$$\delta^{\rho\bar K^*}\equiv\delta^{\rho\bar K^*}_{1/2}-\delta^{\rho\bar K^*}_{3/2}\ ,$$

$$A^{\rho\bar K^*}=\tfrac{2}{3}A^{\rho\bar K^*}_{1/2}=\tfrac{1}{3}[2a_1(m_D+m_{K^*})m_\rho f_\rho-a_2(m_D+m_\rho)m_{K^*}f_{K^*}]\times\frac{G_F}{\sqrt{2}}\cos^2\theta_C A_1(q^2)\ ,$$

and

$$r^{\rho\bar K^*}=A^{\rho\bar K^*}_{3/2}/A^{\rho\bar K^*}_{1/2}=0.44\ . \qquad (32)$$

Incorporating FSI phases to the other amplitudes involving two isospins in the final state leads to the following.

Cabibbo-angle-suppressed decays:

(i)

$$A(D^0\to\rho^+\rho^-)=A^{\rho\rho}e^{i\delta_0^{\rho\rho}}(1+\tfrac{1}{2}r^{\rho\rho}e^{-i\delta^{\rho\rho}})\ ,$$

$$A(D^0\to\rho^0\rho^0)=A^{\rho\rho}e^{i\delta_0^{\rho\rho}}(1-r^{\rho\rho}e^{-i\delta^{\rho\rho}})\ ,$$

where

$$\delta^{\rho\rho}=\delta_0^{\rho\rho}-\delta_2^{\rho\rho}\ ,$$

$$A^{\rho\rho}=\frac{1}{\sqrt{6}}A_0^{\rho\rho}=-\frac{2a_1-a_2}{3}(m_D+m_\rho)\times f_\rho m_\rho\frac{G_F}{\sqrt{2}}\sin\theta_C\cos\theta_C A_1(q^2) \qquad (33)$$

and

$$r^{\rho\rho}=\sqrt{2}\frac{A_2^{\rho\rho}}{A_0^{\rho\rho}}=\frac{2(a_1+a_2)}{2a_1-a_2}=0.48\ (\text{with } a_1=1.2 \text{ and } a_2=-0.5)\ . \qquad (34)$$

$A_0^{\rho\rho}$ and $A_2^{\rho\rho}$ are isospin 0 and 2 amplitudes. If the differences between m_ρ and m_{K^*} and f_ρ and f_{K^*} were ignored, one would get $A^{\rho\rho}=A^{\rho\bar K^*}$ and $r^{\rho\rho}=r^{\rho K^*}$ in conformity with Tables I and II which use SU(3) symmetry.

(ii)

$$A(D^0\to K^{*+}K^{*-})=A^{K^*\bar K^*}e^{i\delta_0^{K^*\bar K^*}}(1+e^{-i\delta^{K^*\bar K^*}})\ ,$$

$$A(D^0\to K^{*0}\bar K^{*0})=A^{K^*\bar K^*}e^{i\delta_0^{K^*\bar K^*}}(1-e^{-i\delta^{K^*\bar K^*}})\ ,$$

where

$$\delta^{K^*\bar K^*}\equiv\delta_0^{K^*\bar K^*}-\delta_1^{K^*\bar K^*}\ , \qquad (35)$$

$$A^{K^*\bar K^*}=\tfrac{1}{2}A_0^{K^*\bar K^*}=\frac{a_1}{2}(m_D+m_{K^*})m_{K^*}f_{K^*}\frac{G_F}{\sqrt{2}}\times\sin\theta_C\cos\theta_C A_1(q^2)\ . \qquad (36)$$

(iii)

$$A(D_s^+\to K^{*+}\rho^0)=B^{D_s\to\rho\bar K^*}e^{i\delta^{\rho K^*}_{1/2}}\times(1-p^{D_s\to\rho K^*}e^{-i\delta^{\rho K^*}})\ ,$$

$$A(D_s^+\to K^{*0}\rho^+)=B^{D_s\to\rho\bar K^*}e^{i\delta^{\rho K^*}_{1/2}}\times(1+\tfrac{1}{2}p^{D_s\to\rho K^*}e^{-i\delta^{\rho K^*}})\ , \qquad (37)$$

where B and p have been used to conform to the notation of Table II and

$$\delta^{\rho K^*}=\delta^{\rho K^*}_{1/2}-\delta^{\rho K^*}_{3/2}\ ,$$

$$B^{D_s\to\rho K^*}=(\tfrac{2}{3})^{1/2}A^{D_s\to\rho K^*}_{1/2}=\frac{-(2a_1-a_2)}{3}(m_{D_s}+m_{K^*})\times m_\rho f_\rho\frac{G_F}{\sqrt{2}}\sin\theta_C\cos\theta_C A_1(q^2)\ ,$$

and

$$p^{D_s\to\rho K^*}=\sqrt{2}A^{D_s\to\rho K^*}_{3/2}/A^{D_s\to\rho K^*}_{1/2}=\frac{2(a_1+a_2)}{(2a_1-a_2)}=r^{\rho\rho}=0.48\ . \qquad (38)$$

In (38), $A^{D_s\to\rho K^*}_{1/2}$ and $A^{D_s\to\rho K^*}_{3/2}$ are the two isospin amplitudes.

Double-Cabibbo-angle-suppressed decays:

(i)

$$A(D^0\to K^{*+}\rho^-)=A^{D^0\to\rho K^*}e^{i\delta^{\rho K^*}_{1/2}}\times(1+\tfrac{1}{2}r^{D^0\to\rho K^*}e^{-i\delta^{\rho K^*}})\ ,$$

$$A(D^0\to K^{*0}\rho^0)=-\frac{1}{\sqrt{2}}A^{D^0\to\rho K^*}e^{i\delta^{\rho K^*}_{1/2}}\times(1-r^{D^0\to\rho K^*}e^{-i\delta^{\rho K^*}})\ , \qquad (39)$$

where

$$A^{D^0\to\rho K^*}=-\frac{\sqrt{2}}{3}A_{1/2}^{D^0\to\rho K^*}$$
$$=-\frac{2a_1-a_2}{3}(m_D+m_\rho)$$
$$\times m_{K^*}f_{K^*}\frac{G_F}{\sqrt{2}}\sin^2\theta_C A_1(q^2)$$

and

$$r^{D^0\to\rho K^*}=-2\frac{A_{3/2}^{D^0\to\rho K^*}}{A_{1/2}^{D^0\to\rho K^*}}=\frac{2(a_1+a_2)}{2a_1-a_2}=r^{\rho\rho}=0.48\ . \tag{40}$$

(ii)

$$A(D^+\to K^{*0}\rho^+)=-C^{D^+\to\rho K^*}e^{i\delta_{1/2}^{\rho K^*}}$$
$$\times(1-\tfrac{1}{2}q^{D^+\to\rho K^*}e^{-i\delta^{\rho K^*}})\ , \tag{41}$$
$$A(D^+\to K^{*+}\rho^0)=-\frac{1}{\sqrt{2}}C^{D^+\to\rho K^*}e^{i\delta_{1/2}^{\rho K^*}}$$
$$\times(1+q^{D^+\to\rho K^*}e^{-i\delta^{\rho K^*}})\ ,$$

where we have used C and q in conformity with the notation of Table III and

$$C^{D^+\to\rho K^*}=-\frac{\sqrt{2}}{3}A_{1/2}^{D^+\to\rho K^*}$$
$$=\tfrac{1}{3}(a_1-2a_2)(m_D+m_\rho)$$
$$\times m_{K^*}f_{K^*}\frac{G_F}{\sqrt{2}}\sin^2\theta_C A_1(q^2)$$

and

$$q^{D^+\to\rho K^*}=\frac{2(a_1+a_2)}{a_1-2a_2}=0.64\ . \tag{42}$$

The calculated branching ratios are shown in Tables X–XII *with* and *without* FSI phases. We have used the values of f_V tabulated by BSW[13] and *we have ignored the annihilation term.* We have shown the branching ratios for three values of $A_1(0)$: 0.5, 0.6, and 0.88. The phase differences $\delta^{\rho\bar{K}^*}$, $\delta^{K^*\bar{K}^*}$, and $\delta^{\rho\rho}$ are all set equal to 60°. In the case of $D\to\rho\bar{K}^*$ decays, this choice agrees with the determination of $\delta^{\rho\bar{K}^*}$ from Mark III data.[7]

We note from Table X that the factorization model describes the Cabibbo-angle-favored $(D,D_s^+)\to VV$ branching ratios well with FSI phases and $A_1(0)=0.5$ rather than 0.88. The latter value of $A_1(0)$ overestimates $D\to\bar{K}^*\rho$ branching ratios by a factor of about 3. $A_1(0)=0.5$ also agrees with a recent measurement of $D^+\to\bar{K}^{*0}e^+\nu_e$ form factors by E-691 Collaboration.[18] These points have been made previously.[7]

Since our model neglects annihilation processes, $D_s^+\to\omega\rho^+$ remains forbidden. However, even in the spectator model, such as we have considered, $\omega\rho^+$ channel could be generated through inelastic FSI involving the G-odd channels, $\phi\rho^+,(K^{*+}\bar{K}^0)^{G=-1}$ and $\omega\rho^+$. Since $B(D_s^+\to\phi\rho^+)$ is quite robust, it is entirely possible to generate $B(D_s^+\to\omega\rho^+)\approx 1\%$ at the expense of $D_s^+\to\phi\rho^+$ channel.

Among the Cabibbo-angle-suppressed modes, the strongest appear to be $D^0\to\rho^+\rho^-$, $D^+\to K^{*+}\bar{K}^{*0}$, $\omega\rho^+$, and $D_s^+\to\rho^+K^{*0}$.

In double-Cabibbo-angle-suppressed decays, we find

$$\frac{B(D^0\to\rho^-K^{*+})}{B(D^0\to\rho^+K^{*-})}=\frac{B(D^0\to\rho^0K^{*0})}{B(D^0\to\rho^0\bar{K}^{*0})}=1.2\ \tan^4\theta_C\ , \tag{43}$$
$$\frac{B(D^+\to\rho^0K^{*+})}{B(D^+\to\rho^+\bar{K}^{*0})}=1.61\ \tan^4\theta_C\ , \tag{44}$$
$$\frac{B(D^+\to\rho^+K^{*0})}{B(D^+\to\rho^+\bar{K}^{*0})}=1.23\ \tan^4\theta_C\ . \tag{45}$$

The factor of 1.2 in (43) arises from SU(3)-symmetry breaking through the masses m_ρ and m_{K^*}. This factor is unity in SU(3) symmetry as we saw in (13). The factors of

TABLE X. Cabibbo-angle-favored $(D,D_s^+)\to VV$ branching ratios in the factorization model. All branching ratios are in percent. $a_1=1.2$, $a_2=-0.5$ used. f_V's from BSW.[13] Other parameters are shown in the heading in the columns.

Mode	No FSI: branching ratio (%)			FSI: $\delta^{\rho\bar{K}^*}=\delta^{K^*\bar{K}^*}=\delta^{\rho\rho}=60°$			Branching ratio experiment (%)
	$A_1(0)=0.5$	$A_1(0)=0.6$	$A_1(0)=0.88$	$A_1(0)=0.5$	$A_1(0)=0.6$	$A_1(0)=0.88$	
$D^0\to K^{*-}\rho^+$	6.55	9.44	20.3	5.59	8.05	17.3	$6.2\pm2.3\pm2.0$ [a]
$\to\bar{K}^{*0}\rho^0$	0.70	1.00	2.16	1.66	2.40	5.15	$1.9\pm0.3\pm0.7$ [a]
$\to\omega\bar{K}^{*0}$	0.66	0.96	2.06	0.66	0.96	2.06	$<2.5\%$ [b]
$D^+\to\bar{K}^{*0}\rho^+$	4.67	6.72	14.46	4.67	6.72	14.46	$4.8\pm1.2\pm1.4$ [a]
$D_s^+\to\rho^+\rho^0$	Forbidden by isospin and angular momentum selection rules						
$\to K^{*+}\bar{K}^{*0}$	1.39	2.00	4.31	1.39	2.00	4.31	7.2 ± 4.8 [c]
$\to\phi\rho^+$	6.35	9.14	19.66	6.35	9.14	19.66	$<5.4\%$ [d]
$\to\omega\rho^+$	Forbidden in absence of inelastic FSI and annihilation process						

[a]Mark III Collaboration, Ref. 7.
[b]E-691 Collaboration, Ref. 20.
[c]ACCMOR Collaboration, Ref. 8. $B(D_s^+\to K^{*+}\bar{K}^{*0})/B(D_s^+\to\phi\pi^+)=2.4\pm1.6$. We have used $B(D_s^+\to\phi\pi^+)=3\%$.
[d]ACCMOR Collaboration, Ref. 8. $B(D_s^+\to\phi\rho^+)/B(D_s^+\to\phi\pi^+)<1.8$. We have used $B(D_s^+\to\phi\pi^+)=3\%$.

TABLE XI. Cabibbo-angle-suppressed $(D,D_s^+)\to VV$ branching ratios (in %) in the factorization model. See caption of Table X for other parameters.

Mode	No FSI: branching ratio in % $A_1(0)=0.5$	$A_1(0)=0.6$	$A_1(0)=0.88$	FSI: $\delta^{\rho\rho}=\delta^{K^*\bar{K}^*}=60°$ $A_1(0)=0.5$	$A_1(0)=0.6$	$A_1(0)=0.88$	Experiment (in %)
$D^0\to\rho^+\rho^-$	0.53	0.76	1.63	0.44	0.64	1.37	
$\to\rho^0\rho^0$	0.05	0.07	0.14	0.13	0.18	0.39	
$\to K^{*+}K^{*-}$	0.24	0.34	0.73	0.18	0.25	0.55	
$\to K^{*0}\bar{K}^{*0}$	0.0	0.0	0.0	0.06	0.08	0.18	0.12 ± 0.08 [a]
$\to\phi\rho^0$	0.026	0.037	0.080	0.026	0.037	0.080	$0.036^{+0.024}_{-0.016}$ [b]
$\to\omega\rho^0$	0.002	0.003	0.006	0.002	0.003	0.006	
$\to\omega\phi$	0.023	0.034	0.072	0.023	0.034	0.072	
$\to\omega\omega$	0.020	0.032	0.068	0.020	0.032	0.068	
$D^+\to\rho^+\rho^0$	0.22	0.32	0.68	0.22	0.32	0.68	
$\to K^{*+}\bar{K}^{*0}$	0.58	0.83	1.80	0.58	0.83	1.80	
$\to\phi\rho^+$	0.13	0.18	0.39	0.13	0.18	0.39	
$\to\omega\rho^+$	0.31	0.44	0.95	0.31	0.44	0.95	
$D_s^+\to\rho^0K^{*-}$	0.045	0.065	0.14	0.13	0.18	0.39	
$\to\rho^+K^{*0}$	0.52	0.75	1.61	0.44	0.63	1.36	$<3.9\%$ [c]
$\to\phi K^{*+}$	0.057	0.082	0.177	0.057	0.082	0.177	
$\to\omega K^{*+}$	0.022	0.032	0.068	0.022	0.032	0.068	

[a]ACCMOR Collaboration, Ref. 8. $B(D^0\to K^{*0}\bar{K}^{*0})/B(D^0\to K^-\pi^+)=0.03\pm0.02$. We have used $B(D^0\to K^-\pi^+)=4\%$.
[b]ACCMOR Collaboration, Ref. 8. $B(D^0\to\phi^0\rho^0)/B(D^0\to K^-\pi^+)=0.009^{+0.006}_{-0.004}$. We have used $B(D^0\to K^-\pi^+)=4\%$.
[c]ACCMOR Collaboration, Ref.8. $B(D_s^+\to\rho^+K^{*0})/B(D_s^+\to\phi\pi^+)=<1.3$. We have used $B(D_s^+\to\phi\pi^+)=3\%$.

1.61 and 1.23 in (44) and (45) arise from SU(3)-symmetry breaking and FSI interference effects in the double-Cabibbo-angle-suppressed modes. Note that the right-hand side of (44) and (45) is quite different from the right-hand side of (14) where the same ratios are calculated. However, we remind the reader that (14) was derived on the assumption that $A=C$ and $r=q$. In the factorization model all these parameters are calculable. In particular, we found that the parameter q $(=0.64)$ in (42) is quite different from the parameter r $(=0.44)$ in (32). This, and the fact that $C^{D^+\to\rho K^*}$ in (42) is not simply related to $A^{\rho\bar{K}^*}$ in (32) through a factor of $\tan^2\theta_C$ results in the different predictions in (14) compared to those in (44) and (45).

The strongest double-Cabibbo-angle-suppressed modes are $D^0\to\rho^-K^{*+}$, $D^+\to\rho^+K^{*0}$, $D^+\to\rho^0K^{*+}$, and $D^+\to\omega K^{*+}$, in agreement with the conclusion drawn from the SU(3)-symmetry scheme.

IV. DISCUSSION AND CONCLUSION

In this paper we have discussed all the $(D,D_s^+)\to VV$ decays in two models: SU(3) symmetry with nonet symmetry and a factorization model, both with the inclusion of final-state-interaction phases. In the latter model we have ignored the annihilation process.

In the following we have contrasted with two models and their predictions. Throughout the following discussion we have adopted $A_1(0)=0.5$.

Cabibbo-angle-favored decays

The parameters of the nonet-symmetry model were determined, as indicated in Table IV, by using $(D,D^+)\to\rho\bar{K}^*$ and $D_s^+\to K^{*+}\bar{K}^{*0}$ data. Once this is done, $B(D^0\to\omega\bar{K}^{*0})$, $B(D_s^+\to\phi\rho^+)$, and $B(D_s^+\to\omega\rho^+)$ become predictions. For either sign of the

TABLE XII. Double-Cabibbo-angle-suppressed $(D,D_s^+)\to VV$ branching ratios in the factorization model. Multiply the figures by 10^{-2} to get branching ratios in percent. See caption of Table X for other parameters.

Mode	No FSI. To get branching ratios in % multiply by 10^{-2} $A_1(0)=0.5$	$A_1(0)=0.6$	$A_1(0)=0.88$	FSI: $\delta^{\rho K^*}=60°$. To get branching ratio in % multiply by 10^{-2} $A_1(0)=0.5$	$A_1(0)=0.6$	$A_1(0)=0.88$
$D^0\to\rho^-K^{*+}$	2.24	3.24	6.97	1.90	2.73	5.88
$\to\rho^0K^{*0}$	0.20	0.28	0.60	0.55	0.79	1.70
$\to\omega K^{*0}$	0.18	0.27	0.58	0.18	0.27	0.58
$D^+\to\rho^+K^{*0}$	0.96	1.38	2.97	1.61	2.32	5.01
$\to\rho^0K^{*+}$	2.76	3.98	8.54	2.10	3.03	6.52
$\to\omega K^{*+}$	2.63	3.80	8.15	2.63	3.79	8.15
$D_s^+\to K^{*+}K^{*0}$	0.76	1.10	2.37	0.76	1.10	2.37

parameter r ($=\pm0.4$) the central value of $B(D_s^+\to\omega\rho^+)$ is predicted to be certainly too high. In contrast $B(D_s^+\to\phi\rho^+)$ is highly suppressed at 0.26%. This is due to the fact that $A(D_s^+\to\phi\rho^+)$ is $-4e/5$ while sextet dominance ensures that $e\ll c$.

In contrast, in the factorization model we have used only two parameters, $A_1(0)$ and $\delta^{\rho\bar{K}^*}$, the QCD coefficients a_1 and a_2 having been fixed from $D\to\pi\bar{K}$ data, to predict all the branching ratios among the Cabibbo-angle-favored modes. In particular, $D\to\rho\bar{K}^*$ branching ratios are predicted to be in excellent agreement with data. In an important, and telling, contrast to the nonet-symmetry scheme $D_s^+\to\omega\rho^+$, large in the nonet-symmetry scheme, is forbidden in the factorization model, if the annihilation process and inelastic FSI's are neglected.

In addition, it could be argued that the annihilation term in $D_s^+\to\omega\rho^+$, ignored here, would be larger than the corresponding annihilation term in $D_s^+\to\omega\pi^+$ decay. The argument goes as follows. In both cases the annihilation term is proportional to $a_1 f_{D_s}$ times the matrix element of the divergence of $(\bar{u}d)$ current between the vacuum and $\omega\rho^+$ or $\omega\pi^+$ state. These two hadronic states have opposite G parities: $G=-1$ for $\omega\rho^+$ and $+1$ for $\omega\pi^+$. The vector part of $(\bar{u}d)$ current, having vanishing divergence due to the conserved-vector-current (CVC) hypothesis, does not contribute to either decay mode. However, the axial-vector part of $(\bar{u}d)$ current, having the quantum numbers of the pion, has odd G parity. Consequently, it can connect the vacuum state to $\omega\rho^+$ state but not to $\omega\pi^+$ state. However, in any case, if the partially conserved axial-vector current (PCAC) hypothesis is used (as much as it can be trusted at $q^2=m_{D_s}^2$) for the divergence of the axial-vector $(\bar{u}d)$ current, the annihilation term will be proportional to m_π^2 and, hence, very small.

The factorization model also generates a significant (6.35%) branching ratio for $D_s^+\to\phi\rho^+$, which in nonet symmetry is strongly suppressed. Since $\phi\rho^+$ and $\omega\rho^+$ states have the same G parity, it is possible that inelastic FSI could generate decays of D_s^+ into $\omega\rho^+$ channel at the expense of $\phi\rho^+$ channel.

Cabibbo-angle-suppressed decays

The nonet-symmetry scheme is characterized by strongly suppressed branching ratios for (D^0,D^+) decays into channels involving ϕ: $D^0\to\phi\rho^0$, $D^0\to\omega\phi$, and $D^+\to\phi\rho^+$. The reason being that the decay amplitudes are proportional to e. Thus it predicts $B(D^0\to\phi\rho^0)=0.0038\%$, $B(D^0\to\omega\phi)=0.0035\%$, and $B(D^+\to\phi\rho^+)=0.044\%$. In the factorization model, all these modes are color suppressed (i.e., amplitudes proportional to a_2); however, the branching ratios are considerably higher: $B(D^0\to\phi\rho^0)=0.026\%$, $B(D^0\to\omega\phi)=0.023\%$, and $B(D^+\to\phi\rho^+)=0.13\%$. ACCMOR data,[8] $B(D^0\to\phi\rho^0)/B(D^0\to K^-\pi^+)=0.009^{+0.006}_{-0.004}$ with[9] $B(D^0\to K^-\pi^+)=4\%$, yields $B(D^0\to\phi\rho^0)$ in agreement with the factorization model.

As much as the nonet-symmetry model suppresses the decay rates of (D,D^+) involving ϕ, it gives a large branching ratio for modes involving ω (and *not* ϕ as the other particle). Thus it predicts $B(D^0\to\omega\rho^0)=0.30\%$ or 0.56% (depending on the sign of r), $B(D^0\to\omega\omega)0.23\%$ or 0.17% and an uncomfortably large $B(D^+\to\omega\rho^+)=1.85\%$ or 3.15%. In contrast, in the factorization model, $B(D^0\to\omega\rho^0)$ is highly suppressed at 0.002%. The reason can be found in the listing of the amplitudes in Table VIII. The opposite signs of the two contributions to the decay amplitude arises from the $q\bar{q}$ contents of ω and ρ^0: $(u\bar{u}+d\bar{d})/\sqrt{2}$ for ω and $(u\bar{u}-d\bar{d})/\sqrt{2}$ for ρ^0. In addition, $B(D^0\to\omega\omega)$ at 0.02% is an order of magnitude smaller than the nonet-symmetry prediction. $B(D^+\to\omega\rho^+)$ at 0.31% is also much smaller than the corresponding nonet-symmetry prediction.

Double-Cabibbo-angle-suppressed decays

Here there are no glaring differences in the predictions of the two models. However, one has to remember that the nonet-symmetry model predictions have large uncertainties and are made in an approximation, $C=A$, $q=r$, which is bound to be inaccurate.

In summary, the factorization model appears to describe data much better than the nonet-symmetry scheme. It uses fewer parameters and affords the means to impose dynamical symmetries such as CVC and PCAC, since it uses matrix elements of currents rather than that of the weak Hamiltonian.

Among the Cabibbo-angle-favored decays, it will be useful to have $B(D_s^+\to\phi\rho^+)$ and $B(D_s^+\to\omega\rho^+)$. The former at 6.35% in the factorization model is already at the upper end of data:[8] $B(D_s^+\to\phi\rho^+)/B(D_s^+\to\phi\pi^+)<1.8$. It will also be useful to have a measurement of $B(D_s^+\to\omega\rho^+)$ for the reason that if this branching ratio is measured to be small ($\leq1\%$), it will clearly rule out the nonet-symmetry scheme and help us to refine the spectator model presented here.

Among the Cabibbo-angle-suppressed rates, measurements of modes involving ϕ and ω in the final state will discriminate between the two schemes.

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