Cabibbo-angle-favored, -suppressed, and -doubly-suppressed $D \to PP$ and $D \to VP$ decays in SU(3) symmetry with final-state interactions

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(Received 13 June 1990)

We have studied Cabibbo-angle-favored, -suppressed, and -doubly-suppressed $D \to PP$ and $D \to VP$ decays in a nonet-symmetry and a broken-nonet-symmetry scheme, with the inclusion of final-state-interaction phases. For $D \to VP$ decays, the implications of sextet dominance are also investigated. In the discussion we have argued that the symmetry approach, as also the diagrammatic approach used by other authors, does not fare as well in describing $D \to VP$ decays as the factorization approach, particularly in describing $B(D_s^+ \to \omega\pi^+)$, $B(D_s^+ \to \phi\pi^+)$, and $B(D_s^+ \to \rho^0\pi^+)$.

I. INTRODUCTION

In this paper we have studied the decays of D mesons into two pseudoscalar mesons ($D \to PP$) and also into a pseudoscalar and a vector meson ($D \to VP$) within SU(3) symmetry with final-state-interaction (FSI) phases. We stress at the outset that this approach is *not* equivalent to a multichannel approach which one of the authors has followed in the recent past.[1] The effort, instead, is to see how far one can go in explaining data, and make predictions, within SU(3) symmetry with a FSI. The latter respects isospin-SU(2) symmetry and breaks flavor-SU(3) symmetry in a particular way. We have looked at all the two-body decays of the kind $D \to PP$ and $D \to VP$, i.e., Cabibbo-angle favored, Cabibbo-angle suppressed, and double Cabibbo-angle suppressed.

In an earlier work[2] we had performed a similar analysis of $D \to PP$ Cabibbo-angle-favored and Cabibbo-angle-suppressed decays. The present work is an extension of the work in Ref. 2; however, it goes further by using new data and analyzing double-Cabibbo-angle-suppressed $D \to PP$ decays which one should be able to observe at the τ-charm or B factories. We have also investigated $D \to VP$ decays.

In Sec. II we discuss $D \to PP$ decays. In Sec. III we analyze $D \to VP$ decays. We discuss the consequences of sextet dominance in Sec. IV. In Sec. V we conclude with a discussion and summary. The Appendix relates our model parameters to those used previously by other authors.

II. $D \to PP$ decays

This section deals with a reanalysis of Cabibbo-angle-favored and Cabibbo-angle-suppressed $D \to PP$ decays using new data and a new analysis of double-Cabibbo-angle-suppressed $D \to PP$ decays within SU(3) symmetry with a FSI. The notation has already been introduced in Ref. 2. We will reproduced some of the essential details.

The general structure of $\Delta C = 1$ decay amplitude for D decays into two pseudoscalar octet mesons is

$$A(D \to P_8 P_8) = c(P_a^m P_m^c P^b) H_{[bc]}^a + d(P_a^m P_m^c P^b) H_{\{bc\}}^a + e(P^m P_m^c P_a^b) H_{\{bc\}}^a \ . \quad (1)$$

Here $P^a = (D^0, D^+, D_s^+)$ belongs to a $\mathbf{3}^*$ representation of SU(3). P_a^b is a *traceless* pseudoscalar octet with diagonal elements

$$\left[\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}}, \frac{-\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}}, \frac{-2\eta_8}{\sqrt{6}} \right] ;$$

η_8 is the $I=0$ member of the octet. $H_{[bc]}^a$ is the weak spurion belonging to the $\mathbf{6}^*$ representation of SU(3), and $H_{\{bc\}}^a$ belongs to the $\mathbf{15}$ representation. For Cabibbo-angle-favored decays $H_{bc}^a = H_{13}^2$, for Cabibbo-angle-suppressed decays $H_{bc}^a = H_{12}^2 - H_{13}^3$ and for double-Cabibbo-angle-suppressed decays $H_{bc}^a = H_{12}^3$. Of course, there will also be the factors $\cos^2\theta_C$, $\cos\theta_C \sin\theta_C$, and $-\sin^2\theta_C$ (θ_C = Cabibbo angle), respectively, for these decays. QCD factors are absorbed in the coefficients c, d, and e. One anticipates $(d,e) \ll c$ due to sextet dominance.

Contrary to the assumption made in Ref. 2, a generalization of (1) to nonet symmetry is *not* obtained simply by allowing P_b^a to a pseudoscalar nonet with diagonal elements

$$\left[\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}}, \frac{-\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}}, \frac{-2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \right] ,$$

where η_1 is an SU(3) singlet. Rather, since in nonet symmetry the representations are not traceless, the trace part of the $\mathbf{27}$ representation of $P \otimes P$ in the last term in (1) makes an additional contribution. The correct generalization of (1) to nonet symmetry involving decays of D into two pseudoscalar nonets is

$$A(D \to PP, \text{in nonet symmetry}) = A(D \to P_9 P_9) - \frac{e}{5}(P_m^m P_a^b P^c) H_{\{bc\}}^a \ , \quad (2)$$

where $A(D \to P_9 P_9)$ is the amplitude used in Ref. 2 with P_9, the pseudoscalar nonet. The number of parameters

TABLE I. Decay amplitudes for double-Cabibbo-angle-suppressed $D \to PP$ modes. Symbols: $A \equiv c+d+e/3$, $C = c-d-e/3$, $r = 4e/3A$, $q = 4e/3C$, $\delta^{\pi K} \equiv \delta^{\pi K}_{1/2} - \delta^{\pi K}_{3/2}$, FSI ≡ final-state interaction. A factor of $-\sin^2\theta_C$ is omitted. θ_P is the $\eta - \eta'$ mixing angle.

Mode [a]	SU(3)-symmetric amplitudes	Amplitudes with FSI
$D^0 \to K^+\pi^-$	$A(1+r/2)$	$A\exp(i\delta^{\pi K}_{1/2})[1+(r/2)\exp(-i\delta^{\pi K})]$
$\to K^0\pi^0$	$-(1/\sqrt{2})A(1-r)$	$-(1/\sqrt{2})A\exp(i\delta^{\pi K}_{1/2})[1-r\exp(-i\delta^{\pi K})]$
$\to K^0\eta$	$-(1/\sqrt{6})A(1-r)\cos\theta_P$ $-(2/\sqrt{3})A(1-r/10)\sin\theta_P$	$-(1/\sqrt{6})A\exp(i\delta^{\eta K}_{1/2})[(1-r)\cos\theta_P$ $+2\sqrt{2}(1-r/10)\sin\theta_P]$
$\to K^0\eta'$	$-(1/\sqrt{6})A(1-r)\sin\theta_P$ $+(2/\sqrt{3})A(1-r/10)\cos\theta_P$	$-(1/\sqrt{6})A\exp(i\delta^{\eta' K}_{1/2})[(1-r)\sin\theta_P$ $-2\sqrt{2}(1-r/10)\cos\theta_P]$
$D^+ \to K^0\pi^+$	$-C(1-q/2)$	$-C\exp(i\delta^{\pi K}_{1/2})[1-(q/2)\exp(-i\delta_{\pi K})]$
$\to K^+\pi^0$	$-(1/\sqrt{2})C(1+q)$	$-(1/\sqrt{2})C\exp(i\delta^{\pi K}_{1/2})[1+q\exp(-i\delta^{\pi K})]$
$\to K^+\eta$	$(1/\sqrt{6})C(1+q)\cos\theta_P$ $+(2/\sqrt{3})C(1+q/10)\sin\theta_P$	$(1/\sqrt{6})C\exp(i\delta^{\eta' K}_{1/2})[(1+q)\sin\theta_P$ $+2\sqrt{2}(1+q/10)\sin\theta_P]$
$\to K^+\eta'$	$(1/\sqrt{6})C(1+q)\sin\theta_P$ $-(2/\sqrt{3})C(1+q/10)\cos\theta_P$	$(1/\sqrt{6})C\exp(i\delta^{\eta' K}_{1/2})[(1+q)\sin\theta_P$ $-2\sqrt{2}(1+q/10)\cos\theta_P]$
$D_s^+ \to K^+K^0$	$2e$	$2e\exp(i\delta^{KK}_1)$

[a]For η, η' modes, nonet symmetry is used.

remains at three. Equation (2) reproduces the results of Refs. 3 and 4. The decay amplitudes resulting from (2) are the same as those listed in Tables I and II of Ref. 2 *except* those involving the SU(3) singlet η_1, where e must be replaced by $\frac{2}{5}e$. Since we had not discussed double-Cabibbo-angle-suppressed decays in Ref. 2, we display all the double-Cabibbo-angle-suppressed amplitudes in Table I.

A nonet-symmetry-breaking model is introduced by adding two new parameters to the model as follows:[5]

TABLE II. Cabibbo-angle-favored $D \to PP$ decays.

		Theoretical branching ratio (%)			
		$\theta_P = -19°$		$\theta_P = -11°$	
Mode	Experimental branching ratio (%)	Nonet symmetry	Broken nonet symmetry	Nonet symmetry	Broken nonet symmetry
$D^0 \to \pi^+K^-$	$4.2 \pm 0.4 \pm 0.14$ [c]	4.2			
				Unaffected	
$\to \pi^0\bar{K}^0$	$1.9 \pm 0.4 \pm 0.2$ [c]	1.9			
$\to \eta\bar{K}^0$	$1.6 \pm 0.6 \pm 0.4$ [d]	0.06	0.70 [a] (1.6) [a]	0.0	0.7 [a] (1.6) [a]
$\to \eta'\bar{K}^0$	<2.7 [d]	3.16	0.80(3.84) [b]	3.20	2.76(12.6)
$D^+ \to \pi^+\bar{K}^0$	$3.2 \pm 0.5 \pm 0.2$ [c] $2.5 \pm 0.4 \pm 0.6$ [e,f]	3.33			
				Unaffected	
$D_s^+ \to K^+\bar{K}^0$	$(1.15 \pm 0.31 \pm 0.19)$ $\times B(D_s^+ \to \phi\pi^+)$ [e,f] $(0.92 \pm 0.32 \pm 0.20)$ $\times B(D_s^+ \to \phi\pi^+)$ [c]	2.25			
$\to \eta\pi^+$	$<2.5B(D_s^+ \to \phi\pi^+)$ [d] $<1.5B(D_s^+ \to \phi\pi^+)$ [e]	7.94	3.09(1.45) [b]	6.93	3.03(1.69)
$\to \eta'\pi^+$	$<1.9B(D_s^+ \to \phi\pi^+)$ [d] $<1.6B(D_s^+ \to \phi\pi^+)$ [e] $(2.5 \pm 0.5 \pm 0.3)$ $\times B(D_s^+ \to \phi\pi^+)$ [g]	1.78	0.14(8.5) [b]	3.58	5.15(18.7)

[a]These numbers are used as input.
[b]Figure in brackets uses $B(D^0 \to \eta\bar{K}^0) = 1.6\%$; the other figure uses $B(D^0 \to \eta\bar{K}^0) = 0.7\%$.
[c]Reference 10.
[d]Reference 11.
[e]Reference 12.
[f]Reference 13.
[g]Reference 15.

$$A(D\to PP)=A(D\to P_8P_8)+(P_1P_k^lP^m)(aH^k_{[lm]}+bH^k_{\{lm\}})\ , \qquad (3)$$

where P_1 is the pseudoscalar SU(3) singlet; a and b are two new parameters and $A(D\to P_8P_8)$ is defined in (1).

There are three parameters (c,d,e) in the nonet-symmetry model and five (a,b,c,d,e) in the nonet-symmetry-breaking model. In addition certain phase parameters enter our model through FSI. They are $\delta_{\pi K}=\delta_{1/2}-\delta_{3/2}$, in πK final states, $\delta_{\pi\pi}=\delta_0-\delta_2$, in $\pi\pi$ final states and $\delta_{K\bar{K}}=\delta_0-\delta_1$, in $K\bar{K}$ final states. The subscripts indicate the isospin of the relevant two-body final state. $\delta_{\pi\pi}$ is varied in the interval (180°–215°) while $\delta_{K\bar{K}}$ is varied in the interval (0°–30°). $\delta_{\pi K}$ is fixed[6,7] at 79°. The sign of δ does not affect the calculated rates. The choice of $\delta_{\pi\pi}$ is dictated by the phase-shift analyses for $\pi\pi$ scattering[8] and $\delta_{K\bar{K}}$ from the observation that both $f_0(975)$ with $I^G=0^+$ and $J^P=0^+$ and a_0 (980) with $I^G=1^-$ and $J^P=0^+$ have coupling to $K\bar{K}$ channel.[9] Consequently the $K\bar{K}$ final state resonates in both $I=0$ and 1 channels. Hence we do not expect δ_0 to be very different from δ_1.

In Tables II–IV we present our results for Cabibbo-angle-favored, Cabibbo-angle-suppressed, and double-Cabibbo-angle-suppressed $D\to PP$ decays. The results are given in both the nonet-symmetry and the nonet-symmetry-breaking schemes for $\eta_1-\eta_8$ mixing angles $-11°$ and $-19°$ where the mixing angle θ_P is defined by

$$\eta'=\eta_1\cos\theta_P+\eta_8\sin\theta_P\ ,$$
$$\eta=\eta_8\cos\theta_P-\eta_1\sin\theta_P\ . \qquad (4)$$

The parameters of the model were determined as follows: In nonet symmetry, c, d, and e were determined by the following three data points ($\delta_{\pi K}$ was fixed at 79°):

$$B(D^0\to K^-\pi^+)=(4.2\pm0.4\pm0.4)\%\quad \text{(Ref. 10)}\ , \qquad (5)$$

$$B(D^0\to\bar{K}^0\pi^0)=(1.8\pm0.2\pm0.2)\%\quad \text{(Ref. 11)}\ , \qquad (6)$$

$$B(D^+\to K^+\bar{K}^0)/B(D^+\to\bar{K}^0\pi^+)=(0.271\pm0.065\pm0.039)\%\quad \text{(Refs. 12 and 13)}\ . \qquad (7)$$

Note that the last ratio in (7) from Mark III[10] was higher, $0.317\pm0.086\pm0.048$. We found $(c,d,e)=(318.1,-34.3,94.6)$ in $10^{11/2}$ MeV$^{1/2}$ sec$^{-1/2}$ in the nonet-symmetry scheme. Once the parameters of the model are fixed, $B(D_s^+\to\eta\pi^+)$, $B(D_s^+\to\eta'\pi^+)$, $B(D^0\to\eta\bar{K}^0)$, and $B(D^0\to\eta'\bar{K}^0)$ become predictions. As we see from Table I, the prediction for $B(D^0\to\eta\bar{K}^0)$ is too low and that for $B(D^0\to\eta'\bar{K}^0)$ too high. The prediction for $B(D_s^+\to\eta\pi^+)$ is too high if the E-691 upper limit[12,14] is used (see Table II) but appears to be barely compatible with Mark III limit.[11] $B(D_s^+\to\eta'\pi^+)$ is pre-

TABLE III. Cabibbo-angle-suppressed $D\to PP$ decays.

Mode	Experimental branching ratio (%)	Theoretical branching ratio (%) $\theta_P=-19°$ Nonet symmetry	$\theta_P=-19°$ Broken nonet symmetry	$\theta_p=-11°$ Nonet symmetry	$\theta_p=-11°$ Broken nonet symmetry
$D^0\to\pi^+\pi^-$	$0.14\pm0.4\pm0.03$ [d]	0.14,0.15 [a]			
$\to\pi^0\pi^0$	<0.3 [d]	0.21,0.19 [a]			
				Unaffected	
$\to K^+K^-$	$0.51\pm0.09\pm0.06$ [d]	0.26,0.25 [b]			
$\to K^0\bar{K}^0$	0.10 ± 0.08 [f]	0.0,0.02 [b]			
	$0.06\pm0.03\pm0.01$ [f]				
$\to\eta\eta$	<1.2 [d]	0.008	0.10(0.22) [c]	0.00	0.10(0.24) [c]
$\to\eta\pi^0$	<0.9 [d]	0.066	0.006(0.00) [c]	0.05	0.007(0.00) [c]
$\to\eta'\pi^0$		0.062	0.05(0.17) [c]	0.09	0.11(0.45) [c]
$\to\eta\eta'$		0.20	0.03(0.17) [c]	0.28	0.17(0.8) [c]
$D^+\to\pi^+\pi^0$	<0.48 [d]	0.09			
$\to K^+\bar{K}^0$	$1.01\pm0.32\pm0.18$ [d]	0.9			
	$0.68\pm0.20\pm0.13$ [e]				
				Unaffected	
$D_s^+\to\pi^0K^+$		0.14			
$\to\pi K^0$	$<0.21B(D_s^+\to\phi\pi^+)$ [f]	0.31			
$\to\eta K^+$		0.052	0.001(0.017) [c]	0.026	0.00(0.015) [c]
$\to\eta'K^+$		0.16	0.07(0.26) [c]	0.18	0.17(0.74) [c]

[a]First number is calculated with $\delta_{\pi\pi}=180°$, second with $\delta_{\pi\pi}=215°$.
[b]First number is calculated with $\delta_{K\bar{K}}=0°$, second with $\delta_{K\bar{K}}=30°$.
[c]Figure in brackets uses $B(D^0\to\eta\bar{K}^0)=1.6\%$ as input; the other figure uses $B(D^0\to\eta\bar{K}^0)=0.7\%$ as input.
[d]Reference 10.
[e]References 12 and 13.
[f]Reference 27.

TABLE IV. Double-Cabibbo-angle-suppressed $D \to PP$ decays.

	Theoretical branching ratio (10^{-5})			
	$\theta_P = -19°$		$\theta_P = -11°$	
Mode	Nonet symmetry	Broken nonet symmetry	Nonet symmetry	Broken nonet symmetry
$D^0 \to K^+\pi^-$	11.8		Unaffected	
$\to K^0\pi^0$	5.3			
$\to K^0\eta$	0.16	1.96(4.5) [a]	0.01	2.0(4.5) [a]
$\to K^0\eta'$	8.8	2.24(10.7) [a]	8.9	7.7(35.3) [a]
$D^+ \to K^0\pi^+$	25.9		Unaffected	
$\to K^+\pi^0$	17.4			
$\to K^+\eta$	0.52	13.8(23.4) [a]	2.6	14.2(24.0) [a]
$\to K^+\eta'$	30.7	3.6(22.0) [a]	29.2	16.8(81.7) [a]
$D_s^+ \to K^+K^0$	3.4		Unaffected	

[a]Figure in parentheses uses $B(D^0 \to \bar{K}^0\eta) = 1.6\%$; the other figure uses $B(D^0 \to \bar{K}^0\eta) = 0.7\%$.

dicted to be within both E-691[12] and Mark III[11] limits. ARGUS has observed[15] $D_s^+ \to \eta'\pi^+$ and finds

$$B(D_s^+ \to \eta'\pi^+)/B(D_s^+ \to \phi\pi^+) = 2.5 \pm 0.5 \pm 0.3 .$$

Next, we broke the nonet symmetry by introducing two new parameters a and b as shown in (3). It so happens that the decays $D^0 \to \eta\bar{K}^0$ and $\eta'\bar{K}^0$ depend[5] on the combination $(a-b)$ while the decays $D_s^+ \to \eta\pi^+$ and $\eta'\pi^+$ depend on $(a+b)$. Thus, fixing the combination $(a-b)$ from the measurement of $B(D^0 \to \eta\bar{K}^0)$ gives us a prediction for $B(D^0 \to \eta'\bar{K}^0)$. Similarly, if there were a measurement of $B(D_s^+ \to \eta\pi^+)$ one could determine $(a+b)$ and predict $B(D_s^+ \to \eta'\pi^+)$. However, there are only upper bounds on $B(D_s^+ \to \eta\pi^+)$ and the recent ARGUS[15] measurement of $B(D_s^+ \to \eta'\pi^+)$ is higher than the upper bound set by E-691.[12] Our data fitting indicates that $b \ll a$. *In the tables we have used* $b=0$. Consequently, fitting $B(D^0 \to \eta\bar{K}^0)$ alone determines the remaining branching ratios $B(D^0 \to \eta'\bar{K}^0)$, $B(D_s^+ \to \eta\pi^+)$, and $B(D_s^+ \to \eta'\pi^+)$. We have used two values of $B(D^0 \to \eta\bar{K}^0)$ in Tables II–IV, 0.7% and 1.6% the former being the lowest value allowed by Mark III data[11] and the latter being the central value. For $B(D^0 \to \eta\bar{K}^0) = 0.7\%$ and $\theta_P = -19°$, we found $a = 216.0$ in $10^{11/2}$ MeV$^{1/2}$ sec$^{-1/2}$ and for $B(D^0 \to \eta\bar{K}^0) = 1.6\%$ with $\theta_P = -19$ we found $a = 441.6$ in the same units. The corresponding numbers for $\theta_P = -11°$ were $a = 354.2$ and 739.0.

In the following, we highlight some of our predictions. Since Cabibbo-angle-favored and -suppressed decays were considered in Ref. 2, only the new results are presented below.

A. Double-Cabibbo-angle-suppressed decays

First, within SU(3) symmetry with a FSI, we find

(a) $B(D^0 \to \pi^-K^+)/B(D^0 \to \pi^+K^-)$

$$= B(D^0 \to \pi^0K^0)/B(D^0 \to \pi^0\bar{K}^0) = \tan^4\theta_C , \quad (8)$$

(b) $B(D^+ \to \pi^+K^0)/B(D^+ \to \pi^+\bar{K}^0)$

$$\approx 7.8 \times 10^{-3} = 2.8 \tan^4\theta_C , \quad (9)$$

(c) $B(D^+ \to \pi^0K^+)/B(D^+ \to \pi^+\bar{K}^0)$

$$\simeq 5.3 \times 10^{-3} = 1.9 \tan^4\theta_C , \quad (10)$$

(d) independent of mixing angle θ_P and both in nonet-symmetry and nonet-symmetry breaking,

$$B(D^0 \to K^0\eta)/B(D^0 \to \bar{K}^0\eta) = B(D^0 \to K^0\eta')/B(D^0 \to \bar{K}^0\eta') = \tan^4\theta_C . \quad (11)$$

The ratios in (9) and (10) depart from $\tan^4\theta_C$ due to the fact that in double-Cabibbo-angle-suppressed decays, $D^+ \to \pi^+K^0$ and π^0K^+, the final state is a mixture of $I=\frac{1}{2}$ and $\frac{3}{2}$ while in Cabibbo-angle-favored decay $D^+ \to \pi^+\bar{K}^0$ the final state is a pure $I=\frac{3}{2}$ state.

Our results displayed in (8) and (11) agree with those in Ref. 7.

B. Cabibbo-angle-suppressed $D \to K\bar{K}$ and $\pi\pi$ rates

In our model,

$$B(D^0 \to \pi^+\pi^-)/B(D^0 \to \pi^0\pi^0) = \frac{2\left[1+\frac{r^2}{4}+r\cos\delta_{\pi\pi}\right]}{1+r^2-2r\cos\delta_{\pi\pi}} , \quad (12)$$

where[2] $r = 4e/3A$, $A \equiv c+d+e/3$. $\delta_{\pi\pi} = \delta_0 - \delta_2$, δ_0 and δ_2 being the phases of the decay amplitudes in $I=0$ and 2 states. With our set of parameters $r \approx 0.4$. Also

$$B(D^0 \to K^+K^-)/B(D^0 \to K^0\bar{K}^0) = \frac{1+\cos\delta_{K\bar{K}}}{1-\cos\delta_{K\bar{K}}} , \quad (13)$$

where $\delta_{K\bar{K}} = \delta_0 - \delta_1$, δ_0, and δ_1 being the phases of the amplitudes in $I=0$ and 1 states.

Choosing (with r fixed at 0.4) $\delta_{\pi\pi}=180°$ minimizes $B(D^0\to\pi^+\pi^-)$ and maximizes $B(D^0\to\pi^0\pi^0)$. Similarly choosing $\delta_{K\bar{K}}=0°$ maximizes $B(D^0\to K^+K^-)$ and suppresses $B(D^0\to K^0\bar{K}^0)$ completely. For example, with $\delta_{K\bar{K}}=0°$ we get $B(D^0\to K^+K^-)=0.26\%$ and $B(D^0\to K^0\bar{K}^0)=0$, while with $\delta_{K\bar{K}}=30°$ we get $B(D^0\to K^+K^-)=0.25\%$ and $B(D^0\to K^0\bar{K}^0)=0.02\%$.

In order to secure a large value of the ratio $B(D^0\to K^+K^-)/B(D^0\to\pi^+\pi^-)$ as Mark III data[10] would require (a ratio of about 3.5 ± 1.1) one would need to maximize the numerator (i.e., choose $\delta_{K\bar{K}}$ close to 0°) and minimize the denominator (i.e., choose $\delta_{\pi\pi}$ close to 180°). The following equation summarizes the value of this ratio for $\delta_{\pi\pi}$ close to 180° and $\delta_{K\bar{K}}$ close to 0°:

$$B(D^0\to K^+K^-)/B(D^0\to\pi^+\pi^-)=\begin{cases}1.92 & (\delta_{\pi\pi}=180°,\ \delta_{K\bar{K}}=0°),\\ 1.80 & (\delta_{\pi\pi}=180°,\ \delta_{K\bar{K}}=30°),\\ 1.73 & (\delta_{\pi\pi}=215°,\ \delta_{K\bar{K}}=0°),\\ 1.63 & (\delta_{\pi\pi}=215°,\ \delta_{K\bar{K}}=30°).\end{cases} \tag{14}$$

Thus, within SU(3) symmetry with a FSI, it is difficult to reach values for this ratio in the vicinity of 3.5. On the other hand, E-691, ARGUS, and CLEO data seem to favor a lower value for this ratio

$$B(D^0\to K^+K^-)/B(D^0\to\pi^+\pi^-)=\begin{cases}2.14\pm0.48\pm0.13 & (E-691,\ \text{Ref. 10})\\ 2.5\pm0.7 & (\text{ARGUS, Ref. 17})\\ 2.2\pm0.5 & (\text{CLEO, Ref. 17}).\end{cases} \tag{15}$$

If $B(D^0\to K^0\bar{K}^0)$ were to eventually turn out to be much larger than 0.02%, which at present is at the lower end of experimental tolerance, $B(D^0\to K^+K^-)$ would have to decrease and, consequently, also the ratio $B(D^0\to K^+K^-)/B(D^0\to\pi^+\pi^-)$.

Note also, from Table III, that $B(D^0\to\pi^0\pi^0)$ is within the experimental bound of 0.3%. We feel quite confident that the eventual measurement will reveal a value quite close to 0.2%.

C. η, η' problem

Both $D^0\to\eta\bar{K}^0$ and $\eta'\bar{K}^0$ depend on the combination $(a-b)$ of the nonet-symmetry-breaking model, Eq. (3). Thus, once $B(D^0\to\eta\bar{K}^0)$ is used to fit this parameter, $B(D^0\to\eta'\bar{K}^0)$ is determined. From Table II, it is clear that the upper limit[11] $B(D^0\to\eta'\bar{K}^0)<2.7\%$ prefers $B(D^0\to\eta\bar{K}^0)$ somewhat lower than the central value[11] of 1.6% for $\theta_P=-19°$. For $\theta_P=-11°$, the largest value of $B(D^0\to\eta\bar{K}^0)$ tolerated is 0.7%. Here, we are critical of a recent paper by Rosen,[18] who uses the broken-nonet-symmetry constraint between the three amplitudes

$$A(D^0\to\eta'\bar{K}^0)=-\cot\theta_P A(D^0\to\eta\bar{K}^0)+\frac{\csc\theta_P}{\sqrt{3}}A(D^0\to\pi^0\bar{K}^0)\ . \tag{16}$$

The phase of $A(D^0\to\pi^0\bar{K}^0)$ is known up to an overall phase of the isospin $\frac{3}{2}$ amplitude in $D^0\to\pi K$ decays. He parametrizes $A(D^0\to\eta\bar{K}^0)$ with an arbitrary phase and then shows that there is a choice of this phase which controls $B(D^0\to\eta'K^0)$ and gives him a value of 1.3%—well within the experimental[11] bounds. We believe that this procedure is flawed for the following reasons.

SU(3) (or broken-nonet-symmetry) relations such as

$$A(D^0\to\pi^+K^-)+\sqrt{2}A(D^0\to\pi^0\bar{K}^0)=A(D^+\to\pi^+\bar{K}^0) \tag{17}$$

are preserved even in the presence of a FSI because a FSI respects isospin SU(2) and the triangular relation in (17) involves only isospin SU(2) multiplets. In contrast, the triangular relation of (16) does not involve the same isospin multiplets. Relation (16) is true only in SU(3) symmetry or broken nonet symmetry. The FSI breaks SU(3) symmetry and gives $A(D^0\to\pi^0\bar{K}^0)$ a phase that bears no relationship to the phases of $A(D^0\to\eta\bar{K}^0)$ and $A(D^0\to\eta'\bar{K}^0)$. Thus, in the presence of a FSI, Eq. (16) breaks down.

Our attitude towards $D_s^+\to\eta\pi^+$ and $\eta'\pi^+$ data and the selection of experiments we trust also needs to be explained. The earlier Mark II[19] and the more recent[20] NA 14′ data are now disputed. These data would make

$$B(D_s^+\to\eta'\pi^+)/B(D_s^+\to\phi\pi^+)\approx5$$

and if $B(D_s^+\to\phi\pi^+)$ were to be in the range of 2–4 %, $B(D_s^+\to\eta'\pi^+)$ would be $\approx(10-20)\%$. In contrast, Mark III quotes[11]

$$B(D_s^+\to\eta'\pi^+)/B(D_s^+\to\phi\pi^+)<1.9$$

and E-691 gives[12] an upper limit of 1.6 for this ratio. There are theoretical reasons too, to which we will return, for doubting Mark II and NA 14′ results. The recent measurement by ARGUS,[15]

$$B(D_s^+\to\eta'\pi^+)/B(D_s^+\to\phi\pi^+)=(2.5\pm0.5\pm0.3)$$

contradicts the E-691 upper bound and is just barely consistent with the Mark III upper limit.

Mark II had also reported[19]

$$B(D_s^+\to\eta\pi^+)/B(D_s^+\to\phi\pi^+)\approx3$$

making $B(D_s^+ \to \eta\pi^+) \approx 6-12\,\%$. Recent Mark III measurements[11,21] give an upper bound of 2.5 for this ratio, which is not in disagreement with the Mark II result;[19] however, the E-691 limit[12] of 1.5 would disagree with Mark II data.[19]

Theoretical studies tend to support Mark III[11] and E-691[12] results. It was shown in Ref. 22 that in a factorization model one could easily explain $B(D_s^+ \to \phi\pi^+) \approx 3\%$, $B(D_s^+ \to \eta\pi^+) \approx 3-5\,\%$ and $B(D_s^+ \to \eta'\pi^+) \approx 2-5\,\%$ with $\theta_P = -19°$. Theoretical models could not be stretched to generate $B(D_s^+ \to \eta'\pi^+)$ at the level of $10-20\,\%$. The results of Ref. 22 are quite compatible with the upper limits set by Mark III[11] and E-691.[12]

In Ref. 5, Mark II data on $D_s^+ \to \eta\pi^+$ and $\eta'\pi^+$ were used as inputs. This necessitated large nonet-symmetry breaking. Since η' is largely an SU(3) singlet this nonet-symmetry breaking forced all rates involving η' upwards. Thus, it was found that $B(D_s^+ \to \eta'\pi^+) \approx 20\%$ would require $B(D_s^+ \to \eta'\bar{K}^0) \approx 20\%$ also. This is now contradicted by Mark III data.[11] This is yet another piece of evidence that Mark II and NA 14′ branching ratios for $D_s^+ \to \eta'\pi^+$ are too high.

In the present paper we have found that a nonet-symmetry-breaking scheme is needed to bring $B(D_s^+ \to \eta\pi^+)$ within E-691 bounds.[12] However, to get $B(D_s^+ \to \eta'\pi^+)$ also within E-691[12] and Mark III[11] limits, we need to have $B(D_s^+ \to \eta\bar{K}^0) < 1.6\%$. The new ARGUS data[15] with a sizable branching fraction for $D_s^+ \to \eta'\pi^+$ would require $B(D_s^+ \to \eta\bar{K}^0)$ closer to 1.6%.

III. $D \to VP$ decays

A. Formulation

In contrast with the case of $D \to PP$ decays discussed in the previous section where the final state is allowed to be only in the $\mathbf{8}_S$ and **27** representations of SU(3), in $D \to VP$ decays the final state can be in $\mathbf{8}_S$, $\mathbf{8}_A$, **10**, **10***, and **27** representations of SU(3). As against three independent parameters needed to describe $D \to PP$ decays in nonet symmetry, one needs 7 parameters to describe $D \to VP$ decays in nonet symmetry. If nonet symmetry is broken one needs to introduce four new parameters. This proliferation of parameters makes it harder to analyze $D \to VP$ decays and make predictions in the symmetry scheme we are discussing. In the cruder limit of sextet dominance one is left with only three parameters in the nonet symmetry and one is then able to make some predictions, which we will discuss in Sec. IV.

The general structure of the decay amplitude for $D \to VP$ decays in a broken-nonet-symmetry scheme is (details and comparison with the notation of Einhorn and Quigg,[3] Kohara,[23] and Rosen[24] are given in the Appendix)

$$\begin{aligned} A(D \to VP) = {} & [a_1(P_l^i V_j^l P^k) + a_2(P_j^l V_l^i P^k) + a_3(P_j^i V_l^k + P_l^i V_j^k)P^l]H^j_{[i,k]} \\ & + [b_1(P_l^i V_j^l P^k) + b_2(P_j^l V_l^i P^k) + b_3(P_j^i V_l^k P^l) + b_4(P_l^i V_j^k P^l)]H^j_{\{i,k\}} \\ & + [c_1(P_j^k V_m^m P^i) + c_2(P_m^m V_j^k P^i)]H^j_{[i,k]} + [d_1(P_j^k V_m^m P^i) + d_2(P_m^m V_j^k P^i)]H^j_{\{i,k\}} \;. \end{aligned} \tag{18}$$

TABLE V. Decay amplitudes for Cabibbo-angle-favored ($\Delta C = \Delta S = 1$) $D \to PV$ modes. (Amplitudes are $\cos^2\theta_C$ times values listed.)

Amplitude	a_1	a_2	a_3	b_1	b_2	b_3	b_4	$3c_1$	$3c_2$	$3d_1$	$3d_2$
$A(D^+ \to \bar{K}^0\rho^+)$	0	0	-2	0	0	1	1	0	0	0	0
$A(D^+ \to \pi^+ K^{*0})$	0	0	2	0	0	1	1	0	0	0	0
$A(D^0 \to K^-\rho^+)$	-1	0	-1	1	0	0	1	0	0	0	0
$\sqrt{2}A(D^0 \to \bar{K}^0\rho^0)$	1	0	-1	-1	0	1	0	0	0	0	0
$\sqrt{6}A(D^0 \to \bar{K}^0 V_8)$	-1	2	-1	1	-2	1	0	0	0	0	0
$\sqrt{3}A(D^0 \to \bar{K}^0 V_1)$	-1	-1	-1	1	1	1	0	1	0	1	0
$A(D^0 \to \pi^+ K^{*-})$	0	-1	1	0	1	1	0	0	0	0	0
$\sqrt{2}A(D^0 \to \pi^0\bar{K}^{*0})$	0	1	1	0	-1	0	1	0	0	0	0
$\sqrt{6}A(D^0 \to \eta_8\bar{K}^{*0})$	2	-1	1	-2	1	0	1	0	0	0	0
$\sqrt{3}A(D^0 \to \eta_1\bar{K}^{*0})$	-1	-1	1	1	1	0	1	0	1	0	1
$\sqrt{2}A(D_s^+ \to \pi^+\rho^0)$	-1	1	0	-1	1	0	0	0	0	0	0
$\sqrt{6}A(D_s^+ \to \pi^+ V_8)$	1	1	-2	1	1	-2	0	0	0	0	0
$\sqrt{3}A(D_s^+ \to \pi^+ V_1)$	1	1	1	1	1	1	0	-1	0	1	0
$\sqrt{2}A(D_s^+ \to \pi^0\rho^+)$	1	-1	0	1	-1	0	0	0	0	0	0
$\sqrt{6}A(D_s^+ \to \eta_8\rho^+)$	1	1	2	1	1	0	-2	0	0	0	0
$\sqrt{3}A(D_s^+ \to \eta_1\rho^+)$	1	1	-1	1	1	0	1	0	-1	0	1
$A(D_s^+ \to K^+\bar{K}^{*0})$	1	0	1	1	0	0	1	0	0	0	0
$A(D_s^+ \to \bar{K}^0 K^{*+})$	0	1	-1	0	1	1	0	0	0	0	0

Here P_l^i and V_l^i are pseudoscalar and vector nonets respectively. c_1, c_2, d_1, and d_1 are independent of a_i and b_i in broken nonet symmetry while in nonet symmetry they are related to a_i and b_i as

$$
\begin{aligned}
c_1 &= -c_2 = (a_3)/3 , \\
d_1 &= -(4b_3 - b_4)/15 , \\
d_2 &= (b_3 - 4b_4)/15 .
\end{aligned} \tag{19}
$$

In our calculations we have assumed ideal mixing for the vector mesons:[9]

$$
\begin{aligned}
\omega &= V_1 \cos\theta_V + V_8 \sin\theta_V , \\
\phi &= V_8 \cos\theta_V - V_1 \sin\theta_V ,
\end{aligned} \tag{20}
$$

with $\sin\theta_V = 1/\sqrt{3}$ and $\cos\theta_V = (\frac{2}{3})^{1/2}$. In Tables V–VII we have listed the amplitudes for the Cabibbo-angle-favored, -suppressed, and -double-suppressed decays. In the following we have summarized some of the relations implied by the listing of these tables for the nonet-symmetric weak Hamiltonian ($6^* + 15$).

1. Cabibbo-angle-favored modes

$$
\begin{aligned}
&A(D_s^+ \to \pi^0 \rho^+) = -A(D_s^+ \to \pi^+ \rho^0) , \\
&A(D^0 \to K^- \rho^+) + \sqrt{2} A(D^0 \to \bar{K}^0 \rho^0) \\
&\qquad = A(D^+ \to \bar{K}^0 \rho^+) , \qquad (21) \\
&A(D^0 \to \pi^+ K^{*-}) + \sqrt{2} A(D^0 \to \pi^0 \bar{K}^{*0}) \\
&\qquad = A(D^+ \to \pi^+ \bar{K}^{*0}) .
\end{aligned}
$$

2. Cabibbo-angle-suppressed modes:

$$
\begin{aligned}
&A(D^+ \to K^+ \bar{K}^{*0}) = -A(D_s^+ \to \pi^+ K^{*0}) , \\
&A(D^+ \to \bar{K}^0 K^{*+}) = -A(D_s^+ \to K^0 \rho^+) , \\
&A(D^0 \to \pi^+ \rho^-) = -A(D^0 \to K^+ \bar{K}^{*-}) \\
&\qquad = \tan\theta_C A(D^0 \to \pi^+ K^{*-}) , \\
&A(D^0 \to \pi^- \rho^+) = -A(D^0 \to K^- K^{*+}) \\
&\qquad = \tan\theta_C A(D^0 \to K^- \rho^+) , \qquad (22)
\end{aligned}
$$

TABLE VI. Decay amplitudes for Cabibbo-angle-suppressed ($\Delta C = -1$, $\Delta S = 0$) $D \to PV$ modes. (Amplitudes are $-\sin\theta_C \cos\theta_C$ times values listed.)

Amplitude	a_1	a_2	a_3	b_1	b_2	b_3	b_4	$3c_1$	$3c_2$	$3d_1$	$3d_2$
$\sqrt{2}A(D^+ \to \pi^+\rho^0)$	−1	1	−2	−1	1	−1	−1	0	0	0	0
$\sqrt{6}A(D^+ \to \pi^+ V_8)$	1	1	4	1	1	1	3	0	0	0	0
$\sqrt{3}A(D^+ \to \pi^+ V_1)$	1	1	1	1	1	1	0	−1	0	1	0
$\sqrt{2}A(D^+ \to \pi^0\rho^+)$	1	−1	2	1	−1	−1	−1	0	0	0	0
$\sqrt{6}A(D^+ \to \eta_8\rho^+)$	1	1	−4	1	1	3	1	0	0	0	0
$\sqrt{3}A(D^+ \to \eta_1\rho^+)$	1	1	−1	1	1	0	1	0	−1	0	1
$A(D^+ \to K^+\bar{K}^{*0})$	1	0	−1	1	0	−1	0	0	0	0	0
$A(D^+ \to \bar{K}^0 K^{*+})$	0	1	1	0	1	0	−1	0	0	0	0
$A(D^0 \to K^0\bar{K}^{*0})$	−1	1	0	1	−1	0	0	0	0	0	0
$A(D^0 \to \bar{K}^0 K^{*0})$	1	−1	0	−1	1	0	0	0	0	0	0
$A(D^0 \to K^- K^{*+})$	1	0	1	−1	0	0	−1	0	0	0	0
$A(D^0 \to K^+ K^{*-})$	0	1	−1	0	−1	−1	0	0	0	0	0
$A(D^0 \to \pi^+\rho^-)$	0	−1	1	0	1	1	0	0	0	0	0
$A(D^0 \to \pi^-\rho^+)$	−1	0	−1	1	0	0	1	0	0	0	0
$2A(D^0 \to \pi^0\rho^0)$	−1	−1	0	1	1	−1	−1	0	0	0	0
$2\sqrt{3}A(D^0 \to \pi^0 V_8)$	1	1	4	−1	−1	−1	3	0	0	0	0
$\sqrt{6}A(D^0 \to \pi^0 V_1)$	1	1	1	−1	−1	−1	0	−1	0	−1	0
$2\sqrt{3}A(D^0 \to \eta_8\rho^0)$	1	1	−4	−1	−1	3	−1	0	0	0	0
$\sqrt{6}A(D^0 \to \eta_1\rho^0)$	1	1	−1	−1	−1	0	−1	0	−1	0	−1
$2A(D^0 \to \eta_8 V_8)$	1	1	0	−1	−1	1	1	0	0	0	0
$\sqrt{2}A(D^0 \to \eta_8 V_1)$	−1	−1	−1	1	1	1	0	1	0	1	0
$\sqrt{2}A(D^0 \to \eta_1 V_8)$	−1	−1	1	1	1	0	1	0	1	0	1
$A(D^0 \to \eta_1 V_1)$	0	0	0	0	0	0	0	0	0	0	0
$A(D_s^+ \to K^0\rho^+)$	0	−1	−1	0	−1	0	1	0	0	0	0
$\sqrt{2}A(D_s^+ \to K^+\rho^0)$	0	−1	−1	0	−1	0	−1	0	0	0	0
$\sqrt{6}A(D_s^+ \to K^+ V_8)$	2	−1	5	2	−1	2	3	0	0	0	0
$\sqrt{3}A(D_s^+ \to K^+ V_1)$	−1	−1	−1	−1	−1	−1	0	1	0	−1	0
$A(D_s^+ \to \pi^+ K^{*0})$	−1	0	1	−1	0	1	0	0	0	0	0
$\sqrt{2}A(D_s^+ \to \pi^0 K^{*+})$	−1	0	1	−1	0	−1	0	0	0	0	0
$\sqrt{6}A(D_s^+ \to \eta_8 K^{*+})$	−1	2	−5	−1	2	3	2	0	0	0	0
$\sqrt{3}A(D_s^+ \to \eta_1 K^{*+})$	−1	−1	1	−1	−1	0	−1	0	1	0	−1

TABLE VII. Decay amplitudes for double-Cabibbo-angle-suppressed ($\Delta C = -\Delta S = -1$) $D \to PV$ modes. (Amplitudes are $-\sin^2\theta_C$ times values listed.)

Amplitude	a_1	a_2	a_3	b_1	b_2	b_3	b_4	$3c_1$	$3c_2$	$3d_1$	$3d_2$
$\sqrt{2}A(D^+\to K^+\rho^0)$	0	1	−1	0	1	−1	0	0	0	0	0
$\sqrt{6}A(D^+\to K^+V_8)$	−2	1	1	−2	1	1	0	0	0	0	0
$\sqrt{3}A(D^+\to K^+V_1)$	1	1	1	1	1	1	0	−1	0	1	0
$A(D^+\to K^0\rho^+)$	0	1	−1	0	1	1	0	0	0	0	0
$A(D^+\to \pi^+K^{*0})$	1	0	1	1	0	0	1	0	0	0	0
$\sqrt{2}A(D^+\to \pi^0K^{*+})$	1	0	1	1	0	0	−1	0	0	0	0
$\sqrt{6}A(D^+\to \eta_8K^{*+})$	1	−2	−1	1	−2	0	1	0	0	0	0
$\sqrt{3}A(D^+\to \eta_1K^{*+})$	1	1	−1	1	1	0	1	0	−1	0	1
$A(D^0\to K^+\rho^-)$	0	−1	1	0	1	1	0	0	0	0	0
$\sqrt{2}A(D^0\to K^0\rho^0)$	0	1	−1	0	−1	1	0	0	0	0	0
$\sqrt{6}A(D^0\to K^0V_8)$	2	−1	−1	−2	1	1	0	0	0	0	0
$\sqrt{3}A(D^0\to K^0V_1)$	−1	−1	−1	1	1	1	0	1	0	1	0
$A(D^0\to \pi^-K^{*+})$	−1	0	−1	1	0	0	1	0	0	0	0
$\sqrt{2}A(D^0\to \pi^0K^{*0})$	1	0	1	−1	0	0	1	0	0	0	0
$\sqrt{6}A(D^0\to \eta_8K^{*0})$	−1	2	1	1	−2	0	1	0	0	0	0
$\sqrt{3}A(D^0\to \eta_1K^{*0})$	−1	−1	1	1	1	0	1	0	1	0	1
$A(D_s^+\to K^+K^{*0})$	0	0	2	0	0	1	1	0	0	0	0
$A(D_s^+\to K^0K^{*+})$	0	0	−2	0	0	1	1	0	0	0	0

$$A(D^+\to\pi^+V_1)=\tan\theta_C\,A(D_s^+\to\pi^+V_1)\,,$$
$$A(D^+\to\rho^+\eta_1)=\tan\theta_C\,A(D_s^+\to\rho^+\eta_1)\,,$$
$$A(D^0\to\pi^0V_1)=-\tan\theta_C\,A(D^0\to\bar{K}^0V_1)\,,$$
$$A(D^0\to\rho^0\eta_1)=-\tan\theta_C\,A(D^0\to\bar{K}^{*0}\eta_1)\,.$$

3. Double-Cabibbo-angle-suppressed modes:

$$\begin{aligned}
&A(D^0\to K^+\rho^-)+\sqrt{2}A(D^0\to K^0\rho^0)=A(D^+\to K^0\rho^+)-\sqrt{2}A(D^+\to K^+\rho^0)\,,\\
&A(D^0\to\pi^-K^{*+})+\sqrt{2}A(D^0\to\pi^0K^{*0})=A(D^+\to\pi^+K^{*0})-\sqrt{2}A(D^+\to\pi^0K^{*+})\,,\\
&A(D_s^+\to K^+(K^0),K^{*0}(K^{*+}))=\tan^2\theta_C\,A(D^+\to\pi^+(\bar{K}^0),\bar{K}^{*0}(\rho^+))\,,\\
&A(D^+\to\pi^+(K^0),K^{*0}(\rho^+))=\tan^2\theta_C\,A(D_s^+\to K^+(\bar{K}^0),\bar{K}^{*0}(K^{*+}))\,,\\
&A(D^0\to K^+(\pi^-),\rho^-(K^{*+}))=\tan^2\theta_C\,A(D^0\to\pi^+(K^-),K^{*-}(\rho^+))\,,\\
&A(D^+\to K^+(\eta_1),V_1(K^{*+}))=\tan^2\theta_C\,A(D_s^+\to\pi^+(\eta_1),V_1(\rho^+))\,,\\
&A(D^0\to K^0(\eta_1),V_1(K^{*0}))=\tan^2\theta_C\,A(D^0\to\bar{K}^0(\eta_1),V_1(\bar{K}^{*0}))\,.
\end{aligned}\tag{23}$$

Each of the last five equalities has to be read as a pair of equalities. For example, the very last equality in (23) implies

$$A(D^0\to K^0V_1)=\tan^2\theta_C\,A(D^0\to\bar{K}^0V_1)$$

and

$$A(D^0\to\eta_1K^{*0})=\tan^2\theta_C\,A(D^0\to\eta_1\bar{K}^{*0})\,.$$

Among the Cabibbo-angle-favored decays only $D\to\bar{K}\rho$ and $D\to\bar{K}^*\pi$ have two isospins, $I=\frac{1}{2}$ and $\frac{3}{2}$ in these cases, in the final state. All other decays listed in Table V involve only a single isospin final state. This includes $D_s^+\to\pi^+\rho^0$, where the final state being charged is forbidden to have $I=0$, and since the weak spurion cannot excite $I=2$, the final state is a pure $I=1$ eigenstate. In fact, the first of (21) is a consequence of the vanishing of $I=2$ decay in $D_s^+\to(\rho\pi)^+$.

Thus, interference effects due to FSI are important only in $D\to\bar{K}\rho$ and $D\to\bar{K}^*\pi$ decays. That is not to say that an inelastic FSI could not affect the magnitudes and phases of the other decay modes but only that interference effects will not enter their description.

B. Amplitude analysis of Cabibbo-angle-favored $D\to\bar{K}\rho$ and $\bar{K}^*\pi$ decays

Mark III group has carried out an amplitude analysis of their data[25]:

$$B(D^0 \to K^- \rho^+) = (10.8 \pm 0.4 \pm 1.7)\% ,$$
$$B(D^0 \to \bar{K}^0 \rho^0) = (0.8 \pm 0.1 \pm 0.5)\% , \tag{24}$$
$$B(D^+ \to \bar{K}^0 \rho^+) = (6.9 \pm 0.8 \pm 2.3)\% ,$$

and

$$B(D^0 \to K^{*-} \pi^+) = (5.3 \pm 0.4 \pm 1.0)\% ,$$
$$B(D^0 \to \bar{K}^{*0} \pi^0) = (2.6 \pm 0.3 \pm 0.7)\% , \tag{25}$$
$$B(D^+ \to \bar{K}^{*0} \pi^+) = (5.9 \pm 1.9 \pm 2.5)\% .$$

With the following definition of the decay amplitudes in terms of final-state isospin $\frac{1}{2}$ and $\frac{3}{2}$ amplitudes,

$$A(D^0 \to K^- \rho^0) = \frac{1}{\sqrt{3}}(A_{3/2} e^{i\delta_{3/2}} + \sqrt{2} A_{1/2} e^{i\delta_{1/2}}) ,$$
$$A(D^0 \to \bar{K}^0 \rho^0) = \frac{1}{\sqrt{3}}(\sqrt{2} A_{3/2} e^{i\delta_{3/2}} - A_{1/2} e^{i\delta_{1/2}}) , \tag{26}$$
$$A(D^+ \to \bar{K}^0 \rho^+) = \sqrt{3} A_{3/2} e^{i\delta_{3/2}} ,$$

they find[25]

for $\bar{K}\rho$ mode:

$$A_{1/2}/A_{3/2} = 3.12 \pm 0.4 ,$$
$$\delta \equiv \delta_{1/2} - \delta_{3/2} = (0 \pm 26)^\circ , \tag{27}$$
$$\cos\delta = 1.0^{+0.0}_{-0.1} ,$$

for $\bar{K}^*\pi$ mode:

$$A_{1/2}/A_{3/2} = 3.22 \pm 0.97 ,$$
$$\delta = (84 \pm 13)^\circ , \tag{28}$$
$$\cos\delta = 0.1 \pm 0.2 .$$

The above analysis agrees with one mode in Ref. 7.

In comparison, a similar analysis of Mark III data on $D \to \bar{K}\pi$ decay yields

$\bar{K}\pi$ mode:

$$A_{1/2}/A_{3/2} = 3.67 \pm 0.27 ,$$
$$\delta = (77 \pm 11)^\circ , \tag{29}$$
$$\cos\delta = 0.2 \pm 0.2 .$$

From the last three equations one finds that the ratio $A_{1/2}/A_{3/2}$ is much the same for $D \to PP$ and $D \to VP$ modes. The relative phase δ is almost the same for $D \to \bar{K}\pi$ and $\bar{K}^*\pi$ modes. The small value of δ in $D \to \bar{K}\rho$ decays is necessitated by the small value of $B(D^0 \to \bar{K}^0\rho^0)$, which demands a large destructive interference between $I = \frac{1}{2}$ and $\frac{3}{2}$ amplitudes in (26).

C. Predictions for Cabibbo-angle-suppressed and -double-suppressed branching ratios

In this subsection we have calculated the branching ratios for some Cabibbo-angle-suppressed and -double-suppressed modes from the constraints of (22) and (23). To do this we have assumed nonet symmetry, see (19), and *ignored the FSI*. Where data are available, the predicted branching ratios compare reasonably well with experiments. This gives us some confidence in the predictions for rates where data are not yet available. We predict the following.

Cabibbo-angle-suppressed modes:

1. $B(D_s^+ \to \pi^+ K^{*0}) = 0.84 B(D^+ \to K^+ \bar{K}^{*0})$;

with $B(D^+ \to K^+ \bar{K}^{*0}) = (0.44 \pm 0.23)\%$ (Mark III, Ref. 10), we get

$$B(D_s^+ \to \pi^+ K^{*0}) = (0.37 \pm 0.19)\% .$$

2. $B(D_s^+ \to K^0 \rho^+) \simeq B(D^+ \to \bar{K}^0 K^{*+})$.

3. $B(D^0 \to K^- K^{*+}) \simeq 0.03 B(D^0 \to K^- \rho^+)$;

with $B(D^0 \to K^- \rho^+) = (10.8 \pm 0.4 \pm 1.7)\%$ (Mark III, Ref. 10), we get

$$B(D^0 \to K^- K^{*+}) = (0.32 \pm 0.05)\% .$$

The measured rate is

$$B(D^0 \to K^- K^{*+})_{\text{expt}} = (0.82 \pm 0.42)\% \quad \text{(E-691, Ref. 12)} .$$

4. $B(D^0 \to \pi^- \rho^+) = 0.076 B(D^0 \to K^- \rho^+) = (0.82 \pm 0.13)\%$.

This number uses Mark III[10] input for $B(D^0 \to K^- \rho^+)$ as in item (3) above. Mark III gives[10] $B(D^0 \to \pi^+ \pi^- \pi^0)_{\text{expt}} = (1.1 \pm 0.4 \pm 0.3)\%$.

5. $B(D^0 \to K^+ K^{*-}) \simeq 0.033 B(D^0 \to \pi^+ K^{*-})$;

with $B(D^0 \to \pi^+ K^{*-}) = (5.2 \pm 0.3 \pm 1.5)\%$ (Mark III, Ref. 10), we get

$$B(D^0 \to K^+ K^{*-}) = (0.17 \pm 0.05)\% .$$

Experiments:

$$B(D^0 \to K^+ K^{*-}) = (0.8 \pm 0.5)\% \text{ (Mark III, Ref. 10)}$$
$$< 0.16\% \text{ (E-691, Ref. 12)}.$$

6. $B(D^0 \to \pi^+ \rho^-) = 0.09 B(D^0 \to \pi^+ K^{*-}) = (0.47 \pm 0.14)\%$.

This number uses Mark III input for $B(D^0 \to \pi^+ K^{*-})$ as in item (5).

Experiment:

$$B(D^0 \to \pi^+ \pi^- \pi^0)_{\text{expt}} = (1.1 \pm 0.4 \pm 0.2)\% \quad \text{(Mark III, Ref. 10)} .$$

Double-Cabibbo-angle-suppressed modes:

7. $B(D^+\to\pi^+K^{*0})=2.73\tan^4\theta_C B(D_s^+\to K^+\bar{K}^{*0})$;

with

$$B(D_s^+\to K^+\bar{K}^{*0})=(0.87\pm0.14)B(D_s^+\to\phi\pi^+)\ \text{(E-691, Refs. 12 and 14)}\ ,$$
$$=(0.84\pm0.30\pm0.22)B(D_s^+\to\phi\pi^+)\ \text{(Mark III, Ref. 26)}\ ,$$

we get

$$B(D^+\to\pi^+K^{*0})=(0.67\pm0.11)10^{-2}B(D_s^+\to\phi\pi^+)$$

using E-691 data.

8. $B(D^+\to K^0\rho^+)=3.18\tan^4\theta_C B(D_s^+\to\bar{K}^0K^{*+})$

with

$$B(D_s^+\to\bar{K}^0K^{*+})=\begin{cases}(0.22\pm0.11)B(D_s^+\to\phi\pi^+)\ \text{(E-691, Ref. 12)},\\(1.2\pm0.21\pm0.13)B(D_s^+\to\phi\pi^+)\ \text{(CLEO, Refs. 14 and 27)}\ ,\end{cases}$$

we get

$$B(D^+\to K^0\rho^+)=\begin{cases}(0.2\pm0.1)10^{-2}B(D_s^+\to\phi\pi^+),\\(1.07\pm0.22)10^{-2}B(D_s^+\to\phi\pi^+)\ .\end{cases}$$

The former number uses E-691 (Ref. 12) data and the latter CLEO (Refs. 14 and 27) data.

9. $B(D^0\to K^+\rho^-)=1.16\tan^4\theta_C B(D^0\to\pi^+K^{*-})$

$$=(1.68\pm0.5)\times10^{-2}\%\ .$$

This number uses Mark III (Ref. 10) input for $B(D^0\to\pi^+K^{*-})$ as in item 5 above.

10. $B(D^0\to\pi^-K^{*+})=0.86\tan^4\theta_C B(D^0\to K^-\rho^+)$

$$=(2.58\pm0.42)\times10^{-2}\%\ .$$

This number uses Mark III (Ref. 10) input for $B(D^0\to K^-\rho^+)$ as in item 3 above.

11. $B(D_s^+\to K^+K^{*0})=0.36\tan^4\theta_C B(D^+\to\pi^+\bar{K}^{*0})$;

with $B(D^+\to\pi^+\bar{K}^{*0})=(5.9\pm1.9\pm2.5)\%$ (Mark III, Ref. 10), we get

$$B(D_s^+\to K^+K^{*0})=(0.59\pm0.34)\times10^{-2}\%\ .$$

12. $B(D_s^+\to K^0K^{*+})=0.31\tan^4\theta_C B(D^+\to\bar{K}^0\rho^+)$;

with $B(D^+\to\bar{K}^0\rho^+)=(6.9\pm0.8\pm2.3)\%$ (Mark III, Ref. 10), we get

$$B(D_s^+\to K^0K^{*+})=(0.61\pm0.21)\times10^{-2}\%\ .$$

In the above predictions, wherever data are available, the predictions are fairly good; for example, see items 3 to 6 above. The disagreement is due to the neglect of phases. This gives us reason for confidence in predictions for other branching ratios.

IV. RESULTS IN SEXTET DOMINANCE

In sextet dominance only the terms proportional to a_1, a_2, a_3, c_1, and c_2 in (18) contribute; all b_i's vanish. Further, in nonet symmetry, c_1 and c_2 are related to a_3 as in (19). The results that follow are in the nonet-symmetry scheme. The decay amplitudes for $D\to VP$ decays can be readoff from Tables V–VII. To simplify the calculations we have used an $\eta_1-\eta_8$ mixing angle given by $\sin\theta_P=-\frac{1}{3}$ or $\theta_P=-19.47°$. Some of the amplitude constraints that emerge are as follows.

Cabibbo-angle-favored modes:

$$\begin{aligned}
&A(D^+\to\pi^+\bar{K}^{*0})=-A(D^+\to\bar{K}^0\rho^+)=-3A(D_s^+\to\phi\pi^+)\ ,\\
&A(D_s^+\to K^+\bar{K}^{*0})=-A(D^0\to K^-\rho^+)\ ,\\
&A(D_s^+\to\bar{K}^0K^{*+})=-A(D^0\to\pi^+K^{*-})\ ,\\
&\sqrt{3}A(D_s^+\to\eta'\rho^+)=A(D_s^+\to\omega\pi^+)\ ;
\end{aligned}\tag{30}$$

Cabibbo-angle-suppressed modes:

$$\begin{aligned}
&\sqrt{2}A(D^0\to\eta\rho^0)=A(D^+\to\eta\rho^+)\ ,\\
&\sqrt{2}A(D_s^+\to K^+\rho^0)=A(D_s^+\to K^0\rho^+)=-\sqrt{2}\tan\theta_C A(D^0\to\pi^0\bar{K}^{*0})\ ,\\
&\sqrt{2}A(D_s^+\to\pi^0K^{*+})=A(D_s^+\to\pi^+K^{*0})=-\sqrt{2}\tan\theta_C A(D^0\to\bar{K}^0\rho^0)\ ,\\
&A(D^0\to\bar{K}^0K^{*0})=-\sqrt{2}\tan\theta_C A(D_s^+\to\pi^+\rho^0)\ ,\\
&2A(D^0\to\omega\pi^0)=2\sqrt{3}A(D^0\to\eta'\rho^0)=\sqrt{6}A(D^+\to\eta'\rho^+)\ ,\\
&\qquad=\sqrt{2}A(D^+\to\omega\pi^+)=\sqrt{3}\tan\theta_C A(D_s^+\to\eta\rho^+)\ ,\\
&\sqrt{2}A(D^0\to\phi\pi^0)=A(D^+\to\phi\pi^+)=-2\tan\theta_C A(D_s^+\to\phi\pi^+)\ ,\\
&\sqrt{2}A(D^0\to\eta'\omega)=-\tan\theta_C A(D_s^+\to\eta'\rho^+)\ .
\end{aligned}\tag{31}$$

Double-Cabibbo-angle-suppressed modes:

$$\begin{aligned}
&A(D_s^+\to K^+K^{*0})=-\tan^2\theta_C A(D^+\to\bar{K}^0\rho^+)\ ,\\
&\sqrt{2}A(D^+\to\rho^0K^+)=A(D^+\to K^0\rho^+)=\sqrt{2}A(D^0\to\rho^0K^0)\\
&\qquad=-\tan^2\theta_C A(D^0\to\pi^+K^{*-})\ ,\\
&\sqrt{2}A(D^+\to\pi^0K^{*+})=A(D^+\to\pi^+K^{*0})=\sqrt{2}A(D^0\to\pi^0K^{*0})\\
&\qquad=-\tan^2\theta_C A(D^0\to K^-\rho^+)\ ,\\
&A(D^0\to K^0\phi)=-A(D^+\to K^+\phi)\ ,\\
&A(D^0\to\omega K^0)=-A(D^+\to\omega K^+)\ .
\end{aligned}\tag{32}$$

The relations in (30) imply the following branching ratios in the Cabibbo-angle-favored decays.

1. $B(D^+\to\pi^+\bar{K}^{*0})/B(D^+\to\bar{K}^0\rho^+)=0.86$.

The Mark III (Ref. 10) value for this ratio is (0.86±0.55) (our errors; the actual error is probably much less due to the cancellation among some systematic errors in the ratio). This is in excellent agreement with the sextet dominance prediction.

2. $B(D_s^+\to K^+\bar{K}^{*0})=0.77B(D^0\to K^-\rho^+)$.

Using Mark III (Ref. 10) value for $B(D^0\to K^-\rho^+)$ given in item 3 of Sec. III C, we get

$$B(D_s^+\to K^+\bar{K}^{*0})=(8.32\pm1.34)\%\ .$$

The experimental number is

$$\begin{aligned}
B(D_s^+\to K^+\bar{K}^{*0})=&(0.84\pm0.30\pm0.22)B(D_s^+\to\phi\pi^+)\ \text{(Mark III, Refs. 10 and 26)}\\
&(0.87\pm0.14)B(D_s^+\to\phi\pi^+)\ \text{(E-691, Refs. 12 and 14)}\ .
\end{aligned}$$

With $B(D_s^+\to\phi\pi^+)$ expected to be in the range of 2–4 %, the prediction for $B(D_s^+\to K^+\bar{K}^{*0})$ is a little too high.

3. $B(D_s^+\to\bar{K}^0K^{*+}=0.90B(D^0\to\pi^+K^{*-})$.

Using $B(D^0\to\pi^+K^{*-})$ from Mark III (Ref. 10) as in item 5 of Sec. III C, we get

$$B(D_s^+\to\bar{K}^0K^{*+})=(4.7\pm1.4)\%\ .$$

Experiments give

$$\begin{aligned}
B(D_s^+\to\bar{K}^0K^{*+})=&(0.22\pm0.11)B(D_s^+\to\phi\pi^+)\ \text{(E-691, Ref. 12)}\\
&(1.2\pm0.21\pm0.13)B(D_s^+\to\phi\pi^+)\ \text{(CLEO, Refs. 14 and 27)}\ .
\end{aligned}$$

The prediction is closer to the CLEO data with $B(D_s^+ \to \phi\pi^+) \approx 2-4\%$.

4. $B(D_s^+ \to \omega\pi^+) = 15.5 B(D_s^+ \to \eta'\rho^+)$.

This prediction is probably too high. E-691 limit[28] is $B(D_s^+ \to \omega\pi^+) < 0.5\%$. We comment on this below.

5. $B(D_s^+ \to \phi\pi^+) = 0.03 B(D^+ \to \bar{K}^0\rho^+)$.

With Mark III (Ref. 10) value of $B(D^+ \to \bar{K}^0\rho^+)$ used in item 12 of Sec. III C, we get

$$B(D_s^+ \to \phi\pi^+) = (0.21 \pm 0.07)\% ,$$

a value too low.[10,29] We comment on this below.

In order to understand the origin of the results in items 4 and 5 above—large $B(D_s^+ \to \omega\pi^+)$ and small $B(D_s^+ \to \phi\pi^+)$—one notes from the listing of the amplitudes in Table V that $A(D_s^+ \to \phi\pi^+)$ is proportional to a_3 while $A(D_s^+ \to \omega\pi^+)$ is proportional to $(a_1 + a_2)$ and $A(D_s^+ \to \rho^0\pi^+)$ proportional to $(a_1 - a_2)$. In order to suppress[10] $B(D_s^+ \to \rho^0\pi^+)$, one has to require $a_1 \approx a_2$. This necessarily enhances $B(D_s^+ \to \omega\pi^+)$. The parameters a_1, a_3, and $\delta_{\rho K}$ are determined from the decay modes $D \to \bar{K}\rho$. a_2 is determined from $D^0 \to \pi^+ K^{*-}$ and $\pi^0 \bar{K}^{*0}$ modes. We find $a_1 \approx a_2 \approx 3a_3$. Because of the smallness of a_3 compared to the other two parameters the decay amplitude for $D_s^+ \to \phi\pi^+$, which is proportional to a_3, is suppressed. In fact in sextet dominance with nonet symmetry we find

$$A(D_s^+ \to \phi\pi^+) = \tfrac{1}{3} A(D \to \bar{K}^0\rho^+) ,$$

which clearly shows the suppression of $B(D_s^+ \to \phi\pi^+)$. Thus sextet dominance symmetry breaks down in describing $D_s^+ \to \omega\pi^+$ and $\phi\pi^+$.

We conclude this section with some general comments on sextet dominance. In sextet dominance the weak spurion is an isovector ($\Delta I = 1$) in the Cabibbo-angle-favored ($\Delta C = \Delta S = -1$) sector. It is an isospinor ($\Delta I = \frac{1}{2}$) in Cabibbo-angle-suppressed ($\Delta C = -1, \Delta S = 0$) sector and an isoscalar ($\Delta I = 0$) in double-Cabibbo-angle-suppressed ($\Delta C = -\Delta S = -1$) sector. As a consequence, (i) D_s^+ decays involve only a single isospin final states, i.e., interferences due to different isospin states do not occur, and (ii) all double-Cabibbo-angle-suppressed decays involves a single isospin final state, and again no interference due to different isospin states occurs.

V. SUMMARY AND DISCUSSION

In this paper we have investigated $D \to PP$ and $D \to VP$ decays in both a nonet-symmetry and a nonet-symmetry-breaking scheme. In $D \to PP$ decays, we have reanalyzed the Cabibbo-angle-favored and -suppressed decays that we had originally discussed in Ref. 2. The double-Cabibbo-angle-suppressed $D \to PP$ decays have been discussed earlier by Chau and Cheng.[7] Our results, with the inclusion of FSI phases, confirm theirs. We have also investigated the issue of the ratio $B(D^0 \to K^+K^-)/B(D^0 \to \pi^+\pi^-)$. FSI phases play a crucial role here. In the symmetry approach a value of about 2.0 for this ratio is easier to secure while a value close to 3.0 is much harder to obtain. We also find that in nonet symmetry it is not possible to understand all four branching ratios: $B(D^0 \to \eta\bar{K}^0)$, $B(D^0 \to \eta'\bar{K}^0)$, $B(D_s^+ \to \eta\pi^+)$, and $B(D_s^+ \to \eta'\pi^+)$. Table II has been prepared with the parameter b of (3) set to zero. Hence the fits are not the best that could be obtained. However, if the ARGUS result[15] for $B(D_s^+ \to \eta'\pi^+)$ is assumed then a value of $B(D^0 \to \eta\bar{K}^0)$ close to 1.6% is favored. If, on the other hand, the E-691 upper limit for $B(D_s^+ \to \eta'\pi^+)$ is assumed then a lower value of $B(D^0 \to \eta\bar{K}^0)$ would be favored.

In this paper we have also described $D \to VP$ decays in a nonet-symmetry scheme. A broken-nonet-symmetry scheme has too many (11) parameters to have any predictive power. Even in a nonet-symmetry scheme, which has seven independent parameters, no new testable constraints are derived for Cabibbo-angle-favored decays. The constraints shown in (21) for these decays are simply isospin constraints. The predictions made for Cabibbo-angle-suppressed decays, discussed in Sec. III C, appear to be in reasonable accord with data where data exist. Consequently, we believe that the predictions for Cabibbo-angle-suppressed and doubly suppressed branching ratios are quite trustworthy.

The discussion in Sec. IV of sextet dominance shows that sextet dominance *with* nonet symmetry fails badly in describing $D_s^+ \to \phi\pi^+$ and $\omega\pi^+$ decays. Within the context of this model relaxing nonet symmetry, i.e., freeing c_1 and c_2 of the constraint (19) might patchup this problem. However, see below for a further discussion.

A word of caution: one should recall that all the predictions of Secs. III and IV have been made ignoring the FSI phases. On the other hand, the amplitude analysis for $D \to \bar{K}\rho$ and $\pi\bar{K}^*$ has shown that the phases $\delta_{\rho K}$ and $\delta_{\pi K^*}$ are substantial and that the FSI cannot be ignored.

The problem of $D \to VP$ decays has been discussed at length in a factorizable model with multichannel FSI in Ref. 1. This problem has also been discussed in a factorizable model *without* FSI in Ref. 30. It was shown in Ref. 1 that even if the model did not have an intrinsic decay amplitude for decays such as $D^0 \to \bar{K}^0\phi$ and $D_s^+ \to \pi^+\rho^0$, these decays could be generated at the experimental level through a multichannel FSI. Further, the low observed limit[28] for $B(D_s^+ \to \omega\pi^+)$ was also understood since there is no spectator amplitude for $D_s^+ \to \omega\pi^+$, and the annihilation amplitude vanishes by the conserved-vector-current (CVC) hypothesis and absence of second-class currents. Thus $D_s^+ \to \omega\pi^+$ would have to be generated by multichannel final-state mixing. The problem with the symmetry approach as adopted in this paper, and also the diagrammatic approach followed by other authors,[6,31] is that concepts such as CVC are not built into the scheme. In the factorization approximation, on the other hand, one deals with matrix elements of *currents*. Imposition of CVC is then an easy matter. Thus the suppression of $D_s^+ \to \omega\pi^+$ is almost natural in the factorization method. Further, $B(D_s^+ \to \phi\pi^+)$ at the 3–4% level was also obtained rather easily in the factorization model of Ref. 30.

Chau and Cheng[31] have discussed $D \to VP$ decays in their diagrammatic approach. Their amplitude for $D_s^+ \to \pi^+ \rho^0$ decay is proportional to the *difference* of two annihilation amplitudes and that for $D_s^+ \to \pi^+ \omega$ decay, to the *sum* of the same two amplitudes. This situation parallels the statement we make in Sec. IV, following item 5. Thus, suppressing $B(D_s^+ \to \pi^+ \rho^0)$ leads to a large $B(D_s^+ \to \pi^+ \omega)$ *unless* one requires that *each* of the annihilation amplitudes is small. This, as we have argued, is almost trivially accomplished in a factorization model. Chau and Cheng[31] had to require that the annihilation amplitude for the creation of $u\bar{u}$ or $d\bar{d}$ pair from the vacuum be suppressed, roughly by a factor of 3, compared to the case where $s\bar{s}$ pair is created from the vacuum. This, however, is contrary to expectation.[32]

We emphasize that a complete description of $D \to VP$ must include multichannel FSI since there are several final-state channels that can couple to each other. However, the present work is an exhaustive analysis within a symmetry scheme.

ACKNOWLEDGMENTS

This work was partly funded by a research grant to A.N.K. from the Natural Sciences and Engineering Research Council of Canada. A.N.K. thanks Fritz De Jongh, Alice Bean, Mike Witherell, and Rafe Schindler for discussions and communications.

APPENDIX

In this appendix we relate the parameters of our model amplitude (18) for $D \to PV$ decays in nonet symmetry to those used in Refs. 23 and 24 and those of Ref. 3 in sextet dominance.

In a notation close to that used by Kohara[23] and Rosen,[24] the decay amplitude for $D \to VP$ decays is written as

$$A(D \to PV) = S_6 \langle (PV)_{8S} | H_w^{6*} | D \rangle + A_6 \langle (PV)_{8A} | H_w^{6*} | D \rangle + D_6 \langle (PV)_{10*} | H_w^{6*} | D \rangle + S_{15} \langle (PV)_{8S} | H_w^{15} | D \rangle + A_{15} \langle (PV)_{8A} | H_w^{15} | D \rangle + D_{15} \langle (PV)_{10} | H_w^{15} | D \rangle + T_{15} \langle (PV)_{27} | H_w^{15} | D \rangle \ . \quad \text{(A1)}$$

The tensor structures of various $(P \times V)$ representations are

$$(PV)_{8A} = (P_k^i V_j^k - P_j^k V_k^i) \ , \quad \text{(A2)}$$

$$(PV)_{8S} = (P_k^i V_j^k + P_j^k V_k^i - \tfrac{2}{3} \delta_j^i P_l^k V_k^l) \ , \quad \text{(A3)}$$

$$(PV)_R = T_{jl}^{ik} - \alpha_R (\delta_j^i T_{ml}^{mk} + \delta_l^i T_{jm}^{mk} + \delta_j^k T_{ml}^{im} + \delta_l^k T_{jm}^{im}) + \beta_R (\delta_j^i \delta_l^k + \delta_l^i \delta_j^k) T_{mn}^{mn} \ , \quad \text{(A4)}$$

where, for $R = 10$,

$$\alpha_{10} = \tfrac{1}{3}, \quad \beta_{10} = 0 \ , \qquad T_{jl}^{ik} \equiv P_j^i V_l^k + P_l^i V_j^k - P_j^k V_l^i - P_l^k V_j^i \ , \quad \text{(A5)}$$

for $R = 10^*$,

$$\alpha_{10^*} = \tfrac{1}{3}, \quad \beta_{10^*} = 0 \ , \qquad T_{jl}^{ik} \equiv P_j^i V_l^k - P_l^i V_j^k + P_j^k V_l^i - P_l^k V_j^i \ , \quad \text{(A6)}$$

and for $R = 27$,

$$\alpha_{27} = \tfrac{1}{5}, \quad \beta_{27} = \tfrac{1}{20} \ , \qquad T_{jl}^{ik} \equiv P_j^i V_l^k + P_l^i V_j^k + P_j^k V_l^i + P_l^k V_j^i \ . \quad \text{(A7)}$$

The trace part of $(PV)_{8S}$ and the double trace part of $(PV)_{27}$ do not contribute to $D \to VP$ decays. The correspondence between the parameters of (A1) and those of (18), in nonet symmetry, is given by

$$\begin{aligned} a_1 &= S_6 + A_6 - \tfrac{2}{3} D_6 \ , \\ a_2 &= S_6 - A_6 + \tfrac{2}{3} D_6 \ , \\ a_3 &= 2D_6 \ , \\ b_1 &= S_{15} + A_{15} + \tfrac{2}{3} D_{15} - \tfrac{2}{5} T_{15} \ , \\ b_2 &= S_{15} - A_{15} - \tfrac{2}{3} D_{15} - \tfrac{2}{5} T_{15} \ , \\ b_3 &= 2(T_{15} + D_{15}) \ , \\ b_4 &= 2(T_{15} - D_{15}) \ . \end{aligned} \quad \text{(A8)}$$

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