Excited charm mesons in semileptonic $\overline{B}$ decay and their contributions to a Bjorken sum rule

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It has recently been shown that hadrons containing a single heavy quark exhibit a new flavor-spin symmetry of QCD. We exploit this symmetry to obtain model-independent predictions for the 14 form factors in weak decays from the ground-state pseudoscalar meson P_{Q_i} of a heavy quark Q_i to the low-lying positive-parity excited states of a heavy quark Q_j in terms of two universal functions of momentum transfer. These predictions are of interest in the study of $\overline{B}\to D_2^*(2460)$, $D_1(2420)$, $D_1(\sim 2360)$, and $D_0^*(\sim 2360)$ semileptonic decays. We also discuss the connection between these results and the slope of the function ξ (which determines the $\overline{B}\to D$ and $\overline{B}\to D^*$ transition form factors) given by a heavy-quark sum rule suggested by Bjorken.

I. INTRODUCTION

The properties of hadrons containing a single heavy quark Q ($m_Q \gg \Lambda_{\rm QCD}$) along with light degrees of freedom are constrained by symmetries which are not manifest in *QCD*.[1] The first of these is a flavor symmetry which arises from the fact that the long-wavelength properties of the light degrees of freedom in such a hadron become independent of m_Q for $m_Q \gg \Lambda_{\rm QCD}$. Thus, for example, the $\overline{B}$ and D mesons can be related by an (approximate) $b\leftrightarrow c$ SU(2) symmetry even though m_b and m_c are very different. The second symmetry pointed out in Refs. 1 is a related spacetime symmetry which arises in QCD because the spin of a heavy quark decouples from the gluon field.[2] This makes $\mathbf{S}_Q$, the heavy-quark spin operator, the generator of another SU(2) group of symmetries applicable to mesons containing a single heavy quark. Thus, for example, the light degrees of freedom in the $\overline{B}$ and $\overline{B}^*$ mesons are in (approximately) the same state since the spin orientation of the b quark does not affect their dynamics. These symmetries are manifest in an effective theory where the heavy quark acts, in its hadron's rest frame, like a spatially static triplet source of color field. In the effective theory the heavy quark's couplings to the gluon degrees of freedom are independent of its mass and spin and described by a Wilson line.[3]

The consequences of these symmetries for the weak decays of $\overline{B}$ and D mesons were worked out in Refs. 1. The existence of conservation laws associated with the symmetries allows one to make absolutely normalized predictions for all $b\to c$ weak form factors between ground state pseudoscalar P and vector V mesons at "zero recoil" (where, in the rest frame of the initial hadron, the final hadron is at rest). The symmetries also give relations between the $P\to P$ and $P\to V$ weak form factors. In addition, the flavor symmetry relates,[1,4] for example, $\overline{B}\to X_u$ and $D\to X_d$ weak transition form factors (X_u and X_d are particular light-hadron final states related by isospin which occur due to the $b\to u$ and $c\to d$ weak transitions). These latter relations may be crucial in the reliable extraction of the Cabibbo-Kobayashi-Maskawa[5] matrix element V_{ub} from experimental data.

In the first of the Refs. 1 we applied these symmetries for static quarks, where a $Q_i\to Q_j$ transition simply substituted one static heavy quark for another. (See also Ref. 6, where the manifest symmetry which exists when $m_{Q_i}\simeq m_{Q_j}$ was applied and the physics at zero recoil was also discussed.) In the second of Refs. 1 we exploited a powerful extension of this method, which makes use of the fact that (in the effective theory) when Q_i at velocity $\mathbf{v}'$ makes a weak transition into Q_j at velocity $\mathbf{v}'$, the amplitude for the light degrees of freedom to make any associated transition is independent of m_{Q_i} and m_{Q_j} if they are sufficiently large. The light degrees of freedom interact only with the (moving) color fields of Q_i and Q_j, which depend only on the Lorentz boosts required of the mass-independent rest frame color fields. In the effective theory the mass of the heavy quark is taken to infinity in such a way that p_Q^μ/m_Q is held fixed, but the four-momentum of the light degrees of freedom are neglected compared with m_Q. In this limit the interactions of the gluons with the heavy quark do not alter its straight world line and are independent of its mass and spin. The interactions of the light degrees of freedom with the heavy quark do, of course, depend on the heavy quark's four-velocity v^μ. The resulting symmetries are therefore somewhat unusual in that they relate states of equal velocity but different mass, and therefore different momentum. For the matrix elements of weak currents in the effective theory

$$\mathbb{V}_\nu^{ji}\equiv\overline{\mathbb{Q}}_j\gamma_\nu\mathbb{Q}_i \ , \tag{1a}$$

$$\mathbb{A}_\nu^{ji}\equiv\overline{\mathbb{Q}}_j\gamma_\nu\gamma_5\mathbb{Q}_i \ , \tag{1b}$$

changes in the heavy-quark velocity and spin occur only due to the actions of the currents. In a typical transition, $H_i(\mathbf{v}=0)\to H_j(\mathbf{v}')$ (where H_n is the hadron containing the single heavy quark Q_n) the form factors will therefore be determined by the product of the amplitude for the heavy quarks to make the transition $Q_i(\mathbf{v}=0)\to Q_j(\mathbf{v}')$ and for the light quarks to be "excited" by the transition from the hadron H_i at rest into the hadron H_j moving with velocity $\mathbf{v}'$.

To apply these symmetries we must know the relationship between the weak currents in the complete theory

$$V_\nu^{ji}\equiv\bar{Q}_j\gamma_\nu Q_i\ , \tag{2a}$$

$$A_\nu^{ji}\equiv\bar{Q}_j\gamma_\nu\gamma_5 Q_i\ , \tag{2b}$$

and those in the effective theory [Eq. (1)]. This relationship has the form (for $J_\nu=V_\nu$ or A_ν, and $\mathbb{J}_\nu=\mathbb{V}_\nu$ or $\mathbb{A}_\nu$)

$$J_\nu^{ji}=C_{ji}\mathbb{J}_\nu^{ji}+\cdots\ , \tag{3}$$

where the ellipsis denotes other possible Lorentz structures which are suppressed by factors of $\alpha_s(m_Q)/\pi$ as well as higher-dimension operators whose physical effects are suppressed by powers of $\Lambda_{\rm QCD}/m_Q$. In the leading-logarithmic approximation,[7,8]

$$C_{ji}(w)=\left[\frac{\alpha_s(m_{Q_i})}{\alpha_s(m_{Q_j})}\right]^{a_I}\left[\frac{\alpha_s(m_{Q_j})}{\alpha_s(\mu)}\right]^{a_L} \tag{4}$$

where, for the $b\to c$ transition,

$$a_I=-\tfrac{6}{25} \tag{5a}$$

and

$$a_L=\frac{8[wr(w)-1]}{27} \tag{5b}$$

with

$$r(w)=\frac{1}{\sqrt{w^2-1}}\ln(w+\sqrt{w^2-1})\ , \tag{6}$$

where w is the dot product of the four-velocity of Q_i,v^μ, with the four-velocity of Q_j,v'^μ. This velocity-dependent contribution (which was missed in Refs. 1) was calculated in Ref. 8. For $v'=v$ (i.e., $w=1$) the currents are not renormalized in the effective theory $[r(1)=1]$ and their matrix elements are independent of the subtraction point μ. This occurs because the quantity $\bar{\mathbb{Q}}_j\gamma_0\mathbb{Q}_i$ (for $v'=v$) is related to a generator for the SU(2)-flavor symmetry in the effective theory, and so its matrix element is a physical quantity.

In addition to Ref. 8, there have been several other recent improvements on the work of Refs. 1. In Ref. 9 a power-counting argument was given to prove (in a particular case) that, to all orders in perturbation theory, matrix elements in the complete theory factorize into a coefficient function (i.e., C_{ji}) times a matrix element in the effective theory where the heavy quark couples as a Wilson line. Also, in Ref. 10 (see also Ref. 11), it was shown how the effective theory can be written as a Lorentz-invariant field theory with a superselection rule for the velocity of the heavy quark. The extension of the analysis of semileptonic $\bar{B}$ decays to multiparticle hadronic final states (and to inclusive decays) was made in Ref. 12. See also Ref. 13 for a discussion of inclusive heavy-quark decay. In Ref. 14 it is shown how the number of independent functions required to describe a given set of matrix elements may easily be counted using conservation of helicity of the light degrees of freedom.

There have also been several recent applications of the heavy-quark symmetry to new processes, including heavy-baryon semileptonic decay,[15] weak hadronic decays of the type $\bar{B}\to D\bar{D}_s$, $D\bar{D}_s^*$, $D^*\bar{D}_s$, $D^*\bar{D}_s^*$,[16] and e^+e^- annihilation into exclusive channels like $D\bar{D}$, $D\bar{D}^*+D^*\bar{D}$, and $D^*\bar{D}^*$.[17]

In this paper we will apply the heavy-quark symmetry to decays of a ground-state pseudoscalar meson P_{Q_i} of a heavy quark Q_i to the positive-parity states expected to constitute the first excited states above the degenerate pseudoscalar P_{Q_j} and vector V_{Q_j} ground states of the heavy quark Q_j. Such predictions are of some interest in their own right as they are expected[18] (and possibly observed[19]) to be produced in a significant fraction of $\bar{B}$ decays. However, they are also interesting because a heavy-quark sum rule suggested by Bjorken in Ref. 12 can be used to relate the rate for such processes to the slope of the universal function[1] $\xi(w)$ controlling $\bar{B}\to D$ and D^* semileptonic decays.

II. POSITIVE-PARITY EXCITED STATES

In the heavy-quark limit $\mathbf{S}_Q$ and $\mathbf{S}_l\equiv\mathbf{S}-\mathbf{S}_Q$ (the spin of the light degrees of freedom) are separately conserved by the strong interaction, so mesons containing a single heavy quark Q can be simultaneously assigned the quantum numbers s_Q, m_Q, s_l, and m_l. Since the dynamics depend only on s_l, the mesons will appear in degenerate multiplets of the total spins s that can be formed from s_Q and s_l. It is accordingly more convenient in the heavy-quark limit to classify states by s_l (and π_l, the parity of the light degrees of freedom).

In the constituent quark model, the first excited states above the ground states P_Q and V_Q would be closely spaced states with relative orbital angular momentum $l=1$ and total spin $s=1$ and 0 corresponding to the ${}^{2S+1}L_J$ states ${}^3P_2$, ${}^3P_1$, ${}^3P_0$, and ${}^1P_1$ with $J^P=2^+$, 1^+, 0^+, and 1^+, respectively. Given this expectation and the empirical evidence, it is safe to assume that the light degrees of freedom with antiquark quantum numbers accompanying Q will have a ground state with $s_l^{\pi_l}=\frac{1}{2}^-$ (leading to P_Q and V_Q when combined with $s_Q^{\pi_Q}=\frac{1}{2}^+$) and as their first excitations two closely spaced states with $s_l^{\pi_l}=\frac{3}{2}^+$ and $\frac{1}{2}^+$. When combined with Q, these excited states lead to degenerate $s_l^{\pi_l}=\frac{3}{2}^+$ multiplets with $J^P=2^+$ and 1^+ and degenerate $s_l^{\pi_l}=\frac{1}{2}^+$ multiplets with $J^P=1^+$ and 0^+; we denote the corresponding states by ${}^{3/2}E_Q2^+$, ${}^{3/2}E_Q1^+$, ${}^{1/2}E_Q1^+$, and ${}^{1/2}E_Q0^+$, respectively. The first and last of these states are obviously the states ${}^3P_2$ and ${}^3P_0$, while

$$|{}^{3/2}E_Q1^+\rangle=+\sqrt{\tfrac{2}{3}}|{}^1P_1\rangle+\sqrt{\tfrac{1}{3}}|{}^3P_1\rangle\ , \tag{7a}$$

$$|{}^{1/2}E_Q1^+\rangle=+\sqrt{\tfrac{1}{3}}|{}^1P_1\rangle-\sqrt{\tfrac{2}{3}}|{}^3P_1\rangle\ . \tag{7b}$$

The importance of these linear combinations of axial-vector states was noted by Rosner,[20] who emphasized in the context of the constituent quark model the separate conservation of $\mathbf{S}_Q$ and $\mathbf{S}_l$ in heavy-quark systems.

Since $\mathbf{S}_Q$ generates the heavy-quark spin symmetry, it is important to know the action of $\mathbf{S}_Q$ on the states E_Q. In what follows we will in particular use the relations

$$S_Q^3|^{3/2}E_Q 2^+(0,2)\rangle = +\tfrac{1}{2}|^{3/2}E_Q 2^+(0,2)\rangle \ , \tag{8a}$$

$$S_Q^3|^{3/2}E_Q 2^+(0,1)\rangle = +\tfrac{1}{4}|^{3/2}E_Q 2^+(0,1)\rangle - \frac{\sqrt{3}}{4}|^{3/2}E_Q 1^+(0,1)\rangle \ , \tag{8b}$$

$$S_{Q+}|^{1/2}E_Q 1^+(0,1)\rangle = 0 \ , \tag{8c}$$

$$S_Q^3|^{1/2}E_Q 1^+(0,0)\rangle = -\tfrac{1}{2}|^{1/2}E_Q 0^+(0)\rangle \ , \tag{8d}$$

where $|^{s_l}E_Q J^P(0,m)\rangle$ denotes a state at rest with third component of spin m.

III. WEAK FORM FACTORS FOR $P_{Q_i} \to E_{Q_j}$ SEMILEPTONIC DECAYS

In this section we discuss the weak form factors that arise in the semileptonic decays $P_{Q_i} \to E_{Q_j}$. Since the heavy-quark symmetries relate states of fixed velocities, it is convenient to define a set of form factors for these transitions that multiply Lorentz invariants formed from the available polarization tensors and the four velocities v and v' (instead of the four-momenta p and p'). It is also convenient to remove an overall factor of $(m_{E_{Q_j}} m_{P_{Q_i}})^{1/2}$ from our meson states which are conventionally normalized to

$$\langle X(\mathbf{v}',\epsilon_\beta)|X(\mathbf{v},\epsilon_\alpha)\rangle = (2\pi)^3 2E\delta_{\beta\alpha}\delta^3(\mathbf{p}'-\mathbf{p}) \ , \tag{9}$$

although they are *labeled* by their velocity. With these conventions, the form factors that can arise in transitions to the $s_l=\frac{3}{2}$ states are

$$\frac{\langle {}^{3/2}E_{Q_j} 2^+(\mathbf{v}',\epsilon)|V_\nu|P_{Q_i}(\mathbf{v})\rangle}{[m_{E_{Q_j}} m_{P_{Q_i}}]^{1/2}} \equiv i\tilde{h}\,\epsilon_{\nu\alpha\beta\gamma}\epsilon^{*\alpha\mu}v_\mu(v+v')^\beta(v-v')^\gamma \ , \tag{10}$$

$$\frac{\langle {}^{3/2}E_{Q_j} 2^+(\mathbf{v}',\epsilon)|A_\nu|P_{Q_i}(\mathbf{v})\rangle}{[m_{E_{Q_j}} m_{P_{Q_i}}]^{1/2}} \equiv \tilde{k}\epsilon^*_{\nu\alpha}v^\alpha + \epsilon^*_{\alpha\beta}v^\alpha v^\beta[\tilde{b}_+(v+v')_\nu + \tilde{b}_-(v-v')_\nu] \tag{11}$$

and

$$\frac{\langle {}^{3/2}E_{Q_j} 1^+(\mathbf{v}',\epsilon)|V_\nu|P_{Q_i}(\mathbf{v})\rangle}{[m_{E_{Q_j}} m_{P_{Q_i}}]^{1/2}} \equiv \tilde{l}_{3/2}\epsilon^*_\nu + \epsilon^*_\alpha v^\alpha[\tilde{c}_{3/2+}(v+v')_\nu + \tilde{c}_{3/2-}(v-v')_\nu] \ , \tag{12}$$

$$\frac{\langle {}^{3/2}E_{Q_j} 1^+(\mathbf{v}',\epsilon)|A_\nu|P_{Q_i}(\mathbf{v})\rangle}{[m_{E_{Q_j}} m_{P_{Q_i}}]^{1/2}} \equiv i\tilde{q}_{3/2}\epsilon_{\nu\alpha\beta\gamma}\epsilon^{*\alpha}(v+v')^\beta(v-v')^\gamma \ , \tag{13}$$

while those for $s_l=\frac{1}{2}$ are

$$\frac{\langle {}^{1/2}E_{Q_j} 1^+(\mathbf{v}',\epsilon)|V_\nu|P_{Q_i}(\mathbf{v})\rangle}{[m_{E_{Q_j}} m_{P_{Q_i}}]^{1/2}} \equiv \tilde{l}_{1/2}\epsilon^*_\nu + \epsilon^*_\alpha v^\alpha[\tilde{c}_{1/2+}(v+v')_\nu + \tilde{c}_{1/2-}(v-v')_\nu] \ , \tag{14}$$

$$\frac{\langle {}^{1/2}E_{Q_j} 1^+(\mathbf{v}',\epsilon)|A_\nu|P_{Q_i}(\mathbf{v})\rangle}{[m_{E_{Q_j}} m_{P_{Q_i}}]^{1/2}} \equiv i\tilde{q}_{1/2}\epsilon_{\nu\alpha\beta\gamma}\epsilon^{*\alpha}(v+v')^\beta(v-v')^\gamma \ , \tag{15}$$

and

$$\frac{\langle {}^{1/2}E_{Q_j} 0^+(\mathbf{v}')|A_\nu|P_{Q_i}(\mathbf{v})\rangle}{[m_{E_{Q_j}} m_{P_{Q_i}}]^{1/2}} \equiv \tilde{u}_+(v+v')_\nu + \tilde{u}_-(v-v')_\nu \tag{16}$$

(the vector matrix element for $P_{Q_i} \to {}^{1/2}E_{Q_j}0^+$ vanishes). In Eqs. (10)–(16) we adopt the conventions $\epsilon_{0123}=1$ and $\epsilon(1) = -(1/\sqrt{2})(1,i,0)$. The relation of these form factors to the more conventionally defined ones of Ref. 18 is given in Appendix A. As shown in Ref. 1, the heavy-quark symmetry relations apply to the combinations

$$\frac{\langle E_{Q_j}(\mathbf{v}',\epsilon)|J_\nu^{ji}|P_{Q_i}(\mathbf{v})\rangle}{C_{ji}[m_{E_{Q_j}} m_{P_{Q_i}}]^{1/2}} \ , \tag{17}$$

which are independent of the masses of the heavy quarks. We begin our derivation of the heavy-quark symmetry relations with the $s_l=\frac{1}{2}$ states. From Eq. (17) we have that

$$\tilde{q}_{1/2}=C_{ji}\tau_{1/2}(w) \tag{18}$$

where $\tau_{1/2}(w)$ is a function of

$$w=v'\cdot v=\frac{p'\cdot p}{m_{E_{Q_j}}m_{P_{Q_i}}}=1+\frac{t_m-t}{2m_{E_{Q_j}}m_{P_{Q_i}}} , \tag{19}$$

which is independent of i and j. Here $t_m=(m_{P_{Q_i}}-m_{E_{Q_j}})^2$ is the maximum momentum transfer corresponding to the zero-recoil point. We can use the commutation relations of the weak currents with $\mathbf{S}_{Q_j}$ (Refs. 1 and 22) to express all the other $s_l=\frac{1}{2}$ form factors in terms of $\tau_{1/2}(w)$. From Eq. (8d) and the commutation relations

$$\langle {}^{1/2}E_{Q_j}1^+(\mathbf{0},0)|A^{ji}_{\pm}|P_{Q_i}(\mathbf{v})\rangle = \mp\langle {}^{1/2}E_{Q_j}0^+(\mathbf{0})|A^{ji}_{\pm}|P_{Q_i}(\mathbf{v})\rangle , \tag{20}$$

which implies that

$$\tilde{u}_+ + \tilde{u}_- = -2C_{ji}\tau_{1/2}(w) . \tag{21}$$

From the relation

$$\langle {}^{1/2}E_{Q_j}1^+(\mathbf{0},1)|V^{ji}_{-}|P_{Q_j}(\mathbf{v})\rangle=0 \tag{22}$$

we have

$$\tilde{c}_{1/2+}+\tilde{c}_{1/2-}=0 , \tag{23}$$

while

$$\langle {}^{1/2}E_{Q_j}1^+(\mathbf{0},1)|V^{ji}_0|P_{Q_i}(\mathbf{v})\rangle = -\langle {}^{1/2}E_{Q_j}1^+(\mathbf{0},1)|A^{ji}_3|P_{Q_i}(\mathbf{v})\rangle \tag{24}$$

gives

$$\tilde{c}_{1/2+}-\tilde{c}_{1/2-}=-2C_{ji}\tau_{1/2}(w) . \tag{25}$$

Next use

$$\langle {}^{1/2}E_{Q_j}1^+(\mathbf{0},0)|V^{ji}_3|P_{Q_i}(\mathbf{v})\rangle = \langle {}^{1/2}E_{Q_j}0^+(\mathbf{0})|A^{ji}_0|P_{Q_i}(\mathbf{v'})\rangle \tag{26}$$

which gives

$$\tilde{l}_{1/2}=-(\tilde{u}_+-\tilde{u}_-)-w(\tilde{u}_++\tilde{u}_-) . \tag{27}$$

To complete the determination of the $s_l=\frac{1}{2}$ form factors in terms of the function $\tau_{1/2}$, we apply the heavy-quark velocity superselection rule. Since in the effective theory only the components of the field Q_i and Q_j with *velocities* v and v' enter into the matrix elements,[10]

$$\gamma\cdot vQ_i=Q_i , \tag{28a}$$

$$\bar{Q}_j\gamma\cdot v'=\bar{Q}_j , \tag{28b}$$

so that $(v-v')$ contracted against a vector-current matrix element and $(v+v')_\mu$ contracted against an axial-vector-current matrix element give zero. This constraint is trivial when applied to (14) but gives two relations when applied to (13) and (15):

$$\tilde{l}_{1/2}=-2(1-w)\tilde{c}_{1/2-} \tag{29}$$

and

$$(\tilde{u}_++\tilde{u}_-)=-(\tilde{u}_+-\tilde{u}_-) . \tag{30}$$

These two new relations overdetermine the form factors so one of them may be used as a consistency check. We finally obtain the results shown in Table I.

TABLE I. The weak form factors for semileptonic $\bar{B}$ decay to excited charmed mesons.

	Form factor	Value in units of $C_{ji}\tau_{s_l}(w)$
$s_l=\frac{1}{2}$	$\tilde{u}_++\tilde{u}_-$	-2
	$\tilde{u}_+-\tilde{u}_-$	$+2$
	$\tilde{l}_{1/2}$	$+2(w-1)$
	$\tilde{c}_{1/2+}+\tilde{c}_{1/2-}$	0
	$\tilde{c}_{1/2+}-\tilde{c}_{1/2-}$	-2
	$\tilde{q}_{1/2}$	$+1$
$s_l=\frac{3}{2}$	$\tilde{h}$	$+\sqrt{3}/2$
	$\tilde{k}$	$+\sqrt{3}(w+1)$
	$\tilde{b}_++\tilde{b}_-$	0
	$\tilde{b}_+-\tilde{b}_-$	$-\sqrt{3}$
	$\tilde{l}_{3/2}$	$-(w^2-1)/\sqrt{2}$
	$\tilde{c}_{3/2+}+\tilde{c}_{3/2-}$	$-3/\sqrt{2}$
	$\tilde{c}_{3/2+}-\tilde{c}_{3/2-}$	$+(w-2)/\sqrt{2}$
	$\tilde{q}_{3/2}$	$-(w+1)/2\sqrt{2}$

The derivation of the form factors relevant to the $s_l=\frac{3}{2}$ states proceeds similarly. In analogy to Eq. (18) we first *define*

$$\tilde{q}_{3/2}=-\frac{(w+1)C_{ji}\tau_{3/2}(w)}{2\sqrt{2}} , \tag{31}$$

where $\tau_{3/2}$ is analogous to $\tau_{1/2}$ and where a factor of $(w+1)/2\sqrt{2}$ has been added for later convenience. From

$$\langle {}^{3/2}E_{Q_j}2^+(\mathbf{0},2)|A^{ji}_{-}|P_{Q_i}(\mathbf{v})\rangle=0$$

we have

$$\tilde{b}_++\tilde{b}_-=0 . \tag{32}$$

Using Eq. (8a) and the commutation relation $[S^3_{Q_j},\mathbb{A}^{ji}_0]=-\frac{1}{2}\mathbb{V}^{ji}_3$ gives

$$\langle {}^{3/2}E_{Q_j}2^+(\mathbf{0},2)|A^{ji}_0|P_{Q_i}(\mathbf{v})\rangle = -\langle {}^{3/2}E_{Q_j}2^+(\mathbf{0},2)|V^{ji}_3|P_{Q_i}(\mathbf{v})\rangle , \tag{33}$$

which means that

$$\tilde{b}_+-\tilde{b}_-=-2\tilde{h} . \tag{34}$$

Using (8b) and $[S^3_{Q_j},\mathbb{A}^{ji}_3]=-\frac{1}{2}\mathbb{V}^{ji}_0$ gives

$$\langle {}^{3/2}E_{Q_j}2^+(\mathbf{0},1)|A^{ji}_3|P_{Q_i}(\mathbf{v})\rangle = \sqrt{3}\langle {}^{3/2}E_{Q_j}1^+(\mathbf{0},1)|A^{ji}_3|P_{Q_i}(\mathbf{v})\rangle \tag{35}$$

so that

$$\tilde{k} = -2\sqrt{6}\tilde{q}_{3/2} \ . \tag{36}$$

From (8b) and $[S^3_{Q_j}, \mathbb{V}^{ji}_\pm] = \pm\frac{1}{2}\mathbb{V}^{ji}_\pm$ we get

$$\langle {}^{3/2}E_{Q_j} 1^+(\mathbf{0},1)|V^{ji}_\pm|P_{Q_i}(\mathbf{v})\rangle = \left[\frac{1\mp 2}{\sqrt{3}}\right]\langle {}^{3/2}E_{Q_j}2^+(\mathbf{0},1)|V^{ji}_\pm|P_{Q_i}(\mathbf{v})\rangle \ , \tag{37}$$

which gives the two relations

$$\tilde{c}_{3/2+}+\tilde{c}_{3/2-} = -\sqrt{6}\,\tilde{h} \tag{38}$$

and

$$\tilde{l}_{3/2} = -\sqrt{2/3}(w^2-1)\tilde{h} \ , \tag{39}$$

and from $[S^3_{Q_j}, \mathbb{V}^{ji}_0] = -\frac{1}{2}\mathbb{A}^{ji}_3$ we have

$$\langle {}^{3/2}E_{Q_j}2^+(\mathbf{0},1)|A^{ji}_3|P_{Q_i}(\mathbf{v})\rangle = \frac{\sqrt{3}}{2}\langle {}^{3/2}E_{Q_j}1^+(\mathbf{0},1)|V^{ji}_0|P_{Q_i}(\mathbf{v})\rangle \tag{40}$$

so that

$$(\tilde{c}_{3/2+}+\tilde{c}_{3/2-})w + (\tilde{c}_{3/2+}-\tilde{c}_{3/2-}) = -\sqrt{2/3}\tilde{k} \ . \tag{41}$$

As before, we finally need to apply heavy-quark current conservation that gives the two relations

$$\tilde{k} = -(1+w)(\tilde{b}_+ - \tilde{b}_-) \tag{42}$$

and

$$\tilde{l}_{3/2} = 2(w-1)\tilde{c}_{3/2-} \ . \tag{43}$$

One is again redundant and may be used as a consistency check on the results displayed in Table I.

IV. COMPARISON WITH A QUARK-MODEL CALCULATION

The results for the fourteen weak transition form factors given in Table I are model independent and hold for any $Q_i \to Q_j$ transition with the same functions $\tau_{1/2}$ and $\tau_{3/2}$ in the limit $m_{Q_i} > m_{Q_j} \gg \Lambda_{\rm QCD}$. For any finite m_{Q_i}, m_{Q_j} these relations will receive corrections of order $\alpha_s(m_Q)/\pi$ and $\Lambda_{\rm QCD}/m_Q$. The perturbative corrections to the results of Table I are easily included using $\bar{\mathbb{Q}}_j\mathbb{Q}_i = v_\mu \bar{\mathbb{Q}}_j\gamma^\mu\mathbb{Q}_i$, $\bar{\mathbb{Q}}_j\gamma_5\mathbb{Q}_i = -v_\mu\bar{\mathbb{Q}}_j\gamma^\mu\gamma_5\mathbb{Q}_i$, and the results of Refs. 8 and 21. The physical mechanisms that are the origin of many of the power corrections are in operation in the constituent quark model, and it is consequently of value to use this phenomenological model as an indicator of the importance of this class of corrections.

To the best of our knowledge, these matrix elements to positive-parity excited states have only been estimated in Ref. 18 using the constituent quark model. Appendix A gives a translation dictionary between the form factors $\tilde{f}$ of Table I and the more conventionally defined form factors f of that paper. In the extreme heavy-quark limit the model's results[23] reduce to those of Table I near $w=1$ (where the model was claimed to be valid in the weak-binding limit) with the identification $\tau_{1/2} = \tau_{3/2} = \tau$, where

$$\tau = \frac{m_d}{\sqrt{6}\beta_{P_{Q_i}}}\left[\frac{\beta_{P_{Q_i}}\beta_{E_{Q_j}}}{\beta^2_{P_{Q_i}E_{Q_j}}}\right]^{5/2}\exp\left[-\frac{m_d^2(w-1)}{2\kappa^2\beta^2_{P_{Q_i}E_{Q_j}}}\right] \ . \tag{44}$$

[Here m_d is the light constituent quark mass, β_r is a variational parameter proportional to the rms momentum in the meson r, $\beta^2_{rs} = \frac{1}{2}(\beta^2_r + \beta^2_s)$, and κ^2 is an *ad hoc* "relativistic correction factor." For details see Ref. 18.] Table II shows the comparison between the form factors predicted (near $w=1$) in the heavy-quark limit and those of this

TABLE II. Comparison of the predictions of the heavy-quark symmetry with those of a quark model near zero recoil as a measure of the importance of a class of $\Lambda_{\rm QCD}/m_Q$ corrections in $b \to c$ transitions; for this comparison we have set $\kappa = 1$ in the Ref. 18 results (see Ref. 23) and have set $t_m - t = 2\tilde{m}_B\tilde{m}_X(w-1)$.

Form factor	Heavy-quark symmetry [in units $C_{ji}\tau_{s_l}(1)$]	Ref. 18 results [in units $\tau(1)$]
$\tilde{u}_+ + \tilde{u}_-$	-2	$-2(0.92)$
$\tilde{u}_+ - \tilde{u}_-$	$+2$	$+2(1.08)$
$\tilde{l}_{1/2}$	$0+2(w-1)$	$-0.02+2(1.06)(w-1)$
$\tilde{c}_{1/2+}+\tilde{c}_{1/2-}$	0	$+0.13$
$\tilde{c}_{1/2+}-\tilde{c}_{1/2-}$	-2	$-2(1.13)$
$\tilde{q}_{1/2}$	$+1$	$+1(0.99)$
$\tilde{h}$	$+\frac{1}{2}\sqrt{3}$	$+\frac{1}{2}\sqrt{3}(1.13)$
$\tilde{k}$	$+2\sqrt{3}$	$+2\sqrt{3}(0.92)$
$\tilde{b}_+ + \tilde{b}_-$	0	$+0.06$
$\tilde{b}_+ - \tilde{b}_-$	$-\sqrt{3}$	$-\sqrt{3}(0.98)$
$\tilde{l}_{3/2}$	$0-\sqrt{2}(w-1)$	$-0.75-\sqrt{2}(1.06)(w-1)$
$\tilde{c}_{3/2+}+c_{3/2-}$	$-\frac{3}{2}\sqrt{2}$	$-\frac{3}{2}\sqrt{2}(0.91)$
$\tilde{c}_{3/2+}-\tilde{c}_{3/2-}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{2}(1.02)$
$\tilde{q}_{3/2}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{2}(1.18)$

quark model calculation for $b \to c$ transitions.

We see from this comparison that the quark model suggests, as it did for the $\bar{B} \to D$ and $\bar{B} \to D^*$ form factors, that most of these form factors will be close to the heavy-quark symmetry limit. However, examples of deviations (such as the intercept of $\tilde{I}_{3/2}$) should serve as a reminder that $\Lambda_{\rm QCD}/m_c$ is not a very small expansion parameter. In addition, we note that the quark model results of Ref. 18 are based on the weak-binding limit and so explicitly exclude corrections of relative order β_r^2/m^2 (where m^2 is the product of any two quark masses). Given that the mass difference between the ${}^{s_l}E_Q$ and P_Q states (roughly 500 MeV) is substantial compared to m_c, conclusions based on Table II must be treated with even more caution than the analogous results for the $\bar{B} \to D$ and $\bar{B} \to D^*$ form factors. Thus, testing the predictions of Table I and determining the universal limiting functions τ_{s_l} will require the careful study of $\Lambda_{\rm QCD}/m_c$ corrections (including those not estimated by the quark model) and the extraction from data of those form factors (or linear combinations of form factors) least sensitive to such corrections.

V. A BJORKEN SUM RULE

Bjorken has shown[12] that the heavy-quark symmetry allows one to derive an analog of the Cabibbo-Radicati sum rule,[24] and thereby to relate the slope of the universal form factor $\xi(w)$ appearing in $\bar{B} \to D$ and $\bar{B} \to D^*$ semileptonic decays[1] to the rates for production of inelastic states. His derivation is based on equal-time current-commutation relations.[25] In this section we rederive this Bjorken sum rule and present an interpretation of it that we believe is correct beyond the parton-model approximation.

The rate for the semileptonic decay $P_{Q_i} \to X_{Q_j} e \bar{\nu}_e$ can be written in the form

$$\frac{d^2\Gamma}{dx\,dy} = |V_{Q_jQ_i}|^2 \frac{G_F^2 m_{P_{Q_i}}^5}{32\pi^2} \left[\frac{\alpha^X}{m_{P_{Q_i}}^2} A(x,y) + \beta_{++}^X B(x,y) + \gamma^X G(x,y) \right] \tag{45}$$

where, with $r \equiv m_{X_{Q_j}}^2 / m_{P_{Q_i}}^2$,

$$A(x,y) = y \;, \tag{46a}$$

$$B(x,y) = 2[2x(1-r^2+y) - 4x^2 - y] \;, \tag{46b}$$

$$G(x,y) = -y(1-r^2-4x+y) \;, \tag{46c}$$

and where $x \equiv E_e/m_{P_{Q_i}}$ and $y \equiv t/m_{P_{Q_i}}^2 = (p-p')^2/m_{P_{Q_i}}^2 = 1+r^2-2rw$, with E_e the electron energy and p and p' the momenta of P_{Q_i} and X_{Q_j}, respectively. In Eq. (45), α^X, β_{++}^X and γ^X are functions of w that expand the hadronic tensor

$$h_{\mu\nu}^X(w) \equiv \sum_s \langle P_{Q_i}(\mathbf{v}) | J_\nu^{ji\dagger} | X_{Q_j}(\mathbf{v}',s) \rangle \langle X_{Q_j}(\mathbf{v}',s) | J_\mu^{ji} | P_{Q_i}(\mathbf{v}) \rangle \tag{47}$$

$$= -\alpha^X g_{\mu\nu} + \sum_{\sigma,\sigma'=\pm} \beta_{\sigma\sigma'}^X (p+\sigma p')_\mu (p+\sigma' p')_\nu + i\gamma^X \epsilon_{\mu\nu\rho\sigma}(p+p')^\rho (p-p')^\sigma \;. \tag{48}$$

Bjorken's sum rule applies to each of the six functions appearing in (48), and also to any choice for the two currents. Although it is therefore redundant, we will for concreteness consider the physical case $J_\mu^{ji} = V_\mu^{ji} - A_\mu^{ji}$ and concentrate on the three functions α, β_{++}, and γ that enter the rate (45) obtained when we ignore the lepton mass. Explicit formulas for those three functions in terms of the weak form factors are given in Ref. 18.

The sum rule is based on the observation that if one sums the rates for all hadronic final states with masses from $m_{P_{Q_j}}$ to $m_{P_{Q_j}} + \mu$, then so long as $\mu \gg \Lambda_{\rm QCD}$ this inclusive rate and its associated hadronic tensor can be computed in perturbative QCD, up to corrections of order $\Lambda_{\rm QCD}/m_{Q_i}$, as an infrared safe heavy-quark transition. If on the other hand we restrict $\mu \ll m_{Q_j}$, then the hadronic tensor for each exclusive channel could also be computed using the heavy-quark effective theory. In the heavy-quark limit $m_Q \to \infty$, the heavy quarks have well-defined four-velocities, and the thresholds of all the states required to build up the low w inclusive rate coincide at $w=1$. So, for w of order unity we have the sum rule

$$h_{\mu\nu}^{Q_i \to Q_j}(w,\mu) = \sum_{m_{X_{Q_j}} - m_{P_{Q_j}} < \mu} h_{\mu\nu}^X(w) \;. \tag{49}$$

At $w=1$ only the "elastic" final states P_{Q_j} and V_{Q_j} will contribute to (49) [giving $\xi(1)=1$]. For larger w these "elastic" contributions will fall with their decreasing from factor $\xi(w)$, but "inelastic" final states (e.g., those considered in this paper) will be excited. [For finite heavy-quark masses, the thresholds for the inelastic states (where the inelastic state is being produced at zero recoil) will occur in an inclusive Dalitz plot at a value of

TABLE III. Contributions to the hadronic tensor using the notation $\epsilon=(w-1)$, $\eta=(w+1)/2$, $\sigma=(r^2-2rw+1)/4r$, and $\delta=[2rw^2-(1+4r+r^2)w+2(1+r^2)]/2r$.

Transition	$C_{ji}^{-2}\alpha/m_{P_{Q_i}}^2$	$C_{ji}^{-2}\beta_{++}$	$C_{ji}^{-2}\gamma$
$Q_i\to Q_j$	$4rw$	2	2
$P_{Q_i}\to P_{Q_j}$	0	$[\eta+\sigma]\lvert\xi\rvert^2$	0
$P_{Q_i}\to V_{Q_j}$	$4rw\eta\lvert\xi\rvert^2$	$[\eta-\sigma]\lvert\xi\rvert^2$	$2\eta\lvert\xi\rvert^2$
$P_{Q_i}\to {}^{1/2}E_{Q_j}0^+$	0	$[2\epsilon+4\sigma]\lvert\tau_{1/2}\rvert^2$	0
$P_{Q_i}\to {}^{1/2}E_{Q_j}1^+$	$8rw\epsilon\lvert\tau_{1/2}\rvert^2$	$[2\epsilon-4\sigma]\lvert\tau_{1/2}\rvert^2$	$4\epsilon\lvert\tau_{1/2}\rvert^2$
$P_{Q_i}\to {}^{3/2}E_{Q_j}1^+$	$4rw\eta^2\epsilon\lvert\tau_{3/2}\rvert^2$	$2\eta[w^2-1-\delta]\lvert\tau_{3/2}\rvert^2$	$2\eta^2\epsilon\lvert\tau_{3/2}\rvert^2$
$P_{Q_i}\to {}^{3/2}E_{Q_j}2^+$	$12rw\eta^2\epsilon\lvert\tau_{3/2}\rvert^2$	$2\eta[w^2-1+\delta]\lvert\tau_{3/2}\rvert^2$	$6\eta^2\epsilon\lvert\tau_{3/2}\rvert^2$
$P_{Q_i}\to P_{Q_j}^{(n)}$	0	$[\eta+\sigma]\epsilon^2\lvert\xi^{(n)}\rvert^2$	0
$P_{Q_i}\to V_{Q_j}^{(n)}$	$4rw\eta\epsilon^2\lvert\xi^{(n)}\rvert^2$	$[\eta-\sigma]\epsilon^2\lvert\xi^{(n)}\rvert^2$	$2\eta\epsilon^2\lvert\xi^{(n)}\rvert^2$

momentum transfer t corresponding to a finite $w_{\rm elastic}-1$, where

$$w_{\rm elastic}-1=\frac{(m_{P_{Q_i}}-m_{P_{Q_j}})^2-t}{2m_{P_{Q_i}}m_{P_{Q_j}}}\ .$$

The impact of these kinematic effects will be discussed below.]

The sum rule of Eq. (49) depends on μ. However, for $\Lambda_{\rm QCD}\ll\mu\ll m_{Q_j}$ it is appropriate to match on to the effective heavy-quark currents to compute both the left- and right-hand sides of the sum rule. Thus each side of the equation contains a factor $|C_{ji}(w,\mu)|^2$ that will cancel out. (On the left-hand side this factor arises from perturbative strong-interaction corrections to quark decay summed in the leading-logarithmic approximation.) After this cancellation, the left-hand side of Eq. (49) is explicitly μ independent. The right-hand side of the equation, which must therefore also be μ independent, contains two compensating sources of μ dependence: the form factors [e.g., $\xi(w)$, $\tau_{1/2}(w)$, and $\tau_{3/2}(w)$] are μ dependent (for $w\neq1$) since they are defined as matrix elements of currents in the effective theory, but their μ dependence is canceled by the μ dependence of the limits of the sum.

Table III gives α, β_{++}, and γ for the heavy-quark transition $Q_i\to Q_j$ and for the low-lying resonant states in the heavy-quark symmetry limit. The form factors required for transitions to P_{Q_j} and V_{Q_j} are given in Ref. 1. Those for ${}^{1/2}E_{Q_j}0^+$, ${}^{1/2}E_{Q_j}1^+$, ${}^{3/2}E_{Q_j}1^+$, and ${}^{3/2}E_{Q_j}2^+$ are from Table II. In addition, the table shows the result of extending Ref. 1 to the transitions to the "radial excitations" $P_{Q_j}^{(n)}$ and $V_{Q_j}^{(n)}$: since $\mathbb{V}_\mu^{ji}$ is a conserved current of the effective theory, the function analogous to $\xi(w)$ for those states must *vanish* for $w=1$ and so may be written in the form $(w-1)\xi^{(n)}(w)$. In the quark model such states would arise as radial excitations of the ground state. Here they can be any excited states whatsoever carrying the quantum numbers $s_l^{\pi_l}=\frac{1}{2}^-$ of the ground states. One can also trivially extend the results of this paper to the "radially excited" states with $s_l^{\pi_l}=\frac{1}{2}^+$ or $\frac{3}{2}^+$: the nth such state will have the form factors displayed in Table II with $\tau_{1/2}\to\tau_{1/2}^{(n)}$ and $\tau_{3/2}\to\tau_{3/2}^{(n)}$ (so that $\tau_{1/2}^{(1)}\equiv\tau_{1/2}$ and $\tau_{3/2}^{(1)}\equiv\tau_{3/2}$). This gives (from the sum rule for any of α, β_{++}, or γ)

$$1=\left[\frac{w+1}{2}\right]|\xi(w)|^2+(w-1)\left[\sum_{n=1}^{n_{\max}(\mu)}\left[\frac{w^2-1}{2}\right]|\xi^{(n)}(w)|^2+2\sum_{m=1}^{m_{\max}(\mu)}|\tau_{1/2}^{(m)}(w)|^2+(w+1)^2\sum_{p=1}^{p_{\max}(\mu)}|\tau_{3/2}^{(p)}(w)|^2\right]+\cdots\ , \tag{50}$$

where the ellipsis denotes possible contributions from the inelastic continua and from resonances with quantum numbers for the light degrees of freedom other than $s_l^{\pi_l}=\frac{1}{2}^-,\frac{1}{2}^+$, and $\frac{3}{2}^+$.

One can dramatically reduce the number of states contributing to the sum rule by expanding Eq. (50) in a power series about $w=1$. Keeping terms up to linear order in $(w-1)$ gives $\xi(1)=1$ and Bjorken's sum rule[12] for the "charge radius" ρ of $\xi(w)$ defined by the expansion

$$\xi(w)=1-\rho^2(w-1)+\cdots\ . \tag{51}$$

The sum rule is

$$\rho^2=\tfrac{1}{4}+\sum_{m=1}^{m_{\max}(\mu)}|\tau_{1/2}^{(m)}(1)|^2+2\sum_{p=1}^{p_{\max}(\mu)}|\tau_{3/2}^{(p)}(1)|^2+\cdots\ , \tag{52}$$

where the ellipsis denotes possible contributions from inelastic continua. [We have already seen explicitly that the "radial excitations" of P_{Q_j} and V_{Q_j} make no contribution to (52); we will see below that there are no other resonant contributions to this sum rule.] Note that the μ dependence of ρ^2 is compensated by the cutoff at the excitation energy μ of the sum over resonances (as well as in the possible continuum term represented by the ellipsis.)

We now show that the states considered in this paper are the only excited resonant states contributing to Eq. (52). If a resonance is to contribute, it must be produced in either an S or a P wave, otherwise its contribution would be of higher order than $(w-1)$. If $f_s(w)$ is any S-wave form factor of an excited stated, then it must vanish at least as fast as $(w-1)$ as $w \to 1$ since all excited states of the light degrees of freedom are orthogonal to the ground state at $w=1$. Thus excited states contributing to the right-hand side of Eq. (52) must have P-wave form factors, i.e., ones proportional to $\mathbf{v}'$ in the frame where $\mathbf{v}=0$. To proceed it is convenient to consider the heavy quarks to be spinless, as we may because of the spin symmetry. We then want to know the conditions on $s_l'^{\pi_l'}$ for $\langle s'^{\pi_l'}(\mathbf{v}')|j(0)|\frac{1}{2}^-(0)\rangle$ to be proportional to $\mathbf{v}'$. There are two possibilities, depending on whether the heavy-quark current is proportional to $\mathbf{v}'Q_j^\dagger Q_i$ or simply $Q_j^\dagger Q_i$. In the former case the matrix element of $Q_j^\dagger Q_i$ must not vanish as $w \to 1$, which means that the light degrees of freedom have remained in their ground state: this case corresponds to a P-wave "elastic" form factor proportional to $\xi(w)$. In the latter case $s_l'^{\pi_l'}(\mathbf{v}')$ must be a P-wave state with $J^P=\frac{1}{2}^-$, i.e., $s_l'^{\pi_l'}$ must be either $\frac{1}{2}^+$ or $\frac{3}{2}^+$. Thus *all new resonant states contributing to* ρ^2 *are "radial excitations" of the states* ${}^{1/2}E_{Q_j}0^+$, ${}^{1/2}E_{Q_j}1^+$, ${}^{3/2}E_{Q_j}1^+$ *and* ${}^{3/2}E_{Q_j}2^+$ *considered here.* In constituent quark models, such states are typically somewhat more than 1 GeV above the ground state.[26]

In the nonrelativistic quark model

$$\tau_{1/2}^{(n)}(1)=\tau_{3/2}^{(n)}(1) \propto \int d^3k\, \phi_{n1}^*(k)k\phi_{00}(k)\ , \tag{53}$$

where ϕ_{nl} is the radial wave function of the nth radial state with orbital angular momentum l. In the harmonic-oscillator model $\tau_{1/2}^{(n)}(1)$ and $\tau_{3/2}^{(n)}(1)$ are identically zero for $n>1$: with other potentials their magnitudes fall rapidly with n due to oscillations in ϕ_{n1}. It would not be unreasonable, therefore, to expect the sum to converge rapidly.

This brings us to the important kinematical consideration mentioned earlier. The physical thresholds in t for a state E_{Q_j} contributing to Eq. (52) will be

$$2m_{P_{Q_i}}(m_{E_{Q_j}}-m_{P_{Q_j}})-(m_{E_{Q_j}}^2-m_{P_{Q_j}}^2)$$

below that for the state P_{Q_j}. Thus in a given heavy-quark transition $Q_i \to Q_j$, the decrease of ξ as a function of (t_m-t) will not be locally compensated by the onset of resonance production. However, wherever the production of the states E_{Q_j} occurs in t, the functions $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$ can be determined (where $w=v\cdot v'$ with v' the four-velocity of the state E_{Q_j}), and it is the threshold behavior of these functions in w (i.e., near $w=1$) that is relevant to the sum rule. Of course as $m_Q \to \infty$ all of the relevant thresholds will occur in a range of (t_m-t) which corresponds to an infinitesimal range of w for the $P_{Q_i} \to P_{Q_j}$ process so that for (t_m-t) well above this range the sum rule (50) would apply directly in the Dalitz plot. In this case the "shift" implied by the above prescription would be unnecessary.

Given the potential corrections to the heavy-quark symmetry limit for $\bar{B} \to {}^{s_l}E_c$ decays, to test the sum rule (52) it will probably be necessary to adopt the strategy mentioned earlier of extracting those form factors which are least sensitive to $\Lambda_{\rm QCD}/m_c$ effects. From these one can most reliably determine the universal limiting function τ_{s_l}, which are related by the sum rule to the slope of the universal limiting function $\xi(w)$.

Finally, we would like to comment on the constant term in Eq. (52). In the nonrelativistic limit [see Eq. (44)], $\frac{1}{4} \ll |\tau_{1/2}(1)|^2+2|\tau_{3/2}(1)|^2$, and the resonant terms give a radius that reproduces the nonrelativistic radius of the form factor $\xi(w)$. Thus the $\frac{1}{4}$, which arises from the $P_{Q_i} \to P_{Q_j}$ and $P_{Q_i} \to V_{Q_j}$ ground-state transitions, is a "relativistic correction," corresponding to a contribution to an elastic "charge radius" in $P_Q \to P_Q$ of $3/4m_Q^2$. This term is not present in the analogous $\Lambda_{Q_i} \to \Lambda_{Q_j}$ form factor (where $s_l=0$). Moreover, in the case of a spinless heavy "quark," the $\frac{1}{4}$ occurs once again for $s_l=\frac{1}{2}$ and not for $s_l=0$. We speculate that it can be associated with *Zitterbewegung* (the origin of the Darwin term in the hydrogen atom[27]) of the light degrees of freedom by $\Delta r_l^2 \sim m_l^{-2}$, which forces a smearing $\Delta r_Q^2 \sim m_Q^{-2}$ of the heavy-quark coordinate to preserve the position of the center of mass.

VI. CONCLUSIONS

Semileptonic $\bar{B}$ decay appears to be saturated by D, D^* and a small ($\sim$20%) contribution from either continuum states or excited resonances.[19] In this paper we have completed the heavy-quark symmetry predictions for the resonance production form factors which are likely to be seen in $\bar{B}$ decays by adding to the results of Ref. 1 the predictions for the positive-parity excited charmed mesons which lie about 500 MeV above the D and D^*. We have also discussed the interpretation of perturbative strong interaction corrections to a sum rule suggested by Bjorken, shown that the resonances contributing to the sum rule are all of the type considered here, and discussed the application of the sum rule in realistic circumstances.

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APPENDIX: RELATIONSHIP BETWEEN THE FORM FACTORS $\tilde{f}$ OF THE TEXT AND CONVENTIONALLY DEFINED FORM FACTORS f

The Lorentz-invariant form factors $\tilde{f}$ of the text are defined as coefficients of kinematic invariants formed out of polarization tensors and the four velocities v and v', as is appropriate to the heavy-quark symmetry limit. They also have a trivial kinematical factor of $(m_{E_{Q_j}} m_{P_{Q_i}})^{1/2}$ divided out. If this factor is multiplied back into Eqs. (10)–(16) and if factors of $m_{P_{Q_i}}$ and $m_{E_{Q_j}}$ are inserted to turn v and v' into p and p', then the form factors $\tilde{f}$ will be converted into the more conventionally defined form factors f of Ref. 18. The relation between these form factors may be expressed in the form

$$\tilde{f}_\alpha = m_{E_{Q_j}}^{n_j/2} m_{P_{Q_i}}^{n_i/2} f_\alpha \ , \tag{A1}$$

where n_i and n_j are the integers given in Table IV. For the 1^+ states one must in addition make use of Eqs. (7).

Since Ref. 18 quotes only the form factors that contribute in the limit of zero lepton masses, we quote here in the notation of that paper the additional formulas required to complete a comparison with that constituent quark-model calculation:

TABLE IV. The integers n_j and n_i of Eq. (A1).

$\tilde{f}$	n_j	n_i
$\tilde{h}$	$+1$	$+3$
$\tilde{k}$	-1	$+1$
$\tilde{b}_+ + \tilde{b}_-$	-1	$+5$
$\tilde{b}_+ - \tilde{b}_-$	$+1$	$+3$
$\tilde{l}_{s_l}$	-1	-1
$\tilde{c}_{s_l+} + \tilde{c}_{s_l-}$	-1	$+3$
$\tilde{c}_{s_l+} - \tilde{c}_{s_l-}$	$+1$	$+1$
$\tilde{q}_{s_l}$	$+1$	$+1$
$\tilde{u}_+ + \tilde{u}_-$	-1	$+1$
$\tilde{u}_+ - \tilde{u}_-$	$+1$	-1

$$b_+ + b_- = F_7 \frac{m_d^2 \beta_X}{4\sqrt{2} m_q m_b \tilde{m}_B \beta_B^2} \left[1 - \frac{m_d \beta_X^2}{2\tilde{m}_B \beta_{BX}^2} \right] , \tag{A2}$$

$$u_+ + u_- = -\sqrt{\tfrac{2}{3}} F_5 \frac{m_d}{\beta_B} \ , \tag{A3}$$

$$c_+ - c_- = F_5 \frac{m_d}{2 m_q \beta_B} \left[1 - \frac{m_d m_q \beta_B^2}{2\tilde{m}_X \mu_- \beta_{BX}^2} \right] , \tag{A4}$$

$$s_+ - s_- = F_5 \frac{m_d}{\sqrt{2} m_q \beta_B} \left[1 - \frac{m_d m_q \beta_B^2}{2\tilde{m}_X \mu_+ \beta_{BX}^2} \right] . \tag{A5}$$

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