Heavy-quark symmetries in form factors at large recoil

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It has recently been shown that hadrons containing a single heavy quark exhibit a new flavor-spin symmetry of QCD, and that this symmetry leads to relations between operator matrix elements involving such hadrons. On examining mechanisms that might break the symmetry at large recoil, I conclude that these relations are probably valid over the full kinematic ranges available in transitions involving $b \to c$, $b \to s$, $b \to u$, $c \to s$, and $c \to d$ currents.

I. INTRODUCTION

The properties of hadrons containing a single heavy quark Q ($m_Q \gg \Lambda_{\rm QCD}$) along with light degrees of freedom are constrained by symmetries which are not manifest in QCD.[1] The first of these is a flavor symmetry which arises from the fact that the long-wavelength properties of the light degrees of freedom in such a hadron become independent of m_Q for $m_Q \gg \Lambda_{\rm QCD}$. Thus, for example, the light degrees of freedom of a $\bar{B}$ and D meson can be related by an (approximate) $b \leftrightarrow c$ SU(2) symmetry even though m_b and m_c are very different (i.e., $m_b - m_c \gg \Lambda_{\rm QCD}$). The second symmetry pointed out in Ref. 1 is a related spacetime symmetry which arises in QCD because the spin of a heavy quark decouples from the gluon field.[2] This makes $\mathbf{S}_Q$, the heavy-quark spin operator, the generator of another SU(2) group of symmetries of the light degrees of freedom of a meson containing a single heavy quark. Thus, for example, the light degrees of freedom in the $\bar{B}$ and $\bar{B}^*$ mesons are in (approximately) the same state since the spin orientation of the b quark does not affect their dynamics. These symmetries are manifest in an effective theory where the heavy quark acts, in its hadron's rest frame, like a spatially static triplet source of color field.[2,3] In the effective theory the heavy quark's couplings to the gluon degrees of freedom are independent of its mass and described by a Wilson line.[3]

The consequences of these symmetries for the weak decays of $\bar{B}$ and D mesons were worked out in Ref. 1. The existence of conservation laws associated with the symmetries allows one to make absolutely normalized predictions for all $b \to c$ weak transition form factors between ground-state pseudoscalar (P) and vector (V) mesons at "zero recoil" (where, in the rest frame of the initial hadron, the final hadron is at rest). The symmetries also give relations between all six of the $P \to P$ and $P \to V$ weak form factors. In addition, the flavor symmetry relates, for example, $\bar{B} \to X_u$ and $D \to X_d$ weak transition form factors (X_u and X_d are particular light-hadron final states related by isospin which occur due to the $b \to u$ and $c \to d$ weak transitions). These latter relations may be crucial in the reliable extraction of the Cabibbo-Kobayashi-Maskawa[4] matrix element V_{ub} from experimental data.

In the first work cited in Ref. 1 these symmetries were applied for static quarks where the $Q_i \leftrightarrow Q_j$ symmetry simply substituted one static heavy quark for another.[5] The second work cited in Ref. 1 exploited a powerful extension of this method which makes use of the fact that (in the effective theory) when Q_i at velocity $\mathbf{v}$ is replaced by Q_j at $\mathbf{v}'$, the amplitude for the light degrees of freedom to make any associated transition is independent of m_i and m_j if they are sufficiently large: the light degrees of freedom interact only with the (moving) color fields of Q_i and Q_j which depend only on the Lorentz boosts required of the (mass-independent) rest frame color fields. In the effective theory the mass of the heavy quark is taken to infinity in such a way that p_Q^μ / m_Q is held fixed, but the four-momentum of the light degrees of freedom is neglected compared with m_Q. In this limit the interactions of the gluons with the heavy quark do not alter its straight world line and are independent of its mass and spin. The interactions of the heavy quark do depend on the heavy quark's four-velocity v^μ. The resulting symmetries are as a result somewhat unusual in that they relate states of equal velocity but different mass, and therefore different momentum.

For example, in matrix elements of

$$\mathbb{V}_\nu^{ji} \equiv \bar{\mathbb{Q}}_j \gamma_\nu \mathbb{Q}_i \ , \tag{1a}$$

$$\mathbb{A}_\nu^{ji} \equiv \bar{\mathbb{Q}}_j \gamma_\nu \gamma_5 \mathbb{Q}_i \tag{1b}$$

(the weak $Q_i \to Q_j$ currents in the effective theory), changes in the heavy-quark velocity and spin occur only due to the actions of the currents. In a typical such transition, $H_i(\mathbf{v}=0) \to H_j(\mathbf{v}')$ where H_n is the hadron containing the single heavy quark Q_n) the form factors will therefore be determined by the product of the amplitude for the heavy quarks to make the transition $Q_i(\mathbf{v}=0) \to Q_j(\mathbf{v}')$ and for the light quarks to be "excited" by the transition from the hadron H_i at rest into the hadron H_j moving with velocity $\mathbf{v}'$. To apply these symmetries we must know the relationship between the weak currents in the complete theory

$$V_\nu^{ij} \equiv \bar{Q}_j \gamma_\nu Q_i \ , \tag{2a}$$

$$A_\nu^{ij} \equiv \bar{Q}_j \gamma_\nu \gamma_5 Q_i \ , \tag{2b}$$

and those in Eq. (1) of the effective theory. This relationship has the form (for $J_\nu = V_\nu$ or A_ν, and $\mathbb{J}_\nu = \mathbb{V}_\nu$ or $\mathbb{A}_\nu$)

$$J_\nu^{ij} = C_{ji} \mathbb{J}_\nu^{ji} + \cdots , \tag{3}$$

where the ellipsis denotes other possible Lorentz structures which are suppressed by factors of $\alpha_s(m_Q)/\pi$ as well as higher-dimension operators whose physical effects are suppressed by powers of $\Lambda_{\rm QCD}/m_Q$. In the leading-logarithmic approximation,[6,7]

$$C_{ji} = \left[\frac{\alpha_s(m_i)}{\alpha_s(m_j)} \right]^{a_I} \left[\frac{\alpha_s(m_j)}{\alpha_s(\mu)} \right]^{a_L} , \tag{4}$$

with

$$a_I = -\frac{6}{33-2N_f} \tag{5}$$

(where in the $b \to c$ transition, for example, the number of flavors $N_f = 4$ for the region between m_b and m_c) and

$$a_L = \frac{8[v' \cdot v \, r(v' \cdot v) - 1]}{33-2N_f} , \tag{6}$$

in which

$$r(v' \cdot v) = \frac{1}{\sqrt{(v' \cdot v)^2 - 1}} \ln[v' \cdot v + \sqrt{(v' \cdot v)^2 - 1}] \tag{7}$$

(where in $b \to c$, $N_f = 3$ for the region below m_c). Note that the factor C_{ij} depends on the dot product of the four-velocity v_μ of Q_i with the four-velocity v'_μ of Q_j. This velocity-dependent contribution (which was missed in Ref. 1) was calculated in Ref. 7. For $v' = v$ (i.e., $v' \cdot v = 1$) the currents are not renormalized in the effective theory $[r(1) = 1]$ and their matrix elements are independent of the subtraction point μ. This occurs because the three quantities $Q^a = \int d^3x \, \mathbb{Q}_j^\dagger \tau_{ji}^a \mathbb{Q}_i$ are generators of the SU(2)-flavor symmetry in the effective theory, and so the matrix elements of Q^a are physical quantities.

The heavy-quark symmetry also relates matrix elements of $Q_i \to q$ and $Q_j \to q$ operators, where q is, for example, a light quark. In this case the weak currents in the effective theory,

$$\mathbb{V}_\nu \equiv \bar{\mathbb{q}} \gamma_\nu \mathbb{Q}_i , \tag{8a}$$

$$\mathbb{A}_\nu \equiv \bar{\mathbb{q}} \gamma_\nu \gamma_5 \mathbb{Q}_i , \tag{8b}$$

are related to those of the complete theory by

$$J_\nu = C(\mu) \mathbb{J}_\nu + \cdots , \tag{9}$$

where

$$C(\mu) = \left[\frac{\alpha_s(m_i)}{\alpha_s(\mu)} \right]^{a_I} . \tag{10}$$

[If there are flavor thresholds between m_i and μ, then $C(\mu)$ must be modified to take into account the fact that N_f is not constant.] The value of the resulting relations in the model-independent determination of V_{ub} was shown in the first work cited in Ref. 1; details are supplied in Ref. 8.

In addition to Ref. 7 there have been several other recent improvements on the work of Ref. 1. In Ref. 9 a power-counting argument was given to prove (in a particular case) that, to all orders in perturbation theory, matrix elements in the complete theory factorize into a coefficient function (i.e., C_{ji}) times a matrix element in the effective theory where the heavy quark couples as a Wilson line. Also, in Ref. 10 (see also Ref. 7), it was shown how the effective theory can be written as a Lorentz-invariant field theory with a superselection rule for the velocity of the heavy quark. Reference 7 and independently Ref. 11 showed how to derive the results of Ref. 1 in a much simpler way.

It is clear from their derivations that the heavy-quark symmetry relations are valid in the region near zero recoil. However, violation of the heavy-quark flavor and spin symmetries arising from the fact that m_i and m_j are finite will, among other things, perturb the light degrees of freedom away from their limiting state. Since an $H_i \to H_j$ transition at high recoil becomes sensitive to small components in the hadronic wave function, it is to be expected that the symmetry relations will eventually fail. Reference 1 gave some qualitative estimates of the range of validity of such relations, based on the momentum scales at which the states of the light degrees of freedom in H_i and H_j would differ from their limiting state. In this paper I examine the limitation on the range of validity of the heavy-quark symmetry relations more carefully.

II. $Q_i \to Q_j$ TRANSITIONS

The heavy-quark symmetry relates the six form factors in $P_i \to P_j$ and $P_i \to V_j$ weak vector and axial-vector transitions to a single universal function $\xi(t)$ with $\xi(0) = 1$. These form factors are in principle all measurable in $B \to D$ and $B \to D^*$ transitions, so these predictions are of some interest. The predictions arise in part from relations between $P_i \to P_i$, $P_i \to P_j$, and $P_j \to P_j$ which depend only on the $Q_i \leftrightarrow Q_j$ flavor symmetry and partly from relations between $P_i \to P_j$ and $P_i \to V_j$ which depend on the heavy-quark spin symmetry of Q_j.

A. A heuristic argument

We begin our consideration of transitions induced by $Q_i \to Q_j$ operators with a simple heuristic argument which contains some of the physics of the more elaborate discussions which follow.

In an $H_i \to H_j$ transition, one can by Lorentz invariance view any operator matrix element in the frame $H_i(-\mathbf{v}_T) \to H_j(+\mathbf{v}_T)$. This frame would correspond to the Breit frame in the case $m_i = m_j$. In the general case the usual hadronic momentum transfer $t = (p_i - p_j)^2$ is given by (we approximate $m_{H_i} \simeq m_i, m_{H_j} \simeq m_j$)

$$t = t_m - 4\gamma_T^2 v_T^2 m_i m_j , \tag{11}$$

where $t_m \equiv (m_i - m_j)^2$ is the maximum four-momentum transfer. However, if the light degrees of freedom of these hadrons have an "effective mass" $m_l \sim \Lambda_{\rm QCD}$, then this frame *is* their Breit frame and they experience a momentum transfer

$$\mathbf{Q}_l=-2m_l\gamma_T\mathbf{v}_T\ , \tag{12a}$$

with

$$-t_l\equiv Q_l^2=4\gamma_T^2v_T^2m_l^2\ . \tag{12b}$$

We note that

$$Q_l^2=\frac{m_l^2}{m_im_j}(t_m-t) \tag{13}$$

is much smaller than t_m-t. In the rest frame of the heavy-quark hadrons such a momentum transfer corresponds to a momentum spread in the direction of $\mathbf{v}_T$ of $\Delta p_T \sim (m_l/m_im_j)(t_m-t)$ in either the initial or final heavy hadron. One would as a result naively expect the heavy-quark relations to hold so long as

$$\frac{t_m-t}{4m_im_j}<\frac{m_j}{m_l}\ ,$$

beyond which point $\Delta p_T\sim m_j$, the quark Q_j is no longer approximately static in its rest frame, and structure in H_j on the scale m_j^{-1} will be revealed. We will see below that this requirement is too strict, but even it suggests that the heavy-quark relations of Ref. 1 will be valid over the full $B\to D$ and $B\to D^*$ Dalitz plots.

B. A valence parton model

With these arguments to guide us, we begin our examination of the $H_i\to H_j$ transition in a frame boosted to "infinite" velocity v along an axis $\hat{\mathbf{z}}$ transverse to the velocities $\mp\mathbf{v}_T$ in the Breit-like frame introduced earlier where $H_i(-\mathbf{v}_T)\to H_j(+\mathbf{v}_T)$. In such a frame, the initial and final four-momenta are

$$p_i^\mu=(\gamma m_{i_T},-m_{i_T}\mathbf{v}_T,\gamma m_{i_T}v)\ , \tag{14a}$$

$$p_j^\mu=(\gamma m_{j_T},+m_{j_T}\mathbf{v}_T,\gamma m_{j_T}v)\ , \tag{14b}$$

where $\gamma=(1-v^2)^{-1/2}$ and $m_{k_T}=m_k(1-\mathbf{v}_T^2)^{-1/2}$ is the "transverse mass" of H_k. Since in general $m_i\neq m_j$, in this frame the transition can involve an "infinite" momentum change, but from the perspective of the heavy-quark symmetries this is irrelevant: the large momentum difference $p_i\to p_j$ is due to the mass difference between the heavy quarks Q_i and Q_j, while the light degrees of freedom whose symmetries interest us are experiencing finite momentum transfers independent of the m_i that are proportional to $\gamma_T\mathbf{v}_T$. This observation, incidentally, suggests that the scaled momentum fraction $u=(m_i/m_l)x$ will be a more useful variable for this problem than the usual Bjorken scaling variable x.

In this section we consider a parton model in which H_i is dominated by its valence-quark structure: Q_i and a light antiquark (taken to be $\bar{d}$ for concreteness). Then

$$|H_i(P_i)\rangle=\left[\frac{2m_im_d}{N_c}\right]^{1/2}\int\frac{dx\,d^2p_T}{\sqrt{x(1-x)}}\phi_i^{s\bar{s}}(x,\mathbf{p}_T)|Q_i^\alpha([1-x]P_i,-\mathbf{p}_T;s)\bar{d}^{\,\alpha}(xP_i,\mathbf{p}_T;\bar{s})\rangle\ , \tag{15}$$

where with

$$\sum_{s\bar{s}}\int dx\,d^2p_T|\phi_i^{s\bar{s}}(x,\mathbf{p}_T)|^2=1\ , \tag{16}$$

and quark states normalized to

$$\langle q^{\alpha'}(\mathbf{p}',s')|q^\alpha(\mathbf{p},s)\rangle=\delta_{\alpha'\alpha}\delta_{s's}\frac{E}{m}\delta^3(\mathbf{p}'-\mathbf{p})\ ,$$

H_i is conventionally normalized to $2E\delta^3(\mathbf{P}'-\mathbf{P})$. In Eq. (15), s and $\bar{s}$ are spins, α is a color index for N_c colors, the $\phi_i^{s\bar{s}}$ are functions which couple the spins and momenta to the quantum numbers of the hadron H, and x is the longitudinal-momentum fraction along $\mathbf{P}_i$. In this naive model, the state of the light degrees of freedom in H_i can depend on Q_i only through ϕ_i: the constitutent antiquark is assumed to be structureless.

In the framework of this model, which is very similar to models considered in Ref. 12 in connection with the pion form factor, one can easily show that the universal function $\xi(t)$ of Ref. 1 is given by

$$\xi(Q_l^2)=\int du\,d^2p_T\Phi_\infty(u,\mathbf{p}_T+u\mathbf{Q}_l)^*\Phi_\infty(u,\mathbf{p}_T)\ , \tag{17}$$

where $\mathbf{Q}_l$ is exactly the "heuristic" momentum transfer to the light degrees of freedom defined in Eq. (12) [which is related to t by Eq. (13)], and Φ_∞ is a universal limiting wave function defined by

$$\Phi_\infty(u,\mathbf{p}_T)=\lim_{m_i\to\infty}\tilde{\phi}_i(u,\mathbf{p}_T)\ , \tag{18a}$$

where

$$\tilde{\phi}_i(u,\mathbf{p}_T)\equiv\left[\frac{m_l}{m_i+m_l}\right]^{1/2}\phi_i\left[\frac{m_l}{m_i+m_l}u,\mathbf{p}_T\right]\ . \tag{18b}$$

The function Φ_∞ expresses the fact that as $m_i\to\infty$, the ϕ_i differ only by a kinematic shift of the momentum fraction of the light degrees of freedom to smaller x and a trivial normalization factor induced by Eq. (16). [We have suppressed the spin indices of Φ_∞ in (18) as well as in (17), where they are implicitly summed.] It should be noted that similar formulas can be derived in other frames, but that such formulas will typically involve additional kinematic factors. Such alternative formulas are as a result especially sensitive to (15) being a solution of the equations of motion of the theory (see Ref. 12).

We now investigate the possible breakdown of the universality of $\xi(Q_l^2)$ from violations of the heavy-quark symmetry in terms of this simple model. There are two basic ways in which the state of the light degrees of freedom represented by the $\bar{d}$ quark in Eq. (15) differs from the limiting case $m_i \to \infty$: (1) the rms transverse momentum in ϕ_i will depend on m_i, approaching a constant value of order Λ_{QCD} as $m_i \to \infty$, and (2) interactions between the $\bar{d}$ and Q_i with a range m_i^{-1} will induce a high p_T tail to ϕ_i with a strength which vanishes as $m_i \to \infty$. (By rotational invariance analogous changes are to be expected in the x distribution: see below.) The simplest example of the first effect is the perturbation about $m_i \to \infty$ from the heavy-quark kinetic energy; in a nonrelativistic bound-state problem this effect can all be absorbed into the two-body reduced mass. The prototype of the second effect is the distortion of the state of the light degrees of freedom produced by short-range color-magnetic effects. The preeminent such effect in low-energy hadron phenomenology is the Fermi-Breit spin-spin interaction which arises from color-magnetic fields of range m_i^{-1} about the position of the heavy quark Q_i. At higher energies this interaction becomes the transverse hard-scattering process which is eventually supposed to lead to an asymptotic power-law behavior of the form factors.[13]

We now estimate the sizes of these two effects. We begin by noting that if $\tilde{\phi}_i$ behaves like $(a_i p_T)^{-n}$ for large p_T, where $a_i \to a_\infty$ as $m_i \to \infty$, then $\delta\tilde{\phi}/\Phi_\infty = -n\,\delta a/a_\infty$ is independent of p_T and of order Λ_{QCD}/m_i. This suggests that a *parametric* change of the rms p_T with m_i may not lead to a failure of the approximate equality of the $\tilde{\phi}_i$ at large p_T. (We will see below that a much less demanding condition is actually required to guarantee the symmetry of form factors.) The existence of short-range interactions of scale m_i can be expected to produce a small long-range tail to $\tilde{\phi}_i$:

$$\delta\tilde{\phi}_i \sim \alpha_s(m_l p_T)\frac{\Lambda_{\text{QCD}}}{m_i}\chi(p_T/m_i) \tag{19}$$

for $p_T \gg \Lambda_{\text{QCD}}$, where χ is a normalized p_T wave function with $\chi(\rho)\to 0$ for $\rho \gg 1$. In contrast to the "reduced mass effect," this effect could, at sufficiently high p_T, make $\tilde{\phi}_i$ very different from Φ_∞: $\delta\tilde{\phi}_i/\Phi_\infty$ will be of order unity when $\tilde{\phi}(p_T)/\tilde{\phi}(0) \sim \alpha_s \Lambda_{\text{QCD}}/m_i$. We now examine the effects of these two kinds of distortions of ξ as given in Eq. (17) more quantitatively by defining

$$\xi_{ji}(Q_l^2) \equiv \int du\, d^2p_T \tilde{\phi}_j(u,\mathbf{p}_T+u\mathbf{Q}_l)^* \tilde{\phi}_i(u,\mathbf{p}_T) , \tag{20}$$

where $\tilde{\phi}_j$ and $\tilde{\phi}_i$ differ from Φ_∞ by the subasymptotic effects noted above. For orientation, consider an example where the p_T dependence is integrable:

$$\tilde{\phi}_i(u,\mathbf{p}_T) = U_i(u)\frac{1}{\sqrt{\pi}\beta_i}e^{-p_T^2/2\beta_i^2} , \tag{21}$$

i.e., where $\tilde{\phi}_i$ is the product of separately normalized functions of u and $\mathbf{p}_T$. Then

$$\xi_{ji}(Q_l^2) = \frac{\beta_i\beta_j}{\beta_{ij}^2}\int du\, U_j^*(u)U_i(u)e^{-u^2Q_l^2/4\beta_{ij}^2} , \tag{22}$$

where $\beta_{ij}^2 \equiv \frac{1}{2}(\beta_i^2+\beta_j^2)$. If the heavy-quark mesons were nonrelativistic objects, m_l would be m_d and the U's would be sharply peaked around $u=1$, giving

$$\xi_{ji}^{\text{NR}}(t) = \frac{\beta_i\beta_j}{\beta_{ij}^2}\exp\left[-\frac{m_d^2(t_m-t)}{4\beta_{ij}^2 m_i m_j}\right] \tag{23}$$

as in the first work cited in Ref. 14. In real heavy-quark mesons we expect the U's to be functions which have most of their strength in the neighborhood of $u=1$, but which have a natural width $\Delta u \sim 1$ corresponding to the fact that light quarks have only one scale Λ_{QCD} determining their momentum distribution. For example, if

$$U_k = \sqrt{2a_k^2 u}\exp(-\tfrac{1}{2}a_k^2u^2) , \tag{24a}$$

then one has

$$\xi_{ji}(t) = \frac{\beta_i\beta_j}{\beta_{ij}^2}\left[\frac{a_{ij}^2}{a_ia_j}+\frac{m_l^2(t_m-t)}{4\beta_{ij}^2a_ia_jm_im_j}\right]^{-1} , \tag{24b}$$

where $a_{ij}^2 = \frac{1}{2}(a_i^2+a_j^2)$. This example illustrates a feature which is quite general: for large (t_m-t), $\xi_{ji}(t)$ will be dominated by the end-point region $u \simeq 0$, and the sharp nonrelativistic drop of the form factor with $t_m - t$ seen in Eq. (23) will be softened. Equations (24) illustrate the remark made earlier on the lack of impact of the parametric dependence of the rms p_T values (here characterized by the β's and a's) on the symmetry of the form factors. It shows that even a Gaussian p_T dependence (which falls faster than our earlier polynomial example and would eventually lead to $\delta\tilde{\phi}/\Phi_\infty > 1$ at high p_T) leads to a soft dependence of the *form factors* on the β's and a's. To see this, expand β_k and a_k around $m_k \to \infty$ $[\beta_k = \beta_\infty(1+\beta' m_l/m_k)$ and $a_k = a_\infty(1+a'm_l/m_k)]$. Then we see that $\Delta\xi_{ji}/\xi = O(m_l/m_j)$ for all values of t. We also note using these expansions that Eq. (24b) gives $\xi_{ji}(t_{\max}) = 1+O(m_l^2/m_j^2)$ as demanded by the Ademollo-Gatto theorem.[15] Examination of models for $\tilde{\phi}(u,\mathbf{p}_T)$ (including ones which do not factorize and ones with other functional forms for the p_T and u dependence) leads one to the conclusion that this qualitative behavior is quite general.

Let us next consider the effect on $\xi_{ji}(t)$ of a high-p_T tail like that of Eq. (19). We begin with an extension of the above example, adding a function corresponding to the right-hand side of Eq. (19) to the p_T wave function in (21). With

$$\tilde{\phi}_k(u,p_T) = U(u)\left[\frac{1}{\sqrt{\pi}\beta}e^{-p_T^2/2\beta^2} + \frac{\alpha_s\Lambda_{\text{QCD}}}{m_k}\frac{1}{\sqrt{\pi}m_k}e^{-p_T^2/2m_k^2}\right] , \tag{25a}$$

if $U(u) = \sqrt{2a^2u}\,\exp(-\frac{1}{2}a^2u^2)$ we obtain

$$\xi_{ji}(t) \simeq \frac{1}{1+\dfrac{m_l^2(t_m-t)}{4a^2\beta^2m_jm_i}} + O\left[\frac{\alpha_s\Lambda_{\text{QCD}}\beta}{m_j^2}\frac{1}{1+m_l^2(t_m-t)/2a^2m_j^3m_i}\right] , \tag{25b}$$

where one can ignore the effect of the tail of $\tilde{\phi}_i$, which will be smaller since $m_i > m_j$. It is easy to see that in this model the leading term in (25b), which is of course the function $\xi(t)$ of Eq. (17), is dominant for all t: for $(t_m - t)/4m_i m_j \lesssim m_j^2/m_l^2$ it is very dominant, while for larger values of $t_m - t$ it is weakly dominant by a power of $\alpha_s(Q_l^2)$. This is the condition originally quoted in Ref. 1.

In drawing this last conclusion we have ignored the low u analogue of the high-p_T tail in the longitudinal wave function $U(u)$. This is consistent because the high-p_T tail does not "need" $u \to 0$ until $Q_l^2 \sim m_j^2$ corresponding to $(t_m - t)/4m_j m_i = O(m_j^2/m_l^2)$. Of course by rotational invariance one could in principle also discuss the above restrictions in terms of the normal p_T wave function being affected by the low-u tail.

Within the context of such a model, this qualitative conclusion on the lack of impact of the high-p_T tails is quite general. Equation (22) is dominated by small u for large Q_l^2/β^2, $\xi(t)$ in this region will depend only on the small u behavior of U, and if $U(u) \sim u^{p/2}$, $\xi(t) \sim [4\beta^2 m_j m_i / m_l^2(t_m - t)]^{(p+1)/2}$. On the other hand, for Q_l^2/m_j^2 small, the high-p_T tail of $\tilde{\phi}_j$ will give a contribution to ξ_{ji} of

$$\frac{2\alpha_s \Lambda_{\rm QCD}\beta}{m_j^2}\left[1 - \frac{m_l^2(t_m - t)}{2m_j^3 m_i}\langle u^2 \rangle\right],$$

where $\langle u^2 \rangle$ is the mean-square value of u in U, i.e., a number of order unity, while for large Q_l^2/m_j^2 it gives a contribution of order

$$\frac{\alpha_s \Lambda_{\rm QCD}\beta}{m_j^2}\left[\frac{m_j^3 m_i}{m_l^2(t_m - t)}\right]^{(p+1)/2}.$$

Thus for $p > 1$ the region in which $\xi_{ji} \simeq \xi$ is

$$\frac{t_m - t}{4m_j m_i} < \left[\frac{m_j^2}{\alpha_s(Q_l^2) m_l^2}\right]^{2/(p+1)}, \tag{26}$$

while for $p < 1$, $\xi_{ji} \simeq \xi$ for all t. Recall that $p \simeq 1$ is especially interesting because, as explained in Ref. 12, it is favored by measurements of the pion structure function. [Note that the condition $p < 1$ is required if one demands $\xi_{ji} \simeq \xi$ for all t for all $m_j > \Lambda_{\rm QCD}$. However, for a given m_j the actual requirement is only that $p < 1 + \ln(1/\alpha_s)/\ln(m_j/m_l)$. Thus, e.g., the high-$p_T$ tail associated with the charm quark would not break the symmetry so long as p is less than about 1.5.]

Following Ref. 12, one can also consider many variants on models for the wave functions (nonfactorizing wave functions, alternative functions of p_T, including $L_z \neq 0$ components, etc.). On doing so one will find no departures from the basic conclusion that, at least in the simple valence parton model picture of this section, the form factors $\xi_{ji}(t)$ can be expected to respect the heavy-quark symmetry so long as a restriction comparable to Eq. (26) is satisfied. This restriction is similar to the one obtained in Sec. II A, but we now see that the simple argument given there was somewhat misleading. The end-point contributions to form factors always have $p_T \sim \Lambda_{\rm QCD}$, so they never ruin the heavy-quark approximations: the real restriction on $t_m - t$ arises from demanding that the soft contributions dominate over hard contributions controlled by the heavy-quark scale.

The models we have just discussed for the high-p_T tail are based on Eq. (19), which was in turn suggested by general arguments on the decoupling of the heavy quark's spin from the light degrees of freedom.[2,3] It is consistent with phenomenological models of hadron structure in which deviations from the heavy-quark symmetry would arise from quark hyperfine interactions. However, we can also discuss such effects in terms of the asymptotic behavior of the form factors predicted in perturbative QCD.[13] Even if one were to accept the argument that these asymptotic predictions are not applicable at available momentum transfers,[12] perturbative QCD at least offers an alternative to the end-point model for the physics controlling the large recoil behavior of the form factors of interest here. In this model, the large recoil which the current imparts in the $Q_i \to Q_j$ transition is transferred to the light "spectator" degrees of freedom by hard (transverse) gluon exchange. Since each such exchange costs a power of α_s and a power of Q^{-2}, the asymptotic behavior is controlled by the valence sector of Fock space. In our case it will be given by the transition $Q_i\bar{d} \to Q_j\bar{d}$ (where now $\bar{d}$ is not a mnemonic for light degrees of freedom with $\bar{d}$ quantum numbers, but rather the $\bar{d}$ current quark). The contributions $\delta f_\pm$ of these processes to the $P_i \to P_j$ form factors $f_\pm$ can be shown to have the form

$$\delta f_\pm|_{p\rm QCD} \sim \eta_\pm \frac{\alpha_s(Q_l^2) m_l}{\sqrt{m_j m_i}} \zeta(Q_l^2), \tag{27}$$

where $\eta_\pm$ are constants of order unity and $\zeta(Q_l^2) \sim m_l^2/Q_l^2$ for $Q_l^2 \gg m_l^2$, in contrast with the predictions[1]

$$f_+ - f_- = \left[\frac{\alpha_s(m_i)}{\alpha_s(m_j)}\right]^{-6/(33-2N_f)} \xi(Q_l^2) \left[\frac{m_i}{m_j}\right]^{1/2}, \tag{28a}$$

$$f_+ + f_- = \left[\frac{\alpha_s(m_i)}{\alpha_s(m_j)}\right]^{-6/(33-2N_f)} \xi(Q_l^2) \left[\frac{m_j}{m_i}\right]^{1/2}, \tag{28b}$$

of heavy-quark symmetry. In terms of our previous more general language, we can say that the form factor ξ in Eq. (28a) receives a high-p_T correction of order $(\alpha_s m_l/m_i)\zeta$ while the ξ in Eq. (28b) receives a correction of order $(\alpha_s m_l/m_j)\zeta$. This is consistent with the general framework introduced above to discuss such effects [compare to Eq. (19)]. If $p = 1$ so that $\xi \sim m_l^2/Q_l^2$ as does ζ, then $\xi_{ji} \simeq \xi$ at all recoils. The more general conclusion analogous to Eq. (26) is that for $p > 1$, $\xi_{ji} \simeq \xi$, so long as

$$\frac{t_m - t}{4m_i m_j} \le \left[\frac{m_j}{\alpha_s m_l}\right]^{2/(p-1)}, \tag{29}$$

which still leads comfortably to the conclusion that the heavy-quark symmetries will apply throughout the $\bar{B} \to D$ and $\bar{B} \to D^*$ Dalitz plots even if $p \neq 1$. Comparison of

Eqs. (29) and (26) shows that while they indicate the same qualitative behavior, a precise statement of the conditions for the breakdown of the universality of the ξ_{ji} at large recoil must await a better understanding of the dynamics responsible for high-p_T exclusive processes.

C. Conclusions on $Q_i \to Q_j$ transitions

The light degrees of freedom of a heavy-quark hadron H_i are much more complicated than they are represented to be by the simple valence parton model we have just discussed. Nevertheless, at low momentum transfers, where the state vector could be constructed of the effective degrees of freedom of QCD cut off at some low scale $\mu \sim 1$ GeV, the valence model has phenomenological justification from the quark model. With a different interpretation, the model also has some justification at large Q_l^2 where we have actually used it. The reason is that one expects these form factors to be dominated by the simplest, i.e., valence, sectors of a Fock-space expansion of the states involved since there is a large penalty to be paid for each extra constituent which must be given a large momentum transfer. In the case of end-point dominance emphasized here, this penalty is that one must find each of the "spectators" at small x so that essentially the full momentum transfer can be delivered by the current.[12,16] In the study of exclusive processes in perturbative QCD,[13] higher components of Fock space are suppressed by the quark-counting rule powers of Q^{-2} [and by powers of $\alpha_s(Q^2)$]. Thus, if we view the valence calculations considered above as the leading term in an expansion of the high-Q_l^2 behavior of the form factors (and appropriately renormalize the probability of this component of Fock space) they can be justified at high Q_l^2.

I conclude that Eq. (26) is a reasonable guide to the range of validity of the heavy-quark symmetry relations. Since for $B \to D$ and $B \to D^*$ transitions the left-hand side of Eq. (26) is $\lesssim \frac{1}{4}$, it therefore seems likely that the $Q_i \to Q_j$ relations deduced in Ref. 1 are valid over the full Dalitz plots for these decays.

III. $Q_i \to q$ TRANSITONS

The heavy-quark symmetry cannot give absolutely normalized predictions for exclusive form factors such as $\bar{B} \to \pi$, $\bar{B} \to \rho$, etc., induced by a heavy-to-light ($Q_i \to q$ where $q = u$, d, or s) operator, but it can relate them[1] to other heavy-to-light transitions (in this case to $D \to \pi$, $D \to \rho$, etc.). Such relations are guaranteed to hold for t near t_m, and their validity in any region near t_m is sufficient, for example, to allow the model-independent determination of V_{ub}. However, it would be convenient in this regard if the relations were to hold everywhere in the $D \to X_q$ Dalitz plots, since this would provide a maximum overlap with the $\bar{B} \to X_q$ processes: of the full range $0 \le (t_m - t)/t_m \le 1$, the range $0 \le (t_m - t)/t_m \lesssim \frac{1}{3}$ would then be related to $D \to X_q$ decays. In other cases it would be more than just a convenience if the heavy-quark relations were to hold over the full kinematic range. The rare $\bar{B}$ decay $\bar{B} \to K^* \gamma$ involves matrix elements that correspond to those in $\bar{B} \to \rho e \bar{\nu}_e$ at $t = 0$ in the SU(3)-flavor limit, and other decays like $\bar{B} \to K e^+ e^-$ and $\bar{B} \to K^* e^+ e^-$ involve matrix elements that are directly related to those in operation in $D \to K e \bar{\nu}_e$ and $D \to K^* e \bar{\nu}_e$ decays.[8]

Heavy-to-light transitions lack the basic simplicity of the heavy-to-heavy transitions where the heavy quark defines the velocity of the hadron, thereby allowing us to work in the Breit frame of the light degrees of freedom. This difficulty is especially apparent in the case of the pion where it is clear that transition form factors such as $\bar{B} \to \pi$ will not primarily be related to the pion's velocity: this form factor should be a stable function of momentum transfer as we take the chiral limit $m_u, m_d \to 0$, but the pion's velocity for a given momentum transfer is not. The dynamical origin of this difference between $Q_i \to Q_j$ and $Q_i \to q$ is clear: in the light hadrons, two "constituent quarks," each with intrinsic scale Λ_{QCD}, bind to each other with energies that have the same scale. One cannot as a consequence identify the "constituent quark" in, e.g., the $\bar{B}$ with one in the π or ρ (although I would speculate that in the latter case this identification might not be too bad), and as a consequence one cannot identify a Breit frame for such transitions. Such a frame can still be defined for elastic form factors such as the pion electromagnetic form factor $F_\pi(t)$. This allows in principle for the calculation of such form factors at all t in terms of the parton-model framework of Sec. II as was done in Ref. 12. One can also define such a frame at asymptotic momentum transfers where $\sqrt{t_m - t}$ is much larger than the mass of the initial and final particles. This latter possibility is of little use to us here, but it is important for the asymptotic calculations of Ref. 13. An explanation of the role of this frame can be found in Ref. 17, which derives the connection between Breit-frame form factors and the Fourier transforms of "charge" distributions. Intuitively, the charge distribution in the pion is stable as one approaches the chiral limit since it is associated with the scale Λ_{QCD}. Scattering in the Breit frame probes this charge distribution with a pure spacelike momentum transfer $q^\mu = (0, \mathbf{Q})$ corresponding to the spatial resolution Q^{-1}. When such a frame is not available and the momentum transfer has a time component, this simple picture is lost.

This elementary discussion suggests that a *qualitative* picture of inelastic form factors involving light hadrons can be obtained by treating each "constituent quark" of effective mass m_l as part of a hadron with effective mass $2m_l$ (i.e., in analogy to heavy-quark hadrons), so that a Breit frame can be defined. The resulting form factors are then independent of the actual hadron mass but will depend on the crudely defined scale m_l. While this method can be expected to produce only qualitative results in those cases where a Breit frame does not exist, this is sufficient for our present limited purposes where we are not trying to *calculate* the heavy-to-light transition form factors, only to understand the ranges over which they are independent of m_i. Our assumption is that this range will not depend very strongly on the nature of the *light* hadron.

Having adopted this framework, we can simply go back to our discussion of the $Q_i \to Q_j$ transition, let $m_j = m_l$, and examine the dependence of form factors on m_i. We begin with the heuristic argument, which now indicates that

$$Q_l^2|_{Q\to q} = \frac{m_l}{m_i}(t_m - t) , \tag{30}$$

corresponding to a momentum spread in the rest frame of the initial heavy-quark hadron H_i of $\Delta p \sim (t_m - t)/m_i$. Since for $t \simeq 0$, $\Delta p \sim m_i$, this naive argument indicates a potential failure of the heavy-quark symmetries at the highest recoils, since at such momenta Q_i will not be a static quark and structure in H_i with the scale m_i^{-1} will be revealed.

From our more detailed study of $Q_i \to Q_j$ transitions we know that the actual situation is more complex: this naive argument does not take into account the fact that end-point dominance always produces a limited Δp so that the real restrictions on $t_m - t$ arise from the competition between such soft processes and hard processes. For the $P_i \to X_q$ transitions with $X_q = \pi, \rho, \ldots$ we will encounter the form factor analogous to ξ_{ji} in Eq. (20), namely,

$$\xi_{X_q i}(Q_l^2) = \int_0^2 du \int d^2p_T \tilde{\phi}^*_{X_q}(u, \mathbf{p}_T + u\mathbf{Q}_l)\tilde{\phi}_i(u, \mathbf{p}_T) , \tag{31}$$

which by Eq. (18a) becomes a function $\xi_{X_q}(Q_l^2)$ independent of i for large m_i as required. Note that since $\tilde{\phi}_{X_q}$ is $\sqrt{\tfrac{1}{2}}\phi_{X_q}(\tfrac{1}{2}x, \mathbf{p}_T)$, the integral over u runs from 0 to 2 instead of 0 to $m_j/m_l \to \infty$. (The tail of $\tilde{\phi}_i$ at $u > 2$ corresponds to spectator $\bar{d}$ quarks with such a large share of the momentum of P_i that their momentum exceeds the total momentum of X_q.) At $\mathbf{Q}_l = 0$ we obtain

$$\xi_{X_q i}(t_{\max}) = \int_0^2 du \int d^2p_T \tilde{\phi}^*_{X_q}(u, \mathbf{p}_T)\tilde{\phi}_i(u, \mathbf{p}_T) . \tag{32}$$

Note that if the "constituent quark" $\bar{d}$ in P_i and X_q could be identified, and if their momentum distributions were identical, then one would have $\xi_{X_q i}(t_{\max}) = 1$. Although this identification cannot be made and although the momentum distributions will differ by not only binding effects but also "reduced mass" effects, Eq. (32) suggests that $\xi_{X_q i}(t_{\max})$ will not only be constant as $m_i \to \infty$ but also large.

Let us now turn to the question of the range in $t_m - t = (m_i/m_l)Q_l^2$ over which $\xi_{X_q i}(t)$ in Eq. (31) remains independent of i. We proceed as before by considering the case where the wave functions factorize with a Gaussian p_T dependence as in Eq. (21) to give

$$\xi_{X_q i}(Q_l^2) = \frac{\beta_i \beta_{X_q}}{\beta_{iX_q}^2} \int_0^2 du\, U^*_{X_q}(u) U_i(u) e^{-u^2 Q_l^2/4\beta_{iX_q}^2} . \tag{33}$$

As before, in the nonrelativistic approximation the U's peak at $u = 1$ and one would obtain a rapidly falling (Gaussian) form factor, but as $Q_l^2/4\beta_{iX_q}^2$ becomes large, the integral will be dominated by the $u \to 0$ end-point region. Thus, although the ansatz (24a) is not appropriate for U_{X_q} for $u > 1$, if the U's $\sim u^{p/2}$ at small u we still have

$$\xi_{X_q i} \sim \left[\frac{4\beta_{X_q}\beta_i}{Q_l^2}\right]^{(p+1)/2} \tag{34}$$

for large Q_l^2. We recognize this as being analogous to the situation in $Q_i \to Q_j$ transitions: since p is of order unity, a weak dependence of the parameters of the wave functions $\tilde{\phi}_i$ on m_i will not destroy the heavy-quark symmetry at any Q_l^2 since these parameters all approach limiting values as $m_i \to \infty$.

We next consider the effect of the high-p_T tail of $\tilde{\phi}_i$ induced by physics with the scale m_i. The discussion is similar to that in $Q_i \to Q_j$ except that now it is the tail of Q_i which is relevant. This produces a contribution to $\xi_{X_q i}$ which has strength $\alpha_s \Lambda_{\rm QCD}\beta/m_i^2$ at low Q_l^2 and which decreases significantly only for $Q_l^2 > m_i^2$. Thus in the physical region for the decay, the form factor $\xi_{X_q i}$ will be independent of i provided

$$\left[\frac{m_l^2}{Q_l^2}\right]^{(p+1)/2} > \alpha_s(Q_l^2)\left[\frac{m_l^2}{m_i^2}\right] \tag{35}$$

or, i.e.,

$$\frac{t_m - t}{m_i^2} < \left[\frac{m_i}{m_l}\right]^{(3-p)/(1+p)} \left[\frac{1}{\alpha_s}\right]^{2/(1+p)} . \tag{36}$$

(I have replaced the β's and $\Lambda_{\rm QCD}$ by m_l for simplicity.) Since in the physical region the left-hand side of Eq. (36) is less than one, this condition is easily met.

As with $Q_i \to Q_j$ decays, one can easily check that these qualitative conclusions are not affected by our assumptions regarding the form of the wave functions. End-point dominance of these form factors in the regions of interest also means that the p_T being probed is always of order $\Lambda_{\rm QCD}$ so that the condition $\not{v}u_Q = u_Q$ on the heavy-quark spinor required to derive relations[8] between some relevant $b \to s$ operators and the vector- and axial-vector-current matrix elements in $c \to s$ and $b \to u$ is satisfied. Thus we can also expect $H_i \to X_q$ relations based on the heavy-quark symmetry to be valid over the full available kinematic range.

One can also consider once again the contributions of perturbative QCD to the form factors at large recoil. In $Q \to q$ transitions, the analogue of Eq. (27) is

$$\delta f_\pm|_{p\rm QCD} \sim \beta_\pm \alpha_s(Q_l^2)\left[\frac{m_l}{m_i}\right]^{1/2} \xi(Q_l^2) , \tag{37}$$

in contrast with the predictions[1]

$$f_+ - f_- = C(m_i, m_l)\xi_-(Q_l^2)\left[\frac{m_i}{m_l}\right]^{1/2} , \tag{38a}$$

$$f_+ + f_- = C(m_i, m_l)\xi_+(Q_l^2)\left[\frac{m_l}{m_i}\right]^{1/2} , \tag{38b}$$

where the $C(m_i, m_l)$ are known and where now $\xi_\pm$ are independent unknown functions. We see that $\delta(f_+ + f_-)$ preserves the heavy-quark symmetry relations. The perturbative contribution to $\delta(f_+ - f_-)$ is down by a factor $\alpha_s m_l/m_i$ with respect to the low recoil form (38a), consistent once again with our general analysis. The perturbative tail from high-p_T components induced in X_q by hard gluon exchange *after* the current acts is clearly independent of Q_i for fixed **v** and **v**′ and so will satisfy the heavy-quark symmetry relations (e.g., the contribution to $f_+ + f_-$ above); the perturbative tail of P_i induced by hard-gluon exchange *before* the current acts will, on the other hand, depend on Q_i and therefore violate the symmetry, but it is suppressed by $\alpha_s m_l/m_i$ relative to the nonperturbatively induced effects at low Q_l^2. In analogy to $Q_i \to Q_j$ decays, if $p=1$ then $\xi_{X_q i}$ is approximately independent of i for all t; the more general condition is that for $p>1$, $\xi_{X_q i} \simeq \xi_{X_q i}$ so long as

$$\left[\frac{m_l^2}{Q_l^2}\right]^{(p+1)/2} > \alpha_s(Q_l^2)\frac{m_l}{m_i} \tag{39a}$$

or, i.e.,

$$\frac{t_m - t}{m_i^2} < \left[\frac{m_i}{m_l}\right]^{(3-p)/(p-1)} \left[\frac{1}{\alpha_s}\right]^{2/(p-1)} \tag{39b}$$

to be compared with (36). Once again, this condition is easily met in the physical region.

IV. OVERVIEW AND CONCLUSIONS

This paper has been devoted to the consideration of various models for the large recoil behavior of weak matrix elements in order to study the region of applicability of form factor relations based on the heavy-quark symmetry of Ref. 1. These investigations indicate that the relations are surprisingly resilient: they should be maintained even at recoils comparable to the heavy-quark mass scale. In the region of end-point dominance (which, according to the picture of Ref. 12, should include the full kinematic regions of interest here), it is more probable to find the "spectators" to the weak decays $Q_i \to Q_j$ and $Q_i \to q$ at low x in both the initial and final hadron with $p_T \sim \Lambda_{\rm QCD}$ than to find them near their average x with a very high p_T. As a result, the mean p_T probed in the hadronic wave functions in the end-point-dominance region is always small: the large recoil is almost all provided by the pointlike operator. This means first of all that a parametric dependence of the p_T distribution on the heavy-quark mass will not upset the heavy-quark symmetry relations. It also means that any effects of symmetry-breaking high-p_T tails in the wave functions are postponed to higher recoils than one would naively expect. To estimate the recoil at which this crossover occurs, I introduced a schematic model for a symmetry-breaking high-p_T tail induced by short-range color-magnetic forces. Since such effects are suppressed by α_s and by powers of m_l/m_Q, and since the end-point mechanism allows the low-p_T wave function to contribute to high recoil, such a high-p_T tail only breaks the heavy-quark symmetry at very large recoils. This general, but schematic, analysis strongly suggests that such symmetry breaking is not important in the cases considered. As a particular example of such a generic symmetry-breaking mechanism, I have also considered the possibility that at the highest recoils the end-point mechanism may be overtaken by the asymptotic one[13] in which it becomes favorable to create high p_T directly in the wave function at the expense of a hard-gluon exchange. In this region, contributions from high p_T in the initial and final wave functions have the same t dependence, but are suppressed by $\alpha_s m_l/m_i$ and $\alpha_s m_l/m_j$, respectively, for a $Q_i \to Q_j$ transition, transition, and by $\alpha_s m_l/m_i$ and α_s, respectively, for a $Q_i \to q$ transition. In the $Q_i \to Q_j$ transitions, the symmetry is broken, but only by terms of order $\alpha_s m_l/m_j$ with respect to the end-point contributions. Such terms are not expected to become dominant in the $\bar{B} \to D$ and $\bar{B} \to D^*$ Dalitz plots of immediate interest here. In the latter case contributions from the high-p_T tail of the light hadron continue to respect the symmetry so that symmetry-breaking terms are of order $\alpha_s m_l/m_i$, and it seems unlikely that such terms ever become important in the $P_i \to X_q$ Dalitz plot.[18] I conclude that the heavy-quark relations will be valid over the full kinematic ranges available in the particular case of $b \to c$ decays as well as in all $Q \to q$ decays including the $b \to s$, $b \to u$, $c \to s$, and $c \to d$ cases of immediate phenomenological interest.

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