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Determining the *CP*-violating and *CP*-conserving form factors in the decay modes $K_S \to \gamma\gamma$ and $K_L \to \gamma\gamma$

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In the decay modes $K_S \to \gamma\gamma \to e^- e^+ \mu^- \mu^+$ and $K_L \to \gamma\gamma \to e^- e^+ \mu^- \mu^+$, the measurements of the decay distributions with respect to the angle between the decay planes of the lepton pairs produced by double internal conversions of the two photons can be used in determining the *CP*-violating and *CP*-conserving form factors in $K_S \to \gamma\gamma$ and $K_L \to \gamma\gamma$.

I. INTRODUCTION

The purpose of this paper is to describe a method which can be used to determine the *CP*-violating and *CP*-conserving form factors in the decay of the neutral kaons K_S and K_L into two photons. The idea is to study the angular behavior of the planes of decay of the two lepton pairs (Dalitz pairs)[1] which are produced by double internal conversions of the two photons in the process $K \to \gamma\gamma \to l^- l^+ L^- L^+$, where l and L are leptons of different species.

The analysis of the variations of the angle between the planes of polarization of the two photons produced by a meson was first done by Yang.[2] Using arguments based entirely on conservation laws, he pointed out that for a scalar meson, the planes of polarization of the photons must be parallel, whereas for a pseudoscalar meson, these planes must be perpendicular. In the decay of the neutral pion into two photons where each of the photons converts into an electron pair, $\pi^0 \to \gamma_1 + \gamma_2 \to (e_1^- + e_1^+) + (e_2^- + e_2^+)$ the decay plane of $(e_1^- + e_1^+)$ contains the electric field vector of photon γ_1 whereas the decay plane of $(e_2^- + e_2^+)$ contains the electric field vector of photon γ_2. Using this concept and quantum electrodynamics, Kroll and Wada[3] derived the angular distribution functions for scalar and pseudoscalar pions. However, in their work, Kroll and Wada included the effect of only one pairing out of the two possible pairings of the four leptons of the same species: $(e_1^- + e_1^+; e_2^- + e_2^+)$ and $(e_1^- + e_2^+; e_2^- + e_1^+)$. Samios *et al.*[4] utilized the analysis of Kroll and Wada to establish experimentally that π^0 is a pseudoscalar.

Miyazaki and Takasugi[5] expanded on the theoretical analysis of Kroll and Wada by including the effects of the two possible pairings of the leptons, when the leptons are of the same species in the decays of π^0, η, and K_L. They also cited that the analysis can be used to obtain information on the form factor of the meson-$\gamma\gamma$ vertex. However, in their application of the analysis to the decay of K_L to two lepton pairs via double internal conversions, they assumed that K_L is K_2, the $CP=-1$ eigenstate of the neutral-kaon system. The included only the second of the two possible couplings of the two photons to the meson: $\Phi F_{\mu\nu} F_{\mu\nu}$, corresponding to $CP=+1$ and $\Phi \epsilon_{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$, corresponding to $CP=-1$.[6]

In this paper, both the decays of the short-lived K_S and long-lived K_L into two different lepton pairs via double internal conversions, $K \to \gamma\gamma \to e^- e^+ \mu^- \mu^+$, will be discussed. This obviates the technical problem that arises when both the possible pairings of the leptons have to be included. In considering the decays into lepton pairs of the same species, $(e^- e^+; e^- e^+)$ or $(\mu^- \mu^+; \mu^- \mu^+)$, the author has encountered formidable difficulty in the integrations involving the matrix element corresponding to the exchange pairing of the leptons. The dependence of this matrix element on ϕ, the angle between the decay planes of the lepton pairs in the center-of-mass frame, is complicated such that even the integrations of the other four independent variables are arduously difficult. The two couplings $\Phi F_{\mu\nu} F_{\mu\nu}$ and $\Phi \epsilon_{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$ of a meson to two photons will be included. The decay spectrum with respect to the angle ϕ, $d\Gamma/d\phi$, will be used to determine the relative strength of these couplings in both K_S and K_L.

The description of the derivation of the decay spectrum with respect to the angle ϕ between the planes of the lepton pairs for K_S and K_L is in the next section, followed by the conclusion.

II. ANGULAR DECAY SPECTRA FOR THE DECAY PROCESSES $K_S \to \gamma\gamma \to e^- e^+ \mu^- \mu^+$ AND $K_L \to \gamma\gamma \to e^- e^+ \mu^- \mu^+$

For the decay process $K \to \gamma\gamma \to l^- l^+ L^- L^+$, where l and L are different lepton species, one considers the Feynman graph shown in Fig. 1. Q_1, Q_2, Q_3, and Q_4 are the physical three-momenta of the leptons l^+, l^-, L^+, and L^-, respectively and m_1, m_2, m_3, and m_4 denote the corresponding masses. The momenta of the two photons are k_1 and k_2. The QED coupling ie is assumed for the $l^- l^+ \gamma$ and $L^- L^+ \gamma$ vertices. It is also assumed that the phenomenological Lagrangian[1,6]

$$L = \frac{iH}{4M_k} \Phi \epsilon_{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} + \frac{iG}{4M_k} \Phi F_{\mu\nu} F_{\mu\nu} \tag{1}$$

holds for the $K\gamma\gamma$ vertex. Here, Φ is the meson field and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, where A_μ is the photon field. H and G are dimensionless form factors that parametrize the dy-

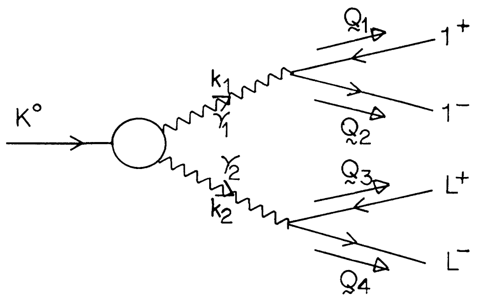

FIG. 1. Feynman diagram for the decay $K \to \gamma\gamma \to l^- l^+ L^- L^+$.

namics of the $K\gamma\gamma$ vertex. In general, they depend on the momenta of the two photons. It will be assumed that their momentum dependence can be neglected within the range of energy that is involved:

$$\begin{aligned} H(k_1^2,k_2^2) &= H(0,0) \ , \\ G(k_1^2,k_2^2) &= G(0,0) \ , \end{aligned} \tag{2}$$

for

$$\begin{aligned} 0 \le -k_1^2 &\le (M_K - 2m_e)^2 \ , \\ 0 \le -k_2^2 &\le (M_K - 2m_e)^2 \ . \end{aligned} \tag{3}$$

Two recent papers[7,8] have reported the measurement of the single Dalitz decay $K_L \to e^+e^-\gamma$ and the observation of an enhancement in the distribution of the invariant electron-positron pair mass. This enhancement has been interpreted as an evidence for a $K_L \to e^+e^-\gamma$ form factor arising from virtual-vector-meson contributions to the photon propagator.[9] Our assumption in this paper excludes these contributions; however, the predictions, based on constant form factors, about the angular distributions $\Sigma_1(\phi)$, $\Sigma_2(\phi)$, $\Delta_1(\phi)$, and $\Delta_2(\phi)$, discussed later in the paper, provide a reference with respect to which one can compare the experimental $\Sigma_1(\phi)$, $\Sigma_2(\phi)$, $\Delta_1(\phi)$, and $\Delta_2(\phi)$. The measured deviations can be used to assess models that are proposed to account for dynamics which are of non-QED origins that are embedded in the momentum dependence of the form factors.

The invariant matrix element then is

$$\mathcal{M}_{r'r,s's} = u_{r'}(Q_2) ie\gamma_\mu v_r(Q_1) O_{\alpha\beta} u_{s'}(Q_4) ie\gamma_\beta v_s(Q_3) \ , \tag{4}$$

where

$$\begin{aligned} O_{\alpha\beta} = \frac{-2i}{M_K} \frac{1}{(Q_1+Q_2)^2} \frac{1}{(Q_3+Q_4)^2} & \\ \times [H\epsilon_{\mu\alpha\rho\beta}(Q_1+Q_2)_\mu (Q_3+Q_4)\rho & \\ + G\delta_{\alpha\beta}(Q_1+Q_2)(Q_3+Q_4)] \ . & \end{aligned} \tag{5}$$

Since this is a four-body decay, five independent variables[10] are needed to parametrize $|\mathcal{M}|^2$. The variables chosen are

$$x_2 = -(Q_1+Q_2)^2, \quad x_3 = -(Q_3+Q_4)^2 \ , \tag{6}$$

$$y_2 = -(Q_1+Q_2+Q_4)^2, \quad y_3 = -(Q_1+Q_3+Q_4)^2 \ , \tag{7}$$

$$w_{23} = -(Q_2+Q_3)^2 \ . \tag{8}$$

In the rest frame of the kaon, it can be shown[10] that w_{23} is linearly related to ϕ, the angle between the plane determined by Q_1 and Q_2 and the plane determined by Q_3 and Q_4:

$$w_{23} = -J_1 \cos\phi + J_2 \ , \tag{9}$$

where

$$J_1 = \frac{8(D_{12}D_{34})^{1/2}}{\lambda(s,x_2,x_3)} \ , \tag{10}$$

$$J_2 = \frac{1}{2s\lambda(s,x_2,x_3)} [\eta(s;z_2,y_3;x_2,x_3;2z_1)\eta(s;z_3,y_2;x_3,x_2;2z_4) - \lambda(s,x_2,x_3)\eta(s;z_2,y_3;z_3,y_2;4(z_2+z_3))] \ , \tag{11}$$

$$D_{12} = \frac{1}{16s}[\lambda(s,x_2,x_3)\lambda(s,z_3,y_2) - \eta^2(s;z_3,y_2;x_3,x_2;2z_4)] \ , \tag{12}$$

$$D_{34} = \frac{1}{16s}[\lambda(s,x_3,x_2)\lambda(s,z_2,y_3) - \eta^2(s;z_2,y_3;x_2,x_3;2z_1)] \ , \tag{13}$$

$$z_1 = m_1^2, \quad z_2 = m_2^2, \quad z_3 = m_3^2, \quad z_4 = m_4^2 \ . \tag{14}$$

λ and η are the generic functions

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \ , \tag{15}$$

$$\eta(x;y,z;u,v;w) = [-x^2 + x(y+z+u+v-w) - (y-z)(u-v)] \ . \tag{16}$$

What is important to note is that J_1 and J_2 are functions of x_2,x_3,y_2,y_3 and, therefore, one can choose ϕ as the fifth independent variable,[10] instead of w_{23}, in writing down the expressions for $|\mathcal{M}|^2$. The result of carrying out the long algebra and summing over all the spin states in evaluating $|\mathcal{M}|^2$ in the center of mass of the kaon is denoted by $\langle |\mathcal{M}|^2 \rangle$:[11]

$$\langle |\mathcal{M}|^2\rangle = \frac{64e^4}{s}\Bigg\{ |H|^2 \left[\frac{1}{(x_2x_3)^2} \left[\frac{16(D_{12})(D_{34})}{\lambda(s,x_2,x_3)}\sin^2\phi - (D_{12})x_2 - (D_{34})x_3 + \tfrac{1}{8}\lambda(s,x_2,x_3)x_2x_3 \right]\right]$$

$$+|G|^2\left[\left[\frac{s-x_2-x_3}{2x_2x_3}\right]^2 [J_1^2\cos^2\phi - (2J_1J_2+J_1A)\cos\phi + J_2^2 + J_2A + B + C$$

$$+\tfrac{1}{2}(z_3+z_4)x_2 + \tfrac{1}{2}(z_2+z_1)x_3]\right]$$

$$+\mathrm{Im}(HG^*)\sin\phi \left[\frac{4(D_{12})(D_{34})}{\lambda(s,x_2,x_3)}\right]^{1/2}\left[\frac{s-x_2-x_3}{(x_2x_3)^2}(2J_1\cos\phi - 2J_2 - A)\right]\Bigg\}, \tag{17}$$

where

$$A=(x_2+x_3+2y_2+2y_3-3s-2z_1-2z_4-4z_2-4z_3)/2\ , \tag{18}$$

$$B=[(x_3+y_2)(x_2+y_3)-(x_2+y_3)(s+z_2+z_3+z_4)-(x_3+y_2)(s+z_3+z_2+z_1)-(y_2+y_3)(z_2+z_3)]/2\ , \tag{19}$$

$$C=[(s+z_2+z_3+z_4)(s+z_3+z_2+z_1)+(z_2+z_3)(s+z_1+z_2+z_3+z_4)]/2\ . \tag{20}$$

In the following discussion, Γ_S, Γ_L, and Γ refer to the rates of the decay modes $K_S\to\gamma\gamma\to e^-e^+\mu^-\mu^+$, $K_L\to\gamma\gamma\to e^-e^+\mu^-\mu^+$, and $K\to\gamma\gamma\to e^-e^+\mu^-\mu^+$, where K is either K_S or K_L. Similarly, $\Gamma(K_S\to\gamma\gamma)$, $\Gamma(K_L\to\gamma\gamma)$, and $\Gamma(K\to\gamma\gamma)$ refer to the decay rates into two photons of K_S, K_L, and K, where K is either K_S or K_L.

The differential decay width for a particle of momentum P, energy ω, and mass M is[11]

$$d\Gamma = \frac{(2\pi)^4}{2\omega}\delta^4\left[P-\sum_{i=1}^{4}Q_i\right]\langle|\mathcal{M}|^2\rangle\prod_{j=1}^{4}\frac{d^3Q_j}{(2\pi)^3 2\omega_j}\ . \tag{21}$$

Integrating out the Dirac δ function in the rest frame of the kaon and expressing the result in the chosen variables, one gets

$$d\Gamma = \frac{1}{128(2\pi)^6 s\sqrt{s}\sqrt{\lambda(s,x_2,x_3)}}\langle|\mathcal{M}|^2\rangle dx_2dx_3dy_2dy_3d\phi\ . \tag{22}$$

Since the objective is to obtain the angular spectrum $d\Gamma/d\phi$, the variables y_3,y_2,x_3,x_2 are integrated out, in that order, subject to the following limits of integrations:[10]

$$y_2^{(\pm)} = \{\eta(x_3;z_3,z_4;s,x_2;0)\pm[\lambda(x_3,z_3,z_4)\lambda(x_3,s,x_2)]^{1/2}\}/(2x_3)\ , \tag{23}$$

$$y_3^{(\pm)} = \{\eta(x_2,z_2,z_1;s,x_3,0)\pm[\lambda(x_2,z_2,z_1)\lambda(x_2,s,x_3)]^{1/2}\}/(2x_2)\ , \tag{24}$$

$$x_2^{(-)}=(m_1+m_2)^2,\quad x_2^{(+)}=(\sqrt{s}-m_3-m_4)^2\ , \tag{25}$$

$$x_3^{(-)}=(m_3+m_4)^2,\quad x_3^{(+)}=(\sqrt{s}-x_2)^2\ . \tag{26}$$

The author has used the symbolic program[12] MACSYMA in carrying out algebraic simplifications and some integrations with respect to y_2 and y_3. The integrations with respect to x_3 and x_2 were performed numerically using the FORTRAN package IMSL.[13] The result is

$$\frac{d\Gamma}{d\phi} = \frac{4\alpha^2}{2(M_K)^5(2\pi)^4}[|H|^2\sigma_1\sin^2\phi + |G|^2\sigma_2\cos^2\phi + \mathrm{Im}(HG^*)\sigma_3\sin\phi\cos\phi + |H|^2\sigma_4 + |G|^2\sigma_5]\ , \tag{27}$$

where $\alpha=e^2/(4\pi)$ is the fine-structure constant and M_K is the mass of the neutral kaon. The values of the coefficients σ_1, σ_2, σ_3, σ_4, and σ_5 are listed in Table I.

In terms of the Lagrangian in Eq. (1), the rate of decay of K to $\gamma\gamma$ is

$$\Gamma(K\to\gamma\gamma) = \frac{1}{16\pi}M_K(|H|^2+2|G|^2)\ . \tag{28}$$

One can express the ratio of Eqs. (27) and (28) in terms of the absolute values of H and G and their relative phase difference. Let

$$H=h\exp(i\psi_h),\quad G=g\exp(i\psi_g),\quad \delta=(\psi_g-\psi_h)\ , \tag{29}$$

so that

$$[\Gamma(K-\gamma\gamma)]^{-1}\frac{d\Gamma}{d\phi}=F\sigma_2[(h/g)^2 s_1\sin^2\phi+s_2\cos^2\phi-(h/g)\sin\delta s_3\sin\phi\cos\phi+(h/g)^2 s_4+s_5]/[(h/g)^2+2]\ , \qquad (30)$$

or

$$[\Gamma(K-\gamma\gamma)]^{-1}\frac{d\Gamma}{d\phi}=F\sigma_1[l_1\sin^2\phi+(g/h)^2 l_2\cos^2\phi-(g/h)\sin\sigma l_3\sin\phi\cos\phi+l_4+(g/h)^2 l_5]/[1+2(g/h)^2]\ , \qquad (31)$$

where

$$F=4\left[\frac{4\alpha^2}{M_K^6}\right]\frac{1}{(2\pi)^3}=2.26\times10^{-22}/(\mathrm{MeV})^6\ ,$$
$$s_i=\sigma_i/\sigma_2,\quad l_i=\sigma_i/\sigma_1\quad \text{for } i=1,5\ . \qquad (32)$$

The values of σ_i, s_i, and l_i are in Table I.

So far, our analysis applies both to K_S and K_L; that is, both Eqs. (30) and (31) apply to either of $K_S\to\gamma\gamma\to e^-e^+\mu^-\mu^+$ or $K_L\to\gamma\gamma\to e^-e^+\mu^-\mu^+$. In the following, we distinguish the two decay modes by using Eq. (30) for K_S and Eq. (31) for K_L; furthermore, we will use a subscript 1 for K_S's parameters and a subscript 2 for K_L's parameters. To show clearly how to isolate the ratio (g/h) or (h/g) and the relative phase δ of G with respect to H, the following expressions are defined:

$$\Sigma_1(\phi)=[F\sigma_2\Gamma(K_S\to\gamma\gamma)]^{-1}\left[\frac{d\Gamma_s(\phi)}{d\phi}+\frac{d\Gamma_s(-\phi)}{d\phi}\right]$$
$$=2[(h_1/g_1)^2 s_1\sin^2\phi+s_2\cos^2\phi+(h_1/g_1)^2 s_4+s_5][(h_1/g_1)^2+2]^{-1}\ , \qquad (33)$$

$$\Delta_1(\phi)=[F\sigma_2\Gamma(K_S\to\gamma\gamma)]^{-1}\left[\frac{d\Gamma_s(\phi)}{d\phi}-\frac{d\Gamma_s(-\phi)}{d\phi}\right]$$
$$=-2[(h_1/g_1)\sin\delta_1 s_3\cos\phi\sin\phi][(h_1/g_1)^2+2]^{-1}\ , \qquad (34)$$

$$\Sigma_2(\phi)=[F\sigma_1\Gamma(K_L\to\gamma\gamma)]^{-1}\left[\frac{d\Gamma_L(\phi)}{d\phi}+\frac{d\Gamma_L(-\phi)}{d\phi}\right]$$
$$=[2l_1\sin^2\phi+(g_2/h_2)^2 l_2\cos^2\phi+l_4+(g_2/h_2)^2 l_5][1+2(g_2/h_2)^2]^{-1}\ , \qquad (35)$$

$$\Delta_2(\phi)=[F\sigma_1\Gamma(K_L\to\gamma\gamma)]^{-1}\left[\frac{d\Gamma_L(\phi)}{d\phi}-\frac{d\Gamma_L(-\phi)}{d\phi}\right]$$
$$=-2[(g_2/h_2)\sin\delta_2 l_3\cos\phi\sin\phi][1+2(g_2/h_2)^2]^{-1}\ . \qquad (36)$$

Figures 2 and 3 show the graphs of $\Delta_1(\phi)$ and $\Delta_2(\phi)$ for various values of δ_1 and δ_2 but for fixed values of $h_1/g_1=0.10$ and $g_2/h_2=0.10$, respectively. One can see that for positive δ_1 and δ_2, $\Delta_1(\phi)$ and $\Delta_2(\phi)$ start out as negative, whereas for negative δ_1 and δ_2, they start out as positive. Furthermore, as δ_1 and δ_2 increase from 0 to $\pi/2$, the amplitudes of $\Delta_1(\phi)$ and $\Delta_2(\phi)$ also increase.

Figures 4 and 5 are graphs of $\Sigma_1(\phi)$ for $h_1/g_1=0$, 0.01, and 0.10. It can be seen that the difference between the amplitudes of $\Sigma_1(\phi)$ for nonzero h_1/g_1 and zero h_1/g_1 increases as the value of h_1/g_1 increases. The graph for nonzero h_1/g_1 recedes downward from that of zero h_1/g_1 as h_1/g_1 increases. Figures 6 and 7 are graphs of $\Sigma_2(\phi)$ for $g_2/h_2=0$, 0.01, and 0.10. Although the difference between the amplitudes of $\Sigma_2(\phi)$ for nonzero g_2/h_2 and zero g_2/h_2 also increases as the ratio g_2/h_2 increases, the graph for nonzero g_2/h_2 recedes upward from that of zero g_2/h_2 as g_2/h_2 increases.

TABLE I. The values of the coefficients σ_i, s_i, and l_i in Eqs. (27), (30), (31), and (33)–(36).

i	σ_i (10^{14} MeV^6)	$s_i=\sigma_i/\sigma_2$	$l_i=\sigma_i/\sigma_1$
1	2.694	0.953	1.000
2	2.826	1.000	1.049
3	5.504	1.948	2.043
4	18.807	6.656	6.982
5	9.067	3.209×10^4	3.366×10^4

By experimentally observing and measuring the quantities $\Sigma_1(\phi)$ and $\Delta_1(\phi)$ for K_S and $\Sigma_2(\phi)$ and $\Delta_2(\phi)$ for K_L and then comparing them with Eqs. (33)–(36), the values of h_1/g_1, δ_1, g_2/h_2, and δ_2 can be extrapolated. Furthermore, by using the known rates for $K_S\to\gamma\gamma$ and $K_L\to\gamma\gamma$, one can also obtain h_1, g_1, h_2, and g_2 as we explain in the following paragraphs.

Suppose that the values of $\Sigma_1(\phi)$ and $\Sigma_2(\phi)$ at $\phi=0$ and $\phi=\pi/2$ and the values of $\Delta_1(\phi)$ and $\Delta_2(\phi)$ at $\phi=\pi/4$ have been obtained experimentally. Then, from Eqs. (33) and (34),

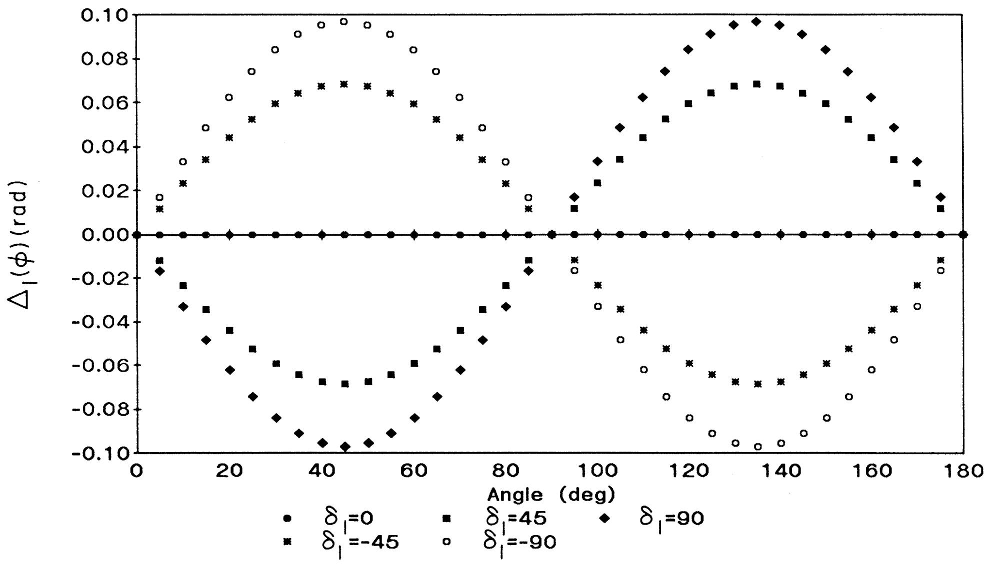

FIG. 2. Angular variations of $\Delta_1(\phi)$ for $h_1/g_1=0.10$ and $\delta_1=0, \pm 45°, \pm 90°$.

$$h_1/g_1=\left[\frac{2[\Sigma_1(0)-s_2-s_5]}{[2s_4-\Sigma_1(0)]}\right]^{1/2}, \tag{37}$$

$$h_1/g_1=\left[\frac{2[\Sigma_1(\pi/2)-s_5]}{[2(s_1+s_4)-\Sigma_1(\pi/2)]}\right]^{1/2}, \tag{38}$$

$$\sin\delta_1=\frac{-[(h_1/g_1)^2+2]\Delta_1(\pi/4)}{s_3(h_1/g_1)}, \tag{39}$$

and putting the values of (h_1/g_1) in Eq. (28), we further get

$$g_1=\left[\frac{16\pi\Gamma(K_S\to\gamma\gamma)}{M_K[2+(h_1/g_1)^2]}\right]^{1/2}, \tag{40}$$

$$h_1=\left[\frac{h_1}{g_1}\right]g_1 . \tag{41}$$

Similarly, from Eqs. (35), (36), and (28), one can derive

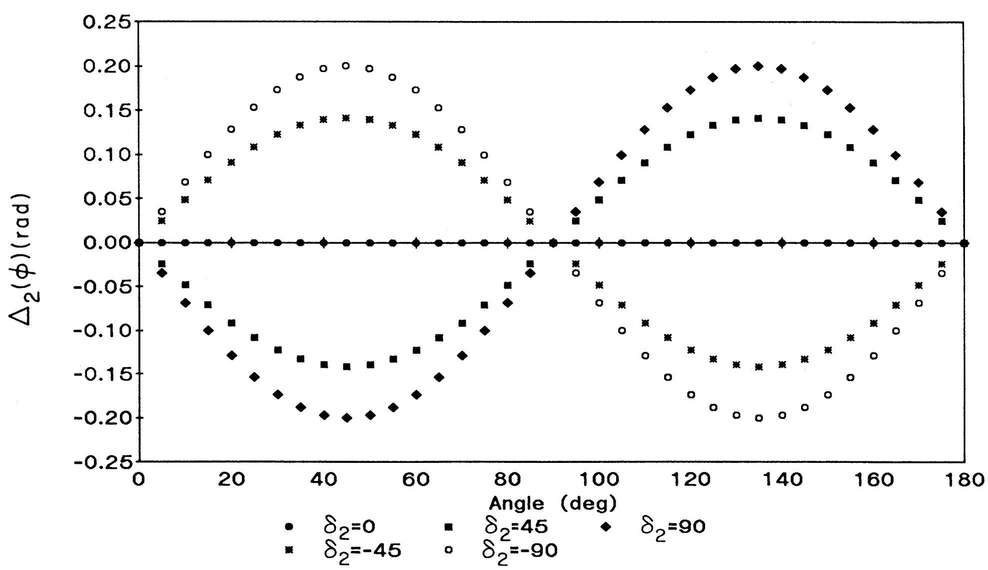

FIG. 3. Angular variations of $\Delta_2(\phi)$ for $g_2/h_2=0.10$ and $\delta_2=0, \pm 45°, \pm 90°$.

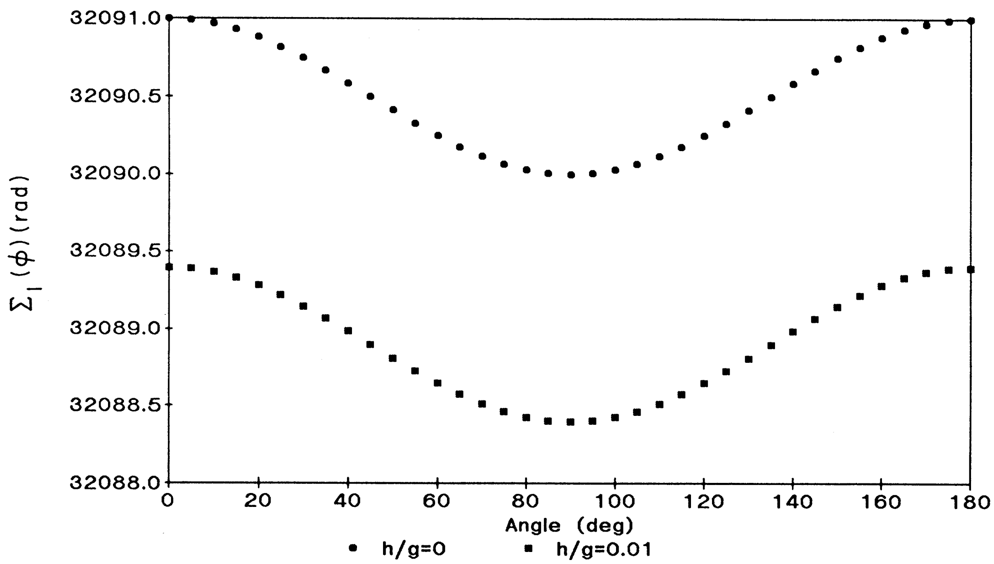

FIG. 4. Angular variations of $\Sigma_1(\phi)$ for $h_1/g_1=0$ and $h_1/g_1=0.01$.

$$g_2/h_2=\left[\frac{2l_4-\Sigma_2(0)}{2[\Sigma_2(0)-l_2-l_5]}\right]^{1/2}, \tag{42}$$

$$g_2/h_2=\left[\frac{2(l_1+l_4)-\Sigma_2(\pi/2)}{2[\Sigma_2(\pi/2)-l_5]}\right]^{1/2}, \tag{43}$$

$$\sin\delta_2=\frac{-[1+2(g_2/h_2)^2]\Delta_2(\pi/4)}{l_3(g_2/h_2)}, \tag{44}$$

$$h_2=\left[\frac{16\pi\Gamma(K_L\to\gamma\gamma)}{M_K[1+2(g_2/h_2)^2]}\right]^{1/2}, \tag{45}$$

$$g_2=\left[\frac{g_2}{h_2}\right]h_2\,. \tag{46}$$

The precision of determining these form factors banks heavily on fine-tuning the measurements of both $\Gamma(K_S\to\gamma\gamma)$ and $\Gamma(K_L\to\gamma\gamma)$. The values of these rates based on the known branching ratios[14,15]

$$\begin{aligned} &B(K_S\to\gamma\gamma)=(2.4\pm1.2)\times10^{-6}\,,\\ &B(K_L\to\gamma\gamma)=(5.70\pm0.23)\times10^{-4}\,, \end{aligned} \tag{47}$$

and the known lifetimes[14] $\tau_S=0.8922\times10^{-10}$ sec and $\tau_L=5.18\times10^{-8}$ sec are

$$\begin{aligned} &\Gamma(K_S\to\gamma\gamma)=(17.7\pm8.8)\times10^{-18}\ \text{MeV}\,,\\ &\Gamma(K_L\to\gamma\gamma)=(7.24\pm0.29)\times10^{-18}\ \text{MeV}\,. \end{aligned} \tag{48}$$

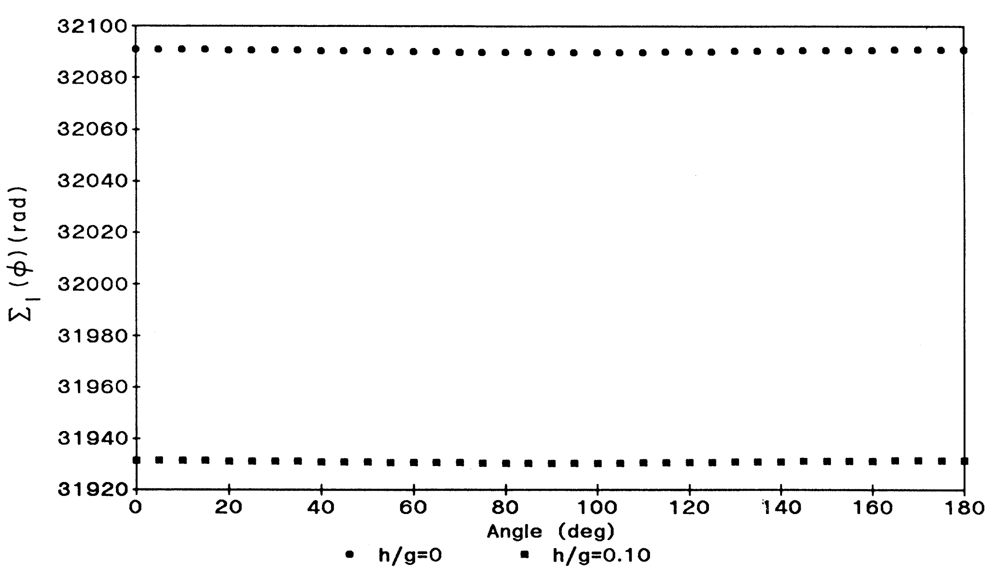

FIG. 5. Angular variations of $\Sigma_1(\phi)$ for $h_1/g_1=0$ and $h_1/g_1=0.10$.

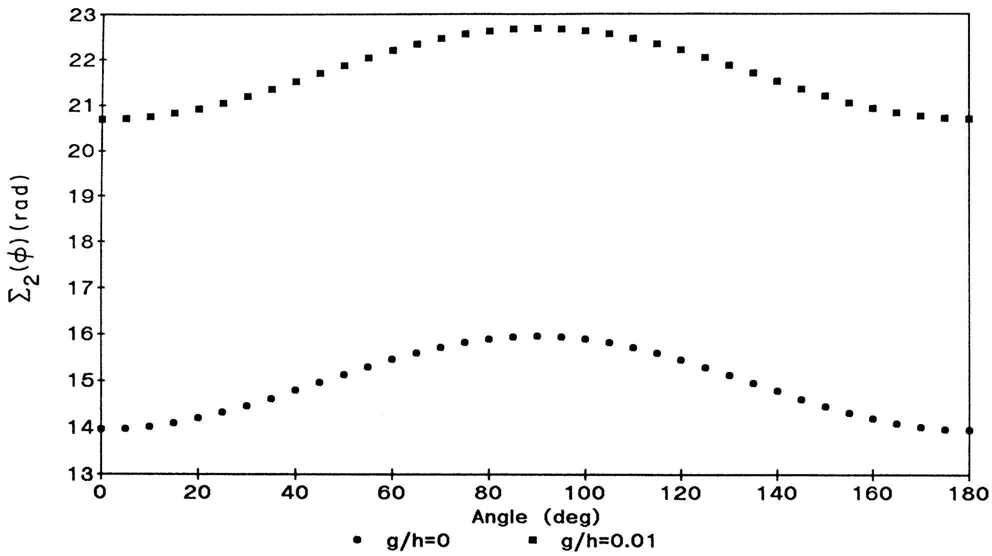

FIG. 6. Angular variations of $\Sigma_2(\phi)$ for $g_2/h_2=0$ and $g_2/h_2=0.01$.

It is also interesting to pursue the consequences of Eqs. (30) and (31) in evaluating the branching ratios of the decays of K_S and K_L into $e^-e^+\mu^-\mu^+$ via double internal conversions in the extreme cases of $h=0$ and $g=0$. Setting $h=0$ in Eq. (30) and $g=0$ in Eq. (31), and integrating over ϕ from 0 to 2π, one gets[16]

$h=0$:

$$\frac{\Gamma(K\to\gamma\gamma\to e^-e^+\mu^-\mu^+)}{\Gamma(K\to\gamma\gamma)}=F\sigma_2\int_0^{2\pi}d\phi(s_2\cos^2\phi+s_5)/2$$
$$=0.644\times10^{-2} \tag{49}$$

and

$g=0$:

$$\frac{\Gamma(K\to\gamma\gamma\to e^-e^+\mu^-\mu^+)}{\Gamma(K\to\gamma\gamma)}=F\sigma_1\int_0^{2\pi}d\phi(l_1\sin^2\phi+l_4)$$
$$=2.862\times10^{-6}\ . \tag{50}$$

Since

$$B(K\to\gamma\gamma\to e^-e^+\mu^-\mu^+)=\frac{\Gamma(K\to\gamma\gamma\to e^-e^+\mu^-\mu^+)}{\Gamma(K\to\gamma\gamma)}B(K\to\gamma\gamma)\ , \tag{51}$$

one can use the branching ratios[14] in Eq. (47) to obtain the following. For $h=0$,

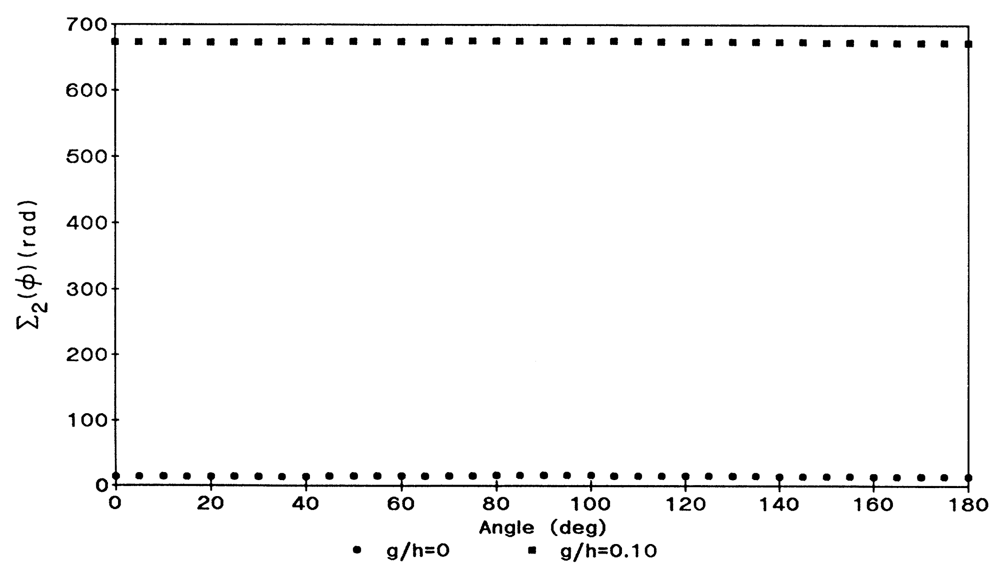

FIG. 7. Angular variations of $\Sigma_2(\phi)$ for $g_2/h_2=0$ and $g_2/h_2=0.10$.

$$B(K_S\to\gamma\gamma\to e^-e^+\mu^-\mu^+)=(1.54\pm0.77)\times10^{-8}\ , \quad (52)$$

$$B(K_L\to\gamma\gamma\to e^-e^+\mu^-\mu^+)=(3.67\pm0.15)\times10^{-6}\ . \quad (53)$$

For $g=0$,

$$B(K_S\to\gamma\gamma\to e^-e^+\mu^-\mu^+)=(6.87\pm3.43)\times10^{-12}\ , \quad (54)$$

$$B(K_L\to\gamma\gamma\to e^-e^+\mu^-\mu^+)=(1.63\pm0.07)\times10^{-9}\ , \quad (55)$$

The branching ratio for K_S ranges from 3.44×10^{-12} (pure $CP=-1$, 100% CP-violating mode) to 2.31×10^{-8} (pure $CP=+1$, 100% CP-conserving mode). There is no known experimental upper bound for the decay mode $K_S\to e^-e^+\mu^-\mu^+$.

For K_L, the branching ratio ranges from 1.56×10^{-9} (pure $CP=-1$, 100% CP-conserving mode) to 3.82×10^{-6} (pure $CP=+1$, 100% CP-violating mode). Balats *et al.*[17] has set an upper limit of 4.9×10^{-6} (at 90% confidence level) to the decay mode $K_L\to e^-e^+\mu^-\mu^+$.

It is of interest to consider how much is the contribution of the 2π intermediate state via the decay $K-\pi^0\pi^0-\mu^+e^-\mu^+e^-$. The branching ratios of $K\to\pi^0\pi^0$ are higher than that of $K\to\gamma\gamma$ for both K_S and K_L: $B(K_S\to\pi^0\pi^0)=31.39\times10^{-2}$ and $B(K_L\to\pi^0\pi^0)=0.0909\times10^{-2}$. However, the process $\pi^0\to\mu^+e^-$, which violates the conservation of lepton number, is extremely small. Reference 14 gives an upper bound of $B(\pi^0\to\mu^+e^-)<7\times10^{-8}$. We, therefore, expect the contribution of the 2π intermediate state to be much smaller compared to that of the 2γ intermediate state.

III. CONCLUSION

We have discussed how the measurements of the angular decay distributions $d\Gamma_S/d\phi$ and $d\Gamma_L/d\phi$ for the decay modes $K_S\to\gamma\gamma\to e^-e^+\mu^-\mu^+$ and $K_L\to\gamma\gamma\to e^-e^+\mu^-\mu^+$, respectively, can be utilized to determine the ratios of the CP-violating and CP-conserving form factors, the absolute values of those form factors, and their relative phases. The significance of obtaining these form factors is that their values are indicators of the presence or absence of CP violations in the decays $K_S\to\gamma\gamma$ and $K_L\to\gamma\gamma$ and are, therefore, constraining factors in the construction of dynamical models[18–21] for these decay modes. Furthermore, they can serve as a guide in a more reliable theoretical treatment of the contribution of the 2γ intermediate state in the decay $K_L\to\mu^-\mu^+$.[22]

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