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Meson-baryon reactions in the three-flavor Skyrme model

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The consequences of the three-flavor Skyrmion model for the reaction $M+B \rightarrow M'+B'$, where M and M' are 0^- octet mesons, B an octet baryon, and B' an octet or decuplet baryon, have been studied by Karliner and Mattis at the level of partial-wave amplitudes. Using their results, I show that the partial-wave sums for certain combinations of spin amplitudes corresponding to definite spin in the t channel may be carried out explicitly. It is found that the 30 independent SU(3) amplitudes (14 for the octet and 16 for the decuplet) may be written as linear combinations of only 10 reduced amplitudes. The relations found acquire a particularly simple form when expressed in terms of t-channel SU(3) amplitudes. A subset of charged-meson elastic-scattering amplitudes may be used to determine seven of the reduced amplitudes; the remaining three correspond to an exotic t channel in decuplet production and are probably small. The resulting scheme is highly predictive.

I. INTRODUCTION

An extremely thorough comparison of the predictions of the three-flavor Skyrme model for the scattering of pseudoscalar octet mesons on octet baryons with the available data has been carried out by Karliner and Mattis.[1] They used data in the form of partial-wave analyses of thirteen different reactions involving pion and kaon beams on nucleon targets, yielding an octet pseudoscalar meson and either a spin-$\frac{1}{2}$ octet or a spin-$\frac{3}{2}$ decuplet baryon in the final state. Their assessment was that their model provides a good description of πN reactions, a poor description of KN reactions, and mixed results for $\bar{K}N$ reactions. The model assumes SU(3) symmetry, which is certainly not exact, and it expresses all partial-wave amplitudes in terms of a set of "reduced partial-wave amplitudes"

$$\tau_{KL'L}^{\{IY\}} .$$

Here L' and L denote the final and initial orbital angular momenta, the pair $\{I,Y\}$ takes on the values of the standard octet, $\{1,0\}$, $\{0,0\}$, $\{1/2,1\}$, and $\{1/2,-1\}$, respectively, and the index K is such that the triangle inequalities involving both (KIL) and (KIL') are satisfied. If the final baryon has spin $\frac{1}{2}$, then $L'=L$, and there are eight different reduced amplitudes, while, if it has spin $\frac{3}{2}$, there exist two additional amplitudes with $L'=L\pm 2$, $K=L\pm 1$, and $\{IY\}=\{1,0\}$. These reduced amplitudes are functions of energy, and may be computed numerically by solving a potential scattering problem. The overall relative success of the model was considered encouraging by the authors, despite a few areas of disagreement.

In subsequent work on the two-flavor Skyrme model, it was observed by the present author that the predictions of the model appeared simpler when expressed in terms of t-channel rather than s-channel isospin amplitudes.[2] In addition, the particular form of the Skyrme expression for the partial-wave amplitudes was such that the sum over J, the total angular momentum, could be carried out explicitly, yielding the various spin amplitudes as linear combinations of a smaller number of reduced amplitudes.[3] Even if one assumes the reduced amplitudes are unknown (which is my own viewpoint), the model is highly predictive, and rather successful. Mattis and Mukerjee[4] discovered the selection rule $I_t=J_t$, which requires that a combination of partial-wave amplitudes corresponding to a given t-channel isospin I_t, and what they termed the t-channel spin J_t, vanishes unless the two are equal. I then showed[5] that their quantity J_t may be interpreted in terms of the maximal spin-flip at the baryon vertex. An additional prediction of the two-flavor model is that the reactions $\pi N \rightarrow \pi N$ and $\pi N \rightarrow \pi\Delta$ are described by the same reduced amplitude for $I_t=J_t=1$, the corresponding cross sections being in the ratio of 1:2. For these reactions there exists one reduced amplitude with $I_t=J_t=0$ (πN only), another with $I_t=J_t=1$ (both πN and $\pi\Delta$), and three with $I_t=J_t=2$ ($\pi\Delta$ only), for a total of five. This is to be compared with two spin amplitudes for each isospin in πN, and four for each isospin in $\pi\Delta$, for a total of twelve. Although these predictions are not perfectly satisfied, they do provide a reasonable description of πN reactions.

A natural question is whether anything similar happens in the three-flavor model of Karliner and Mattis. This paper provides the affirmative reply, which is obtained by computing fixed J_t amplitudes using the tables presented in Ref. 1. There are four reduced SU(3) amplitudes with $J_t=0$ (final baryon in octet), three with $J_t=1$ (final baryon in the octet or decuplet), and three with $J_t=2$ (final baryon in the decuplet). This is to be contrasted with seven allowed SU(3) amplitudes for each spin possibility for the octet (14), and four allowed SU(3) amplitudes for each spin possibility for the decuplet (16). Thus a total of thirty amplitudes may be expressed in terms of ten reduced amplitudes.

Although this may be seen to occur simply by forming the s-channel SU(3) amplitudes of definite J_t, considerable conceptual simplification occurs upon crossing to t-

channel SU(3) amplitudes. The constraints of the model are that certain t-channel amplitudes are zero, and for those with $J_t=1$, the spin-$\frac{1}{2}$ amplitudes are proportional to those of spin $\frac{3}{2}$. There exists thus a strong parallel between the two- and three-flavor results. However, some predictive power of the two-flavor model is lost. For example, the $I_t=J_t$ selection rule of the two-flavor model is predictive for πN elastic scattering; the $I_t=0$ amplitude is non-spin-flip, while the $I_t=J_t=1$ amplitude is spin-flip. In the three-flavor model this property is lost; the four amplitudes are unconstrained. Nevertheless, the four πN amplitudes determine two of the four $J_t=0$ SU(3) amplitudes, and two of the three $J_t=1$ SU(3) amplitudes. This leaves only two $J_t=0$ SU(3) amplitudes and one $J_t=1$ SU(3) amplitude to be determined, if one limits the discussion to final spin-$\frac{1}{2}$ baryons. Once these remaining amplitudes are determined, using some KN scattering amplitudes, for example, the three-flavor model becomes highly predictive.

The paper is organized as follows. In Sec. II the amplitudes of definite J_t are introduced and the results of Karliner and Mattis are used to establish the ten reduced s-channel amplitudes. The SU(3) crossing matrices of Rebbi and Slansky[6] are employed to express the results in terms of t-channel SU(3) amplitudes in Sec. III. In Sec. IV the four non-spin-flip and the three spin-flip reduced t-channel amplitudes are related to the elastic amplitudes for charged-meson scattering, and the predicted constraints among the latter are displayed. The conclusions are presented in Sec. V. All of the relevant results have been collected in numerous tables, since my aim is to offer easy access to the reader who is more interested in the results than in their obtention.

II. AMPLITUDES OF DEFINITE J_t

To a large extent I shall employ the notation of Karliner and Mattis,[1] although some departures will be necessary. The standard formulas relating the partial-wave amplitudes to the spin-projection amplitudes are taken from Goldberger and Watson,[7] except that I use 3-j rather than Clebsch-Gordan coefficients. The partial-wave amplitudes

$$T^{J_s}_{L's'Ls}$$

are indexed by the total angular momentum J_s, L and L', the initial and final angular momenta, respectively, and by s and s', the initial and final baryon spins (here $s=\frac{1}{2}$). Parity conservation in this case requires that $L'-L$ be even. The spin-projection amplitudes are denoted $T_{v'v}$, where v' (v) is the z component of the spin of the final (initial) baryon (the choice of the axes in the c.m. is arbitrary but the axes in the baryon rest frames are related to the c.m. axes by pure Lorentz transformations or "boosts"). They are functions of the c.m. energy and production angles, although this dependence will not be explicitly displayed. The $T_{v'v}$ may be written in terms of the partial-wave amplitudes as

$$T_{v'v}=\sum_{\substack{LL'MM'\\ M_sJ_s}}(2J_s+1)(-1)^{L-L'+s'-s}\begin{pmatrix}L' & s' & J_s\\ M' & v' & -M_s\end{pmatrix}\begin{pmatrix}L & s & J_s\\ M & v & -M_s\end{pmatrix}T^{J_s}_{L's'Ls}\,Y^{M'}_{L'}(\theta'\phi')Y^{M}_{L}(\theta\phi)^{*}\ , \tag{1}$$

where the standard 3-j symbols and spherical harmonics Y^M_L are used. The c.m. directions of the incident and outgoing mesons are given by the spherical polar angles (θ,ϕ) and (θ',ϕ'), respectively. I introduce the following linear combinations of these amplitudes:

$$T^{J_t}_{M_t}=\sum_{vv'}(-1)^{s'+M_t-v}\begin{pmatrix}s' & s & J_t\\ v' & -v & M_t\end{pmatrix}T_{v'v}\ , \tag{2}$$

where J_t is coupled to the spins of the initial and final baryons via a 3-j symbol. It corresponds, in this case, to

TABLE I. The amplitudes $\hat{T}^{J_t}_{L's'L(1/2)}$ of Eq. (4) in terms of the partial-wave amplitudes $T^{J_s}_{L's'L(1/2)}$ for $L=L'$.

S'	J_t	$(-1)^L\sqrt{(2J_t+1)(2L+1)}\hat{T}^{J_t}_{L's'L(1/2)}$
$\frac{1}{2}$	0	$\sqrt{2}[(L+1)T^{L+1/2}_{L(1/2)L(1/2)}+LT^{L-1/2}_{L(1/2),L(1/2)}]$
$\frac{1}{2}$	1	$\sqrt{2L(L+1)}(T^{L+1/2}_{L(1/2)L(1/2)}-T^{L-1/2}_{L(1/2)L(1/2)})$
$\frac{3}{2}$	1	$\sqrt{(2L+3)(L+1)}T^{L+1/2}_{L(3/2)L(1/2)}+\sqrt{L(2L-1)}T^{L-1/2}_{L(3/2)L(1/2)}$
$\frac{3}{2}$	2	$\sqrt{(2L-1)(L+1)}T^{L+1/2}_{L(3/2)L(1/2)}-\sqrt{L(2L+3)}T^{L-1/2}_{L(3/2)L(1/2)}$

TABLE II. The amplitudes $\hat{T}^{2}_{L'(3/2)L(1/2)}$ of Eq. (4) in terms of the partial-wave amplitudes $T^{J_s}_{L'(3/2)L(1/2)}$ for $L=L'\pm 2$.

$L=L'+2$	$\sqrt{5}\hat{T}^{2}_{L'(3/2)L(1/2)}=(-1)^{L+1}(2L'+4)^{1/2}T^{L'+3/2}_{L'(3/2)L+2,1/2}$
$L=L'+2$	$\sqrt{5}\hat{T}^{2}_{L'(3/2)L(1/2)}=(-1)^{L}(2L'-2)^{1/2}T^{L'-3/2}_{L'(3/2)L-2,1/2}$

the variable introduced by Mattis and Mukerjee in Ref. 4. By using a standard identity relating 3-j and 6-j symbols,[8] one may write these amplitudes in the form

$$T^{J_t}_{M_t}=\sum_{\substack{LL'\\MM'}}(-1)^{M}\begin{bmatrix}L' & L & J_t\\ M' & -M & -M_t\end{bmatrix}\times\hat{T}^{J_t}_{L's'Ls}Y^{M'}_{L'}(\theta'\phi')Y^{M}_{L}(\theta\phi)^{*}\ , \tag{3}$$

where I have introduced the quantities $\hat{T}^{J_t}_{L's'Ls}$, which are defined by

$$\hat{T}^{J_t}_{L's'Ls}=\sum_{J_s}(2J_s+1)(-1)^{s'+J_s}\begin{Bmatrix}L' & s' & J_s\\ s & L & J_t\end{Bmatrix}T^{J_s}_{L's'Ls}\ , \tag{4}$$

where the standard 6-j symbol has been used. When $s=\frac{1}{2}$, one may use expressions for the 6-j symbols given in Ref. 8 to obtain the explicit form of the $\hat{T}^{J_t}_{L's'L(1/2)}$ for $s'=\frac{1}{2}$ and $\frac{3}{2}$. They are shown in Table I when $L=L'$ and in Table II when $L=L'\pm 2$.

In the remainder of the paper I shall choose the incident meson to proceed along the positive z axis, which makes

$$Y^{M}_{L}(\theta,\phi)=[(2L+1)/4\pi]^{1/2}\delta_{M0}\ .$$

With this choice the summation over M and M' in Eq. (3) reduces to the single term with $M'=M_t$. The arguments (hereafter suppressed) of all spherical harmonics which occur henceforth are understood to be (θ',ϕ'), the angles of the outgoing meson. With this choice, those amplitudes with M_t negative are related to those with the same, but positive value, by parity.

Then, using the numerical coefficients given by Karliner and Mattis in their Tables XV and XVII, one may calculate the predictions of the three-flavor model for the $T^{J_t}_{M_t}$ in terms of their reduced partial-wave amplitudes $\tau^{\{IY\}}_{KL'L}$. The results may be summarized as follows.

For $J_t=M_t=0$, the *seven* s-channel SU(3) amplitudes $T^{0}_{0}(\mu)$ [here the final baryon is in the octet and μ denotes the various SU(3) representations], may be written as linear combinations of *four* reduced amplitudes which I call $f^{\{IY\}}$:

$$T^{0}_{0}(\mu)=\sum_{\{IY\}}C^{\{IY\}}(\mu)f^{\{IY\}}\ . \tag{5}$$

The four $f^{\{IY\}}$ have the following partial-wave expan-

TABLE III. The coefficients $C^{\{IY\}}(\mu)$ of Eq. (5) expressing the s-channel SU(3) amplitudes $T^{0}_{0}(\mu)$ in terms of the $f^{\{IY\}}$.

$T^{0}_{0}(\mu)/\sqrt{2}$	$f^{\{0,0\}}$	$f^{\{1,0\}}$	$f^{\{1/2,1\}}$	$f^{\{1/2,-1\}}$
$\langle 27\|27\rangle$	$\frac{2}{15}$	$\frac{14}{135}$	$\frac{4}{9}$	$\frac{1}{9}$
$\langle \overline{10}\|\overline{10}\rangle$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{2}{5}$	$\frac{1}{5}$
$\langle 10\|10\rangle$	0	$\frac{4}{15}$	0	$\frac{1}{5}$
$\langle 8_1\|8_1\rangle$	$\frac{1}{20}$	$\frac{3}{20}$	0	$\frac{1}{2}$
$\langle 8_2\|8_2\rangle$	$\frac{1}{4}$	$\frac{1}{12}$	0	$\frac{1}{2}$
$\langle 8_1\|8_2\rangle$	$\frac{-1}{4\sqrt{5}}$	$\frac{1}{4\sqrt{5}}$	0	$\frac{-1}{2\sqrt{5}}$
$\langle 1\|1\rangle$	0	0	0	1

TABLE IV. The coefficients $C^{\{IY\}}(\mu)$ of Eq. (7) expressing the s-channel SU(3) amplitudes $T^{1}_{1}(\mu)$ in terms of the $g^{\{IY\}}$, when the final baryon has spin $\frac{1}{2}$.

$\sqrt{3}T^{1}_{1}(\mu)$	$g^{\{1,0\}}$	$g^{\{1/2,1\}}$	$g^{\{1/2,-1\}}$
$\langle 27\|27\rangle$	$\frac{6}{135}$	$\frac{4}{27}$	$\frac{1}{135}$
$\langle \overline{10}\|\overline{10}\rangle$	$\frac{-1}{15}$	$\frac{-2}{5}$	$\frac{1}{15}$
$\langle 10\|10\rangle$	$\frac{2}{15}$	0	$\frac{1}{15}$
$\langle 8_1\|8_1\rangle$	$\frac{-3}{20}$	0	$\frac{1}{10}$
$\langle 8_2\|8_2\rangle$	$\frac{-1}{12}$	0	$\frac{-1}{6}$
$\langle 8_1\|8_2\rangle$	$\frac{-1}{4\sqrt{5}}$	0	$\frac{-1}{2\sqrt{5}}$
$\langle 1\|1\rangle$	0	0	-1

TABLE V. The coefficients $C^{\{IY\}}(\mu)$ of Eq. (7) expressing the s-channel SU(3) amplitudes $T_1^1(\mu)$ in terms of the $g^{\{IY\}}$, when the final baryon has spin $\frac{3}{2}$.

$\sqrt{3}T_1^1(\mu)$	$g^{\{1,0\}}$	$g^{\{1/2,1\}}$	$g^{\{1/2,-1\}}$
$\langle 27\|27\rangle$	$\frac{-1}{6\sqrt{5}}$	$\frac{-4\sqrt{5}}{27}$	$\frac{2}{27\sqrt{5}}$
$\langle 10\|10\rangle$	$\frac{-\sqrt{10}}{12}$	0	$\frac{-2}{3\sqrt{10}}$
$\langle 8\|8_1\rangle$	$\frac{-1}{4\sqrt{5}}$	0	$\frac{-1}{\sqrt{5}}$
$\langle 8\|8_2\rangle$	$\frac{-1}{12}$	0	$\frac{1}{3}$

sions in terms of the reduced partial-wave amplitudes of Ref. 1:

$$4\pi f^{\{0,0\}}=\sum_L (2L+1)\tau^{\{0,0\}}_{LLL}P_L \ , \tag{6a}$$

$$4\pi f^{\{1,0\}}=\sum_L [(2L-1)\tau^{\{1,0\}}_{L-1,LL}+(2L+1)\tau^{\{1,0\}}_{LLL} +(2L+3)\tau^{\{1,0\}}_{L+1,LL}]P_L \ , \tag{6b}$$

$$4\pi f^{\{1/2,\pm1\}}=\sum_L [L\tau^{\{1/2,\pm1\}}_{L-1/2,LL}+(L+1)\tau^{\{1/2,\pm1\}}_{L+1/2,LL}]P_L \ , \tag{6c}$$

where P_L represents the standard Legendre polynomial whose argument is $\cos\theta'$. The numerical coefficients $C^{\{IY\}}(\mu)$ are given in Table III.

For $J_t=M_t=1$, the *seven* s-channel SU(3) amplitudes $T_1^1(\mu)$ (when the final baryon is in the octet), and the *four* $T_1^1(\mu)$ (when the final baryon is in the decuplet) may be written as linear combinations of *three* reduced amplitudes $g^{\{IY\}}$:

$$T_1^1(\mu)=\sum_{\{I,Y\}} C^{\{I,Y\}}(\mu)g^{\{I,Y\}} \ . \tag{7}$$

The three $g^{\{IY\}}$ have the following partial-wave expansions:

$$(4\pi)^{1/2}g^{\{1,0\}}=\sum_L \left[\frac{2L-1}{L}\tau^{\{1,0\}}_{L-1,LL}+\frac{2L+1}{L(L+1)}\tau^{\{1,0\}}_{LLL} -\frac{2L+3}{L+1}\tau^{\{1,0\}}_{L+1,LL}\right] \times\left[\frac{L(L+1)}{2L+1}\right]^{1/2}Y_L^1 \ , \tag{8a}$$

$$(4\pi)^{1/2}g^{\{1/2,\pm1\}}=\sum_{L'} (\tau^{\{1/2,\pm1\}}_{L-1/2,LL}-\tau^{\{1/2,\pm1\}}_{L+1/2,LL}) \times\left[\frac{L(L+1)}{2L+1}\right]^{1/2}Y_L^1 \ . \tag{8b}$$

The numerical coefficients $C^{\{IY\}}(\mu)$ are given in Table IV for $s'=\frac{1}{2}$, and in Table V for $s'=\frac{3}{2}$.

For $J_t=2, M_t$ arbitrary (it is sufficient to choose $M_t\geq 0$), the *four* different s-channel SU(3) amplitudes $T_M^2(\mu)$ (when the final baryon is in the decuplet) are multiples of *one* reduced amplitude $h_M^{\{1,0\}}$:

$$T_M^2(\mu)=C^{\{1,0\}}(\mu)h_M^{\{1,0\}} \ . \tag{9}$$

Note that the numerical coefficients are independent of the index M. The partial-wave expansions of the $h_M^{\{1,0\}}$ for $M=0$, 1, and 2 are

$$4\pi h_0^{\{1,0\}}=\sum_{L'} P_{L'}\{3[\sqrt{(L'+1)(L'+2)}\tau^{\{1,0\}}_{L'+1,L'L'+2}+\sqrt{L'(L'-1)}\tau^{\{1,0\}}_{L'-1,L'L'-2}] -[(L'+1)\tau^{\{1,0\}}_{L'-1,L'L'}-(2L'+1)\tau^{\{1,0\}}_{L'L'L'}+L'\tau^{\{1,0\}}_{L'+1,L'L'}]\} \ , \tag{10a}$$

$$\left[\frac{8\pi}{3}\right]^{1/2}h_1^{\{1,0\}}=\sum_{L'}\left[\frac{L'(L'+1)}{2L'+1}\right]^{1/2}Y_{L'}^1\left\{2\left[\left[\frac{L'+2}{L'+1}\right]^{1/2}\tau^{\{1,0\}}_{L'+1,L'L'+2}-\left[\frac{L'-1}{L'}\right]^{1/2}\tau^{\{1,0\}}_{L'-1,L'L'-2}\right] +[(L'+1)\tau^{\{1,0\}}_{L'-1,L'L'}-(2L'+1)\tau^{\{1,0\}}_{L'L'L'}+L'\tau^{\{1,0\}}_{L'+1,L'L'}]/[L'(L'+1)]\right\} , \tag{10b}$$

$$\left[\frac{8\pi}{3}\right]^{1/2}h_2^{\{1,0\}}=\sum_{L'}\left[\frac{L'(L'-1)(L'+1)(L'+2)}{2L'+1}\right]^{1/2}Y_{L'}^2 \times\{[(L'+1)(L'+2)]^{-1/2}\tau^{\{1,0\}}_{L'+1,L'L'+2}+[L'(L'-1)]^{-1/2}\tau^{\{1,0\}}_{L'-1,L'L'-2} +[(L'+1)\tau^{\{1,0\}}_{L'-1,L'L'}-(2L'+1)\tau^{\{1,0\}}_{L'L'L'}+L'\tau^{\{1,0\}}_{L'+1,L'L'}]/[L'(L'+1)]\} \ . \tag{10c}$$

TABLE VI. The coefficients $C^{\{1,0\}}(\mu)$ of Eq. (9) expressing the s-channel SU(3) amplitudes $T_M^2(\mu)$ in terms of the $h_M^{\{1,0\}}$, when the final baryon has spin $\frac{3}{2}$.

T_M^2	$h_M^{\{1,0\}}$
$\langle 27\|27\rangle$	$\frac{-1}{135\sqrt{2}}$
$\langle 10\|10\rangle$	$\frac{-1}{30}$
$\langle 8\|8_1\rangle$	$\frac{1}{10\sqrt{2}}$
$\langle 8\|8_2\rangle$	$\frac{1}{6\sqrt{10}}$

The numerical coefficients $C^{\{1,0\}}(\mu)$ are given in Table VI.

At this point the main point of the paper has been established by direct computation, namely, that the thirty SU(3) amplitudes are predicted to be linear combinations of ten reduced amplitudes. Given a table of SU(3) coupling coefficients, one may use my tables to work out the predictions of the three-flavor model for any reaction involving octet pseudoscalar mesons and octet or decuplet final baryons. (It should be pointed out that the results of Ref. 1 were derived using the "baryon first" convention for the SU(3) isoscalar factors. If one wishes to use my results as they stand, one must respect the same convention.) Since there are far fewer amplitudes than SU(3) alloys, many new linear relations among amplitudes for different reactions may be derived. However, as I show in the next section, it is preferable to use t-channel rather than s-channel SU(3) representations.

For completeness, the relations among my t-channel spin amplitudes $T_{M_t}^{J_t}$ and the spin-projection amplitudes $T_{v'v}$ are given in Table VII.

III. THE t-CHANNEL SU(3) AMPLITUDES

The fact that the two-flavor model is much simpler when expressed in terms of amplitudes having definite t-channel isospin suggests that the three-flavor model might be simpler when expressed in terms of amplitudes having definite SU(3) quantum numbers in the t channel. Since the relevant SU(3) crossing matrices may be found in the compilation of Rebbi and Slansky,[6] it is easy to cast the results presented in the previous section into t-channel language. The only caveat is that Karliner and Mattis have used the convention "baryon first" in deriving their results, whereas Rebbi and Slansky present crossing matrices relating the reaction $M+B\rightarrow M'+B'$ to $M+M'\rightarrow\bar{B}+B'$, which is "baryon second." The phase factors necessary to convert the s-channel SU(3) amplitudes of Ref. 1 to the "baryon second" convention may be found in Table II of Ref. 6. For the reaction

$$8+8\rightarrow 8+8$$

it amounts to changing the sign of the $\langle 8_1|8_2\rangle$ amplitude. For the reaction

$$8+8\rightarrow 8+10$$

the signs of the $\langle 8|8_2\rangle$ and the $\langle 27|27\rangle$ s-channel amplitudes must be changed. With these modifications, one may use the results of Rebbi and Slansky to derive the results displayed in Tables VIII–XI. In these tables, the SU(3) index μ is now to be understood as denoting the various t-channel SU(3) representations (the $\bar{B}B'$ representation on the left, the MM' on the right). In the t

TABLE VII. The amplitudes $T_{M_t}^{J_t}$ of Eq. (2) expressed as linear combinations of the spin-projection amplitudes $T_{v'v}$.

Final baryon spin $s'=\frac{1}{2}$	Final baryon spin $s'=\frac{3}{2}$
$T_0^0=\sqrt{2}T_{(1/2)(1/2)}$	
$\sqrt{3}T_1^1=T_{(-1/2)(1/2)}$	$\sqrt{3}T_1^1=\frac{1}{2}(T_{(-1/2)(1/2)}+\sqrt{3}T_{(-3/2)(-1/2)})$
	$\sqrt{5}T_0^2=\sqrt{2}T_{(1/2)(1/2)}$
	$\sqrt{5}T_1^2=\frac{1}{2}(\sqrt{3}T_{(-1/2)(1/2)}-T_{(-3/2)(-1/2)})$
	$\sqrt{5}T_2^2=T_{(-3/2)(1/2)}$

TABLE VIII. The coefficients $C^{\{IY\}}(\mu)$ of Eq. (5) expressing the t-channel SU(3) amplitudes $T_0^0(\mu)$ in terms of the $f^{\{IY\}}$. The SU(3) notation is $\langle\bar{B}B'|MM'\rangle$.

$T_0^0(\mu)/\sqrt{2}$	$f^{\{0,0\}}$	$f^{\{1,0\}}$	$f^{\{1/2,-1\}}$	$f^{\{1/2,-1\}}$
$\langle 27\|27\rangle$	$\frac{-1}{15}$	$\frac{-1}{135}$	$\frac{2}{45}$	$\frac{2}{45}$
$\langle 10\|10\rangle$	0	0	0	0
$\langle 8_1\|8_1\rangle$	$\frac{1}{10}$	$\frac{-1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\langle 8_2\|8_1\rangle$	$\frac{-1}{2\sqrt{5}}$	$\frac{1}{2\sqrt{5}}$	$\frac{-1}{2\sqrt{5}}$	$\frac{-1}{2\sqrt{5}}$
$\langle 8_2\|8_2\rangle$	0	0	$\frac{-1}{2}$	$\frac{1}{2}$
$\langle 8_1\|8_2\rangle$	0	0	$\frac{1}{2\sqrt{5}}$	$\frac{-1}{2\sqrt{5}}$
$\langle 1\|1\rangle$	1	1	2	2

TABLE IX. The coefficients $C^{\{IY\}}(\mu)$ of Eq. (7) expressing the t-channel SU(3) amplitudes $T_1^1(\mu)$ in terms of the $g^{\{IY\}}$, when the final baryon has spin $\frac{1}{2}$.

$\sqrt{3}T_1^1(\mu)$	$g^{\{1,0\}}$	$g^{\{1/2,1\}}$	$g^{\{1/2,-1\}}$
$\langle 27\|27\rangle$	0	$\frac{8}{135}$	$\frac{-8}{135}$
$\langle 10\|10\rangle$	$\frac{1}{15}$	$\frac{-2}{15}$	$\frac{-2}{15}$
$\langle 8_1\|8_1\rangle$	0	$\frac{3}{10}$	$\frac{-3}{10}$
$\langle 8_2\|8_1\rangle$	0	$\frac{1}{2\sqrt{5}}$	$\frac{-1}{2\sqrt{5}}$
$\langle 8_2\|8_2\rangle$	$\frac{-1}{6}$	$\frac{-1}{6}$	$\frac{-1}{6}$
$\langle 8_1\|8_2\rangle$	$\frac{-1}{2\sqrt{5}}$	$\frac{-1}{2\sqrt{5}}$	$\frac{-1}{2\sqrt{5}}$
$\langle 1\|1\rangle$	0	0	0

TABLE X. The coefficients $C^{\{IY\}}(\mu)$ of Eq. (7) expressing the t-channel SU(3) amplitudes $T_1^1(\mu)$ in terms of the $g^{\{IY\}}$, when the final baryon has spin $\frac{3}{2}$.

$\sqrt{3}T_1^1(\mu)$	$g^{\{1,0\}}$	$g^{\{1/2,1\}}$	$g^{\{1/2,-1\}}$
$\langle 27\|27\rangle$	0	$\frac{2}{27\sqrt{5}}$	$\frac{-2}{27\sqrt{5}}$
$\langle 10\|10\rangle$	$\frac{1}{3\sqrt{10}}$	$\frac{-2}{3\sqrt{10}}$	$\frac{-2}{3\sqrt{10}}$
$\langle 8\|8_1\rangle$	0	$\frac{1}{\sqrt{5}}$	$\frac{-1}{\sqrt{5}}$
$\langle 8\|8_2\rangle$	$\frac{-1}{3}$	$\frac{-1}{3}$	$\frac{-1}{3}$

channel, the constraint of time-reversal invariance for $\mathbf{8}+\mathbf{8}\to\mathbf{8}+\mathbf{8}$ is that the $\mathbf{10}$ and $\overline{\mathbf{10}}$ amplitudes are identical, and I show only the former. The t-channel amplitudes $\langle 8_1|8_2\rangle$ and $\langle 8_2|8_1\rangle$ are distinct, in contrast to the s channel, where time-reversal invariance requires their equality.

A casual inspection of these new tables is sufficient to reveal the remarkable simplifications that occur when the predictions of the model are presented in the t-channel form. Here is a summary.

First, for the T_0^0 amplitudes, the following relations hold:

$$\langle 8_2|8_2\rangle=-\sqrt{5}\langle 8_1|8_2\rangle\ , \tag{11a}$$

$$\langle 8_2|8_1\rangle=-\sqrt{5}\langle 8_1|8_1\rangle\ , \tag{11b}$$

$$\langle 10|10\rangle=0\ . \tag{11c}$$

Those amplitudes with (1), (27), diagonal (8_1), and (8_2) are unconstrained, and one may express the four $f^{\{IY\}}$ amplitudes in terms of them.

Then, for the T_1^1 amplitudes (spin-$\frac{1}{2}$ final baryon), the following equations hold:

$$\langle 8_2|8_2\rangle=\frac{\sqrt{5}}{3}\langle 8_1|8_2\rangle\ , \tag{12a}$$

$$\langle 8_2|8_1\rangle=\frac{\sqrt{5}}{3}\langle 8_1|8_1\rangle\ , \tag{12b}$$

$$\langle 27|27\rangle=\frac{16}{81}\langle 8_1|8_1\rangle\ , \tag{12c}$$

$$\langle 1|1\rangle=0\ . \tag{12d}$$

The amplitudes with (10), diagonal (8_1), and (8_2) are unconstrained, and one may express the three $g^{\{IY\}}$ amplitudes in terms of them.

For spin-$\frac{3}{2}$ baryons (decuplet), the T_1^1 amplitudes are simply related to their counterparts for spin $\frac{1}{2}$. One observes

spin $\frac{3}{2}$ spin $\frac{1}{2}$

$$\langle 8|8_1\rangle=2\langle 8_2|8_1\rangle\ , \tag{13a}$$

$$\langle 8|8_2\rangle=2\langle 8_2|8_2\rangle\ , \tag{13b}$$

$$\langle 27|27\rangle=\frac{\sqrt{5}}{4}\langle 27|27\rangle\ , \tag{13c}$$

$$\langle 10|10\rangle=\frac{\sqrt{5}}{\sqrt{2}}\langle 10|10\rangle\ . \tag{13d}$$

For the T_M^2 amplitudes things are quite simple. The three amplitudes corresponding to the (27) representation in the t channel are free (one for each M) while all others (nine) are zero. This is reminiscent of the $I_t=J_t$ selection rule, inasmuch as only the (27) has any $I_t=2$ content.

These relations establish the main premise of this work. The symmetry implied by the three-flavor Skyrme model is most apparent when expressed in terms of amplitudes of definite t-channel spin J_t and definite t-channel SU(3) representations.

TABLE XI. The coefficients $C^{\{1,0\}}(\mu)$ of Eq. (9) expressing the t-channel SU(3) amplitudes $T_M^2(\mu)$ in terms of the $h_M^{\{1,0\}}$, when the final baryon has spin $\frac{3}{2}$.

T_M^2	$h_M^{\{1,0\}}$
$\langle 27\|27\rangle$	$\frac{\sqrt{2}}{27}$
$\langle 10\|10\rangle$	0
$\langle 8\|8_1\rangle$	0
$\langle 8\|8_2\rangle$	0

IV. ELASTIC SCATTERING AND THE REDUCED t-CHANNEL AMPLITUDES

The relations I have established among the reduced t-channel SU(3) amplitudes have a large number of experimentally verifiable consequences. Since elastic-scattering cross section and polarization measurements involving charged π and K beams have been performed over a wide range of energies, it appears judicious to see what the model says about them before confronting inelastic reactions. It turns out that using the elastic amplitudes, one may determine the four reduced non-spin-flip, and the three reduced spin-flip amplitudes. In addition, five constraints among the elastic amplitudes are found. Meshkov and Yodh[9] have provided a table of elastic-scattering amplitudes in terms of t-channel SU(3) quantum numbers, from which one may derive the following results. If one forms, for example, the six combinations of elastic-scattering non-spin-flip amplitudes $\pi^+p\pm\pi^-p$, $K^+p\pm K^-p$, and $K^+n\pm K^-n$, which correspond to $C=\pm1$ exchange in the t channel, then the three even-C combinations may be used to determine the diagonal (1), (8_1), and (27) reduced t-channel non-spin-flip amplitudes. One finds

$$2\langle 1|1\rangle=(K^+p+K^-p)+3(\pi^+p+\pi^-p)+4(K^+n+K^-n)\ , \tag{14a}$$

$$2\langle 27|27\rangle=5(K^+p+K^-p)-(\pi^+p+\pi^-p)-4(K^+n+K^-n)\ , \tag{14b}$$

$$2\langle 8_1|8_1\rangle=-(\pi^+p+\pi^-p)+(K^+n+K^-n)\ , \tag{14c}$$

where the expressions on the right-hand side refer to the corresponding combinations of non-spin-flip elastic amplitudes.

For the three $C=-1$ combinations, there is only one t-channel SU(3) amplitude, the diagonal 8_2 representation, and one finds

$$2\langle 8_2|8_2\rangle = -3(K^+p - K^-p) \tag{15a}$$

$$= -15(\pi^+p - \pi^-p) \tag{15b}$$

$$= -\tfrac{15}{4}(K^+n - K^-n)\ . \tag{15c}$$

These are *not* the Johnson-Treiman[10] relations of SU(6); but a linear combination of them is the SU(6) sum rule of Barger and Rubin:[11]

$$(K^+p - K^-p) = (\pi^+p - \pi^-p) + (K^+n - K^-n)\ , \tag{16}$$

which holds whenever the (10) amplitude is zero. At this point the four t-channel non-spin-flip amplitudes are determined, and two constraints among the elastic-scattering amplitudes have been established. The optical theorem then enables one to compare these predictions directly to experiment, without using phase-shift analyses.

The three independent t-channel spin-flip amplitudes may also be determined using the elastic-scattering amplitudes. Again the even- and odd-C combinations must be formed, and one finds the following results. Spin-flip even C:

$$2\langle 8_1|8_1\rangle = \tfrac{81}{19}(K^+p + K^-p) \tag{17a}$$

$$= \tfrac{81}{5}(\pi^+p + \pi^-p) \tag{17b}$$

$$= -\tfrac{81}{22}(K^+n + K^-n)\ . \tag{17c}$$

Spin-flip, odd C:

$$\langle 10|10\rangle = -2(K^+p - K^-p) + 2(\pi^+p - \pi^-p) + 2(K^+n - K^-n)\ , \tag{18a}$$

$$\langle 8_2|8_2\rangle = -(K^+p - K^-p) - \tfrac{1}{2}(\pi^+p - \pi^-p) - \tfrac{1}{2}(K^+n - K^-n)\ , \tag{18b}$$

with the constraint

$$(\pi^+p - \pi^-p) = 3(K^+p - K^-p) + 4(K^+n - K^-n)\ . \tag{19}$$

One sees that, assuming that the six elastic non-spin-flip and spin-flip amplitudes for the six elastic reactions are available, there are five verifiable predictions, and the seven t-channel SU(3) amplitudes are determined. Subsequently, the amplitudes for any reaction leading to a final baryon in the octet are predicted by the model. In addition, the $J_t=1$ amplitudes for reactions with the final baryon in the decuplet are also unambiguously predicted. It should be noted that the successful model of Stodolsky and Sakurai[12] for the reaction $\pi N \to \pi\Delta$ amounts to keeping only the $J_t=1$ amplitude. One may conclude that the most significant amplitude for decuplet baryons is thus predicted once the elastic amplitudes are known. Note also that while the πN amplitudes may be used to determine four of the seven reduced amplitudes, they are not constrained by the model, in contrast with the two-flavor case.

Finally I should comment on the $J_t=2$, $M_t=0$, 1, and 2 amplitudes, which the model predicts to be zero in the nonexotic $\mathbf{8}_1$ and $\mathbf{8}_2$ t-channel representations, and relegates to the exotic **27** t-channel amplitude. Remembering that the reaction $K^-p \to K^+\Xi^{*-}$ is a typical **27** t-channel reaction, one may conclude that the amplitudes $h_M^{\{1,0\}}$ are probably quite small, except perhaps at threshold.

V. CONCLUSIONS

The three-flavor Skyrmion model of the Karliner and Mattis has been shown to be highly predictive for the reaction $M+B \to M'+B'$, where M and M' are octet pseudoscalar mesons, B is an octet spin-$\frac{1}{2}$ baryon and B' an octet spin-$\frac{1}{2}$ or decuplet spin-$\frac{3}{2}$ baryon. The thirty different amplitudes which describe these reactions in the limit of SU(3) symmetry are predicted to be linear combinations of only ten reduced t-channel amplitudes. Of these, seven may be determined using amplitudes measured in elastic-scattering reactions. The remaining three contribute only to the $J_t=2$ amplitudes in decuplet production; they correspond to (27) in the t channel and are probably small. The model thus asserts that with a small amount of experimental input, all the other reactions can be predicted. This is a fairly bold claim, but the exciting possibility of describing two-body hadronic reactions using a theory which claims kinship with quantum chromodynamics justifies further investigation.

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[1]M. Karliner and M. P. Mattis, Phys. Rev. D **34**, 1991 (1986).
[2]J. T. Donohue, Phys. Rev. Lett. **58**, 3 (1987).
[3]J. T. Donohue, Phys. Rev. D **37**, 631 (1988).
[4]M. P. Mattis and M. Mukerjee, Phys. Rev. Lett. **61**, 1344 (1988).
[5]J. T. Donohue, Phys. Rev. D **40**, 1679 (1989).
[6]C. Rebbi and R. Slansky, Rev. Mod. Phys. **42**, 68 (1970).
[7]M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964).
[8]M. Rotenberg, R. Bivins, N. Metropolis, and J. K. Wooten, Jr., *The 3-j and 6-j Symbols* (The Technology Press–Massachusetts Institute of Technology, Cambridge, MA, 1959).
[9]S. Meshkov and G. B. Yodh, Phys. Rev. Lett. **19**, 603 (1967).
[10]K. Johnson and S. B. Treiman, Phys. Rev. Lett. **14**, 189 (1965).
[11]V. Barger and M. H. Rubin, Phys. Rev. **140**, B1365 (1965).
[12]L. Stodolsky and J. J. Sakurai, Phys. Rev. Lett. **11**, 90 (1963).