Study of nuclear dependence of scaling in 800-Gev proton interactions with emulsion nuclei

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Experimental results on the scaled multiplicity distribution of secondary particles produced in interactions of 800-GeV protons with emulsion nuclei are presented and discussed. The validity of Koba-Nielsen-Olesen scaling has been observed in interactions with nuclei. It has been found that the negative-binomial distribution describes well the scaled multiplicity distribution of secondary particles in proton-emulsion interactions.

I. INTRODUCTION

When Koba, Nielsen, and Olesen' (KNO) predicted asymptotic scaling in multiplicity distributions in hadron collisions, this triggered an enormous amount of experimental and theoretical activity, which is presently continuing. KNO scaling, derived on the assumption of the validity of Feynman² scaling, states that the ratio of the topological cross section σ_{n_s} to the inelastic cross section σ_{inel} multiplied by the mean multiplicity $\langle n_{s} \rangle$ is only a function of Z, $\langle n_s \rangle \sigma_{n_s}/\sigma_{\text{inel}} = \psi(Z)$ where $Z = n_s / \langle n_s \rangle$ is the scaled multiplicity. It was found^{3,4} that when diffractive and nondiffractive events were considered separately, the validity of KNO scaling was observed from \sqrt{s} = 1.5 GeV up to a CERN ISR energy of \sqrt{s} = 62 GeV.

However, it was observed by the UA5 Collaboration⁵ that the KNO function in $\bar{p}p$ collisions is broadened considerably with increasing energy. It was pointed out⁶ that one of the important sources of this effect can be the influence of energy and momentum conservation laws which tend to suppress the large-multiplicity tail. As the suppression is obviously stronger at lower energies, the distribution is expected to broaden with increasing energy, as observed. A comprehensive analysis of the data by the UA5 Collaboration led to a new empirical regularity for multiplicity distributions in place of KNO scaling. A remarkable agreement of the UA5 data for the pseudorapidity interval and for the full phase space is obtained by the negative-binomial distribution (NBD) which is derived on the assumption that the final-state particles obey Bose-Einstein statistics.⁷ The NBD is of the form

$$
P(n_s; \langle n_s \rangle, K) = \begin{bmatrix} n_s + K - 1 \\ n_s \end{bmatrix} \left[\frac{\langle n_s \rangle / K}{1 + \langle n_s \rangle / K} \right]^{n_s}
$$

$$
\times \left[\frac{1}{1 + \langle n_s \rangle / K} \right]^K \quad \text{for } K > 0 . \tag{1}
$$

For $K \rightarrow \infty$, the NBD reduces to a Poisson distribution. For negative K , the NBD becomes an ordinary binomial distribution (narrower than the Poisson distribution). At lower energies, NBD is considered to be the correct formula to which KNO scaling is an asymptotic approximation. 8 In the NBD parametrization, the shape of the distribution is determined by the parameter K and the position of the maximum by $\langle n_{s} \rangle$. The variable K is related to the mean multiplicity $\langle n_s \rangle$ and the dispersion D by

$$
D^2/\langle n_s \rangle^2 = \frac{1}{\langle n_s \rangle} + \frac{1}{K} \ . \tag{2}
$$

In the ISR energy range this ratio $D^2/\langle n_s \rangle^2$ is constant, not because of KNO scaling in the distribution, but appears to be simply an accidental local effect resulting from the interplay of decreasing $\langle n_s \rangle$ ⁻¹ and an increasng K^{-1} with energy.⁹ Further, asymptotic KNO scaling requires a constant K^{-1} . Instead K^{-1} increases linearly with \sqrt{s} . ¹⁰ This logarithmic increase of K^{-1} with \sqrt{s} violates KNO scaling in exactly the same way as Feynman scaling is violated by the logarithmic rise of the rapidity plateau with energy. Further, the near constancy of nultiplicity moments¹¹ from a few GeV up to ISR energy is attributed to the fact that the cross section for highmultiplicity events for this energy range is small. However, at the collider energy, the higher multiplicity moments which are more sensitive to the larger-multiplicity region increase significantly.

To parametrize the scaled multiplicity distribution, several forms of the scaling function $\psi(Z)$ for hadronhadron inelastic collisions^{4, 10-13} and for e^-e^+ annihila- \int ion^{14,15} have been obtained in a variety of phenomeno ogical models with rather different theoretical inputs, such as the geometric model, 16 unitary uncorrelated clussuch as the geometric model, ¹⁶ unitary uncorrelated ϕr model, ¹⁷ fireball model.^{18,19} and the clan model.¹⁹

The purpose of this work is to determine if KNO scaling holds in the present interactions and to investigate whether nuclear effects modify the scaling behavior. Further, the "universal" appeal of the NBD to explain the multiplicity distributions in high-energy hadron-hadron interactions can be augmented by checking its validity in hadron-nucleus interactions. In the present work, Sec. II contains the experimental details. Section III gives a discussion of the results on the basis of KNO scaling, C_q moments, the negative-binomial distribution, and the clan model. Conclusions are given in Sec. IV.

A stack consisting of G5 emulsion pellicles of dimensions 10 cm \times 8 cm \times 0.06 cm was exposed to a proton beam of energy 800 GeV at Fermilab. The beam flux was 8.7×10^4 particles/cm² and the distribution of the primary energy was $< 0.05\%$. The emulsion plates were carefully area scanned for inelastic events. All the events were scanned twice by each observer and the average efficiency for detecting events was found to be \sim 96%. Scanning was done at a distance of ¹ cm from the leading edge of emulsion. In order to select the events due to the primary protons, the following criteria were followed. (a) The primary of each interaction was followed up to the entry point in emulsion and there should be no interaction due to a secondary track; and (b) the primary particle should make an angle of $\langle 2^{\circ}$ with the mean beam direction. Events lying up to 25 μ m from the surface or the glass side of the emulsion pellicle were not considered. Taking the above criteria into account, a total of 2407 events were obtained. Following the usual emulsion terminology, 20 the secondary particles having $\beta(v/c) \ge 0.7$ $(I \le 1.4I_0)$ and $\beta < 0.7$ $(I > 1.4I_0)$ were designated as the shower and heavy tracks respectively, where I and I_0 are the ionization of the secondary and primary tracks, respectively. The value of I_0 obtained in the present work is 28 blobs/100 μ m. The total number of shower tracks obtained here is \sim 50000. The multiplicity of shower and heavy tracks is designated by n_s and N_h , respectively, and N_h is the sum of black N_b $(I > 10I_0)$ and grey $N_g(1.4I_0 < I \le 10I_0)$ tracks. The shower tracks are due to fast particles, produced in elementary nucleon-nucleon collisions, the grey tracks are due to medium-energy particles and are due to knock-on nucleons, and black tracks are due to evaporation of the nucleus. The value of N_g has been considered²¹ as a monitor of the number of collisions of the primary particles (v) with the nucleus.

It has been shown²⁰ that the target nucleus can be broadly identified on the basis of values of N_h . Events having $N_h = 0.1$ are mostly due to interactions with the hydrogen target or efFectively with single nucleons of the nuclei. Events having $N_h = 2-5$ were classified as belonging to the C,N,O group of nuclei, while those with $N_h > 9$ were unambiguous interactions with the Ag, Br group of nuclei. Events with $N_h = 6-8$ were not considered as they are ambiguous with respect to the type of target. Events having $N_h \geq 2$ are mostly proton-emulsion nucleus interactions whose number was found to be 2003. Following the above procedure we have found the number of interactions belonging to targets of nucleon; C, N, O; and Ag, Br as 404; 635; and 1016, respectively. The topological cross section may be presented in a variety of ways, but because of growing evidence for scaling in highenergy interactions it was thought necessary to find a variable in terms of which the topological cross section would show the approximate energy independence. Hence the variable multiplicity has been chosen as it is in easily measurable variable at high energies and also it can be measured with a high amount of experimental accuracy.

II. EXPERIMENTAL DETAILS **III. RESULTS AND DISCUSSION**

The scaling has been investigated from many dynamical points of view; the phenomena itself seems much more precise than any theoretical explanation yet brought forth. In addition, the detailed shape of the distribution is not satisfactorily understood although several functional forms have been proposed. $4,7,16-19$

An interesting and comprehensive way of checking the validity of KNO scaling is via the study of C_q moments, defined as

$$
C_q = \langle n_s^q \rangle / \langle n_s \rangle^q .
$$

Zajc 22 has made an interesting observation that Feynman scaling does not predict that the reduced moments C_a should be energy invariant but the factorial moments are expected to be energy independent. The factorial moments are given by

$$
G_q = \frac{\langle n_s(n_s-1)\cdots(n_s-q+1)\rangle}{\langle n_s\rangle^q}
$$

and, by assuming $\langle n_s \rangle \gg q$, KNO extended this result to the invariance of C_q . This result is interesting because the NB distribution at fixed K has this property. The behavior of C_q approaches the behavior of G_q at very high energies, this characteristic being more pronounced at lower values of q. Even the difference in the values of C_q and G_q decrease as the primary energy is increased (Fig. 1 of Ref. 22). The C_q moments for p-emulsion interactions at present energy along with those at lower energies for $q=2$, 3, and 4 are shown in Fig. 1. It is clear that the C_q moments for *p*-nucleus interactions are approximately constant from 20 to 800 GeV, thus supporting the hypothesis of KNO scaling.

Negative-binomial distribution (NBD)

Although KNO scaling holds for $p - A$ interactions also at the present energy, the departure from KNO scaling in the high-multiplicity region prompted us to check the validity of the NBD in the present interactions. Further, the attractiveness of the NBD lies in its universal applicability to interactions up to collider energies. The predictions of the negative-binomial distribution (NBD) have been fitted to the multiplicity distributions for interactions with (i) all emulsion nuclei, (ii) C, N, 0, and (iii) Ag, Br and are shown in Figs. $2(a)-2(c)$ by a solid line. The average multiplicity $\langle n_{s} \rangle$ from experiment is taken to be the input for fitting the NBD to the data and the value of K is obtained by the CERN computer program MINUIT. The values of K obtained are 3.71 ± 0.08 ; 4.55 ± 0.18 ; and 3.89 ± 0.12 for interactions with emulsion nucleus; C, N, O; and Ag, Br targets, respectively, with $\chi^2/N_{\text{DF}} \sim 1$ for all the three distributions. In all cases, the values of K agree within a few percent with the values calculated from Eq. (2). Thus, we find that the NBD reproduces well the scaled multiplicity distribution for *p*-emulsion interactions.

Several physical interpretations have been proposed for the successful representation of data by the NBD. 19,23

FIG. 1. C_q moments for p-emulsion-nucleus interactions at different primary energies.

FIG. 2. The normalized multiplicity (N_{n_s}/N_T) , where N_{n_s} is the number of shower particles in n_s bins and N_T is the total number of events) distribution for interactions with targets of (a) emulsion nucleus, (b) C, N, O, and (c) Ag, Br, respectively.

We discuss here only the most widely accepted and most successful approach, namely, the clan model. 19,24 Van Hove and $Giovannini¹⁹$ in their detailed studies of the clan model concluded that clans are bremsstrahlung gluon jets, i.e., jets that are emitted from the source independently. If the number \bar{n}_c of particles in an average clan (clan size) has a logarithmic distribution, then the convolution of the Poissonian and logarithmic distributions yields the NBD for the multiplicity distribution of the final-state particles.

The average clan multiplicity \overline{N} and average clan size \bar{n}_c are related to the NBD parameters, namely, $\langle n_s \rangle$ and K by

$$
\overline{N} = K \ln \left[1 + \frac{\langle n_s \rangle}{K} \right], \tag{3}
$$

$$
\overline{n}_c = \frac{\langle n_s \rangle}{K} / \ln \left[1 + \frac{\langle n_s \rangle}{K} \right] = \frac{\langle n_s \rangle}{\overline{N}} \ . \tag{4}
$$

The value of $1/K$ is a measure of the particle aggregation $(1/K=0$ for $\bar{n}_c=1$). The clan model is meaningful for $1/K=0$ for $\bar{n}_c=1$). The clan model is meaningful $1/K>0$ ($\bar{N} < \langle n_s \rangle$, $\bar{n}_c > 1$), i.e., for the genuine NBD.

The values of \overline{N} and \overline{n}_c as calculated from Eqs. (3) and (4) and the values of K (for interactions with emulsion nuclei) are shown in Table I along with the corresponding values for p-p interactions at different primary energies. It is seen that the NBD parameters $\langle n_s \rangle$ and K show an A dependence. Whereas the values of $\langle n_s \rangle$ increase with A , those of K decrease as we go from proton to nuclear targets. The clan multiplicity \overline{N} is independent of both the target size and primary energy.^{19(b)} We find that the clan size for p-emulsion interactions is slightly higher compared to that for p - p interactions at the same primary energy. Thus the clan size shows nuclear dependence, however small. This can be understood from the fact that the clan size \bar{n}_c is a function of average particle multiplicity ($\langle n_s \rangle$) and average clan multipicity (\overline{N}). Since \overline{N} remains nearly constant for p-p and p-A interactions, the slight increase of $\langle n_s \rangle$ with the mass number of target results in the corresponding increase of \bar{n}_c with A.

The clan parameters \overline{N} and \overline{n}_c have been determined in forward and backward hemispheres for different rapidity windows for proton-nucleus interactions at 200 GeV Refs. 26–28) and 360 GeV.²⁹ Because of low statistics, we could not study the variation of \overline{N} and \overline{n}_c for different rapidity windows. However, we have studied the variation of the above parameters in forward and backward regions as well as in central and fragmentation regions for the whole rapidity space. The values obtained are given in Table II along with those at 200 and 360 GeV. It is seen that the average number of clans \overline{N} remain invariant with respect to primary energy as well as in forward and backward regions. The values of the clan size \bar{n}_c are higher in the backward as compared to the forward hemisphere, which could be due to higher value of $\langle n_{s} \rangle$ in the former in comparison to the latter hemisphere. The higher value of $\langle n_s \rangle$ in the backward hemisphere compared to that in the forward hemisphere could be due to multinucleon collisions in the former region. At 800 GeV, the average clan size is the same for both the fragmentation and central regions; however, the average clan multiplicity in the former region is lower as compared to that in the latter region. The reason for the latter observation is that there is a much larger contribution to multiplicity from the central region as compared to the fragmentation region.

The data for p-emulsion interactions at 800 GeV has been fitted with the Slattery parametrization,⁴ the geometric model,¹⁶ and the unitary uncorrelated cluster

TABLE II. Values of average multiplicity ($\langle n_s \rangle$), 1/K, average clan multiplicity (\overline{N}), and average clan size (\overline{n}_c) for p-nucleus collisions at different primary energies.

Primary energy (GeV)	Interaction type	Forward (F)/ backward (B) region	Average multipli- city $\langle n_s \rangle$	1/K	Average clan multiplicity \bar{N}	Average clan size \bar{n}_c	$\chi^2/N_{\rm DF}$	Ref.
200	p -Xe	F \bf{B}	5.26 ± 0.14 5.78 ± 0.10	0.110 ± 0.024 0.077 ± 0.014	4.15 ± 0.021 4.78 ± 0.16	1.27 ± 0.05 1.21 ± 0.04	10.2/13 12.7/16	26
	$p-Ar$	$\mathbf F$ \bf{B}	9.22 ± 0.40 14.00 ± 0.53	0.487 ± 0.047 0.648 ± 0.038	3.50 ± 0.17 3.57 ± 0.13	2.64 ± 0.16 3.93 ± 0.18	12.7/31 21.1/48	26
360	$p-A$	${\bf F}$ \bf{B}	6.55 ± 0.70 12.68 ± 2.33	0.25 ± 0.12 0.64 ± 0.20	3.87 ± 0.41 3.45 ± 0.63	1.69 ± 0.18 3.67 ± 0.67	2.4/2 0.7/5	29
	p -Au	${\bf F}$ \bf{B}	7.14 ± 0.31 19.17 ± 0.99	0.15 ± 0.05 0.62 ± 0.07	4.85 ± 0.21 4.12 ± 0.21	1.47 ± 0.06 4.65 ± 0.24	30.9/15 56.6/42	29
800	<i>p</i> -emulsion	$\mathbf F$ \bf{B}	8.91 ± 0.21 10.23 ± 0.24	0.25 ± 0.01 0.59 ± 0.02	4.73 ± 0.11 3.31 ± 0.12	1.88 ± 0.44 3.09 ± 0.10	56.8/29 54.7/40	Present work
		Central $2 < \eta < 4.5$ Fragmenta- tion	12.22 ± 0.34 5.63 ± 0.16	0.26 ± 0.01 0.81 ± 0.02	5.52 ± 0.15 2.10 ± 0.05	2.21 ± 0.06 2.68 ± 0.07	40.9/32 59.4/27	Present work

Parametrized function	Primary energy (GeV)	Interaction type	Parameter values	$\chi^2/N_{\rm DE}$	Ref.
$\psi(Z) = (AZ + BZ^3 + CZ^5 + DZ^7)e^{-EZ}$	$50 - 300$	$p-p$	$A=3.79$, $B=33.72$, $C=-6.64$ $D=0.33, E=3.04$		4
	360	$p-Au$	$A=3.79$, $B=33.72$, $C=-6.04$ $D=0.33, E=3.04$		29
	800	<i>p</i> -emulsion	$A=1.61, B=30.71, C=-4.01$ $D=0.33, E=3.65$	1.05	Present work
Barshay geometric model					
$\psi(Z) = AZe^{-BZ^2}$	18-1980 800	$p-p$ <i>p</i> -emulsion	$A=3.14, B=10.78$ $A = 1.76 \pm 0.65$ $B = 0.89 \pm 0.27$	1.2	16(c) Present work
Unitary uncorrelated cluster model					
$\psi(Z) = A \frac{Z}{[\Gamma(Z+1)]^2} e^{-BZ}$	$12 - 300$	$p-p$	$A=1.33, B=0.89$		17
	800	<i>p</i> -emulsion	$A = 3.15 \pm 0.04$ $B = 0.41 \pm 0.16$	1.02	Present work

TABLE III. Values of parameters χ^2/N_{DF} obtained by fitting different empirical functions for $\psi(Z)$ for p-p and p-nucleus interactions at different primary energies.

model.¹⁷ It is seen that the values of the parameter (Table III) for p-emulsion interactions are not too different from those in p - p interactions. The data has also been fitted with single-, double-, and triple-fireball models as given by Chou, Liu, and Meng³⁴ but only the twofireball model is found to yield a good fit for $n_s > 7$ events with $\chi^2/N_{\text{DF}}=1.2$. The single- and triple-fireball model predictions yielded a high value of $\chi^2/N_{\text{DF}} \sim 3$. This observation is in agreement with the earlier work^{35,36} that the two-fireball model explains particle production at the present energy and at 400 GeV.

IV. CONCLUSIONS

The values of C_q moments show a clear energy independence from 67 to 800 GeV in p-emulsion interactions. This implies the validity of KNO scaling in the above interactions.

Our results show that the multiplicity distribution of secondary hadrons in p-emulsion interactions at 800 GeV are well described by NBD. This extends the already wide range of reactions for which a similar observation has been made. Thus the NBD seems to be an empirical law of remarkably wide applicability.

The clan size for *p*-emulsion interactions is slightly higher in comparison to that for p - p collisions. The average clan multiplicity remains independent of the target mass number as well as primary energy. Further, the values of clan size are higher in the backward region as compared to the forward region. The values of clan size do not differ significantly in the central and fragmentation regions but the clan multiplicity is definitely higher in the central region as compared to the fragmentation region.

The form of the multiplicity distribution for the p emulsion interaction is not different from that of the $p-p$ interaction, implying that the scaled KNO function $\psi(Z)$ is universal up to the present energy. Since the KNO scaling function of a given collision process reflects its reaction mechanism and hadron-nucleus interaction yields information on the space-time development of hadronic reaction, the above-observed analysis shows that most of the nucleons act as spectators in p -emulsion interactions at the present energy.

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