Hard part of the γ-gluon cross section in deep-inelastic scattering off polarized target

Lech Mankiewicz*
Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195
(Received 24 May 1990)

We analyze in detail the problem of separation of the hard γ-gluon cross section in polarized deep-inelastic scattering, closely related to the current dispute over the existence of the so-called anomalous gluon contribution to the flavor-singlet part of the nucleon polarized structure function $g_1(x)$. We find that maintaining the factorization property of the full γ-nucleon cross section beyond the leading order in α_s gives an unambiguous definition of the gluon contribution to the first moment of $g_1(x)$, independent of the value of soft momentum scales, such as quark and gluon masses, involved in the problem. Consideration of the soft contribution, which cannot be calculated reliably in perturbation theory, hints at the possible violation of a naive, constituent-quark-model pattern in polarized quark distributions for light flavors, as advocated earlier by Jaffe and Manohar. We argue that the latter conclusion should have an important impact on the current phenomenology of polarized quark distributions in the nucleon.

Recently, there has been a considerable debate about the existence of the so-called anomalous gluon contribution to the nucleon spin.[1-7] The theoretical conjecture was first made by Efremov and Teryaev,[1] but unfortunately their paper contained some mistakes. Later the idea was reformulated by Altarelli and Ross,[2] Carlitz, Collins, and Mueller,[3] and Altarelli and Stirling.[4] They pointed out that in deep-inelastic scattering of a photon off a longitudinally polarized nucleon target the effective γ^*-gluon interaction via the box graph induces a contribution to the first moment of the singlet part of the g_1^N structure function which is proportional to the gluon polarization. Moreover, while at first glance it is of a higher order in $\alpha_S(Q^2)$, it does not vanish when $Q^2 \to \infty$ because the gluon polarization $\Delta G(Q^2)$ grows like $\ln Q^2$. Such behavior is somewhat different from what we know about deep-inelastic scattering from an unpolarized target, where contributions of this sort are suppressed by an inverse power of $\ln Q^2$. In the language of operator-product expansion this term is interpreted[2-4] as arising from the gluon anomaly in the divergence of the U(1) axial-vector current;[8] hence its name "anomalous gluon contribution." Its existence may help to explain the surprising result of the recent European Muon Collaboration (EMC) experiment,[9] i.e., a surprisingly low value of the first moment of g_1^p, the polarized structure function for a longitudinally polarized proton. This result has been interpreted by many authors as an indication that only a very small fraction of the nucleon's helicity is actually carried by quarks.[10] The anomalous gluon correction changes this interpretation in the sense that now the EMC measurement is to be related to the combination.

$$\Gamma = \sum_{f=u,d,s} \Delta q_f - \frac{3\alpha_s}{2\pi} \Delta G \ , \tag{1}$$

instead of (twice) the total quark helicity, $\sum_f \Delta q_f$. In Eq. (1) it was assumed that at the scale of the EMC experiment $Q^2 \sim 10$ GeV^2, only lightest flavors are active. For large and positive ΔG this formula can help to bring the quark contribution on the left-hand side of (1) closer to its value ~ 1, preferred by phenomenological considerations.

This interpretation has been seriously criticized by the authors of Refs. 11 and 12. Their position can be summarized in the statement that, in the leading twist, there is no local gauge-invariant gluon operator which could contribute to the operator-product expansion for the first moment of $g_1(x)$. Our idea is that the only way to decide which party is right to go deeper from the level of formal arguments into actual derivations, and hope that sooner or later at least one path must end up with a contradiction.

Our aim in this paper is to analyze carefully a derivation of Eq. (1) within perturbation theory, i.e., the calculation of graphs depicted in Fig. 1. The main problem is that in QCD one has to take care to select only those contributions that are manifestly insensitive to long-distance physics. This is usually done by restricting the phase-space integration to the regions where quark propagators are far offshell, i.e., both m^2-k^2, $m^2-(k+q-p)^2 > \mu_{\text{fact}}^2$, where μ_{fact} is the factorization scale taken to be large enough that the perturbative expansion in $\alpha_s(\mu_{\text{fact}}^2)$ is meaningful. Maintaining the factorization property of the total cross section to the order of α_s requires, in general, the contribution from virtualities smaller than μ_{fact}^2 to be identified with the polarized quark distributions. We will discuss this issue in more detail in the second part of the paper. Our result is that if certain care is taken about this factorization there is no ambiguity in the existence of the hard-gluon contribution to the nucleon helicity, independent of the particular values of the low-momentum scales, such as quark and gluon masses, involved in the problem. By "certain care" we mean that the adopted regularization mechanism does not modify the chiral structure of the perturbation theory. More precisely, we require that a factorization scheme does not

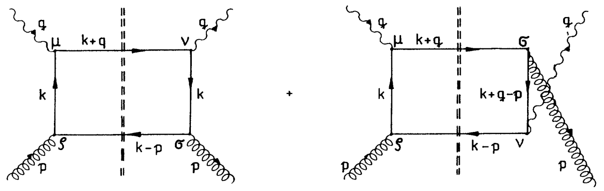

FIG. 1. The elementary graphs leading to a photon-gluon coupling. g_μ is the photon momentum p_μ is the gluon momentum.

lead to any breaking of helicity conservation in the quark-gluon vertex, additional to chiral-symmetry breaking that is already present in the theory because of nonzero values of quark masses.

Most probably, almost none of the statements we make here are new to an expert. Arguments similar to ours have been scattered over the current literature on the problem;[2-4,10] but nevertheless we find it useful (and hopefully the reader will agree with us) to bring them together.

In this paper we are interested only in what happens in the Bjorken limit, when $Q^2=-q^2\to\infty$. Investigations related to possible scaling violations will be reported in a forthcoming paper.

Our starting point is the calculation of the cross section for the scattering of γ^* on a polarized gluon. In the lowest order in α_s there is a contribution from the two graphs depicted in Fig. 1. We take both a quark mass different from zero and a gluon four-momentum squared $p^2=-\mu^2$ (not to be confused with the factorization scale squared $\mu^2_{\rm fact}$) in the anticipation of the fact that a gluon in a nucleon is typically off shell by an amount of a few hundreds MeV. As we keep all terms proportional to μ^2 and m^2, the resulting expressions are lengthy and complicated and we are not going to quote them here; they coincide exactly with what was found, for example, in Ref. 13 if we take their regulator scale to zero, as discussed below.

To isolate the hard contribution we have to introduce a cutoff. Actually, if either m^2 or μ^2 is different from zero, the expression for the cross section is finite, but is does not yet mean that it comes entirely from the perturbative domain. In fact, one can show that the minimal virtuality of the quark line in Fig. 1 is given by

$$m^2-k^2|_{\min}=m^2-(k+q-p)^2|_{\min}=\mu^2x+\frac{m^2}{1-x}\ , \tag{2}$$

where $x=Q^2/(2p\cdot q)$ and $Q^2=-q^2$. It is seen that keeping a gluon mass different from zero may not be sufficient, because no matter how large μ^2 is the small-x region is not in the hard part. Moreover, when the diagrams in Fig. 1 are considered as part of a larger process, the γ^*-N scattering, the gluon virtuality becomes an integration variable, which has no fixed value. Finally, we would like to be able to see what happens for light quarks, such as u, d, or s. To avoid difficulty we look for a cutoff procedure independent of quark and gluon characteristics. We choose to discuss three choices, which we propose to call the $k_\perp$, the λ, the light-cone cutoffs, respectively. Note also that because we cut off the sum of two graphs, not each graph separately, our procedure does not induce a violation of gauge invariance.

To perform actual calculations we go to a special reference frame, namely, the γ^*g center-of-mass frame, and fix the direction of the photon and gluon three-momentum along the z axis. Such a choice does not violate Lorentz invariance, provided that we will be able to express all our results by Lorentz invariants. In this frame, the $k_\perp$ cutoff is simply the requirement that the square of the perpendicular component of the momentum k^μ which flows in the loop in Fig. 1 is larger than some scale $k^2_{\perp\min}=\mu^2_{\rm fact}$. Because in our frame $k_\perp=(k+q-p)_\perp$, see Fig. 1, the minimal virtuality of the quark propagators in Fig. 1 is constrained by

$$\begin{aligned} m^2-k^2|_{\min}&=m^2-(k+q-p)^2|_{\min}\\ &=\mu^2x+\frac{m^2+k^2_{\perp\min}}{1-x}\ . \end{aligned} \tag{3}$$

It is worthwhile to note at this point that, contrary to popular belief, the $k_\perp$ cutoff procedure is Lorentz invariant. To see this it is sufficient to note that in the γ^*g system the $k_\perp$ is just a projection of k^μ, in a covariant, Minkowskian sense, off the plane spanned by four-vectors p and q. Indeed, following Ref. 14 we can define the projector

$$R^{\mu\nu}=-g^{\mu\nu}+\frac{(q\cdot p)(q^\nu p^\nu+q^\mu p^\nu)-q^2p^\mu p^\nu-p^2q^\mu q^\nu}{(q\cdot p)^2-q^2p^2}\ , \tag{4}$$

and denote by $k^{\perp\mu}$ the covariant vector $k^{\perp\mu}=R^{\mu\nu}k_\nu$. Then, in the Bjorken limit, the $k_\perp$ cutoff corresponds to the requirement that

$$-k^{\perp\mu}k^{\perp}_{\mu}\geq k^2_{\perp\min}\ . \tag{5}$$

For the second cutoff we follow Ref. 13; namely, we demand simply that

$$m^2-k^2|_{\min}=m^2-(k+q-p)^2|_{\min}\geq\lambda^2=\mu^2_{\rm fact}\ . \tag{6}$$

Using this cutoff, the authors of Ref. 13 arrived at the conclusion that the first moment of the hard part of the polarized gluon splitting function is equal to $-\frac{1}{2}\alpha_s/2\pi$ per flavor, not $-\alpha_s/2\pi$, as was found in Refs. 2–4. This mysterious result partially motivated the current research. We will see later that the discrepancy is due to the fact that the λ cutoff procedure does not respect the helicity conservation of the quark-gluon coupling.

The choice of the form of the third cutoff is dictated by consideration of the light-cone wave function of a helicity-one gluon. For reasons which will become clear soon we take it in the form

$$\mu^2+\frac{k_\perp^2+m^2}{x(1-x)}\geq\Lambda^2=\mu^2_{\rm fact}\ . \tag{7}$$

To isolate properly the hard contribution we consider all cutoffs to be "large enough;" i.e., $\mu^2_{\rm fact}$ must be $\gg$

than the confinement size, or Λ_{QCD}. After performing some algebra and taking the Bjorken limit, i.e., neglecting terms proportional to μ^2/Q^2 and m^2/Q^2 compared to unity, the resulting expressions still depend on the dimensionless ratios $\mu^2/\mu^2_{\mathrm{fact}}$, $m^2/\mu^2_{\mathrm{fact}}$, and $\mu^2_{\mathrm{fact}}/Q^2$. For perturbative QCD to be a reliable tool we need μ_{fact} to be much larger than all soft-momentum scales involved, and therefore from now on we assume that $m^2,\mu^2 \ll \mu^2_{\mathrm{fact}} \ll Q^2$. Note that we now discuss explicitly the case of light flavors. With the above assumption we obtain the following expressions for polarized gluon splitting functions corresponding to different cutoff schemes:

$$\hat{A}^{k\perp}(x)=C(2x-1)\left[\ln\frac{Q^2}{\mu^2_{\mathrm{fact}}}+\ln\frac{1-x}{x}-1\right], \tag{8a}$$

$$\hat{A}^{\lambda}(x)=C(2x-1)\left[\ln\frac{Q^2}{\mu^2_{\mathrm{fact}}}+\ln\frac{1}{x}-1\right], \tag{8b}$$

and

$$\hat{A}^{L-C}(x)=C(2x-1)\left[\ln\frac{Q^2}{\mu^2_{\mathrm{fact}}}+\ln\frac{1}{x^2}-1\right], \tag{8c}$$

where $C=\alpha_s N_f/2\pi$, N_f is the number of active flavors, the caret over $\hat{A}$ denotes the hard, perturbatively calculable part, and the superscripts on the left-hand side of (8a)–(8c) indicate the cutoff scheme used.

Several comments are now in order. First, we note that the results (8a)–(8c) do not depend on the value of m^2 and μ^2, provided that they both are much smaller than μ^2_{fact}. It is as it should be; both m^2 and μ^2 represent soft scales, and therefore by definition cannot influence the hard part. What happens in the other physically interesting case where $m^2 \gg \mu^2_{\mathrm{fact}}$, which presumably is the case for heavy flavors, was discussed in details in Ref. 15 and we will come back to this question at the end of the paper. Now, the expression for $A^{k\perp}(x)$ coincides with that obtained by Altarelli and Ross, Ref. 2. There has been discussion in the literature[12] about the validity of their calculation because some terms proportional to m^2 were explicitly neglected there, and therefore gauge invariance was violated. In accordance with previous finding,[3,4,15,16] what we find here by including all terms, but cutting off quark virtualities in a gauge-invariant fashion, is that the terms neglected by Altarelli and Ross do not contribute to the hard part of the $\gamma^* g$ cross section, defined with the help of $k_\perp$ cutoff. Finally, expression (9b) corresponds to the cutoff procedure introduced in Ref. 13.

Now, we compute the first moment of the $\hat{A}$'s, denoted by $\langle \hat{A} \rangle$. We might be surprised to see that while (8a) and (8c) give $\langle \hat{A} \rangle = -C$, from (8b) it follows that $\langle \hat{A} \rangle$ actually equals $-\frac{1}{2}C$. At first glance there is nothing which differentiates the λ cutoff from the other two. To understand what has really happened we have to resort to another type of reasoning, namely, to look at arguments which allow one to maintain the factorization property beyond the leading order.[17,18]

In the following few paragraphs we reiterate the reasoning which helps one to organize corrections to deep-inelastic scattering which are of higher order in α_s into factorizable form. We choose to follow closely the arguments of Ref. 18. In the case of γ^*-nucleon scattering the factorization theorem states that in the Bjorken limit the physical cross section factorizes in the form

$$\sigma_N^{\gamma^*}(x,Q^2,m_N)=\hat{\sigma}_q^{\gamma^*}(x,Q^2,\mu^2_{\mathrm{fact}})\otimes f_{q/N}(x,\mu^2_{\mathrm{fact}},m_N) + \hat{\sigma}_g^{\gamma^*}(x,Q^2,\mu^2_{\mathrm{fact}})\otimes f_{g/N}(x,\mu^2_{\mathrm{fact}},m_N)\ , \tag{9}$$

where $\otimes$ denotes convolution, m_N stands for all possible low-energy scales, including m and μ, and sum over quark flavors in implicitly assumed. The physical meaning of the above equation is that after fixing a suitable factorization scale μ^2_{fact}, all large-distance contributions are organized into quark and gluon distributions in a nucleon, i.e., $f_{q/N}$ and $f_{g/N}$. The hard $\gamma^* q$ and $\gamma^* g$ cross sections, represented by $\hat{\sigma}$ in (9) are by definition free of long-distance contributions and thus calculable perturbatively.

While hard γ-quark scattering exists already in the leading, zero order, the hard γ-gluon cross section starts only at the order of α_s. To identify it from the full cross section in this order, represented by the hard part of the graphs in Fig. 1, it is sufficient to notice that the full γ-gluon cross section has the same factorization property: namely, that[17,18]

$$\sigma_g^{\gamma^*}(x,Q^2,m_N)=\hat{\sigma}_q^{\gamma^*}(x,Q^2,\mu^2_{\mathrm{fact}})\otimes f_{q/g}(x,\mu^2_{\mathrm{fact}},m_N) + \hat{\sigma}_g^{\gamma^*}(x,Q^2,\mu^2_{\mathrm{fact}})\otimes f_{g/g}(x,\mu^2_{\mathrm{fact}},m_N)\ , \tag{10}$$

where, as previously, we collectively denote by m_N all relevant low-energy scales.

Now, we consider Eq. (10) in the lowest nontrivial order of perturbation theory. The γ-gluon cross section on the left-hand side starts at order α_s, and obviously the same is true for its hard part $\hat{\sigma}_g^{\gamma^*}(x,Q^2,\mu^2_{\mathrm{fact}})$. Therefore, to maintain consistency we have to identify $f_{g/g}(x,Q^2,\mu^2_{\mathrm{fact}})$ with its zero-order form, $f_{g/g}(x,Q^2,\mu^2_{\mathrm{fact}})=\delta(1-x)$. The perturbatively calculated $f_{q/g}(x,\mu^2_{\mathrm{fact}} m_N)$ is of the first order, and so we take for the hard γ-quark cross section its lowest-order form, representing the cross section for hard-photon scattering off a pointlike fermion. After collecting all the above factors into (10) we obtain

$$\hat{\sigma}_g^{\gamma^*}(x,Q^2,\mu^2_{\mathrm{fact}})=\sigma_g^{\gamma^*}(x,Q^2,m_N) - \hat{\sigma}_q^{\gamma^*}(x,Q^2,\mu^2_{\mathrm{fact}})\otimes f_{q/g}(x,\mu^2_{\mathrm{fact}},m_N)\ , \tag{11}$$

which defines the first-order hard γ-gluon cross section.

In the lowest order of perturbation theory there is no difficulty in identifying $\hat{\sigma}_q^{\gamma^*}(x,Q^2,\mu^2_{\mathrm{fact}})$; it is actually a constant, the cross section for hard-photon scattering off a pointlike fermion. For consistency, however, we should include in (9) also the first-order corrections to the hard γ-quark cross section. They are known[2] to be suppressed in the first moment of $g_1(x)$ by a factor of $\alpha_s(\mu_{\rm fact}^2)$, and therefore should be negligible for a sufficiently large factorization scale. After inserting $\hat{\sigma}_g^{\gamma^*}(x,Q^2,\mu_{\rm fact}^2)$ into Eq. (9) we obtain the parton-model formula valid to the first order in α_s: namely,

$$\sigma_N^{\gamma^*}(x,Q^2,m_N)=\hat{\sigma}_q^{\gamma^*}(x,Q^2,\mu_{\rm fact}^2)\otimes f_{q/N}(x,\mu_{\rm fact}^2,m_N) + [\sigma_g^{\gamma}(x,Q^2,m_N)-\hat{\sigma}_q^{\gamma^*}(x,Q^2,\mu_{\rm fact}^2)\otimes f_{q/g}(x,\mu_{\rm fact}^2,m_N)]\otimes f_{g/N}(x,\mu_{\rm fact}^2,m_N) \ . \tag{12}$$

We see explicitly that, because the low-energy scales m_N cancel in the brackets on the right-hand side of (12), the above formula has the form of Eq. (9); so, factorization is maintained through first order. Therefore, by comparing with (9) and identifying factors convoluted with hard-quark and hard-gluon cross sections as quark and gluon distribution functions, respectively, we have the unambiguous prescription for an identification of possible gluonic contributions. One may ask, however, what if terms in (12) are regrouped, so that we have

$$\sigma_N^{\gamma^*}(x,Q^2,m_N)=\sigma_g^{\gamma}(x,Q^2,m_N)\otimes f_{g/N}(x,\mu_{\rm fact}^2,m_N) + \hat{\sigma}_q^{\gamma^*}(x,Q^2,\mu_{\rm fact}^2)\otimes [f_{q/N}(x,\mu_{\rm fact}^2,m_N)-f_{q/g}(x,\mu_{\rm fact}^2,m_N)\otimes f_{g/N}(x,\mu_{\rm fact}^2,m_N)] \tag{13}$$

instead of (12)? In QED, where soft physics is well understood, it makes no difference, as regrouping a finite number of terms cannot change the final result. In QCD there is a big difference: the factorization property as given by Eq. (9) is part of the definition of quark distributions. To see this note that otherwise there could not be any theoretical input (at least at the present stage) to the calculation of the total cross section, as perturbation theory allows us to compute only $\hat{\sigma}$. But what we now would like to identify as "gluonic contribution" to the total cross section, the first term on the right-hand side of (13) explicitly fails to obey factorization of large- and low-momentum scales, and therefore cannot be regarded as a correct expression. It follows that, within presented scheme, there is no ambiguity in the definition of the gluonic contribution in (9) induced by hard γ-gluon scattering.

Arguments, which lead to (12), are of course based on perturbation theory. Once factorization of scales has been achieved we can safely assume that they hold beyond perturbation theory,[17,18] and we can use Eq. (11) as a definition of the hard cross section $\hat{\sigma}_g^{\gamma^*}(x,Q^2,\mu_{\rm fact}^2)$ and therefore compute the hard part of the polarized gluon splitting function. Indeed, while on the right-hand side of Eq. (11) we have quantities sensitive to nonperturbative physics, the difference is free of any long-distance contributions and therefore can be reliably computed in perturbation theory. To do that we have to know, according to Eq. (11), the first-order perturbative distribution of quarks in a gluon. It is given by the $k_\perp$ integral of the squared norm of the light-cone $q\bar{q}$ wave function of the gluon $\Psi_{q\bar{q}}^g(x,k_\perp)$:[19]

$$f_{q/g}(x,\mu_{\rm fact}^2,m_N)=\int^{O(\mu_{\rm fact}^2)}[d^2k_\perp]|\Psi_{q\bar{q}}^g(x,k_\perp)|^2 \ . \tag{14}$$

Note the factorization scale $\mu_{\rm fact}$ which in general enters here as a cutoff for the perpendicular momentum integration. In light-cone perturbation theory[20] $\Psi_{q\bar{q}}^g(x,k_\perp)$ is given by[3,19,20]

$$\Psi_{q\bar{q}}^g(x,k_\perp,\rho,\rho')=g\frac{\bar{u}_\rho(xk^+,k_\perp)\gamma\cdot\epsilon\, v_{\rho'}((1-x)k^+,-k_\perp)}{2\sqrt{(2\pi)^3}x(1-x)[-\mu^2-(k_\perp^2+m^2)/(x(1-x))]} \ . \tag{15}$$

As previously, we take the gluon mass $=-\mu^2$; ρ and ρ' denote quark helicities and ϵ is the gluon polarization vector.[20] As we are interested in polarized scattering, $f_{q/g}$ equals the difference of positive-helicity quark distributions in helicity $+1$ and -1 gluons. By parity invariance argument it can be expressed as the difference between positive- and negative-helicity quark distributions in helicity $+1$ gluon:

$$f_{q/g}(x,\mu_{\rm fact}^2,m_N)\equiv q^{\uparrow}(x,\mu_{\rm fact}^2,m_N)-q^{\downarrow}(x,\mu_{\rm fact}^2,m_N) \ , \tag{16}$$

where again m_N denotes low energy scales, such as quark and gluon invariant masses. The relevant matrix elements are readily computed with the help, for example, of Table III in Appendix B of Ref. 20. For the reader's convenience we have collected them in Table I. After some algebraic manipulations, according to Eqs. (14)–(16), and integration over the azimuthal angle in the $k_\perp$ plane of Eq. (14), we finally find

$$f_{q/g}(x,\mu_{\rm fact}^2,m_N) = \frac{\alpha_s}{4\pi}\int^{O(\mu_{\rm fact}^2)} dk_\perp^2 \frac{(2x-1)k_\perp^2+m^2}{[\mu^2x(1-x)+m^2+k_\perp^2]^2} \ . \tag{17}$$

The cutoff present in (14) and (17) guarantees that the $k_\perp$ integration probes only regions in the phase space where the $q\bar{q}$ pair is soft; i.e., it is off shell by the amount

TABLE I. Matrix elements of the $\bar{v}_{\rho'}((1-x)k^+,-k_\perp)\gamma\cdot\epsilon u_\rho(xk^+,k_\perp)$ vertex, which defines $q\bar{q}$ perturbative wave function of helicity +1 gluon (Refs. 3, 17, and 18). ϵ^μ is the gluon polarization vector, and ρ and ρ' denote helicity of quark and antiquark, respectively.

ρ	ρ'	$\bar{v}_{\rho'}((1-x)k^+,-k_\perp)\gamma\cdot\epsilon u_\rho(xk^+,k_\perp)$
$\uparrow$	$\uparrow$	$-\left[\frac{2}{x(1-x)}\right]^{1/2} m$
$\downarrow$	$\downarrow$	0
$\uparrow$	$\downarrow$	$-\left[2\frac{x}{1-x}\right]^{1/2}(k^x+ik^y)$
$\downarrow$	$\uparrow$	$\left[2\frac{1-x}{x}\right]^{1/2}(k^x+ik^y)$

smaller than a quantity of the order of μ^2_{fact}. To proceed further we have to choose an explicit form of the cutoff. The light-cone cutoff (7) means that we simply allow only for $q\bar{q}$ pair virtuality, given by the energy denominator in Eq. (13), smaller than μ^2_{fact}. Two other cutoffs, defined by Eqs. (3) and (6), do approximately the same.

For the moment let us assume, following Ref. 3, that the quark mass m is much smaller than μ, as it is for light quarks in QCD. With this assumption, after evaluating explicitly the integral in (17) and adding (equal) contributions from antiquarks we arrive at the following expressions for the *soft* part of polarized gluon splitting functions:

$$A^{k_\perp}(x,\mu^2_{\text{fact}},\mu) = \frac{\alpha_s}{2\pi}(2x-1)\left[\ln\frac{\mu^2_{\text{fact}}}{\mu^2}-\ln x(1-x)-1\right], \quad (18)$$

$$A^{L-C}(x,\mu^2_{\text{fact}},\mu) = \frac{\alpha_s}{2\pi}(2x-1)\left[\ln\frac{\mu^2_{\text{fact}}}{\mu^2}-1\right], \quad (18b)$$

$$A^{\lambda}(x,\mu^2_{\text{fact}},\mu) = \frac{\alpha_s}{2\pi}(2x-1)\left[\ln\frac{\mu^2_{\text{fact}}}{\mu^2}+\ln\frac{1}{x}-1\right]. \quad (18c)$$

We want to remind the reader that formulas (18a)–(18c) alone make no sense in QCD. They have been derived using perturbation theory in a region, where it does not apply. They make sense only when subtracted from the "splitting function" $A^0(x,Q^2,m_N)$ inferred from the full γ-gluon cross section. Fortunately we do not need to compute A^0 here; it is given for the case when $\mu^2 \gg m^2$ by the formula derived in Ref. 3, which for one flavor reads

$$A^0(x,Q^2,m_N) = \frac{\alpha_s}{2\pi}(2x-1)\left[\ln\frac{Q^2}{\mu^2}+\ln\frac{1}{x^2}-2\right]. \quad (19)$$

Now, subtracting $A(x,\mu^2_{\text{fact}},m_N)$ for three light flavors from $A^0(x,Q^2,m_N)$ to obtain the hard part $\hat{A}(x,Q^2,\mu^2_{\text{fact}})$, we get exactly expressions (8a)–(18c) which is, we believe, a rather nontrivial check of the whole line of reasoning. Note that any nonperturbative mechanism, related to, e.g., chiral-symmetry breaking, should influence A^0 and A in the same way, and therefore should be absent in $\hat{A}$. We note of course that the form of $\hat{A}$ depends on the particular regularization we use; therefore, an unambiguous identification of the hard part is not possible for moments higher than the first. For the first moment, however, we arrived at the perfectly well-defined expression for $\langle \hat{A} \rangle$, manifestly free of any long-distance contribution. We believe this corresponds to the statement made in Refs. 1–4. We must note, however, that our analysis is explicitly carried out only at the tree level. To recover the full physical content of Eq. (1) we have to assume that leading radiative corrections to the hard part can be organized into the running coupling constant $\alpha_s(\mu^2_{\text{fact}})$ still maintaining the separation of scales seen in the lowest order. At the moment we see no mechanism preventing this from happening, but certainly this fact deserves explicit demonstration. We note that recently strong arguments in favor of this conjecture have been presented both in operator language[6] and in perturbation theory.[7]

At the same time we note that the "soft" part of the γ^*g cross section, corresponding to (18a)–(18c), which we have to identify with the polarized quark distributions, hints that for light quarks nonperturbative effects in QCD may result in polarized quark distributions which would be far from naive intuition. For example, a nonzero value for strange-quark polarization in a nucleon[9] should not be regarded as a surprise. In this point we agree with the authors of Ref. 11 that the same reasoning which leads to the so-called anomalous gluon contribution to (1) suggests that there is no reason in QCD to expect Δq's to be close, e.g., to their SU(6) values. But we see no reason to expect that for light quarks these two contributions cancel either. They do, certainly, for the case of heavy quarks, i.e., when $m^2 \gg \mu^2_{\text{fact}}$, as discussed in Refs. 3, 15, and 16 and at the end of this paper.

And what about the fact that in the case of λ regularization we find $\langle \hat{A} \rangle = -\frac{1}{2}C$? It is interesting to see now what has really happened. While

$$\langle A^{k_\perp}(x,\mu^2_{\text{fact}},m_N)\rangle = 0 = \langle A^{L-C}(x,\mu^2_{\text{fact}}m_N)\rangle ,$$

for A^λ we have (for one flavor)

$$\langle A^\lambda(x,\mu^2_{\text{fact}},m_N)\rangle = -\frac{1}{2}\frac{\alpha_s}{2\pi} .$$

Note that $A(x,\mu^2,m_N)$ has been defined by Eqs. (14) and (17) in such a way that its first moment measures the quark's helicity content of the gluon. The quark-gluon coupling in the QCD Lagrangian respects helicity conservation, and therefore one can expect that, as a consequence of this symmetry, the first moment of $A(x,\mu^2,m_N)$ should be zero, unless there is a specific mechanism which breaks the symmetry. Clearly, the role of such a mechanism is played here by the λ cutoff prescription. Note that a calculation which leads to Eq. (18c) has been done at the tree level, which suggests that an appearance of the nonzero value of $\langle A^\lambda(x,\mu^2_{\text{fact}},m_N)\rangle$ in this case has nothing to do with the Adler-Bell-Jackiw anomaly.[8] It is a subtle problem why, if at all, one should avoid the λ cutoff or other regularization procedures which lead to similar results. It is a potentially dangerous situation because, in the absence of any good argument which discriminates one set of cutoffs in Eq. (17) against others, there is an obvious question of the uniqueness of the value of $\langle \hat{A} \rangle$.

In fact, it is also relatively easy to see what happens when the quark mass is large.[3,15,16] If it is much larger than μ^2, but still much smaller than a minimal reasonable value of μ_{fact}^2 (a likely case for charm quarks), the calculation of the $A(x,\mu_{\text{fact}}^2,m_N)$ gives

$$A^{k_\perp}(x,\mu_{\text{fact}}^2,m_N)=\frac{\alpha_s}{2\pi}\left[(2x-1)\left[\ln\frac{\mu_{\text{fact}}^2}{m^2}-1\right]+1\right] \tag{20a}$$

and

$$A^{L-C}(x,\mu_{\text{fact}}^2,m_N)=\frac{\alpha_s}{2\pi}\left[(2x-1)\left[\ln\frac{\mu_{\text{fact}}^2}{m^2}+\ln x(1-x)-1\right]+1\right] . \tag{20b}$$

The full cross section A^0 can be taken from the corresponding calculation in Ref. 12. For one flavor we have

$$A^0(x,Q^2,m_N)=\frac{\alpha_s}{2\pi}\left[(2x-1)\left[\ln\frac{Q^2}{m^2}+\ln\frac{1-x}{x}-1\right]-2(1-x)\right] . \tag{21}$$

Now, subtracting (20) from (21) we get

$$\hat{A}^{k_\perp}(x,Q^2,\mu_{\text{fact}}^2)=\frac{\alpha_s}{2\pi}(2x-1)\left[\ln\frac{Q^2}{\mu_{\text{fact}}^2}+\ln\frac{1-x}{x}-1\right] \tag{22a}$$

and

$$\hat{A}^{L-C}(x,Q^2,\mu_{\text{fact}}^2(x)=\frac{\alpha_s}{2\pi}(2x-1)\left[\ln\frac{Q^2}{\mu_{\text{fact}}^2}+\ln\frac{1}{x^2}-1\right] , \tag{22b}$$

which again agrees exactly with (8a) and (8c), respectively. This agreement is indeed a good indication that different momentum scales have been properly factorized, and that low-momentum scales do not contribute to Eqs. (8). It also demonstrates that, as long as μ_{fact}^2 remains much larger than m^2, the hard part of A^0 remains insensitive to the chiral-symmetry violation induced by the nonzero value of the quark mass. As previously mentioned, although $\hat{A}$ depends on the regularization scheme under consideration, its first moment is scheme independent and equals $-\alpha_s/2\pi$ per quark flavor, in agreement with Refs. 2–4.

For $m^2 \gg \mu_{\text{fact}}^2$ the results are quantitatively different.[3,15,16] In this case it is sufficient to note that if this condition is satisfied and simultaneously μ_{fact}^2 is sufficiently large, expression (21) can be regarded as a reliable result for the hard cross section. As the first moment of (21) is equal to zero, there is no contribution from heavy quarks. It is as it should be, because for large quark masses the anomaly is still there, but its contribution is exactly canceled by the mass term.[16,21] In other words, if we neglect the gluon virtuality, the first moment of the box diagram receives the contribution $-\alpha_s/2\pi$ from the quark virtualities $\sim Q^2$ and the contribution $+\alpha_s/2\pi$ from virtualities $\sim m^2$. For large m^2, much larger than Λ_{QCD}, it is a reliable statement, and there is no net effect if the sum of the two is taken into account. We want once more to contrast this situation with the case of small m^2 where this argument does not work anymore because there is no way we can reliably use perturbation theory at small energy scales. The only way to save factorization, and therefore the partonic interpretation, is to hide the large-distance contribution where it really belongs, i.e., in the polarized quark distribution. However, as pointed out by Jaffe and Manohar[11] the existence of such a term in perturbation theory may suggest that there are potentially large, nonperturbative effects, which influence polarized quark distributions in a nucleon and therefore one should not be surprised if there are experimental indications that polarized quark distributions do not follow an SU(6)-inspired pattern. One may try to argue that because the EMC experiment measured the combination (1), the soft and hard parts should cancel in this case as they do perturbatively. We see no reason for such an argument to be valid, because the long-distance contribution is computed equal to $+\alpha_s/2\pi$ in perturbation theory, and so this result is certainly unreliable in QCD.

What follows for phenomenology from this discussion is that probably one should avoid intuition with the principle of helicity conservation in the background while constructing models of low-energy-scale quark and gluon distribution functions. Difficulties with a phenomenological interpretation of the EMC measurement,[22] known as the "spin crisis," may indicate simply that an as-yet unspecified, but very likely present, nonperturbative mechanism which violates helicity conservation at low-energy scales has been missed in the analysis. An example of such a mechanism has been discussed recently in Ref. 23.

In summary, we have discussed an issue of separation of scales which is crucial to maintaining the factorization of the cross section and hence partonic interpretations of deep-inelastic scattering off a polarized nucleon target. We have found that maintaining factorization beyond the leading order unambiguously defines the gluonic contribution to the first moment of the structure function $g_1(x)$ in the form proposed in Refs. 2–4, provided that a factorization procedure respects the helicity conservation of tree-level QCD. This result holds independently of the soft sector and is not sensitive to the ratio of quark and gluon masses, provided both are chosen much smaller than the factorization scale. We have noted, however, that in accordance with suggestions made in Ref. 11 perturbation theory results demonstrate the possibility of a strong violation of naive quark-model intuition for polarized quark distributions. A possible interpretation of the EMC results would be that there is a moderate gluonic contribution to Eq. (1) and a nonzero value of the strange-quark polarization. Finally, we have also discussed briefly the mechanism leading to the cancellation of the gluonic contribution to the first moment of $g_1(x)$ in the case of heavy quarks.

We are grateful to S. J. Brodsky, L. Frankfurt, R. L. Jaffe, A. Manohar, Z. Ryzak, A. Schäfer, and M. Strikman for insightful discussions, and to E. Henley for a careful reading of the manuscript. This paper was completed when the author was a visitor to the Institute for Nuclear Theory at the University of Washington in Seattle, and the support from the Institute is gratefully acknowledged. This work has been supported in part by the Polish Government Research Grant No. C.P.B.P. 01.03, and by the Australian Research Council.

*Permanent address: Nicolaus Copernicus Astronomical Centre, Bartycka 18, PL-00-716 Warsaw, Poland.

[1]A. V. Efremov and O. V. Teryaev, in *Proceedings of the International Hadron Symposium,* Bechyne, Czechoslovakia, 1988, edited by Fischer, Kolay, and Kundrat (Czechoslovakian Academy of Science, Prague, 1988), p. 302.

[2]G. Altarelli and G. G. Ross, Phys. Lett. B **212**, 391 (1988).

[3]R. D. Carlitz, J. C. Collins, and A. H. Mueller, Phys. Lett. B **214**, 229 (1988); in *'88 Electroweak Interactions and Unified Theories,* proceedings of the 24th Rencontre de Moriond, Les Arcs, France, 1989, edited by J. Tran Thanh Van (Editions Frontiéres, Gif-sur-Yvette, 1988).

[4]G. Altarelli and W. J. Stirling, Part. World **1**, 40 (1989).

[5]S. Forte, Phys. Lett. B **224**, 189 (1989); Nucl. Phys. **B331**, 1 (1990).

[6]Z. Ryzak, Harvard University Report No. HUTP-90/B003, 1990 (unpublished).

[7]G. Altarelli and B. Lampe, Z. Phys. C **47**, 315 (1990).

[8]S. L. Adler, Phys. Rev. **177**, 2426 (1969). J. S. Bell and R. Jackiw, Nuove Cimento **60A**, 47 (1969).

[9]EMC Collaboration, J. Ashman *et al.*, Phys. Lett. B **206**, 364 (1988); Nucl. Phys. **B328**, 1 (1989).

[10]For a recent review see T. P. Cheng and L. F. Li, Carnegie-Mellon University Report No. CMU-HEP-90-2, 1990 (unpublished).

[11]R. L. Jaffe and A. Manohar, Nucl. Phys. **B337**, 509 (1990).

[12]G. T. Bodwin and J. Qiu, Phys. Rev. D **41**, 2755 (1990).

[13]S. D. Bass, N. N. Nikolaev, and A. W. Thomas, Adelaide University Report No. ADP-133-T80, 1990 (unpublished).

[14]V. M. Budnev, I. F. Ginzburg, G. V. Meledin, and V. G. Serbo, Phys. Rep. **15**, 181 (1975).

[15]L. Mankiewicz and A. Schäfer, Phys. Lett. B **242**, 455 (1990).

[16]A. V. Efremov, J. Soffer, and N. A. Törnqvist, Phys. Rev. Lett. **64**, 1495 (1990).

[17]J. C. Collins, D. E. Sopper, and G. Sterman, Stony Brook Report No. ITP-SB-89-31, 1989 (unpublished).

[18]W. K. Tung, Int. J. Mod. Phys. A **2**, 1369 (1987).

[19]S. J. Brodsky and G. P. Lepage, SLAC Report No. SLAC-PUB-2447, 1979 (unpublished).

[20]G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).

[21]A. Ambjørn, J. Greensite, and C. Peterson, Nucl. Phys. **B221**, 381 (1983).

[22]Z. Dziembowski, L. Mankiewicz, A. Szczepaniak, and H. J. Weber, Phys. Rev. D **39**, 3277 (1989).

[23]B. L. Ioffe and M. Karliner, SLAC Report No. SLAC-PUB-5235, 1990 (unpublished).