Equation of state of strange quark matter and strange star

Somenath Chakrabarty
Saha Institute of Nuclear Physics, 92 Acharya Prafulla Chandra Road, Calcutta 700 009, West Bengal, India
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Using a relativistic version of the Landau theory of a Fermi liquid and a density-dependent quark-mass approach to confinement, the equation of state of strange quark matter has been derived. This equation of state is used to study the stability and some global properties of a strange quark star.

Properties of strange quark matter have become a topic of current high interest, particularly after the speculation of Witten[1] some years ago that strange quark (SQ) matter may be an absolute ground state of bulk hadronic matter near the normal nuclear density. This speculation was investigated by a number of authors[2,3] using the most popular phenomenological bag model.[4] In a recent work,[5] using a dynamical density-dependent quark-mass approach to confinement, it has been shown that SQ matter may be the true ground state of bulk matter only at a very high density ($7-8n_0$, $n_0=0.17\ \mathrm{fm}^{-3}$ is the normal nuclear density). This work differs significantly from the results obtained by earlier authors. SQ nuggets may be produced at the time of first-order cosmic phase transition in the early Universe; however, this idea was criticized by Applegate and Hogan[6] and it has been shown[2] that as the Universe cools down to the temperature $T\sim 10$ MeV, SQ matter evaporates completely.

The other possibility of SQ matter formation is at the core of a neutron star (density $>5n_0$) or if the density of the neutron star is sufficiently high the whole star may be converted to a quark star. Since SQ matter is energetically favorable, light quarks are converted to strange quarks through the weak processes

$$\begin{aligned}&u+d\leftrightarrow u+s\ ,\\ &s\to u+e^-+\bar{\nu}_e\ ,\\ &d\to u+e^-+\bar{\nu}_e\ ,\\ &u+e^-\to d+\nu_e\ ,\end{aligned}\tag{1}$$

and a SQ star is formed. In the weak reactions (1) a dynamical chemical equilibrium is established.

In this Brief Report, following Plümer *et al.*,[7] we shall derive an equation of state for SQ matter using the relativistic version of Landau theory of a Fermi liquid, which was originally developed by Baym and Chin[8] and use a density-dependent quark-mass model of confinement. We shall use this equation of state to study some global properties of the SQ star. Since Landau theory is applicable for a system in which there is a one-to-one correspondence between the bare particle states of the original system when there was no interaction and the dressed or quasiparticle states of interacting system, it is therefore applicable to the SQ system only when a dynamical equilibrium is established.

Lacking a trustworthy description of confinement of quark matter at very large baryon densities, most authors have fallen back on the phenomenological bag model which *a priori* assumes that, within the bag, quarks are asymptotically free. But the recent results from lattice calculations[9] show that quark matter does not become asymptotically free immediately after the phase transition; it approaches the free gas equation of state rather slowly; in this context the bag model is thus an inadequate description of confinement. There exist in the literature, however, other phenomenological descriptions of confinement, namely, the dynamical density-dependent quark-mass model,[5,7,10] the chromodielectric model,[11] the chiral chromodielectric model,[12] etc.

In this Brief Report we use the first alternative model of confinement, which has been widely discussed in the literature; for the sake of completeness, though, let us review very briefly the basic idea. The first step in this direction was originally taken by Pati and Salam,[13] who pictured confinement as a quark having a small mass inside and a very large mass outside. Confinement is mimicked through the requirement that the mass of an isolated quark becomes infinitely large so that the vacuum is unable to support it.

To obtain the equation of state [$P=P(\epsilon)$] of SQ matter, we parametrize the variation of masses for nonstrange (u and d) and strange (s) quarks with the baryon-number density of the system in the following manner:[5]

$$m_{u,d}=\frac{B}{3n_B}\ ,\tag{2a}$$

$$m_s=m_s^0+\frac{B}{3n_B}\ ,\tag{2b}$$

where B is the constant energy density in the zero density limit ($n_q\to 0$). We assume that the effective masses of light quarks become negligible in accordance with our expectation from asymptotic freedom and restoration of chiral symmetry at very high density; on the other hand, the strange quarks have a non-negligible current mass m_s^0 ($=150-300$ MeV).

Then we have the chemical potential of the species i ($=u$, d, or s):

$$\mu_i=(p_{F_i}^2+m_i^2+\Delta p_{F_i})^{1/2}\ , \tag{3}$$

where p_{F_i} is the Fermi momentum of the species i and

$$\Delta p_{F_i}=\int_0^{p_{F_i}}\left[\frac{6B\pi^2}{p^4}C_i\left[\frac{(p_{F_i}^2-m_i^2)}{(p_{F_i}^2+m_i^2)^{1/2}}+m_i\right]\right]dp_{F_i}\ ; \tag{4}$$

$$C_i=n_i/n_q \tag{5}$$

is the fraction of the species i (u, d, or s) present in the system, n_q is the total quark number density $=3n_B$.

Electrons are assumed to be noninteracting and massless; then

$$\mu_e=p_{F_e}\ . \tag{6}$$

Now the number density of the species i (u, d, s, or e) is given by

$$n_i=\frac{g_i p_{F_i}^3}{6\pi^2}\ , \tag{7}$$

where $g_i=6$ for quarks and 2 for electrons; then we have

$$m_i=\frac{BC_i\pi^2}{p_{F_i}^3} \tag{8a}$$

for $i=u$ or d,

$$m_i=m_s^0+\frac{BC_i\pi^2}{p_{F_i}^3} \tag{8b}$$

for $i=s$.

Since a chemical equilibrium is established among the participants we have, from Eq. (1),

$$\mu_d=\mu_u+\mu_e\ , \tag{8c}$$

$$\mu_s=\mu_u+\mu_e\ . \tag{8d}$$

From the overall charge neutrality of the system, we have

$$2n_u=n_d+n_s+3n_e\ , \tag{9}$$

and finally the baryon-number density is given by

$$n_B=\tfrac{1}{3}(n_u+n_d+n_s)\ , \tag{10}$$

which remains constant throughout the processes.

We can solve Eqs. (8c)–(10) numerically to get p_{F_i} ($i=u$, d, s, or e) for different values of n_B and fixed B and m_s^0. Then the energy density of the SQ system is given by

$$\epsilon_{SQ}=\sum_{i=u,d,s}\frac{3}{\pi^2}\int_0^{p_{F_i}}e_i(p,n_B)p^2dp+\frac{P_{F_e}^4(n_B)}{4\pi^2}\ , \tag{11}$$

where $e_i(p,n_B)$ is the single-particle energy of the flavor i, which is given by

$$e_i=[p^2+m_i^2(n_B)+\Delta_i(p,n_B)]^{1/2} \tag{12}$$

and

$$\Delta_i(p,n_B)=\int_0^p\left[\frac{6B\pi^2C_i}{p^4}\left[\frac{p^2-m_i^2}{p^2+m_i^2}\mu_i(n_B)+m_i(n_B)\right]\right]dp\ . \tag{13}$$

Then the pressure is given by

$$P_{SQ}(n_B)=3n_B\sum_{i=u,d,s}\mu_iC_i+\mu_en_e-\epsilon_{SQ}\ . \tag{14}$$

Hence for a given n_B, one can express $P_{SQ}=P_{SQ}(\epsilon_{SQ})$ for fixed B and m_s^0, which is the equation of state for SQ matter at $T=0$. Now the equation of state should locally satisfy the restriction imposed by special relativity; the energy density ϵ_{SQ} should be positive and the speed of sound

$$c_s^2=\left|\frac{\partial P}{\partial\epsilon}\right| \tag{15}$$

should not exceed the speed of light.

For large values of n_B it becomes constant for both ud and uds (SQ) matter and is $\sim 1/\sqrt{3}$.

Using this model of confinement, we have investigated the stability of noninteracting SQ matter at zero temperature (see Ref. 5). We have seen that SQ matter may be an absolute ground state of matter only at very high density ($\sim 8n_0$). Furthermore, to make the nonstrange quark mattcr (ud) just unbound (ϵ_{ud}/n_B slightly greater than M_N), we have taken $B^{1/4}=265$ MeV (see the solid curves of Fig. 1 in Ref. 5). Similarly we have chosen $B^{1/4}=235$ MeV, for which the ud matter becomes just bound (see the dotted curves of Fig. 1 in Ref. 5).

In the case of interacting SQ matter, to reproduce the qualitative nature of the solid curve (as mentioned above), we have to take $B^{1/4}\approx 197$ MeV for which the solid curves are almost reproduced and the interacting non-

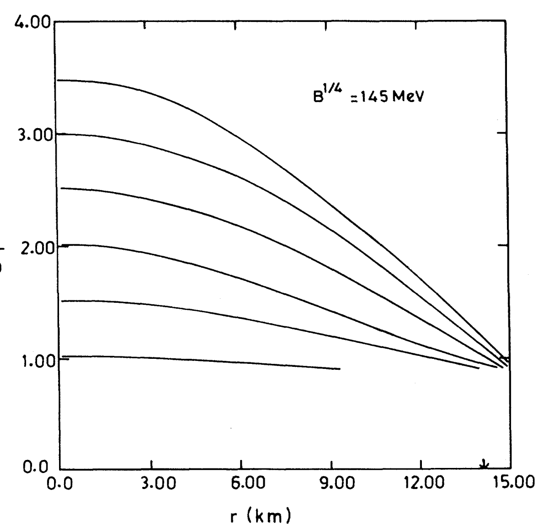

FIG. 1. Density profiles for different central densities with $B^{1/4}=145$ MeV and $m_s^0=150$ Mev. Arrow indicates the radius of the star.

strange quark matter becomes just unbound. The density for which ϵ_{uds}/n_B becomes minimum (of course $< M_N$) is $\sim 5n_0$. Since in the present context the dotted curves are not so interesting, we have not studied the stability of interacting nonstrange quark matter.

To study some global properties of a SQ star we consider it to be a spherically symmetric object, which corresponds to a nonrotating SQ star, and its stability is governed by the general-relativistic equation of hydrostatic equilibrium for a spherical configuration of strange matter, which is given by the Tolman-Oppenheimer-Volkov equation[14]

$$\frac{dP}{dr} = -\frac{Gm(r)\rho(r)}{r^2}\left[1+\frac{4\pi r^3 P(r)}{m(r)c^2}\right]\frac{\left[1+\frac{P(r)}{\rho(r)c^2}\right]}{\left[1-\frac{2Gm(r)}{rc^2}\right]} \tag{16}$$

and the subsidiary condition

$$\frac{dm}{dr} = 4\pi r^2\rho(r) , \tag{17}$$

where G is the gravitational constant, $\rho(r)$ is the local mass density, $P(r)$ is the pressure, and $m(r)$ is the mass of a spherical object of radius r and uniform density $\rho(r)$. Using the equation of state mentioned before these two equations can be solved numerically for a given central density ρ_c (hence P_c) and fixed values of B and m_s^0 and obtain $\rho=\rho(r)$, $P=P(r)$. The radius of a SQ star is determined by $P(r=R)=0$ and the total mass $M=m(r=R)$. Repeating the same exercise for different ρ_c (for fixed B and m_s^0) we obtain different density profiles $\rho(r)$ which are shown in Fig. 1. Since the equation of state is a function of B (for fixed m_s^0) only, the minimum density ρ_s at the surface for which $P=P_s=0$ is independent of the central density ρ_c. From this calculation we can also get $M=M(\rho_c)$ and $R=R(\rho_c)$. M first increases with ρ_c ($dM/d\rho_c>0$), reaches a maximum value $M_{\max}$, and then falls slowly. The central density corresponding to the maximum value of M is the acceptable largest value for ρ_c, above which $dM/d\rho_c<0$ and the system becomes unstable. Figure 2 shows the variation of M with R for three different values of B ($B^{1/4}=145$, 180, and 200 MeV). For the sake of comparison we have plotted the M-R curve for a neutron star also using the Bethe-Johnson equation of state.[14] The qualitative nature of the M-R curve for a SQ star in this model is exactly the same as that obtained from bag-model calculations.[2,3] Unlike a neutron star, in the case of a SQ star there is no lower limit for the radius R. For $M \ll M_\odot$ the major role in the stability of a SQ star is played by the QCD interaction, unless the surface effect becomes more important as in the case of strangelets.[15] In this case,

$$R \sim M^{1/3} . \tag{18}$$

As M increases and becomes $>1M_\odot$, gravity starts to dominate over QCD; the curve bends upward and ultimately it becomes gravitationally unstable.

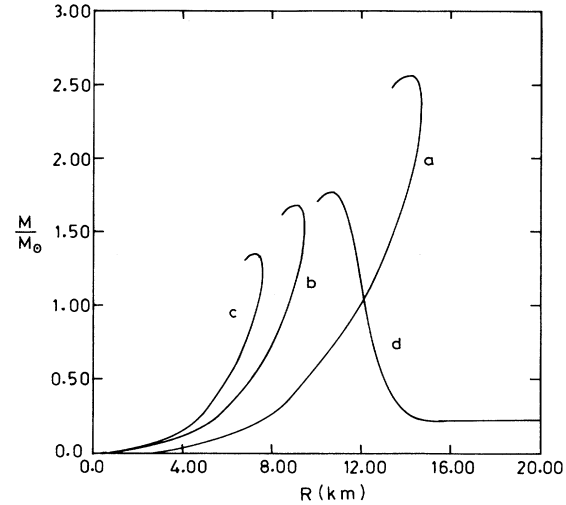

FIG. 2. M-R curves for a strange star: curve (a) $B^{1/4}=145$ MeV, (b) $B^{1/4}=180$ MeV, and (c) $B^{1/4}=200$ MeV and $m_s^0=150$ MeV, whereas curve (d) is for a neutron star.

Although the results obtained for SN 1987A are reported to be wrong,[16] one can calculate the rotational frequencies of a SQ star using the relation

$$\Omega = 2.4\times 10^5\left[\frac{M}{M_\odot}\Big/\frac{R^3}{(\mathrm{km})^3}\right]^{1/2}\ \mathrm{sec}^{-1} , \tag{19}$$

which gives $\Omega=1.023\times 10^4\ \mathrm{sec}^{-1}$ (for $B^{1/4}=145$ MeV, $\rho_c=4\rho_0$, $M_{\max}=2.55M_\odot$, and $R=14.05$ km), $\Omega=1.15\times 10^4\ \mathrm{sec}^{-1}$ (for $B^{1/4}=180$ MeV, $\rho_c=10.69\rho_0$, $M_{\max}=1.664M_\odot$, and $R=8.982$ km), and $\Omega=1.443\times 10^4\ \mathrm{sec}^{-1}$ (for $B^{1/4}=200$ MeV, $\rho_c=16.5\rho_0$, $M_{\max}=1.34M_\odot$, and $R=7.178$ km).

Another important quantity for a SQ star is the adiabatic index Γ, given by

$$\Gamma = \frac{P+\rho c^2}{Pc^2}\frac{dP}{d\rho} , \tag{20}$$

which determines the stability of a gaseous sphere in general relativity, and one can show that

$$\Gamma > \tfrac{4}{3} + k\frac{\mathcal{R}_S}{R} , \tag{21}$$

where $\mathcal{R}_S=2GM/c^2$ is the Schwarzschild radius and k is a constant which depends on the parameters of the equilibrium configuration. For large ρ, $\Gamma\sim\frac{4}{3}$ and there is a minimum value of $\rho=\rho_s$, the surface value below which Γ becomes infinitely large.

Since the baryon density decreases from $n_B^{(c)}$ at the center ($r=0$) to $n_B^{(s)}$ at the surface ($r=R$), in this case the self-consistent solutions of Eqs. (8)–(10) give $p_{F_i}=p_{F_i}(r)$ and as a result electron density $n_e=n_e(r)$. At the core ($r \ll R$), $\mu_e\sim 0$ and therefore $n_u\sim n_d\sim n_s$ (an almost flavor-symmetric mixture). The electron density is maximum at the surface but is very low compared to the baryon-number density ($\sim 10^{-3}n_0$). Since the SQ star has a very sharp periphery, the electron layer at the surface is very thin (~ 100 fm) and is strongly bound to the positively charged-quark core by the Coulomb potential $V_C^{(s)} \approx 3\mu_e/4 \approx 27$ MeV, which is very large and is also a function of the equation of state considered (i.e., depends on B and m_s^0).

Since the electronic crust is very thin and the quark matter inside the star behaves like a superconductor (charged quarks are fermions and form Cooper pairs), as a result the vortex magnetic fields will get pushed towards the crust region of the star almost instantaneously. In the case of a SQ star, the time scale for Ohmic dissipation of the magnetic field, given by[17]

$$\tau = \frac{4\sigma\Delta^2}{\pi c^2}, \tag{22}$$

is extremely short, $\sim 10^{-13}$ sec, where $\sigma \sim 10^{25}$ sec^{-1}, the electrical conductivity of crust matter, and $\Delta \sim 100$ fm is the thickness of the electronic layer, which shows that if a SQ star exists at all its surface magnetic field should be very small.

Thus we can conclude that the equation of state of strange quark matter obtained by using a relativistic version of the Landau theory of a Fermi liquid with a density-dependent quark-mass model of confinement can explain the stability of a SQ star and can also predict some theoretical estimate of the parameters which can be measured experimentally (if they exist).

[1]E. Witten, Phys. Rev. D **30**, 272 (1984).
[2]C. Alcock and E. Farhi, Phys. Rev. D **32**, 1273 (1985); C. Alcock, E. Farhi, and A. Olinto, Astrophys. J. **310**, 261 (1986).
[3]P. Haensel, J. L. Zdunik, and R. Schaeffer, Astron. Astrophys. **160**, 251 (1986).
[4]A. Chodos *et al.*, Phys. Rev. D **9**, 3471 (1974); T. DeGrand *et al.*, *ibid.* **12**, 2060 (1975).
[5]S. Chakrabarty, S. Raha, and B. Sinha, Phys. Lett. B **229**, 112 (1989).
[6]J. Applegate and C. Hogan, Phys. Rev. D **31**, 3037 (1985).
[7]M. Plümer, S. Raha, and R. M. Weiner, Phys. Lett. **139B**, 198 (1984).
[8]G. Baym and S. A. Chin, Phys. Lett. **62B**, 241 (1976).
[9]A. Ukawa, Nucl. Phys. **A498**, 227c (1989).
[10]G. N. Fowler *et al.*, Z. Phys. C **9**, 271 (1981); K. A. Olive, Nucl. Phys. **B196**, 483 (1981); D. H. Boal *et al.*, Phys. Rev. D **26**, 3245 (1982).
[11]H. B. Nielsen and A. Patkos, Nucl. Phys. **B195**, 137 (1982); A. Schuh and H. J. Pirner, Phys. Lett. B **173**, 19 (1986).
[12]W. Broniowski *et al.*, Phys. Rev. D **41**, 285 (1990).
[13]J. C. Pati and A. Salam, in *Proceedings of the International Neutrino Conference 1976,* Aachen, West Germany, edited by H. Faissner, H. Reithler, and P. Zerwas (Vieweg, Braunschweig, 1977), p. 589.
[14]S. L. Shapiro and S. A. Teukolsky, *Black Holes, White Dwarfs and Neutron Stars, the Physics of Compact Objects* (Wiley, New York, 1983).
[15]S. Chakrabarty, Phys. Scr. (to be published).
[16]G. Christopher Anderson, Nature **343**, 679 (1990); New Sci. **125**, 20 (1990).
[17]E. Flowers and M. A. Ruderman, Astrophys. J. **215**, 302 (1977).