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BRIEF REPORTS

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Energy spectrum of a uniformly accelerated detector at finite temperature

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The energy spectrum of the zero-point field of a massless scalar field in the presence of both uniform acceleration and temperature is obtained by the use of the Wightman function. It has an intricate form. However, in two limiting cases, (a) the temperature is approaching zero and (b) the acceleration is approaching zero, it has physically appropriate forms.

In the last decade there has been considerable investigation of quantum phenomena in curved space-times or in uniformly accelerated frames. In particular, a uniformly accelerated particle detector has attracted much attention in connection with Hawking's black-hole radiation effect.[1,2] As is well known, Unruh has shown that, in the case of a free, massive scalar field, the thermal bath of Rindler quanta in the Minkowski vacuum excited a model particle detector at rest in Rindler space-time.[3] A uniformly accelerating observer in the Minkowski vacuum would see a thermal radiation; i.e., the response function of a uniformly accelerated particle detector is identical with that of the same detector at rest in a thermal bath at temperature $T_a = a\hbar/2\pi c k_B$, where a is the proper acceleration of the detector and k_B is Boltzmann's constant.[4,5]

Recently, it has been demonstrated that the gravitational mass and the inertial mass are different at finite temperature.[6] In the nonrelativistic and weak-field limit, the acceleration induced by the Newtonian potential is different for particles of different mass. Even in the relativistic case,[7] the thermal corrections to the gravitational acceleration in the weak-field limit are not a function of the kinetic energy of the test body, and a constant shift of the ratio of gravitational to inertial mass is produced, depending only on temperature T and the rest mass of the particle at $T=0$.

In this paper we follow the zero-point field method[8] and extend this to a more realistic problem: the response function of a particle detector in the presence of both constant acceleration a and temperature T. We restrict our attention to massless scalar fields in four-dimensional space-time. From now on we will take units with $\hbar = c = k_B = 1$.

At finite temperature the thermal Wightman function[9] evaluated at two points $x(\tau+\frac{1}{2}\sigma)$ and $x'(\tau-\frac{1}{2}\sigma)$ on the particle detector's world line (τ is the proper time) is given by the form

$$\begin{aligned} D_\beta^\pm(x,x') &= {}_\beta\langle 0|\phi(x(\tau\pm\tfrac{1}{2}\sigma))\phi(x'(\tau\mp\tfrac{1}{2}\sigma))|0\rangle_\beta , \\ &= \sum_{n=-\infty}^{\infty} \frac{1}{4\pi^2}[|\mathbf{x}-\mathbf{x}'|^2-(t+in\beta-t'\mp i\epsilon)^2]^{-1} , \\ &= \int \frac{d^3k}{4\pi^2 2\omega_k}(1+f_k)e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')-i\omega(x_0-x_0')}+f_k e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')+i\omega(x_0-x_0')} , \end{aligned} \tag{1}$$

where $\phi(x)$ is a massless scalar field, $|0\rangle_\beta$ is the thermal vacuum in Minkowski space-time, and

$$f_k = \frac{1}{e^{\beta\omega_k}-1}, \quad \beta = \frac{1}{k_B T} . \tag{2}$$

The world line of uniformly accelerated detector can be represented as

$$t = \frac{1}{a}\sinh a\tau, \quad x = \frac{1}{a}\cosh a\tau, \quad y = z = 0 , \tag{3}$$

where a is the magnitude of the proper acceleration in the frame of the detector. The thermal Wightman functions for a uniformly accelerated detector are obtained by inserting Eq. (3) into (1):

$$D_\beta^{\pm}(\tau+\tfrac{1}{2}\sigma,\tau-\tfrac{1}{2}\sigma)=\frac{-a^2}{16\pi^2\sinh^2\frac{a}{2}(\sigma\mp i\epsilon)}+\frac{a}{i16\pi^2\beta\sinh a\tau}\sum_{\substack{n=-\infty,\\ n\neq 0}}^{\infty}\frac{1}{n}\left[\frac{1}{\pm\sinh\frac{a}{2}(\sigma\mp i\epsilon)+\frac{ia\beta}{2}e^{a\tau}n}-\frac{1}{\pm\sinh\frac{a}{2}(\sigma\mp i\epsilon)+\frac{ia\beta}{2}e^{-a\tau}n}\right]. \quad (4)$$

The particle density $f_\beta(\omega,\tau)$ and the energy density per mode $(de/d\omega)_\beta$ are given as[8]

$$f_\beta(\omega,\tau)=\frac{1}{(2\pi)^2\omega}[\tilde{D}_\beta^+(\omega,\tau)-\tilde{D}_\beta^-(\omega,\tau)] , \quad (5)$$

$$\left[\frac{de}{d\omega}\right]_\beta=\frac{\omega^2}{\pi}[\tilde{D}_\beta^+(\omega,\tau)+\tilde{D}_\beta^-(\omega,\tau)] , \quad (6)$$

where $\tilde{D}_\beta^{\pm}$ are the Fourier transforms of the thermal Wightman functions and are defined as

$$\tilde{D}_\beta^{\pm}=\int_{-\infty}^{\infty}d\sigma\, e^{i\omega\sigma}D_\beta^{\pm}(\tau+\tfrac{1}{2}\sigma,\tau-\tfrac{1}{2}\sigma) , \quad (7)$$

where ω is the frequency measured by a detector with proper time τ. The Fourier transforms of the thermal Wightman functions for a uniformly accelerated detector are given by

$$\tilde{D}_\beta^{\pm}(\omega,\tau)=\int_{-\infty}^{\infty}ds\, e^{iWs}\left[\frac{-a}{8\pi^2\sinh^2(s\mp i\epsilon)}+\frac{1}{i8\pi^2\beta\sinh a\tau}\sum_{\substack{n=-\infty,\\ n\neq 0}}^{\infty}\frac{1}{n}\left[\frac{1}{\pm sihn(s\mp i\epsilon)+ib_1}-\frac{1}{\pm\sinh(s\mp i\epsilon)+ib_2}\right]\right] , \quad (8)$$

where $W=2\omega/a$, $s=(a/2)\sigma$, $b_1=(a\beta/2)e^{a\tau}n$, and $b_2=(a\beta/2)e^{-a\tau}n$. We can perform integration (8) and obtain, after some calculations,

$$\tilde{D}_\beta^+(\omega,\tau)=\frac{\omega}{2\pi}\left[\frac{e^{2\pi\omega/a}}{e^{2\pi\omega/a}-1}\right]+\frac{1}{4\pi\beta\sinh a\tau}\frac{e^{-\pi W}}{1-e^{-2\pi W}}\sum_{n=1}^{\infty}\frac{1}{n}[f(b_1)-f(b_2)] , \quad (9)$$

where

$$f(b_i)=\begin{cases}\exp[-\tfrac{1}{2}\pi W+iW\ln(b_i-\sqrt{b_i^2-1})-\tfrac{1}{2}\pi i]/\sqrt{b_i^2-1}, & b_i>1 \text{ for } i=1,2 ,\\ \dfrac{e^{-W\arcsin(b_i)}}{\sqrt{1-b_i^2}}, & 0<b_i<1\end{cases} . \quad (10)$$

Similarly, we obtain

$$\tilde{D}_\beta^-(\omega,\tau)=\tilde{D}_\beta^+(\omega,\tau)-\frac{\omega}{2\pi} . \quad (11)$$

Hence, the particle number density (5) at a finite temperature is given by

$$f_\beta(\omega,\tau)=\frac{1}{(2\pi)^2\omega}[\tilde{D}_\beta^+(\omega,\tau)-\tilde{D}_\beta^-(\omega,\tau)]=\frac{1}{(2\pi)^3}=f_0(\omega,\tau) , \quad (12)$$

where $f_0(\omega,\tau)$ is the particle number density at zero temperature. This implies that there are no newly created particles. Using Eqs. (6), (9), and (11), we obtain the energy density of a uniformly accelerated detector:

$$\left[\frac{de}{d\omega}\right]_\beta=\frac{\omega^3}{2\pi^2}\left[\frac{1}{2}+\frac{1}{e^{2\pi\omega/a}-1}+\frac{1}{2\omega\beta\sinh a\tau}\frac{e^{-2\pi\omega/a}-1}{1-e^{-4\pi\omega/a}}\sum_{n=1}^{\infty}\frac{1}{n}[f(b_1)-f(b_2)]\right] . \quad (13)$$

The first term in (13) is the energy of the zero-point field when both acceleration and temperature are zero; the second term is exactly what one would observe in a thermal bath with temperature $T_a=a/2\pi$, and the third term has an intricate form, which contains the effect due to both acceleration and finite temperature T in the Minkowski space-time. Therefore, the energy density does not seem thermal.

We consider two limiting cases: (a) the acceleration a is approaching zero; (b) the temperature T is approaching zero.

In the case of small acceleration ($a \rightarrow 0$), the energy density becomes

$$\left[\frac{de}{d\omega}\right]_\beta \rightarrow \frac{\omega^3}{\pi^2}\left[\frac{1}{2}+\frac{1}{e^{\beta\omega}-1}+\frac{a^2}{24\omega}\{4\tau^2\beta^2 f^{(2)}(\beta\omega)\omega^3+[-\beta^3 f^{(3)}(\beta\omega)-12\tau^2\beta f^{(1)}(\beta\omega)]\omega^2 + 6\beta^2 f^{(2)}(\beta\omega)\omega - 6\beta f^{(2)}\} + O(a^4)\right], \quad (14)$$

where $f^{(n)}(x)$ is defined as

$$f^{(n)}(x)=(-1)^n\left[\frac{d}{dx}\right]^n\left[\frac{1}{e^x-1}\right]. \quad (15)$$

In the other limiting case, $\beta \rightarrow \infty$ ($T \rightarrow 0$),

$$\left[\frac{de}{d\omega}\right]_\beta \rightarrow \frac{\omega^3}{\pi^2}\left[\frac{1}{2}+\frac{1}{e^{2\pi\omega/a}-1}+\frac{T^2}{a\omega}\operatorname{arccosh}\frac{\pi}{a}\omega\frac{\sinh(a+i2\omega)\tau}{\sinh a\tau}\sum_{n=1}^{\infty}\frac{\exp(-i2\omega/a\ \ln a\beta n-\frac{1}{2}\pi i)}{n^2}\right]. \quad (16)$$

Equations (14) and (16) give expected results. The second term in (14) is a thermal distribution for a boson system at temperature $T(=1/\beta)$, in Minkowski space-time. The third term in (14) shows that small acceleration a of the detector affects the energy density through an a^2 term. Note that the contribution of the proper acceleration a on the energy is not a Planckian form. The second term in (16) is a Planckian distribution at temperature T_a, due to uniform acceleration. The third term includes all the effects of small temperature.

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[1] S. W. Hawking, Nature (London) **248**, 30 (1974); Commun. Math. Phys. **43**, 199 (1975).
[2] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Spaces* (Cambridge University Press, Cambridge, England, 1982).
[3] W. Rindler, Am. J. Phys. **34**, 1174 (1966).
[4] W. G. Unruh, Phys. Rev. D **14**, 870 (1975).
[5] T. H. Boyer, Phys. Rev. D **21**, 2137 (1980); D. W. Sciama, P. Candelas, and D. Deutsch, Adv. Phys. **30**, 327 (1981).
[6] J. F. Donoghue, B. R. Holstein, and R. W. Robinett, Phys. Rev. D. **30**, 2561 (1984); **34**, 1208 (1986); Gen. Relativ. Gravit. **17**, 207 (1985).
[7] M. Gasperini, Phys. Rev. D **36**, 617 (1987).
[8] S. Hacyan, A. Sarmiento, G. Cocho, and F. Soto, Phys. Rev. D **32**, 914 (1985).
[9] C. Itzykson and J. B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).