πNN form factor for explaining sea-quark distributions in the nucleon

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We reinvestigate a soft πNN form factor with a monopole cutoff parameter $\Lambda_1 < 0.5$ GeV obtained by Frankfurt, Mankiewicz, and Strikman for explaining sea-quark distributions in the nucleon. It is found that their estimate is small in comparison to the limit $\Lambda_1 < 0.65-0.7$ GeV estimated in this investigation. Studying processes involving πNN and $\pi N\Delta$ vertices, we find that the limit for the dipole form factor is $\Lambda_2 < 0.95$ GeV (which corresponds to $\Lambda_1 < 0.6$ GeV). Therefore, a typical πNN form factor with $\Lambda_1 \sim 0.6$ GeV in quark models could be consistent with deep-inelastic experimental data at this stage. However, it is softer than a πNN form factor with $\Lambda_1 \sim 1$ GeV widely used in nuclear physics. Using the cutoff Λ_2 =0.95 GeV, we investigate pionic contributions to a $SU(2)_f$ -breaking distribution $\bar{u}(x) - \bar{d}(x)$ in the nucleon. We find that the pionic contributions to $\overline{u}(x) - d(x)$ are negative and a contribution to the deviation from the Gottfried sum rule is estimated to be -0.041 , which could explain a part of the discrepancies indicated by recent New Muon Collaboration experiments.

Using Sullivan's formalism¹ for the leptonic scattering processes in Figs. 1(a) and 1(b), Thomas² suggested that the size of the πNN form factor could be estimated by comparing theoretical results with experimental seaquark distributions in the nucleon. Using this approach, Frankfurt, Mankiewicz, and Strikman³ (FMS) suggested that the πNN form factor for explaining deep-inelastic data is much softer than those usually used in nuclear physics. They set a limit for the momentum cutoff parameter in the monopole form factor as $\Lambda_1 < 0.5$ GeV, which is much smaller than a "normal" value $\Lambda_1 \sim 1 \text{ GeV}$ in nuclear physics. There is no definite agreement what the cutoff is in different nuclear-physics processes. However, if their finding is right, we have a lot of problems in muclear physics,⁴ because a cutoff of 1 GeV or larger is by E615

essential for explaining some fundamental nuclear-

physics phenomena,⁵ for example, deuteron properties the cutor

(nuclear tensor force,⁶ 1.0 < Λ essential for explaining some fundamental nuclearphysics phenomena,⁵ for example, deuteron properties (nuclear tensor force,⁶ 1.0 < Λ_1 < 1.4 GeV) and *N-N* scattering data^{7,8} (0.9 < Λ ₁ < 1.4 GeV). Furthermore, the **FMS** soft form factor is even softer than those of cloudy⁹ (chiral) bag $(\Lambda_1 \approx 0.6$ GeV) and flux-tube^{10,11} $(0.7 < \Lambda_1 < 1.0$ GeV) quark models.

Because the FMS conclusion is very important in nuclear physics and is quite different from the usual expectations, we try to reexamine their analysis of deepinelastic experiments by using new data obtained by the E615 Collaboration at the Fermi National Laboratory

 $f_{\pi}^{(\pi)}$

and a recent parametrization by Harriman, Martin, Roberts, and Stirling¹² (HMRS). The FMS analysis uses parametrized sea-quark distributions given by Eichten, Hinchliffe, Lane, and Quigg¹³ (EHLQ), which are based on CERN-Dortmund-Heidelberg-Saclay (CDHS) (1983) lata.¹⁴ In comparison to other experimental data, such as the CERN-Hamburg-Amsterdam-Rome-Moscow CHARM) neutrino experiment,¹⁵ deep-inelastic lepton scattering experiments by $SLAC-MIT¹⁶$ and the European Muon Collaboration¹⁷ (EMC), and a neutrino experiment by Caltech-Columbia-Fermilab-Rochester- $Rockefeller¹⁸$ (CCFRR), the EHLQ sea-quark distribution is believed to be underestimated.¹³ This underesti $mate¹⁹$ is also indicated by a recent Drell-Yan experiment by E615 Collaboration²⁰ and the HMRS parametrization. Therefore, it is not appropriate to set an upper limit for the cutoff parameter Λ_1 based on underestimated seaquark distributions.

The pionic contribution to an antiquark distribution in the nucleon is given^{$1-3$} by a pion momentum distribution $f_{\pi}(y)$ in an infinite-momentum frame and an antiquark distribution in the pion, $\overline{q}_{\pi}(x, Q^2)$:

$$
x\overline{q}_N(x,Q^2) = \int_x^1 dy f_\pi(y) \frac{x}{y} \overline{q}_\pi(x/y,Q^2) , \qquad (1)
$$

where $f_{\pi}(y) = f_{\pi}^{(\pi NN)}(y) + f_{\pi}^{(\pi N\Delta)}(y)$ and they are given by

$$
N(N)(y) = 3 \frac{g_{\pi NN}^2}{16\pi^2} y \int_{-\infty}^{t_{\text{max}}} dt \frac{-t}{(-t + m_{\pi}^2)^2} [F_{\pi NN}(t)]^2,
$$
\n(2a)

$$
f_{\pi}^{(\pi NN)}(y) = 3 \frac{g_{\pi NN}^2}{16\pi^2} y \int_{-\infty}^{t_{\max}} dt \frac{-t}{(-t + m_{\pi}^2)^2} [F_{\pi NN}(t)]^2 ,
$$

$$
f_{\pi}^{(\pi N\Delta)}(y) = 2 \frac{1}{24\pi^2} \left[\frac{g_{\pi N\Delta}}{2m_N} \right]^2 y \int_{-\infty}^{t_{\max}'} dt \frac{[F_{\pi N\Delta}(t)]^2}{(-t + m_{\pi}^2)^2} [(m_N + m_\Delta)^2 - t] \left[\frac{(m_N^2 - m_\Delta^2 - t)^2}{4m_\Delta^2} - t \right].
$$
 (2b)

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FIG. 1. Pionic contributions in deep-inelastic scattering from (a) πNN process and (b) $\pi N\Delta$ process.

In the above equations, t is the four-momentum square of the pion; t_{max} and t'_{max} are the maximum t given by $t_{\text{max}} = -m_N^2y^2/(1-y)$ and $t'_{\text{max}} = m_N^2y - m_N^2y/(1-y)$; and the πNN coupling constant is given by $g_{\pi NN} = 13.5$. The $\pi N\Delta$ coupling constant is given by the Δ decay width (Γ_{Λ}) by^{1,21}

$$
g_{\pi N\Delta} = 2m_N \left[\frac{12\pi m_\Delta \Gamma_\Delta}{|\mathbf{p}_\pi|^3 (m_N + E_N)} \right]^{1/2}, \qquad (3)
$$

where $|\mathbf{p}_{\pi}| = 225.4$ MeV and $E_N = 966.6$ MeV. Equations (2b) and (3) are derived by taking the $\pi N\Delta$ coupling as $g_{\pi N\Delta}/(2m_N)\tilde{\phi}^*_{\pi} \cdot \tilde{T}F_{\pi N\Delta}(p_{\pi}^2)\bar{\psi}^{\mu}_{\mu}p_{\mu}^{\nu} \psi_N$, where ψ^{μ}_{Δ} is the Rarita-Schwinger spinor²² and \tilde{T} is the transition iso-Rarita-Schwinger spinor²² and \tilde{T} is the transition iso-
spin.¹¹ We assume $F_{\pi N\Delta}(t) = F_{\pi NN}(t)$ in Eq. (2b) for simspin.¹¹ We assume $F_{\pi N\Delta}(t) = F_{\pi NN}(t)$ in Eq. (2b) for simplicity. The isospin factors,¹¹ $|\tilde{\phi}_{\pi}^* \cdot \tilde{\tau}|^2 + |\tilde{\phi}_{\pi}^* \cdot \tilde{\tau}|^2 = 3$ in the πNN process by taking the proton as the initial nucleon and $|\tilde{\phi}_{\pi^+}^* \cdot \tilde{T}|^2 + |\tilde{\phi}_{\pi^0}^* \cdot \tilde{T}|^2 + |\tilde{\phi}_{\pi^-}^* \cdot \tilde{T}|^2 = 2$ in the $\pi N\Delta$ process, are included in Eqs. (2) by assuming $\overline{q}_{\pi^+} = \overline{q}_{\pi^0} = \overline{q}_{\pi^-}$ in Eq. (1). This assumption is satisfied as explained in the following paragraphs, because the seaexplained in the following paragraphs, because the sea-
quark distribution, $\overline{q} = (\overline{u} + \overline{d})/2 - \overline{s}$, is studied in this investigation. Equation (1) indicates that the antiquark distribution $[\bar{q}_N(x, Q^2)]$ in the nucleon is a convolution of a probability $[f_\pi(y)]$ of finding a pion with a fraction y of the nucleon momentum with a probability $[\bar{q}_\pi(x/y, Q^2)]$ of finding an antiquark with a fraction x of the pion momentum.

We now discuss what the appropriate distribution (\bar{q}_N) is for investigating the pionic effect. There are significant contributions to the strange-quark distributions from the gluon splitting into a $q\bar{q}$ pair at large Q^2 . Since the sea/valence quark ratios in the nucleon are measured²⁰ in

the Q^2 range, $20 < Q^2 < 70$ GeV², the gluon-splitting process is significant in nucleon's sea-quark distributions. In order to learn about the πNN vertex, we should exclude this contribution. Assuming that the sea quarks from the gluon splitting are flavor independent at large Q^2 , we try to investigate an antiquark distribution, $x\overline{q}_N \equiv x(\overline{u})$ $+\overline{d}$)/2-xs in Eq. (1), where we expect that the gluonic splitting contribution is subtracted out. Then, the $x\overline{q}_N$ given by this equation could be partly identified as the pionic contribution to the antiquark distribution given by Eq. (1). Namely, we are trying to attribute the $SU(3)_{\text{flavor}}$ breaking of the sea to mesonic effects at large Q^2 .

We need a pion structure function and a πNN form factor for evaluating the pionic contribution in Eq. (1). There exist three experimental data $NA3^{23}$, NA10²⁴, E615²⁵) on the pion structure function, all of which have been measured by Drell-Yan experiments. There is, however, a disagreement of the order of 20% among these experimental data. The major reason for this discrepancy lies in the uncertainties in the normalization factor which strongly depends on the unmeasured pion structure functions at small x_{π} , as explained in Fig. 15 of Ref. 25. In order to show the effect of the uncertainties in the pion structure function, we use all three structure functions in our calculations. The antiquark distribution $x\overline{q}_{\pi} = x(\overline{u}+\overline{d})/2 - x\overline{s}$ for the pion is given by the valence-antiquark distribution²³⁻²⁵ as $x\bar{q}_{\pi^+} = x\bar{q}_{\pi^0} = x\bar{q}_{\pi^-} = xV_{\pi}/2$ (note $xV_{\pi} = F_{\pi}^v$ in Ref. 25) by using relations²⁶ $\overline{d}_{\pi^+} = \overline{u}_{\pi^-} = V_{\pi} + S_{\pi}, \ \overline{u}_{\pi^+} = \overline{d}_{\pi^-}$ $=S_{\pi}$, $\bar{u}_{\pi^0} = \bar{d}_{\pi^0} = V_{\pi}/2 + S_{\pi}$, and $S_{\pi} = \bar{S}_{\pi} = S_{\pi}$. We do not use Q^2 -evolved pion structure functions from $\langle Q^2 \rangle$ =25 GeV², at which they are measured, because our evolution program²⁷ for the nonsinglet distribution indicates that changes due to Q^2 evolution in the range $(20 < Q² < 70 \text{ GeV}²)$ are at most 10%. This is smaller than the differences (20%) in the pion-structure-function measurements among different groups. In the pion structure function, 2^{3-25} the SU(3)_f is assumed for sea quarks in the pion. Although it may seem contradictory to the $SU(3)_f$ -breaking physics which we try to investigate in the nucleon, we find this is not a problem in Eq. (1) in the following way. The nucleon's sea distribution at $x \sim 0.15$ is to be investigated and the pion momentum distribution is peaked around $y \sim 0.25$, so the pion structure function at $x/y \sim 0.6$ (essentially valence distribution) contributes to the nucleon's sea significantly, and the pion's sea does not contribute unless x is very small.

For the πNN form factor in nuclear physics, we often

For the *MNN* form factor in nuclear physics, we often
\nuse a phenomenological approach and parametrize it as
\nmonopole, dipole, and exponential forms:
\n
$$
F_{\pi NN}^{(1)}(t) = \frac{1 - m_{\pi}^2 / (\Lambda_1)^2}{1 - t / (\Lambda_1)^2},
$$
\n
$$
F_{\pi NN}^{(2)}(t) = \frac{[1 - m_{\pi}^2 / (\Lambda_2)^2]^2}{[1 - t / (\Lambda_2)^2]^2},
$$
\n(4)
\n
$$
F_{\pi NN}^{(0)}(t) = e^{(t - m_{\pi}^2) / (\Lambda_0)^2},
$$

where the form factors are normalized by $F_{\pi NN}(m_\pi^2)=1$. These form factors have different behaviors at large $|t|$; however, these differences are not particularly important in our research problem. The cutoff parameters for them could be related simply by $F_{\pi NN}^{(1)}(t_0) = F_{\pi NN}^{(2)}(t_0)$ $=F_{\pi NN}^{(0)}(t_0)=0.4$. Solving this equation, ²⁸ we have the following relation among various parameters:

$$
\Lambda_1 = 0.62\Lambda_2 = 0.78\Lambda_0 \ . \tag{5}
$$

If we relate the parameters in this way, antiquark distributions obtained from Eq. (1) are almost independent of the form of the πNN form factor as shown in Fig. 2.

We now show experimental results and compare them with our calculations. Three sets of "experimental data" are shown in Fig. 3. One is the EHLQ parametrization I,¹³ and the others are recent measurements (E615) of the sea/valence ratio,²⁰ $(u_s + d_s) / [2(u_v + d_v)]$ and the HMRS parametrization E^{12} The E615 data are modified to obtain $x\overline{q}_N$ as defined in this paper by $x\overline{q}_N$ = (sea/valence) $x(u_n + d_n) - x\overline{s}$. Here $x(u_n + d_n)$ and $x\overline{s}$ are taken from the HMRS-E parametrization at Q^2 given by the average invariant mass square of the muon pair, $\langle m_{\mu\nu}^2 \rangle$ = 11.1+164.0x GeV² in the E615 experiment.²⁰ We note that the EHLQ parametrization is much different from other two "experimental data." Namely, the E615 measurements of sea/valence and the HMRS-E distribution are 50% larger than the EHLQ-I values in $0.1 < x < 0.2$, even though they are larger than the EHLQ-I values by only 30% in the sea/valence ratios.²⁰ The enhancement (30% \rightarrow 50%) of the differences is due to the $(\text{strange } \text{sea})/(ud \text{ sea})$ ratio, $2\overline{s}/(\overline{u}+\overline{d})=0.43$ (at $Q^2=5 \text{ GeV}^2$).^{13,29} The 50% enhancement in $x\overline{q}_N$ could change results in estimating the πNN form factor, because FMS used the EHLQ parametrization to obtain Λ_1 < 0.5 GeV.

Using the pion structure functions measured by NA3, NA10, and E615 Collaborations and the monopole form factors with $\Lambda_1 = 0.5$, 1.0, 1.5 GeV, we obtain "theoretical" antiquark distributions [from the πNN process in Fig. 1(a)] shown in Fig. 3. We do not show theoretical results in the small-x region $(x < 0.1)$, because the convo-

FIG. 2. Antiquark distributions in the nucleon obtained by using different πNN form factors and the E615 pion structure function for the πNN process. The solid (dashed, dotted) curves are results from the monopole (dipole, exponential) form factor. The cutoff parameters Λ_2 and Λ_0 are given by $\Lambda_2 = 1.61 \Lambda_1$ and Λ_0 =1.28 Λ_1 [see Eq. (5)].

FIG. 3. Theoretical results for the πNN process with the monopole form factor are compared with experimental data. The solid (dashed, dotted) curves are results from using the E615 (NA10, NA3) pion structure function. The XXXXX (dashed-dotted) curve is the distribution by the EHLQ-I Experimental data by the E615 Collaboration (with modifications to obtain $x\bar{q}_N$ $=x[(\bar{u}+\bar{d})/2-\bar{s}]$) are shown with error bars.

ution model is problematic in such a small x region due o shadowing phenomena, 30 and the assumption of $SU(3)_f$ for sea-quark distributions in the pion is also not without problems, as discussed earlier. Figure (3) indicates that the present limit for the monopole cutoff parameter should be $\Lambda_1 < 0.8$ GeV in the region $(1.0 < x < 0.18)$ and $\Lambda_1 < 0.6$ GeV in 0. 18 $< x < 0.30$ by considering experimental errors. Taking an average of these limits, we have an estimate for the monopole cutoff parameter as Λ_1 < 0.7 GeV by the πNN process.

Results in Fig. 3 could be compared with the FMS results (Fig. 6 of their paper) as follows. Because they investigated $\overline{d} - \overline{s}$ distribution, the isospin times pion-structure-function factor is given by $|\tilde{\phi}_{\pi^+}\cdot\tilde{\tau}|^2[\vec{d} - \bar{s}]_{\pi^0}\cdot\tilde{\tau}|^2[\vec{d} - \bar{s}]_{\pi^0} = \frac{5}{2}V_{\pi}$. On the other hand, we have $\left[\tilde{\phi}_{\pi^+}\right]_{\pi^0}^{(1)} = \frac{2}{3}\pi^0$, $\frac{2}{3}\pi^0$, π^+ , $\frac{2}{3}\pi^0$, and is larger by 67% and $\overline{d}-\overline{s}=(\overline{u}+\overline{d})/2-\overline{s}$ for the nucleon if we use the EHLQ or HMRS parametrization, we could find tighter restricion for the cutoff parameter.³¹ The theoretical curve with Λ_1 =0.5 GeV multiplied by $\frac{5}{3}$ agrees with the EHLQ parametrization in Fig. 3; however, it is still below the other two sets of experimental data. Therefore, a reasonable estimate by the $\bar{d} - \bar{s}$ is $\Lambda_1 < 0.65$ GeV in $0.10 < x < 0.18$ which is larger than the FMS estimate, Λ_1 < 0.5 GeV in 0.1 < x < 0.2.

Next, we add contributions from the $\pi N\Delta$ process shown in Fig. 1(b). As it is obvious from Eq. (2b), the integral is logarithmically divergent³ if we take the monopole form factor. Therefore, results with the dipole form factor are shown in Fig. 4. We find that theoretical curves with the NA3 pion structure function agree with the FMS results in Fig. ⁵ of Ref. 3 within 10-15%. However, the limit for Λ_2 is slightly different from their conclusion because of the underestimation of the EHLQ pa-

FIG. 4. Summations of theoretical results for the πNN process and results for the $\pi N\Delta$ with the dipole form factor are compared with experimental data. See Fig. 3 for notations.

rametrization. Reasonable estimates are Λ_2 < 1.05 GeV in $0.10 < x < 0.18$ and $\Lambda_2 < 0.90$ GeV in $0.18 < x < 0.30$ by considering experimental errors. Therefore, an estimate for the limit is Λ_2 < 0.95 GeV by the πNN and $\pi N\Delta$ processes. In order to compare with the limit for the monopole cutoff obtained from Fig. 3, we simply use Eq. (5) to convert the monopole cutoff even though it is not exactly right because of the logarithmically divergent problem. Then, we obtain $\Lambda_1 < 0.6$ GeV by the πNN and $\pi N\Delta$ processes.

In this investigation, we obtain upper limits of the cutoff parameters as

 Λ_1 <0.6 (0.7) GeV, Λ_2 <0.95 (1.1) GeV, (6) Λ_0 < 0.75 (0.9) GeV,

by studying the πNN and $\pi N\Delta$ process contributions to sea-quark distributions. In the above equation, the values in the parentheses are the results obtained by studying only the πNN process, and Eq. (5) is used to relate different cutoff parameters. A typical πNN form factor with Λ_1 ~ 0.6 GeV in quark models^{9,11} and soft form factors used in πN interactions³² could be consistent with deep-inelastic experimental data at this stage. However, it is softer than a πNN form factor with $\Lambda_1 \sim 1$ GeV widely used in nuclear physics. Especially, it is much softer than the form factor with $\Lambda_1 \sim 1.4$ GeV used in oneboson-exchange-potential models. Tighter restriction for the size of the πNN form factor could be obtained by investigating other processes involving ωNN , ρNN , etc., vertices, which are our next projects to be investigated.

Now, we discuss briefly about the pionic contributions to a SU(2)_f-breaking distribution $\bar{u}(x) - \bar{d}(x)$. Using the dipole cutoff Λ_2 =0.95 GeV, which gives a good fit to the data in the range $(0.10 < x < 0.15)$ in Fig. 4, we estimate $SU(2)_f$ -breaking effects, because there is a recent interest

FIG. 5. Pionic contributions to the $SU(2)_f$ breaking in the nucleon's sea quarks. The distributions $x(\bar{u}-\bar{d})$ are calculated by using the dipole form factor with the cutoff, $\Lambda_2 = 0.95$ GeV. and the E615 pion structure function. πNN indicates contributions from the process in Fig. 1(a) and $\pi N\Delta$ indicates contributions from the process in Fig. 1(b). $\pi NN + \pi N\Delta$ indicate the summation of these values. Curves in $0 < x < 0.1$ are shown by dashed curves because of problems associated with the convolution formula and the pion structure function.

in the Gottfried sum rule.³³ Noting $(\bar{u} - \bar{d})_{\pi^+}$
= $-V_{\pi}$, $[\bar{u} - \bar{d}]_{\pi^0} = 0$, and $[\bar{u} - \bar{d}]_{\pi^-} = +V_{\pi}$, we can use the same formalism in Eqs. (1) and (2) with simple modifications. Namely, we should modify the isospin³⁴ times pion-structure-function factors to $-2V_{\pi}$ in the πNN case and $+\frac{2}{3}V_{\pi}$ in the $\pi N\Delta$ case. In this way, we find that we have negative contribution from the πNN process and have smaller positive contribution from the $\pi N\Delta$ process shown in Fig. 5. Calculating the integral, $\frac{2}{3} \int dx (\bar{u} - \bar{d})$, we find that the pionic contribution to the deviation from the Gottfried sum rule $\left[\int (dx/x)(F_2^{ep} - F_2^{en})\right] = \frac{1}{3}$ is -0.041, which could explain a part of the discrepancies indicated by recent NMC experiments.³³ Although this is a rough estimate due to problems associated with the convolution formu la^{30} and the pion structure function³⁵ in the small-x region $(x < 0.1)$, it is encouraging for further investigation, especially comparisons with experiments. This research is in progress and will be submitted for publication.

This research was supported by the U.S. Department of Energy under Contract No. DE-FG02-87ER40365. S.K. thanks Dr. F. E. Close, Dr. J. S. Conway, Dr. J. G. Heinrich, Dr. B. K. Jain, Dr. V. R. Pandharipande, and people at the U.S. National Institute for Nuclear Theory for discussions and suggestions. S.K. thanks Dr. R. G. Roberts for sending their parametrization program and thanks Dr. L. L. Frankfurt and Dr. M. I. Strikman for sending their letter of comments on this research.

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- ³⁴The isospin factors are $|\tilde{\phi}_{\pi^+}^* \cdot \tilde{\tau}|^2 = 2$, $|\tilde{\phi}_{\pi^+}^* \cdot \tilde{\tau}|^2 = \frac{1}{3}$, and $|\tilde{\phi}_{\pi^-}^* \cdot \tilde{\tau}|^2 = 1$.
- ³⁵As explained in the paragraph just above Eq. (4), the SU(2)_f assumption for sea quarks in the pion becomes problematic, because sea quarks in the pion could contribute to the distribution $(\bar{u} - \bar{d})$ at small x (<0.1) if we consider the SU(2)_f breaking in the pion's sea.