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Lowest-order mass corrections for a (1+1)-dimensional Yukawa model in light-front perturbation theory

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In this work we *explicitly* demonstrate the equivalence of covariant (Feynman) perturbation theory with noncovariant light-front perturbation theory (LFPT) for lowest-order self-energy corrections in the $(1+1)$-dimensional Yukawa model. We also perform calculations in old-fashioned perturbation theory in the infinite-momentum frame (OFPT_∞) to elucidate the differences between LFPT and OFPT_∞.

I. INTRODUCTION

Interest in the study of quantum-field-theory models in the light-front formalism has recently grown. Most studies employ Hamiltonian diagonalization in a Fock-space basis. The realization of nonperturbative renormalization in a manifestly noncovariant formalism is an essential feature of such studies. To tackle this issue, it is prudent to first understand perturbative renormalization in the light-front formalism. In this work we perform the lowest-order fermion and boson mass corrections in the $(1+1)$-dimensional Yukawa model using light-front perturbation theory and show their equivalence with the corresponding covariant results. Surprisingly, such calculations have never been performed in a straightforward manner.

The light-front formalism was invented by Dirac[1] and rediscovered by Weinberg[2] in the guise of old-fashioned perturbation theory in the infinite-momentum frame (OFPT_∞). For scalar field models light-front perturbation theory (LFPT) and OPFT_∞ are equivalent. For theories involving fermions, there are subtle differences as was first pointed out by Susskind.[3] Chang and Ma[4] were the first to discuss second-order self-energies and the magnetic moment in QED in covariant perturbation theory using light-cone variables. They did not, however, carry out their calculation in light-front field theory. Drell, Levy, and Yan[5] discussed lowest-order fermion mass and wave-function renormalization for a $(3+1)$-dimensional pseudoscalar Yukawa model in the framework of OFPT_∞. Chang and Yan[6] discuss second-order self-energies and vertex correction for the $(3+1)$-dimensional Yukawa model in the context of light-front Feynman rules. They take the field operators to be normal ordered and hence do not get contributions from the instantaneous interactions. In this case the unrenormalized amplitudes are noncovariant and more divergent than corresponding Feynman amplitudes. We comment on this below. Brodsky, Roskies, and Suaya[7] have performed lowest-order calculations in QED in the framework of OFPT_∞. Their calculation, similar to that of Drell, Levy, and Yan,[5] involves subtle limiting procedures that do not clearly generalize to higher orders. Bouchiat, Fayet, and Sourlas[8] have discussed lowest-order radiative corrections in the light-front perturbation theory. However, their regularization involves the use of an indefinite metric (Pauli-Villars type), whereas we regularize without invoking the indefinite-metric formalism. Their renormalization procedure *cannot,* to our knowledge, be employed in nonperturbative calculations. Further, their evaluation of the instantaneous contribution appears to be somewhat involved compared to our straightforward calculations. To the best of our knowledge, a straightforward demonstration of the equivalence of covariant perturbation theory and LFPT for unrenormalized amplitudes is lacking in the literature.

The light-front Hamiltonian exhibits spurious light-front infrared singularities in contrast with its equal-time counterpart. Thus even the tree-level Hamiltonian needs regularization in the light-front formalism. First, we provide a method for regularization. In the regulated theory separate contributions are noncovariant, but the total contribution is covariant once the instantaneous interactions are taken into account. For the unrenormalized amplitudes to be the same in Feynman theory and LFPT, infrared and ultraviolet regulators must be of a special form, which can be easily deduced from the equal-time theory. In a renormalizable theory one is free to allow cutoffs to approach their limits in any desired manner without inducing any ambiguities other than those always encountered in renormalization.

II. SPURIOUS LIGHT-FRONT SINGULARITIES AND THEIR REGULARIZATION

The light-front Hamiltonian in the plane-wave basis for the $(1+1)$-dimensional Yukawa model presented in the Appendix exhibits singularities. It is worthwhile to remember that in deriving this Hamiltonian we have already introduced an infrared cutoff to eliminate the point $k^+=0$. Thereby, we have eliminated the vertices responsible for the occurrence of disconnected vacuum diagrams. Normal ordering the Hamiltonian, we have generated self-energy contributions at the tree level, which have been named "self-induced inertias" in the literature.[9] Let us now discuss how to handle the singularities we encounter in the light-front Hamiltonian. We introduce a regularization *in the continuum version.* This method has been utilized before in the study of two-dimensional ϕ^4 $[(\phi^4)_2]$ theory.[10]

Let us recall the formal definition of k^+:

$$k^+=\frac{1}{\sqrt{2}}(k^0+k) \ . \tag{2.1}$$

For a free particle of mass m, $k^0(m^2+k^2)^{1/2}$, where k is the momentum. Thus $k^+=(1/\sqrt{2})[(m^2+k^2)^{1/2}+k]$. In the equal-time formulation, the field operator for a boson field, for example is given by

$$\begin{aligned}\phi(x)&=\int_{-\infty}^{\infty}\frac{dk}{2\pi 2\omega_k}[a(k)e^{-ikx}+a^\dagger(k)e^{ikx}]\\ &=\lim_{\Lambda\to\infty}\int_{-\Lambda}^{\Lambda}\frac{dk}{2\pi 2\omega_k}[a(k)e^{-ikx}+a^\dagger(k)e^{ikx}] \ ,\end{aligned} \tag{2.2}$$

where we have introduced the ultraviolet cutoff Λ. Thus, for the light-front momentum k^+, its limiting values are given by

$$k^+_{\max}=\lim_{\Lambda\to\infty}\frac{1}{\sqrt{2}}\left[2\Lambda+\frac{m^2}{2\Lambda}\right] \tag{2.3a}$$

and

$$k^+_{\min}=\lim_{\Lambda\to\infty}\frac{1}{\sqrt{2}}\left[\frac{m^2}{2\Lambda}\right] \ . \tag{2.3b}$$

The limit $\Lambda\to\infty$ is to be taken *after* performing integrations over the loop momenta. By this procedure we have removed the divergence due to the boson and fermion "zero mode." We are now left with the singularities in the instantaneous inertias of the form $1/(k_1^+-k_2^+)$. This singularity has the principal-value prescription.

With our prescriptions we have regulated the *spurious* divergences in the light-front theory. We are left with the original divergences which are present in the covariant theory. There is no need to relate the two light-front cutoffs as in Eq. (2.3), if one does not want to compare divergences in LFPT and covariant perturbation theory. However, it is necessary to relate cutoffs which regulate self-induced inertias and those regulating divergent loop integrations. The consideration of chiral symmetry puts further constraints on the cutoffs that are beyond the scope of this article.

It is important to note that covariant theory also has divergences associated with disconnected vacuum diagrams. Our main motivation is to investigate in a Tamm-Dancoff spirit[11] the spectra of field theories that do not involve the phenomena of symmetry breaking. To implement this numerically it is *essential* to eliminate the disconnected vacuum diagrams. The regulator prescription we use achieves this aim for massive theories under present consideration.

III. LOWEST-ORDER LOOP CORRECTIONS

In this section we evaluate the lowest-order loop corrections to the fermion and boson masses in light-front perturbation theory and demonstrate their equivalence with Feynman perturbation theory results. Two major ingredients in this demonstration are (1) using Chang's prescription for regulating the spurious light-front infrared divergence[10] and (2) keeping the extra terms (i.e., the self-induced inertias) arising from operator ordering in the interaction terms.

A. Fermion mass correction

1. Feynman calculation

To lowest order the fermion self-energy is

$$\Sigma(k)=i\lambda^2\int\frac{d^2q}{(2\pi)^2}G_f^0(q)\Delta^0(k-q) \ , \tag{3.1}$$

where

$$G_f^0(q)=\frac{1}{\not q-m_f+i\epsilon} \ , \tag{3.2}$$

and

$$\Delta^0(k)=\frac{1}{k^2-m_b^2+i\epsilon} \ . \tag{3.3}$$

The lowest-order mass correction is given by

$$\delta m_f=\bar{u}(k)\Sigma(k)u(k) \ , \tag{3.4}$$

where $u(k)$ is an on-mass-shell spinor. A straightforward calculation leads to

$$\delta m_f=\frac{-\lambda^2}{4\pi}\frac{1}{m_f}\int_0^1 dx\frac{(1+x)m_f^2}{(1-x)^2m_f^2+xm_b^2} \ . \tag{3.5}$$

We note that we have removed the cutoffs, and this is a covariant, finite result.

2. LFPT calculation

The interaction P_V^- (see the Appendix) used in second order gives the contribution coming from one-boson–one-fermion intermediate states. A straightforward evaluation leads to

$$\delta m_f^2(a)=-\frac{\lambda^2}{4\pi}m_f^2\int\frac{dx}{x}\frac{(1+x)^2}{m_f^2(1-x)^2+m_b^2x} \ . \tag{3.6}$$

The lower limit of integration is $x_{\min}=(1/\sqrt{2})(m_f^2/2\Lambda P^+)$, and the upper limit of integration is $x_{\max}=1-(1/\sqrt{2})(m_b^2/2\Lambda P^+)$, where P^+ is the momentum of the initial fermion and Λ is the ultraviolet cutoff. Using the fact that

$$\delta m_f^2(a)=(m_f+\delta m_f)^2-m_f^2=2m_f\delta m_f \ , \tag{3.7}$$

we get

$$\delta m_f(a)=-\frac{\lambda^2}{4\pi}\frac{1}{2m_f}\int\frac{dx}{x}\frac{m_f^2(1+x)^2}{m_f^2(1-x)^2+m_b^2x} \ , \tag{3.8}$$

which can be rewritten as

$$\delta m_f(a)=-\frac{\lambda^2}{4\pi}\frac{1}{2m_f}\int_0^1 dx\frac{4m_f^2-m_b^2}{m_f^2(1-x)^2+m_b^2x}+\frac{\lambda^2}{4\pi}\frac{1}{2m_f}\ln\frac{m_f^2}{2\sqrt{2}\Lambda P^+}, \quad (3.9)$$

where we have separated out the noncovariant divergent term. P^+ is the momentum of the initial fermion. However, this is not the whole story. To order λ^2 we also have the contribution coming from the fermion self-inertia, which when evaluated using our prescriptions for regularization is given by

$$\delta m_f(b)=\frac{\lambda^2}{4\pi}\frac{1}{2m_f}\mathcal{P}\int\frac{dx}{x}\frac{1}{1-x}. \quad (3.10)$$

The lower limit of integration is $x_{\min}=(1/\sqrt{2})(m_b^2/2\Lambda P^+)$, and the upper limit of integration is $x_{\max}=(1/\sqrt{2}P^+)(2\Lambda+m_b^2/2\Lambda)$. This yields

$$\delta m_f(b)=-\frac{\lambda^2}{4\pi}\frac{1}{2m_f}\ln\frac{m_b^2}{2\sqrt{2}\Lambda P^+}. \quad (3.11)$$

Using

$$\ln\frac{m_b^2}{m_f^2}=\int_0^1 dx\frac{m_b^2-2(1-x)m_f^2}{m_f^2(1-x)^2+m_b^2x}, \quad (3.12)$$

now

$$\delta m_f=\delta m_f(a)+\delta m_f(b)=-\frac{\lambda^2}{4\pi}\frac{1}{m_f}\int_0^1 dx\frac{m_f^2(1+x)}{m_f^2(1-x)^2+m_b^2x}, \quad (3.13)$$

which agrees with the covariant calculation.

It is important to note that the individual contributions to fermion mass corrections are noncovariant in LFPT. They are also *divergent*. Thus, if we had ignored the self-induced inertia, we would need a *noncovariant, divergent* counterterm to make the fermion mass finite.

B. Boson mass correction

1. *Feynman calculation*

To lowest order the boson self-energy is

$$\Pi(q)=-\lambda^2\int\frac{d^2k}{(2\pi)^2}\frac{\mathrm{Tr}(\not k+m_f)(\not k+\not q+m_f)}{(k^2-m_f^2+i\epsilon)[(k+q)^2-m_f^2+i\epsilon]}. \quad (3.14)$$

The lowest-order boson mass shift is given by

$$\delta m_b^2=\Pi(q^2=m_b^2)=-\frac{\lambda^2}{2\pi}\ln\frac{\Lambda^2}{m_f^2}-\frac{\lambda^2}{2\pi}\int_0^1 dx\frac{2m_f^2-\frac{1}{2}m_b^2}{x(1-x)m_b^2-m_f^2}. \quad (3.15)$$

Thus the lowest-order shift is logarithmically divergent, but covariant.

2. *LFPT calculation*

The interaction P_V^- used in second order gives the contribution from intermediate states containing one fermion-antifermion pair. A straightforward calculation gives

$$\delta m_b^2(a)=\frac{\lambda^2}{4\pi}\int\frac{dx}{x(1-x)}\frac{(1-2x)^2}{m_b^2x(1-x)-m_f^2}, \quad (3.16)$$

where the lower limit of integration is $x_{\min}=(1/\sqrt{2})(m_f^2/2\Lambda P^+)$ and the upper limit of integration is $x_{\max}=1-x_{\min}$. This yields

$$\delta m_b^2(a)=\frac{\lambda^2}{4\pi}\int_0^1 dx\frac{m_b^2-4m_f^2}{m_b^2x(1-x)-m_f^2}+\frac{\lambda^2}{2\pi}\ln 2\frac{m_f^2}{\sqrt{2}\Lambda P^+}. \quad (3.17)$$

Here P^+ is the light-cone momentum of the initial boson. Thus the contribution is noncovariant and divergent. To the same order in λ^2, we also have the contribution from the boson self-inertia:

$$\delta m_b^2(b)=\frac{\lambda^2}{4\pi}\mathcal{P}\int dx\left[\frac{1}{1-x}-\frac{1}{1+x}\right], \quad (3.18)$$

with the lower limit of integration $x_{\min}=(1/\sqrt{2})(m_f^2/2\Lambda P^+)$ and the upper limit of integration $x_{\max}=(1/\sqrt{2}P^+)(2\Lambda+m_f^2/2\Lambda)$. Thus

$$\delta m_b^2(b)=-\frac{\lambda^2}{2\pi}\ln\frac{\Lambda}{2\sqrt{2}P^+}. \quad (3.19)$$

This is again a noncovariant contribution. Adding the two contributions, we get

$$\delta m_b^2=\delta m_b^2(a)+\delta m_b^2(b)=\frac{\lambda^2}{2\pi}\left[\ln\frac{m_f^2}{\Lambda^2}-\int_0^1\frac{2m_f^2-\frac{1}{2}m_b^2}{m_b^2x(1-x)-m_f^2}\right], \quad (3.20)$$

which is exactly the Feynman result.

Note that the individual contributions in LFPT are noncovariant, whereas the sum is covariant as we expect. If we had ignored the self-induced inertia, we would need a divergent *noncovariant* counterterm to make the boson mass finite.

Thus we have explicitly seen that LFPT is able to reproduce the covariant results. In Sec. IV we contrast these calculations with those in the OFPT_∞.

IV. OFPT_∞ CALCULATIONS

The Hamiltonian for the (1+1)-dimensional Yukawa model in equal-time formalism is presented in the Appendix. In this section we calculate the second-order fermion and boson mass shifts in OFPT using this Hamiltonian. We then take the infinite-momentum limit of these expressions[5] to compare and contrast OFPT_∞ with LFPT. One of our main interests is the fate of disconnected vacuum diagrams in the infinite-momentum frame. We find that they do not vanish in the infinite-momentum limit precisely because they are frame independent. Further, the OFPT_∞ expressions are treacherous, and there are various subtle cancellations. The number of Fock-space states needed to generate covariant results is larger in OFPT_∞ than in LFPT. These facts crucially differentiate between LFPT and OFPT_∞ for practical nonperturbative calculations.

A. Fermion mass

The interaction Hamiltonian H_V in second order gives a contribution to the energy shift of the fermion coming from one-boson–one-fermion intermediate states:

$$\delta E_{f-b}=\frac{\lambda^2}{2\pi}\frac{m_f}{E_p}\lim_{\Lambda\to\infty}\int_{-\Lambda}^{\Lambda}dk\frac{1}{2\omega_{P-k}}\frac{m_f}{E_k}\times\frac{\bar u(P)u(k)\bar u(k)u(P)}{E_P-E_k-\omega_{P-k}} . \quad (4.1)$$

Here P is the momentum of the initial nucleon, $E_k=(m_f^2+k^2)^{1/2}$ and $\omega_k=(m_b^2+k^2)^{1/2}$. Note that this is a noncovariant but *finite* result. Now we let $P\to\infty$. Put $k=xP$ with $-\Lambda/P<x<\Lambda/P$. Note that we have to take the limit $\Lambda\to\infty$ before we take the limit $P\to\infty$. We split the integral into several pieces and evaluate each separately:[5]

$$\int_{-\Lambda/P}^{\Lambda/P}=\int_{-\Lambda/P}^{-\epsilon}+\int_{-\epsilon}^{\epsilon}+\int_{\epsilon}^{1}+\int_{1}^{\Lambda/P} .$$

(a) $-\Lambda/P<x<-\epsilon$:

$$\delta m_f=\frac{\lambda^2}{4\pi}\frac{1}{2m_f}\int_{-\Lambda/P}^{-\epsilon}dx\frac{1}{x(1-x)} .$$

(b) $-\epsilon<x<\epsilon$:

$$\delta m_f=-\frac{\lambda^2}{4\pi}\frac{1}{2m_f}\lim_{P\to\infty}P\int_{-\epsilon}^{\epsilon}dx\frac{1}{(m_f^2+x^2P^2)^{1/2}} .$$

(c) $\epsilon<x<1$:

$$\delta m_f=-\frac{\lambda^2}{4\pi}\frac{1}{2m_f}\int_{\epsilon}^{1}\frac{dx}{x}\frac{m_f^2(1+x)^2}{m_f^2(1-x)^2+m_b^2x} .$$

(d) $1<x<\Lambda/P$:

$$\delta m_f\to 0 .$$

Thus taking the limit $P\to\infty$ has made the contribution from one-fermion–one-boson states singular in various kinematic domains. However, this is not the whole story. Because of bda and $a^\dagger d^\dagger b^\dagger$ terms in the Hamiltonian, a second-order contribution also comes from two-fermion–one-antifermion–one-boson intermediate states. This contribution is

$$\delta E_{ff\bar f b}=\delta E_{ff\bar f b}(\text{disconnected})+\delta E_{ff\bar f b}(\text{connected}) ,$$

$\delta E_{ff\bar f b}$(disconnected)

$$=\frac{-\lambda^2}{\pi}\int dk\int dk'\frac{m_f}{E_k}\frac{m_f}{E_{k'}}\frac{1}{2\omega_{k+k'}}\times\frac{\bar v(k')u(k)\bar u(k)v(k')}{\omega_{k+k'}+E_k+E_{k'}}\delta(0) . \quad (4.2)$$

Note that this is highly divergent but independent of P (a covariant contribution) and survives the $P\to\infty$ limit. The connected piece is

$\delta E_{ff\bar f b}$(connected)

$$=\frac{\lambda^2}{\pi}\frac{m_f}{E_p}\int_{-\Lambda}^{\Lambda}dk\frac{1}{2\omega_{P+k}}\frac{m_f}{E_k}\frac{\bar v(k)u(P)\bar u(P)v(k)}{E_P+E_k+\omega_{P+k}} . \quad (4.3)$$

This contribution is evidently noncovariant. It is important to note that $\delta E_{ff\bar f b}$(connected) and $\delta E_{ff\bar f b}$(disconnected) individually violate the Pauli exclusion principle, but their sum does not. $\delta E_{ff\bar f b}$(disconnected) is a purely vacuum correction, which is to be removed by readjusting the vacuum energy. This is easily done as long as one is performing analytic calculations. The $\delta E_{ff\bar f b}$(connected) contribution can be evaluated as follows. Put $k=-xP$ and again split up the integral, remembering that we have to let $\Lambda\to\infty$ before we let $P\to\infty$:

$$\int_{-\Lambda/P}^{\Lambda/P}=\int_{-\Lambda/P}^{0}+\int_{0}^{1-\epsilon}+\int_{1-\epsilon}^{1+\epsilon}+\int_{1+\epsilon}^{\Lambda/P} .$$

(a) $-\Lambda/P<x<0$:

$$\delta m_f\to 0 .$$

(b) $0<x<1-\epsilon$:

$$\delta m_f=\frac{\lambda^2}{4\pi}\frac{1}{2m_f}\int_{0}^{1-\epsilon}dx\frac{1}{1-x} .$$

(c) $1-\epsilon<x<1+\epsilon$:

$$\delta m_f=\frac{\lambda^2}{4\pi}\frac{1}{2m_f}P\int_{-\epsilon}^{\epsilon}dx\frac{1}{(m_b^2+x^2P^2)^{1/2}} .$$

(d) $1+\epsilon<x<\Lambda/P$:

$$\delta m_f=-\frac{\lambda^2}{4\pi}\frac{1}{2m_f}\int_{1+\epsilon}^{\Lambda/P}dx\frac{1}{x(1-x)} .$$

Adding all the contributions and using

$$\int_{-\epsilon}^{\epsilon}dx\frac{1}{(x^2+m_b^2/P^2)^{1/2}}=\ln\frac{4\epsilon^2P^2}{m_b^2} ,$$

we recover the covariant result.

B. Boson mass

The contribution to the energy shift of the one-boson state coming from one-fermion–one-antifermion intermediate states is

$$\delta E_{f\bar{f}}=\frac{\lambda^2}{2\pi}\frac{1}{2\omega_P}\lim_{\Lambda\to\infty}\times\int_{-\Lambda}^{\Lambda}dk\frac{m_f}{E_k}\frac{m_f}{E_{P-k}}\times\frac{\bar{v}(P-k)u(k)\bar{u}(k)v(P-k)}{\omega_P-E_k-E_{P-k}}\ . \tag{4.4}$$

Here P is the momentum of the initial boson. Note that this contribution is noncovariant. By multiplying $E_{f\bar{f}}$ by $2\omega_P$ we obtain the mass shift δm_b^2. Now we wish to take the $P\to\infty$ limit. Note that we take the limit $\Lambda\to\infty$ first and then let $P\to\infty$. Put $k=xP$ with $-\Lambda/P<x<\Lambda/P$.

(a) $-\Lambda/P<x<-\epsilon$:

$$\delta m_b^2=\frac{\lambda^2}{4\pi}\int_{-\Lambda/P}^{-\epsilon}dx\frac{1}{x}\ .$$

(b) $-\epsilon<x<\epsilon$:

$$\delta m_b^2=-\frac{\lambda^2}{4\pi}\lim_{P\to\infty}\int_{-\epsilon}^{\epsilon}dx\frac{1}{(m_f^2+x^2P^2)^{1/2}}\ .$$

(c) $\epsilon<x<1-\epsilon$:

$$\delta m_b^2=\frac{\lambda^2}{4\pi}m_f^2\int_{\epsilon}^{1-\epsilon}\frac{dx}{x(1-x)}\frac{(2x-1)^2}{m_b^2x(1-x)-m_f^2}\ .$$

(d) $1-\epsilon<x<1+\epsilon$:

$$\delta m_b^2=-\frac{\lambda^2}{4\pi}P\int_{-\epsilon}^{+\epsilon}dx\frac{1}{(m_f^2+x^2P^2)^{1/2}}\ .$$

(e) $1+\epsilon<x<\Lambda/P$:

$$\delta m_b^2=\frac{\lambda^2}{4\pi}\int_{-\Lambda/P}^{-\epsilon}dx\frac{1}{x}\ .$$

We get a second-order contribution coming from one-fermion–one-antifermion–two-boson intermediate states. Again, we get a *connected* contribution and a *disconnected* contribution. The disconnected contribution is the same one we encountered in the fermion calculation. Again, we need to readjust the vacuum energy to take care of this contribution. The mass shift coming from the connected contribution is

$$\delta m_b^2=-\frac{\lambda^2}{2\pi}\lim_{P\to\infty}\lim_{\Lambda\to\infty}\times\int_{-\Lambda}^{\Lambda}dk\frac{m_f}{E_k}\frac{m_f}{E_{P+k}}\frac{\bar{v}(k')u(k)\bar{u}(k)v(k')}{\omega_P+E_k+E_{k'}}\ , \tag{4.5}$$

where $k'=-(P+k)$. Put $k=-xP$.

(a) $-\Lambda/P<x<-\epsilon$:

$$\delta m_b^2=-\frac{\lambda^2}{4\pi}\int_{-\Lambda/P}^{-\epsilon}dx\frac{1}{1-x}\ .$$

(b) $-\epsilon<x<\epsilon$:

$$\delta m_b^2\to 0\ .$$

(c) $\epsilon<x<1-\epsilon$:

$$\delta m_b^2\to 0\ .$$

(d) $1-\epsilon<x<1+\epsilon$:

$$\delta m_b^2\to 0\ .$$

(e) $1+\epsilon<x<\Lambda/P$:

$$\delta m_b^2=-\frac{\lambda^2}{4\pi}\int_{1+\epsilon}^{\Lambda/P}dx\frac{1}{x}\ .$$

Adding all the contributions, the frame dependence drops out, and we recover the covariant result.

Note that (1) the OFPT_∞ result agrees with the covariant result only after several delicate cancellations, and (2) the disconnected vacuum contributions of time-ordered perturbation theory survive the $P\to\infty$ limit precisely because they are independent of the frame of reference. Thus the readjustment of the vacuum energy is still needed in the OFPT_∞ calculation.

V. CONCLUSIONS

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APPENDIX

Equal-time case

The Hamiltonian is given by

$$H=H_{\text{free}}+H_V\ , \tag{A1}$$

$$H_{\text{free}}=\int_{-\infty}^{\infty}\frac{dk}{2\pi}\frac{m_f}{E_k}E_k[b^\dagger(k)b(k)+d^\dagger(k)d(k)]+\int_{-\infty}^{\infty}\frac{dk}{2\pi 2\omega_k}\omega_k a^\dagger(k)a(k)\ , \tag{A2}$$

$$H_V=\frac{\lambda}{2\pi}\int dk_1\frac{m_f}{E_{k_1}}\int dk_2\frac{m_f}{E_{k_2}}\int\frac{dk_3}{2\pi 2\omega_{k_3}}[b^\dagger(k_1)b(k_2)a(k_3)\bar{u}(k_1)u(k_2)\delta(k_1-k_2-k_3)$$
$$+b^\dagger(k_1)b(k_2)a^\dagger(k_3)\bar{u}(k_1)u(k_2)\delta(k_1-k_2+k_3)$$
$$-d^\dagger(k_2)d(k_1)a(k_3)\bar{v}(k_1)v(k_2)\delta(k_1-k_2+k_3)$$
$$-d^\dagger(k_2)d(k_1)a^\dagger(k_3)\bar{v}(k_1)v(k_2)\delta(k_1-k_2-k_3)$$
$$+b^\dagger(k_1)d^\dagger(k_2)a(k_3)\bar{u}(k_1)v(k_2)\delta(k_1+k_2-k_3)$$
$$+d(k_1)b(k_2)a^\dagger(k_3)\bar{v}(k_1)u(k_2)\delta(k_1+k_2-k_3)$$
$$+b^\dagger(k_1)d^\dagger(k_2)a^\dagger(k_3)\bar{u}(k_1)v(k_2)\delta(k_1+k_2+k_3)$$
$$+d(k_1)b(k_2)a(k_3)\bar{v}(k_1)u(k_2)\delta(k_1+k_2+k_3)]\ . \tag{A3}$$

Note that H_V changes particle number by 1.

Light-front case

The light-front Hamiltonian is

$$P^-=P_M^-+P_V^-+P_F^-+P_S^-\ . \tag{A4}$$

Here

$$P_M^-=\int\frac{dk_1^+}{2\pi 2k_1^+}a^\dagger(k_1^+)a(k_1^+)\frac{1}{2k_1^+}\left[m_b^2+\frac{\lambda^2}{4\pi}\alpha(k_1^+)\right]+\int\frac{dk_1^+}{2\pi k_1^+}b^\dagger(k_1^+)b(k_1^+)\frac{1}{2k_1^+}\left[m_f^2+\frac{\lambda^2}{4\pi}\beta(k_1^+)\right]$$
$$+\int\frac{dk_1^+}{2\pi k_1^+}d^\dagger(k_1^+)d(k_1^+)\frac{1}{2k_1^+}\left[m_f^2+\frac{\lambda^2}{4\pi}\gamma(k_1^+)\right], \tag{A5}$$

with

$$\alpha(k_1^+)=\mathcal{P}\int dk_2^+\left[\frac{1}{k_1^+-k_2^+}-\frac{1}{k_1^++k_2^+}\right], \tag{A6}$$

$$\beta(k_1^+)=\mathcal{P}\int dk_2^+\frac{k_1^+}{k_2^+}\frac{1}{k_1^+-k_2^+}\ , \tag{A7}$$

$$\gamma(k_1^+)=\int dk_2^+\frac{k_1^+}{k_2^+}\frac{1}{k_1^++k_2^+}\ . \tag{A8}$$

The terms arising from normal ordering, α, β, and γ, have been called the "self-induced inertias" in the literature.[9] The second term of P^- is

$$P_V^-=\frac{m_f\lambda}{4\pi}\int\frac{dk_1^+}{(k_1^+)^{1/2}}\int\frac{dk_2^+}{2\pi 2k_2^+}\int\frac{dk_3^+}{(k_3^+)^{1/2}}$$
$$\times\left[[b^\dagger(k_1^+)b(k_3^+)a(k_2^+)+b^\dagger(k_3^+)b(k_1^+)a^\dagger(k_2^+)]\delta(k_1^+-k_2^+-k_3^+)\left[\frac{1}{k_1^+}+\frac{1}{k_3^+}\right]\right.$$
$$+[d^\dagger(k_1^+)d(k_3^+)a(k_2^+)+d^\dagger(k_3^+)d(k_1^+)a^\dagger(k_2^+)]\delta(k_1^+-k_2^+-k_3^+)\left[\frac{1}{k_1^+}+\frac{1}{k_3^+}\right]$$
$$\left.+[b^\dagger(k_1^+)d^\dagger(k_3^+)a(k_2^+)+d(k_3^+)b(k_1^+)a^\dagger(k_2^+)]\delta(k_1^++k_3^+-k_2^+)\left[\frac{1}{k_1^+}-\frac{1}{k_3^+}\right]\right]\ . \tag{A9}$$

Note that P_V^- changes particle number by 1. The third term of P^- is

$$P_F^- = \frac{\lambda^2}{4\pi}\int \frac{dk_1^+}{(k_1^+)^{1/2}} \int \frac{dk_2^+}{2\pi 2k_2^+} \int \frac{dk_3^+}{2\pi 2k_3^+} \int \frac{dk_4^+}{(k_4^+)^{1/2}}$$
$$\times \Bigg[[b^\dagger(k_1^+)b(k_4^+)a(k_2^+)a(k_3^+) + b^\dagger(k_4^+)b(k_1^+)a^\dagger(k_3^+)a^\dagger(k_2^+)] \frac{\delta(k_3^+ + k_4^+ - k_1^+ + k_2^+)}{k_3^+ + k_4^+}$$
$$+ [d^\dagger(k_4^+)d(k_1^+)a(k_2^+)a(k_3^+) + d^\dagger(k_1^+)d(k_4^+)a^\dagger(k_3^+)a^\dagger(k_2^+)] \frac{\delta(k_4^+ - k_3^+ - k_1^+ - k_2^+)}{k_4^+ - k_3^+}$$
$$+ b^\dagger(k_1^+)d^\dagger(k_4^+)a^\dagger(k_3^+)a(k_2^+)\delta(k_3^+ + k_4^+ - k_2^+ + k_1^+) \left[\frac{1}{k_1^+ - k_2^+} + \frac{1}{k_1^+ + k_3^+} \right]$$
$$+ d(k_4^+)b(k_1^+)a^\dagger(k_2^+)a(k_3^+)\delta(k_3^+ + k_4^+ - k_2^+ + k_1^+) \left[\frac{1}{k_1^+ - k_2^+} + \frac{1}{k_1^+ + k_3^+} \right] \Bigg] . \tag{A10}$$

Note that P_F^- changes particle number by 2. The fourth term of P^- is

$$P_S^- = \frac{\lambda^2}{4\pi}\int \frac{dk_1^+}{(k_1^+)^{1/2}} \int \frac{dk_2^+}{2\pi 2k_2^+} \int \frac{dk_3^+}{2\pi 2k_3^+} \int \frac{dk_4^+}{(k_4^+)^{1/2}}$$
$$\times \Bigg[b^\dagger(k_1^+)b(k_4^+)a^\dagger(k_2^+)a(k_3^+)\delta(k_2^+ - k_4^+ - k_3^+ + k_1^+) \left[\frac{1}{k_4^+ - k_2^+} + \frac{1}{k_3^+ + k_4^+} \right]$$
$$+ d^\dagger(k_4^+)d(k_1^+)a^\dagger(k_2^+)a(k_3^+)\delta(k_4^+ - k_3^+ - k_1^+ + k_2^+) \left[\frac{1}{k_1^+ - k_2^+} + \frac{1}{k_2^+ + k_4^+} \right]$$
$$+ [b^\dagger(k_1^+)d^\dagger(k_4^+)a(k_2^+)a(k_3^+) + d(k_4^+)b(k_1^+)a^\dagger(k_3^+)a^\dagger(k_2^+)] \frac{\delta(k_3^+ - k_4^+ - k_1^+ + k_2^+)}{k_3^+ - k_4^+} \Bigg] . \tag{A11}$$

P_S^- changes particle number by 0. Note that if we had defined P^- to be a normal-ordered Hamiltonian to begin with (just as the free-particle Hamiltonian), the "self-induced inertias" would be absent from the Hamiltonian.

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