Effects of a gravitomagnetic field on pure superconductors

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We report the results of an investigation of the effects of a pure superconductor on external gravitomagnetic and magnetic fields $\mathbf{B}_{g,0}$ and $\mathbf{B}_0$, respectively. We find that the internal fields are given by $\mathbf{B}(z) \approx -(m^2\mu_g/q^2\mu)\mathbf{B}_0 - (m/q)\mathbf{B}_{g,0}$, $\mathbf{B}_g(z) \approx \mathbf{B}_{g,0} + (\mu_g m/q\mu)\mathbf{B}_0$, where μ_g and μ are the gravitomagnetic and magnetic permeabilities of the superconductor. These results show that a small residual uniform magnetic field will pervade the superconductor and that the external fields mutually "induce" additional small internal perturbation fields. The sum of the fields $\mathbf{B} + (m/q)\mathbf{B}_g$ falls exponentially to zero over a characteristic distance λ, which is consistent with previous findings that $\mathbf{B} + (m/q)\mathbf{B}_g = 0$ inside a pure superconductor.

A magnetic-type gravitational field referred to as the gravitomagnetic field, which arises from moving masses in the same way that a moving charge produces a magnetic field, was suggested by Forward.[1] Ross[2] investigated the effects of a gravitational field in a superconductor. He formulated the gravitomagnetic field equations written in terms of the parametrized-post-Newtonian (PPN) parameters of Braginsky[3] and derived modified London equations. It is the purpose of this paper to solve the coupled general relativity, Maxwell, and London equations to obtain quantitative formulas for each of the magnetic and gravitomagnetic fields inside a superconductor.

For a weak-gravity and low-velocity system such as Cooper pairs in a superconductor, the general relativistic field equations assume a form almost identical to Maxwell's equations, and the geodesic equation is identical to the Lorentz force law (see, e.g., Forward,[1] Thorne,[4] and Mashhoon, Paik, and Will[5]):

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + m(\mathbf{E}_g + \mathbf{v} \times \mathbf{B}_g) , \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} , \tag{2}$$

$$\nabla \times \mathbf{B} = \mu \mathbf{j}_e + \mu\epsilon \frac{\partial \mathbf{E}}{\partial t} , \tag{3}$$

$$\nabla \times \mathbf{E}_g = -\frac{\partial \mathbf{B}_g}{\partial t} , \tag{4}$$

$$\nabla \times \mathbf{B}_g = -\mu_g \mathbf{j}_m + \mu_g \epsilon_g \frac{\partial \mathbf{E}_g}{\partial t} , \tag{5}$$

where m and q are the gravitational and the electric charges of a Cooper pair in a superconductor, $\mathbf{E}_g$ and $\mathbf{E}$ denote the gravitoelectric and electric fields, respectively. Just as the magnetic permeability μ and the electric permittivity ϵ characterize the response of a superconductor to the applied fields $\mathbf{E}$ and $\mathbf{B}$ in the electromagnetic case, so the gravitomagnetic permeability μ_g and the gravitoelectric permittivity ϵ_g characterize the response of a superconductor to the applied fields $\mathbf{E}_g$ and $\mathbf{B}_g$ in the gravitoelectromagnetic case.

A dimensional analysis of Eq. (1) suggests that it is insightful to use the mks units, since $\mathbf{B}$ and μ have the dimensions $[m/qt]$ and (ml/q^2), thus, $\mathbf{B}_g$ has the dimensions of the angular velocity $[1/t]$, while the dimensions for μ_g are $[l/m]$, and we note that the dimensions of μ_g/μ are $[q^2/m^2]$, so the quantity $\mu_g m^2/\mu q^2$ is dimensionless. Expressing these fields in terms of potentials such that

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} , \tag{6}$$

$$\mathbf{E}_g = -\nabla\phi_g - \frac{\partial \mathbf{A}_g}{\partial t} , \tag{7}$$

$$\mathbf{B} = \nabla \times \mathbf{A} , \tag{8}$$

$$\mathbf{B}_g = \nabla \times \mathbf{A}_g , \tag{9}$$

Eq. (1) can be expressed in the alternative form

$$\mathbf{F} = -\nabla[q(\phi - \mathbf{v}\cdot\mathbf{A}) + m(\phi_g - \mathbf{v}\cdot\mathbf{A}_g)] - \frac{d}{dt}(q\,\mathbf{A} + m\,\mathbf{A}_g) , \tag{10}$$

where $d/dt = \partial/\partial t + \mathbf{v}\cdot\nabla$. The potential energy

$$U = q(\phi - \mathbf{v}\cdot\mathbf{A}) + m(\phi_g - \mathbf{v}\cdot\mathbf{A}_g) \tag{11}$$

is easily obtained. Then, the canonical momentum expressed by $-i\nabla$ is computed from the Lagrangian for a single Cooper pair

$$L = \tfrac{1}{2}mV^2 - q(\phi - \mathbf{v}\cdot\mathbf{A}) - m(\phi_g - \mathbf{v}\cdot\mathbf{A}_g) . \tag{12}$$

One finds

$$-i\nabla = m\mathbf{v} + q\,\mathbf{A} + m\,\mathbf{A}_g . \tag{13}$$

Electric and mass currents must be induced by the vector potential fields $\mathbf{A}$ and $\mathbf{A}_g$ in a superconductor, and the magnitudes of these currents will be

$$\mathbf{j}_e = -\frac{1}{\mu\lambda_l^2}\left[\mathbf{A} + \frac{m}{q}\mathbf{A}_g\right] \tag{14}$$

and

$$\mathbf{j}_m = -\frac{1}{\mu\lambda_l^2}\frac{m}{q}\left[\mathbf{A}+\frac{m}{q}\mathbf{A}_g\right], \tag{15}$$

where $\mathbf{j}_e = nq\mathbf{v}$ and $\mathbf{j}_m = nm\mathbf{v}$ are the electric and mass current densities, respectively, n is the pair concentration, and $\lambda_l^2 = m/\mu n q^2$ is the London penetration length. On the other hand, Eqs. (14) and (15) can be expressed in the form

$$\frac{\partial \mathbf{j}_e}{\partial t} = \frac{1}{\mu\lambda_l^2}\left[\mathbf{E}+\frac{m}{q}\mathbf{E}_g\right], \tag{16}$$

$$\frac{\partial \mathbf{j}_m}{\partial t} = \frac{1}{\mu\lambda_l^2}\frac{m}{q}\left[\mathbf{E}+\frac{m}{q}\mathbf{E}_g\right]. \tag{17}$$

Equations (14) and (16) are consistent with Ross's modified London equations, whereas Eqs. (15) and (17) are the new results about the relation between the mass current and the fields.

These electric and mass currents arise from the induced motion of the Cooper pairs instantly generating their own magnetic and gravitomagnetic fields. In what follows we solve the coupled field equations for **B** and $\mathbf{B}_g$ in a superconductor. Substituting Eqs. (14) and (15) into Eqs. (3) and (5), respectively, and assuming stationary conditions are established, we obtain

$$\nabla^2\mathbf{B} = \frac{1}{\lambda_l^2}\left[\mathbf{B}+\frac{m}{q}\mathbf{B}_g\right], \tag{18}$$

$$\nabla^2\mathbf{B}_g = -\frac{m^2\mu_g}{q^2\mu\lambda_l^2}\left[\mathbf{B}_g+\frac{q}{m}\mathbf{B}\right]. \tag{19}$$

Equation (18) indicates that a very small magnetic field

$$\mathbf{B}' = \frac{m}{q}\mathbf{B}_g \tag{20}$$

is produced inside the superconductor by the gravitomagnetic field $\mathbf{B}_g$. Equation (19) indicates that a gravitomagnetic field is produced by the internal magnetic field

$$\mathbf{B}_g' = \frac{q}{m}\mathbf{B}. \tag{21}$$

The penetration depth of $\mathbf{B}_g$ is approximately $q^2\mu/m^2\mu_g$ times larger than that of the field **B**.

Rigorous solutions for Eqs. (18) and (19) can be obtained from the equations

$$\nabla^2\mathbf{B}_1 - \frac{1}{\lambda^2}\mathbf{B}_1 = 0, \tag{22}$$

$$\nabla^2\mathbf{B}_2 = 0, \tag{23}$$

where

$$\mathbf{B}_1 = \mathbf{B}+\frac{m}{q}\mathbf{B}_g, \quad \mathbf{B}_2 = \mathbf{B}+\frac{q\mu}{m\mu_g}\mathbf{B}_g,$$

and

$$\lambda^2 = \lambda_l^2 \Big/ \left[1-\frac{m^2\mu_g}{q^2\mu}\right].$$

Considering the simplest boundary condition for a half-infinite long superconductor in the presence of a homogeneous applied magnetic field $\mathbf{B}_0 = (0, B_0, 0)$ and a gravitomagnetic field $\mathbf{B}_{g,0} = (0, B_{g,0}, 0)$, the solutions of Eqs. (22) and (23) are obtained

$$\mathbf{B}_1(z) = \left[\left[B_0+\frac{m}{q}B_{g,0}\right]+\left[\mu\sigma_{e,0}-\frac{m}{q}\mu_g\sigma_{m,0}\right]\right]\mathbf{e}_y e^{-z/\lambda}, \tag{24}$$

$$\mathbf{B}_2(z) = \left[B_0+\frac{\mu q}{\mu_g m}B_{g,0}\right]\mathbf{e}_y + \left[\mu\sigma_{e,0}-\frac{q\mu}{m\mu_g}\mu_g\sigma_{m,0}\right]\mathbf{e}_y, \tag{25}$$

where $\sigma_{e,0}$ and $\sigma_{m,0}$ represent the surface electric and mass current densities, respectively. Then, the fields in the superconductor are given by

$$\mathbf{B}(z) = \frac{1}{1-\dfrac{\mu_g m^2}{\mu q^2}}\left[-\frac{m}{q}(1-e^{-z/\lambda})(B_{g,0}-\mu_g\sigma_{m,0}) - \left[\frac{\mu_g m^2}{\mu q^2}-e^{-z/\lambda}\right](B_0+\mu\sigma_{e,0})\right]\mathbf{e}_y, \tag{26}$$

$$\mathbf{B}_g(z) = \frac{1}{1-\dfrac{q^2\mu}{m^2\mu_g}}\left[-\frac{q}{m}(1-e^{-z/\lambda})(B_0+\mu\sigma_{e,0}) + \left[-\frac{\mu q^2}{\mu_g m^2}+e^{-z/\lambda}\right]\times(B_{g,0}-\mu_g\sigma_{m,0})\right]\mathbf{e}_y. \tag{27}$$

When $z \gg \lambda$, inside the superconductor, these fields reduce to the constant values

$$\mathbf{B}(z) = \frac{1}{1-\dfrac{q^2\mu}{m^2\mu_g}}\left[\left[B_0+\frac{\mu q}{\mu_g m}B_{g,0}\right]+\mu\left[\sigma_{e,0}-\frac{q}{m}\sigma_{m,0}\right]\right]\mathbf{e}_y, \tag{28}$$

$$\mathbf{B}_g(z) = \frac{1}{1-\dfrac{m^2\mu_g}{q^2\mu}}\left[\left[B_{g,0}+\frac{\mu_g m}{\mu q}B_0\right]-\mu_g\left[\sigma_{m,0}-\frac{m}{q}\sigma_{e,0}\right]\right]\mathbf{e}_y, \tag{29}$$

which yield the result

$$\mathbf{B}(z)+\frac{m}{q}\mathbf{B}_g(z)=\left[\left[B_0+\frac{m}{q}B_{g,0}\right]\right.$$
$$\left.+\left[\mu\sigma_{e,0}-\frac{m}{q}\mu_g\sigma_{m,0}\right]\right]\mathbf{e}_y e^{-z/\lambda} . \tag{30}$$

The combination fields $\mathbf{B}+(m/q)\mathbf{B}_g$ falls exponentially and is appreciable only within a layer of thickness λ and indeed vanishes inside the superconductor as pointed out by DeWitt.[6] This result is a more rigorous representation of the Meissner effect in the presence of the gravitomagnetic field.

More importantly, Eqs. (28) and (29) indicate that constant residual magnetic and gravitomagnetic fields

$$\mathbf{B}(z)\approx-\frac{m^2\mu_g}{q^2\mu}\mathbf{B}_0-\frac{m}{q}\mathbf{B}_{g,0} , \tag{31}$$

$$\mathbf{B}_g(z)\approx\mathbf{B}_{g,0}+\frac{\mu_g m}{\mu q}\mathbf{B}_0 , \tag{32}$$

will exist within a pure superconductor. Note $\sigma_{m,0}-(m/q)\sigma_{e,0}=0$. These equations are similar in form to the results derived independently by Keiser[7] and Will.[8] The results reported here differ from the previous London theory and the Meissner effect. The magnetic field inside a superconductor although very small no longer vanishes. This nonzero internal magnetic field raises an interesting consequence regarding the internal gravitomagnetic field. The magnetically produced gravitomagnetic field is order of 10^{11} times the internal magnetic field. In other words, the degree to which the gravitomagnetic field is perturbed by the external magnetic field is determined by the term of $(\mu_g m/\mu q)B_0$. This perturbation term is crucially dependent on the material properties of the superconductor used, namely, on the value of μ_g/μ. Knowledge of the value of μ_g/μ is crucial for a correct prediction of expected effects, and in a follow-up paper[9] we discuss how it is possible for the ratio μ_g/μ to assume a value larger than $\mu_{g,0}/\mu_0$, which is the value for μ_g/μ in free space, without impacting the validity of general relativity.

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