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Wormhole solution in $1/N$ expansion scheme of quantum gravity

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Wormhole solutions are found in the $1/N$ expansion scheme of quantum gravity, where the loop corrections of the large number (N) of matter fields are included in the action. Two types of wormhole solutions are seen depending on N and the renormalized parameter. It is also pointed out that the wormhole induced by the axion field found by Giddings and Strominger could survive in a modified form even if the large loop corrections were present.

I. INTRODUCTION

Recently, important quantum effects caused by the wormhole instanton configuration have been studied by many authors. The most novel point is that many universes, if they exist, can interact with each other via wormhole instantons. It has been pointed out[1] that this interaction between universes could control the parameters in the field theory of our Universe. In particular, the smallness of the cosmological constant in our Universe could be explained as the wormhole effect. After this important indication, several authors[2] have proceeded to determine other parameters according to this framework, called the big fix.

The wormhole effect is quantum tunneling from a big universe to a big universe accompanying a baby universe and vice versa. The size of the baby universe is of the order of the Planck size, where quantum fluctuation of gravitation is of course not small and must be included in the calculation of wormhole effects. In the calculation of the tunneling amplitude, the quantum fluctuations around the wormhole solution usually should be taken into account. The same thing is needed in the big fix around the de Sitter background. However, we do not know any consistent method or theory to calculate the effects of the gravitational quantum fluctuations around some background metric. Nevertheless, several effective theories have been proposed to perform the loop expansion around some background manifold. Among them, the $1/N$ expansion scheme proposed by Tomboulis[3] which is renormalizable, seems to be useful and guarantees unitarity order by order at least in the perturbation. In this scheme, the loop correction of a large number (N) of matter fields and the higher-derivative terms are included in the effective action. A wormhole solution has been found previously[4] in higher-derivative gravity, in which the loop corrections of matter are not included. On the other hand, it has previously been shown[5] in the two-dimensional CP^{N-1} model that the instanton disappears in the presence of the large number of loop corrections. Witten[5] suggested that such a washing out of the instanton due to the loop corrections of the other fields would occur. It is therefore meaningful to see whether the wormhole configurations still remain in the solutions of the Einstein equation in the $1/N$ expansion scheme. If they exist, what kind of wormhole can we see? The same problem is considered also for other wormhole solutions.[6,7] These solutions were obtained by adding the matter field, which condenses to the nontrivial classical configuration and is responsible for the resulting wormhole configuration of the metric.

The purpose of this paper is to search for wormhole solutions in the $1/N$ expansion scheme. It is shown here that wormhole solutions continue to exist even if a large number of loop corrections are included in the action. Rather, these correction terms are responsible for the existence of the wormhole solutions. Wormhole solutions found in this way are induced by quantum correction of matter fields, and so they are different from those induced by the classical configuration of a special matter field. It can be shown that wormholes in the latter case coexist with the former in spite of the presence of extra gravitational terms in the Lagrangian. So we can expect that several types of wormholes are realized, and their effect is also important in the $1/N$ expansion scheme.

In Sec. II the effective action of the $1/N$ expansion scheme in quantum gravity is given. In this scheme the equation of motion can be written in terms of local field operators only. This is because of the conformal-invariant coupling of matter fields with gravity. Another reason is that the background classical metric is assumed to be the conformal flat metric. In Sec. III the equation of the motion is written for the Robertson-Walker metric. This is further simplified through an ansatz for the solution. Using this simplified equation, the characteristic features of wormhole solutions are examined. The possible wormhole solutions are classified by their asymptotic behavior at a large and small radius of S^3. These asymptotic types of behavior are given analytically. In Sec. IV numerical analyses are given. The distribution maps of the two types of typical solutions are given for the three parameter planes in the four-dimensional parameter space of our theory. From each map we can see the characteristic property of the loop corrections of each type of matter. Several wormhole solutions are also shown. It is also assured that the wormhole induced by the axion field survives in a modified form. In Sec. VI the problem of the cosmological constant is examined, and its smallness is obtained if the Coleman mechanism works. The conclusion is given in Sec. VII.

II. EQUATION OF MOTION IN THE 1/N EXPANSION SCHEME

We consider here three kinds of matter fields: scalar, spinor, and vector. These types of matter are restricted such that their coupling with gravity is conformally invariant and there is no self-coupling. Then the Lagrangian used in the $1/N$ expansion is written as

$$\int d^4x\,\mathcal{L}=\int d^4x\,\mathcal{L}_{\rm HD}+\sum_j W_j\ , \tag{1}$$

$$W_j=iN_j\sigma_j{\rm Tr}\ln(D_j)\ , \tag{2}$$

where N_j (D_j) denotes the number (the quadratic operator) of the jth kind of matter field ($j=1$, 2, and 3 for scalar, spinor, and vector, respectively). σ_j is $\frac{1}{2}$ (-1) if the jth field is the boson (fermion). The D_j are given as $D_1=(1/\sqrt{g}\,)\partial_\mu\sqrt{g}g^{\mu\nu}\partial^\nu+\frac{1}{6}R$, $D_2=\gamma^\mu(\partial_\mu+\frac{1}{2}\omega_\mu^{ab}\sigma_{ab})$, and $D_3^{\mu\nu}=g^{\mu\nu}D^\sigma D_\sigma-D^\nu D^\mu$ for $j=1$, 2, and 3, respectively, where ω_μ^{ab} is the spin connection and $\sigma_{ab}=\frac{1}{4}[\gamma_a,\gamma_b]$. Here W_j is the renormalized one. The ultraviolet divergences were subtracted in terms of the counterterms in $\mathcal{L}_{\rm HD}$, and its renormalized form is

$$\mathcal{L}_{\rm HD}=\sqrt{g}\,[\kappa^2R+\alpha(R_{\mu\nu}R^{\mu\nu}-\tfrac{1}{3}R^2)+\tfrac{1}{3}\beta R^2+\Lambda]\ , \tag{3}$$

which is the most general form of higher-derivative gravity.

Then the Einstein equation is given as

$$-\kappa^2(R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R)+\tfrac{1}{2}g_{\mu\nu}\Lambda =-\tfrac{1}{2}\sum_i T^i_{\mu\nu}-\tfrac{1}{3}\beta[\tfrac{1}{2}g_{\mu\nu}R^2 +2(g_{\mu\nu}\Box R-D_\mu D_\nu R-RR_{\mu\nu})]+\cdots\ , \tag{4}$$

where the ellipsis represents the part coming from the conformal part in $\mathcal{L}_{\rm HD}$, $R_{\mu\nu}R^{\mu\nu}-\frac{1}{3}R^2$, which does not contribute to the classical equation of motion considered hereafter; so we drop it. D_μ denotes the covariant derivative. The terms from the matter loops $T^i_{\mu\nu}$ are obtained from W^i as follows:

$$T^i_{\mu\nu}=\frac{2}{\sqrt{g}}\frac{\delta W^i}{\delta g_{\mu\nu}}\ ,$$

Here we used the conventions $R^\mu_{\alpha\beta\nu}=\partial_\beta\Gamma^\mu_{\alpha\nu}-\cdots$, $R_{\mu\nu}=R^{\alpha}_{\mu\nu\alpha}$, and the signature is given by $(-+++)$.

Here the wormhole solutions are examined in terms of the Robertson-Walker metric [Eq. (7)], which is conformally flat. In this case $T^i_{\mu\nu}$ can be written in terms of the local field operators[8] for each matter field, which couple to the gravity in a conformally invariant way, as follows:

$$T^i_{\mu\nu}=-\frac{N_i}{16\pi^2}(\tfrac{1}{9}\gamma^iH^{(1)}_{\mu\nu}+2\delta^iH^{(3)}_{\mu\nu})\ , \tag{5a}$$

where γ^i and δ^i are given[9] in Table I, and

$$H^{(1)}_{\mu\nu}=2D_\mu D_\nu R-2g_{\mu\nu}\Box R-\tfrac{1}{2}g_{\mu\nu}R^2+2RR_{\mu\nu}\ , \tag{5b}$$

TABLE I. Coefficients γ^i and δ^i in Eq. (5a). Note that only the case of $i=3$ (spin $=1$, vector) has a negative γ^i.

i	Spin	γ^i	δ^i
1	0	$\frac{1}{120}$	$-\frac{1}{360}$
2	$\frac{1}{2}$	$\frac{1}{40}$	$-\frac{11}{720}$
3	1	$-\frac{3}{20}$	$-\frac{31}{180}$

$$H^{(3)}_{\mu\nu}=R^{\lambda}_{\mu}R_{\lambda\nu}-\tfrac{2}{3}RR_{\mu\nu}-\tfrac{1}{2}R_{\lambda\sigma}R^{\lambda\sigma}g_{\mu\nu}+\tfrac{1}{4}R^2g_{\mu\nu}\ . \tag{5c}$$

The term coming from the loop correction $T^i_{\mu\nu}$ consists of the two terms. It can be seen from Eqs. (4) and (5a) that to add the first term $H^{(1)}_{\mu\nu}$ is equivalent to modifying the parameter β in Eq. (4) as follows:

$$\beta'=\beta+\sum_i N_i\gamma^i/96\pi^2\ . \tag{6}$$

The second term $H^{(3)}_{\mu\nu}$, however, cannot be obtained through a functional derivative of some higher-derivative term. So this term is the newly appeared one due to the loop corrections.

A wormhole solution has previously been found[4] for higher-derivative gravity with the Lagrangian $\mathcal{L}_{\rm HD}$, where the parameter β was restricted to be positive. This restriction was imposed because of the convergence of the Euclidean path integral.

In spite of the existence of $H^{(3)}_{\mu\nu}$ in Eq. (4), we can see the wormhole solution, which has been seen in higher-derivative gravity as far as $\beta'>0$. For $\beta'<0$, but $\beta>0$, we can see another type of solution which was seen in higher-derivative gravity with negative β. This is seen for the $i=3$ case because of the negative value of γ^3 (see Table I). Then we can say that the values of δ^i, the coefficients of $H^{(3)}_{\mu\nu}$, do not affect the qualitative features of the solutions.

III. WORMHOLE SOLUTION

We search for the Euclidean solution of Eq. (4) by restricting the metric to the form

$$ds^2=d\tau^2+a^2(\tau)d\Omega_3^2\ , \tag{7}$$

where $d\Omega_3^2$ denotes the metric of S^3 and τ is Euclidean time. Here we define the function $f(a)$[4] as $f(a)=\dot{a}^2-1$, and the higher derivatives of $a(\tau)$ with respect to τ are replaced by the derivatives of $f(a)$ with respect to a. This corresponds to rewriting the Einstein equation from the equation of $a(\tau)$ with the variable τ to the one of $f(a)$ with the variable a. The latter is written by the second-order differential equation, which is more tractable than the original higher-order differential equation of $a(\tau)$. This is the reason to use $f(a)$ instead of $a(\tau)$.

For the metric Eq. (7), the time-time component of Eq. (4) is written in terms of $f(a)$ as

$$f-f_0=\sum_{i=0}^{3}\left[\frac{r_i}{a}\right]^2[-\tfrac{1}{4}(f+\tfrac{1}{2}af')(f-\tfrac{1}{2}af')+(1+f)(f-\tfrac{1}{4}af'-\tfrac{1}{4}a^2f'')+c_if^2]\ , \tag{8}$$

where $f'=df/da$ and $f''=d^2f/da^2$. f_0 is the solution, which is given below by Eq. (9a), in the absence of the right-hand side of Eq. (8). In the right-hand side of Eq. (8), the terms of $i=0$ are those coming from $\frac{1}{3}\beta R^2$ in $\mathcal{L}_{\rm HD}$. Other terms corresponding to $i=1-3$ represent T^i_{00}. The parameters r_i and c_i, which are the rescaled values of γ_i and δ_i, are given in the Table II.

The criterion for $f(a)$ as a wormhole solution is that f crosses -1 twice at large and small values of a, say, a_m and a_b ($a_m \gg a_b$), and $f(\geq -1)$ varies smoothly between a_m and a_b. The typical form is seen in Fig. 2. Since $a(\tau)$ is real, $f(a)$ is meaningful in the range $a_b \leq a \leq a_m$ $[f(a)\geq -1]$. And this is just the Euclidean configuration connecting a baby universe (with a size of the Planck length $\sim a_b$) and a large mother universe (with a size of a_m).

In the absence of the right-hand side of the Eq. (8), the solution is $f=f_0$, which is given here as

$$f_0=-\left[\left[\frac{a_0}{a}\right]^4+(Ha)^2\right]\ , \tag{9a}$$

$$H^2=\frac{\Lambda}{6\kappa^2}\ , \tag{9b}$$

where the first term of (9a) could be added if the axion field were included in the Lagrangian given by Eq. (1) and the ansatz, which is used by Giddings and Strominger[6] for the axion configuration, were imposed. For $a_0\neq 0$, (9a) is the typical wormhole solution, which crosses -1 twice at $a\sim a_0$ and $a\sim H^{-1}$. H^{-1} is the radius of the de Sitter mother universe, and a_0 is the radius of the baby universe. In the absence of the axion ($a_0=0$), (9a) denotes the usual Euclidean de Sitter solution, and this fact means that there is no wormhole solution in the case of the pure Einstein terms $\sqrt{g}\,(\kappa^2R+\Lambda)$.

Next, we consider the case where the right-hand side of Eq. (8) is present. As a special case, we have previously found a wormhole solution in higher-derivative gravity[4] [the case of $i=0$ in Eq. (8)] even if the axion field were absent. Here we consider the general case, in which Eq. (8) is rewritten as

$$f-f_0=\left[\frac{r}{a}\right]^2[cf^2+\tfrac{1}{16}(af')^2+(1+f)(f-\tfrac{1}{4}af'-\tfrac{1}{4}a^2f'')]\ , \tag{8'}$$

where

$$r^2=\sum_{i=0}^{3}r_i^2,\quad \sum_{i=0}^{3}r_i^2c_i=(c+\tfrac{1}{4})r^2\ .$$

TABLE II. Coefficients c_i and r_i^2 in Eq. (8) in which $A=1/2880\pi^2$.

i	0	1	2	3
c_i	0	$-\frac{1}{4}$	$-\frac{11}{24}$	$\frac{31}{36}$
r_i^2	$8\beta/\kappa^2$	$2A/\kappa^2$	$6A/\kappa^2$	$-36A/\kappa^2$

Equation (8′) is equivalent to the equation of higher-derivative gravity if $c=-\frac{1}{4}$. Then the point to be examined here is whether the wormhole solution found for $c=-\frac{1}{4}$ remains if c deviates from $-\frac{1}{4}$. Another point is how the axionic instanton was modified if it were surviving in spite of the presence of the right-hand side of Eq. (8). Although another type of nonaxionic wormhole solutions was found[6,7] by adding other matter fields, we study here only the two cases $a_0=0$ and $a_0\neq 0$ (with and without axion) to clear the effects of the loop corrections of the matter.

Before giving the numerical analysis, we study the characteristic property of the solution of Eq. (8) by using Eq. (8′). The asymptotic behavior of $f(a)$ at large and small values of a is examined by assuming the power-series expansion

$$f(a)=a^p\sum_i c_ia^i\ , \tag{10}$$

where $i=0$ to $-\infty$ (0 to ∞) for large (small) a. The results are as follows.

At large a, $p=2$ and the asymptotic solution is $f\propto -a^2$ for $\Lambda\neq 0$. This means that all the nontrivial solutions approach the de Sitter solution, which is given by $f=-\bar{\gamma}a^2$ with some appropriate constant $\bar{\gamma}$, at large a. It can be seen that there are two ways $f(a)$ approaches the de Sitter space at large a. To see this, we consider the case of $a_0=0$ and at the limit $\Lambda\to 0$, where the de Sitter radius is infinite or $\bar{\gamma}\to 0$ in $f=-\bar{\gamma}a^2$. So f is very small. Then Eq. (8′) can be approximated by the linearized one with respect to f as follows:

$$f=\left[\frac{r}{a}\right]^2(f-\tfrac{1}{4}af'-\tfrac{1}{4}a^2f'')\ , \tag{11}$$

where f_0 is neglected because $H\sim 0$. This is further rewritten as follows for $r^2>0$:

$$f''+\frac{1}{x}f'+\left[1-\frac{4}{x^2}\right]f=0\ , \tag{12}$$

where $x=2a/r$, $f'=df/dx$, and $f''=d^2f/dx^2$. Equation (12) is Bessel's differential equation of second order, and its general solution is given as

$$f=\gamma_1J_2(x)+\gamma_2N_2(x)\ , \tag{13}$$

where γ_1 and γ_2 are the small arbitrary constants because f must be small.

The second case is obtained for $r^2<0$. In this case the solution of Eq. (11) is given as follows by the absolute value of x:

$$f=\delta_1I_2(|x|)+\delta_2K_2(|x|)\ , \tag{14}$$

where $I_2(|x|)$ and $K_2(|x|)$ are the modified Bessel functions. Since $I_2(|x|)$ diverges at large $|x|$, we must choose $\delta_1=0$ and a small value of δ_2 because of the consistency with the approximation that f is small.

From Table II it can be expected that the asymptotic solution Eq. (13), which oscillates around $f\sim-\bar{\gamma}a^2$ with an amplitude $\gamma_{1,2}/\sqrt{a}$, could be obtained for the scalar and the spinor cases $i=1$ and 2. On the other hand, the behavior represented by Eq. (14) will be seen in the case of the inclusion of the vector loop correction (the case of $i=3$).

On the other hand, the asymptotic behavior of f at small a can be classified by the exponent p, which must be an integer to get a consistent solution in the form of Eq. (10), as follows. (i) $p>0$. The allowed value is $p=4$ or 2; the latter case corresponds to the de Sitter solution. (ii) $p=-2$. This is realized only for the $i=0$ case (higher-derivative gravity). (iii) $p=-4[(1+c)/3]^{1/2}\leq-3$. (iv) $f=-1/(1+c)=\text{const}$. This might be a wormhole solution if $c<0$ $(f<-1)$. (v) $p=-1$, which is realized only for $a_0\neq0$, and this solution corresponds to the axionic wormhole solutions modified by the presence of higher-derivative terms.

The analyses given above are based on the form of $f(a)$, which is restricted to the polynomial type [Eq. (10)]. We should consider a more general form of $f(a)$ to examine the properties of the solution. However, the analysis given here is enough to assure the existence of several types of wormhole solutions of Eq. (8).

After all, the wormhole solutions would be characterized by the asymptotic types of behavior at small a [(ii), (iii), (iv), and (v) given above] and large a [Eqs. (13) and (14)]. They are given explicitly in Sec. IV by numerical analysis, and the solutions are classified in terms of the combinations of the asymptotic behavior at small and large a.

IV. NUMERICAL ANALYSIS AND THE TYPICAL SOLUTION

We could not give the analytic form of the expected wormhole solutions in the full range of a, but we can show its existence by solving numerically the differential equation (8). Here the Runge-Kutta-Gill method was used to solve this equation.

Our procedure to find the wormhole solution is as follows. At first, consider the case of $a_0=0$. In this case the de Sitter solution $(f=-\bar{\gamma}a^2)$ is the exact solution even if the right-hand side of Eq. (8) is present. This solution corresponds to the flat solution $(f=0)$ for the case of $\Lambda=0$. However, $\bar{\gamma}$ is slightly different from H^2 except for the $i=0$ case, and it depends on the parameters in the right-hand side. It is given as $\bar{\gamma}=[-1+(1+4\alpha' H^2)^{1/2}]/2\alpha'$ and $\alpha'=r^2(c+\frac{1}{4})$ for Eq. (8′). For enough small H, $\bar{\gamma}\sim H^2$. So we first find the de Sitter solution $f=-\bar{\gamma}a^2$ for small H. This is equivalent to giving the value of $\bar{\gamma}$. Then we choose slightly different values for f and f' from those of this de Sitter solution at some large a as the initial condition in order to get another solution numerically. This solution will deviate from the de Sitter solution, and it varies with a according to the asymptotic behavior at small and large a given above. In this way we have found the expected wormhole solutions. Although the de Sitter solution $f=-\bar{\gamma}a^2$ is not the exact solution for $a_0\neq0$, it is still a good approximation at large a. So we performed the same procedure as in the case of $a_0=0$ to find the wormhole solution.

Since four parameters β and N_i $(i=1-3)$ are contained in Eq. (8), we should examine this equation in four-dimensional parameter space. However, the numerical analysis was performed here in the following restricted three parameter planes: (a) $\beta-\bar{r}_1$, (b) $\beta-\bar{r}_2$, and (c) $\beta-\bar{r}_3$, where $\bar{r}_1=(r_1\kappa)^2/2$, $\bar{r}_2=(r_2\kappa)^2/6$, and $\bar{r}_3=-(r_3\kappa)^2/36$. This restriction is economical and is suited to our purpose to see the characteristic property of each loop correction separately.

The solutions found here numerically are classified by the following combinations of the asymptotic types of behavior at small a [three types (ii), (iii), and (v) given in Sec. III] and at large a [Eqs. (13) and (14)]; (A) (ii) Eq. (13), (B) (iii) Eq. (14), and (C) (v) Eq. (13). Other types of wormhole solutions could exist, but we could not find them.

The distribution of A-type (B-type) solutions are shown in Fig. 1 by the squares and dots (triangles) for a special boundary conditions, $f=-0.26$ and $f'=-0.0500839$ at $a=10.02$ for $\Lambda=0.015\kappa^2$ and $\kappa^2=2.0$. From Fig. 1 it is seen clearly as mentioned in Sec. II that the distributions of the two types of solutions are separated by the critical line $\beta'=0$, where β' corresponds to β in the higher-derivative gravity and is given by Eq. (6) above. For each parameter plane $\beta-\bar{r}_i$ these lines are explicitly given as $\beta=-\frac{1}{4}\bar{r}_1$, $\beta=-\frac{3}{4}\bar{r}_2$, and $\beta=\frac{9}{2}\bar{r}_3$ for Figs. 1(a)–1(c), respectively. For the cases of Figs. 1(a), the scalar, and 1(b), the spinor, the gradients of the $\beta'=0$ line are negative; then the region of the A-type solution is extended to the space of negative β. On the other hand, Fig. 1(c) shows that a B-type solution exists even if β is positive for the vector case.

The typical solutions chosen from the point in the distribution maps (Fig. 1) are shown for the three cases in Fig. 2(a)–2(c). The solutions given at the top of Figs. 2(a)–2(c) are the wormhole solutions chosen from the points above the $\beta'=0$ line in Figs. 1(a)–1(c), respectively. The corresponding points are $(\bar{r}_i,\beta)=(4.0,1.0)$, $(0.5,1.5)$, and $(0.25,2.5)$ for $i=1-3$, respectively. Other parameters are the same as those given in Fig. 1. The lower two solutions of each part of Fig. 2 are those which were obtained by slightly changing the initial value of f at $a=10.05$ as $f=-0.30$ and -0.35 for the middle and bottom graphs, respectively, and the other parameters and the value of f' are the same as those at the top of each figure.

From these results we can see clearly that A-type solutions oscillate around de Sitter space at large a according to Eq. (13). Figure 2(d) is the one for higher-derivative gravity $(\bar{r}_i,\beta)=(0.0,1.5)$. The parameters are assigned as in the case of Figs. 2(a)–2(c) for the top and lower two solutions. It can be seen that they all show similar behavior in the whole range of a as in the $\bar{r}_i\neq0$ cases.

Under the critical lines $\beta'<0$ B-type solutions appear in all the parameter spaces [Figs. 1(a)–1(c)]. Typical solutions are shown in Fig. 3. The asymptotic behavior at large a of these solutions is characterized by Eq. (14), and they do not oscillate. The behavior at small a is characterized by the power index p (≤ -3) defined in Eq. (10). This solution is classified as the category (iii) in Sec. III $-\infty < p < -3$. The solutions of $p=-3$ and -4 are obtained on the lines l_i and m_i ($i=1-3$), respectively. These lines are shown in Fig. 1. It should be noted that the line of the solutions for $p=-\infty$ approaches the $\beta'=0$ line. However, no wormhole solution exists just on this line $\beta'=0$, where we obtain the de Sitter solution

$$f=-\frac{a^2}{2c_i r_i^2}[1-(1-4c_i r_i^2 H^2)^{1/2}] .$$

So we will obtain the solution of $p=$(large negative integer) just under this critical line. Then this solution will rapidly change with a. In fact, we can see from Fig. 3(c$_4$) that B-type solutions obtained near this critical line vary rapidly. These B-type solutions, Fig. 3(a$_1$)–3(c$_4$) are essentially equivalent to those shown in Figs. 3(d$_1$) and 3(d$_2$), the solutions of higher-derivative gravity for $\beta<0$.

Another feature of the B-type solution is that it has a large wormhole radius a_b, the smaller value of a where $f=-1$, and a_b is comparable to the one of the mother universe a_m. Although the size of a_b decreases with increasing r_3, it does not shrink to the Planck size as the A-type solution. It can be also seen that the value of $Z_0=\exp(-S_E)$ for the B-type wormhole solution is very

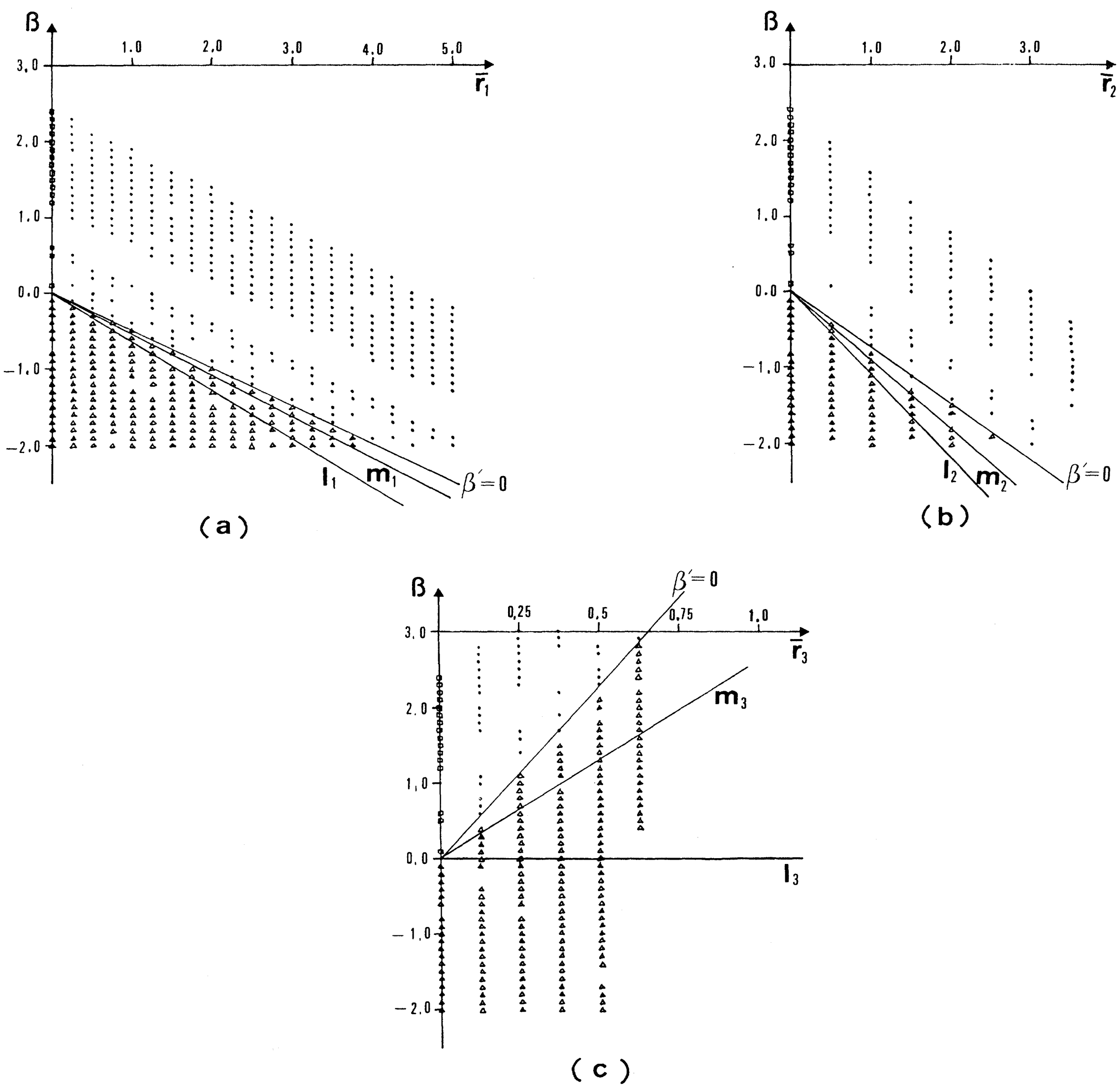

FIG. 1. Distribution of the wormhole solutions found under the special boundary condition $(f,f')=(-0.26,-0.0500839)$ at $a=10.02$, for $\kappa^2=2.0$ and $\Lambda=0.015\kappa^2$. (a) The spinor case (spin$=\frac{1}{2}$), (b) the scalar case (spin$=0$), and (c) the vector case (spin$=1$). The dots and squares represent the A-type wormholes, and the triangles represent the B-type wormholes. Here $\bar{r}_1=\frac{1}{2}\kappa^2 r_1^2$, $\bar{r}_2=\frac{1}{6}\kappa^2 r_2^2$, and $\bar{r}_3=-\frac{1}{36}\kappa^2 r_3^2$. The critical lines of $\beta'=0$ are shown in each figure.

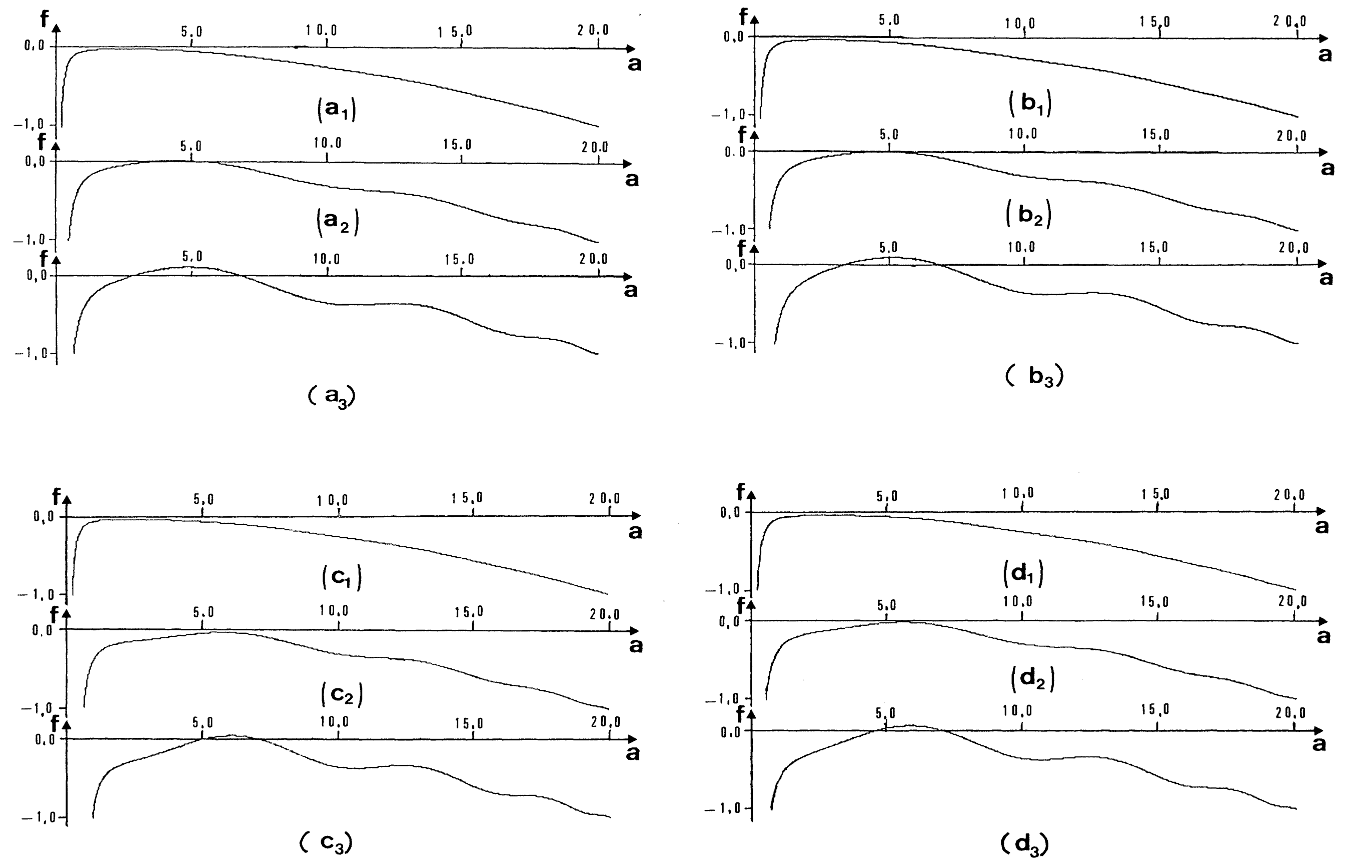

FIG. 2. Typical A-type wormholes are shown. All graphs have the same value of $f' = -0.0500839$ at $a = 10.02$. The values of $f(10.02)$ are assigned for the top, middle, and bottom graphs of (a), (b), and (c) as follows: $f = -0.26$, -0.30, and -0.35, respectively. Other parameters are as follows: (a) $r_1 = 2.0$, $\beta = 1.5$ for the scalar, (b) $r_2 = 0.5$, $\beta = 1.8$ for the spinor, (c) $r_3 = 0.1$, $\beta = 1.4$ for the vector, and (d) $r_i = 0$, $\beta = 1.5$ for the pure higher-derivative terms only.

small compared to the one of the A-type solution. For example, we obtain the ratio $Z_0^B / Z_0^A = \exp(-25\,900)$ for the solution represented in Fig. 2(d_1) [Fig. 3(d_1)] as the A-type (B-type) one. So it is questionable to consider that the B-type solution is responsible for the effective quantum tunneling between the universes with different topologies.

Both A- and B-type solutions mentioned above were obtained in the absence of the axion ($a_0 = 0$). The next problem is to see whether the wormhole induced by the axion field could survive when the right-hand side of Eq. (8) is present. This is seen by searching for wormhole solutions for some nonzero value of a_0. As a result of our analysis, we can say that the axion-induced wormhole is not washed out as the instanton in QCD, and it remains in a modified form. This is assigned as the C-type wormhole solution. This solution is clearly seen above the critical line where the A-type solution is absent occasionally. When the A-type solution exists, the effect of the axion field is small and masked, and so we cannot see it. Typical solutions are shown in Fig. 4

In Figs. 4(a)–4(c) we show the combinations of the two solutions taken from the three-parameter spaces 1(a)–1(c). The upper solution of each combination is the nonwormhole solution with $a_0 = 0$, and the lower is the wormhole solution with the same parameters except for the value of a_0, which is taken as 0.5. In this way we can find the C-type wormhole solution if we switch on a_0 to the nonzero value in the region, where the A-type solution is not realized. This means that the C-type wormhole solution was produced by the axion field condensation. The asymptotic form of the C-type solution is given as $f \propto a^{-1}$ at small a and Eq. (13) at large a, as mentioned above.

There will be another type of solution which approaches the constant less than -1 at $a = 0$. This solution could become a new type of wormhole solution. However, the parameter region, where this solution is allowed, is wide but restricted. The forbidden region of these wormhole solutions in each parameter plane is given as $-\frac{1}{2} < \beta/\bar{r}_1 < -\frac{1}{4}$, $-\frac{17}{8} < \beta/\bar{r}_2 < -\frac{3}{4}$, and $-\frac{99}{8} < \beta/\bar{r}_3 < \frac{9}{2}$ for the parameter spaces (a), (b), and (c), respectively. Then both above and under the critical line $\beta' = 0$ they are allowed. However, we cannot reproduce this type of wormhole solution numerically. This is because of the difficulty of finding a precise initial condition

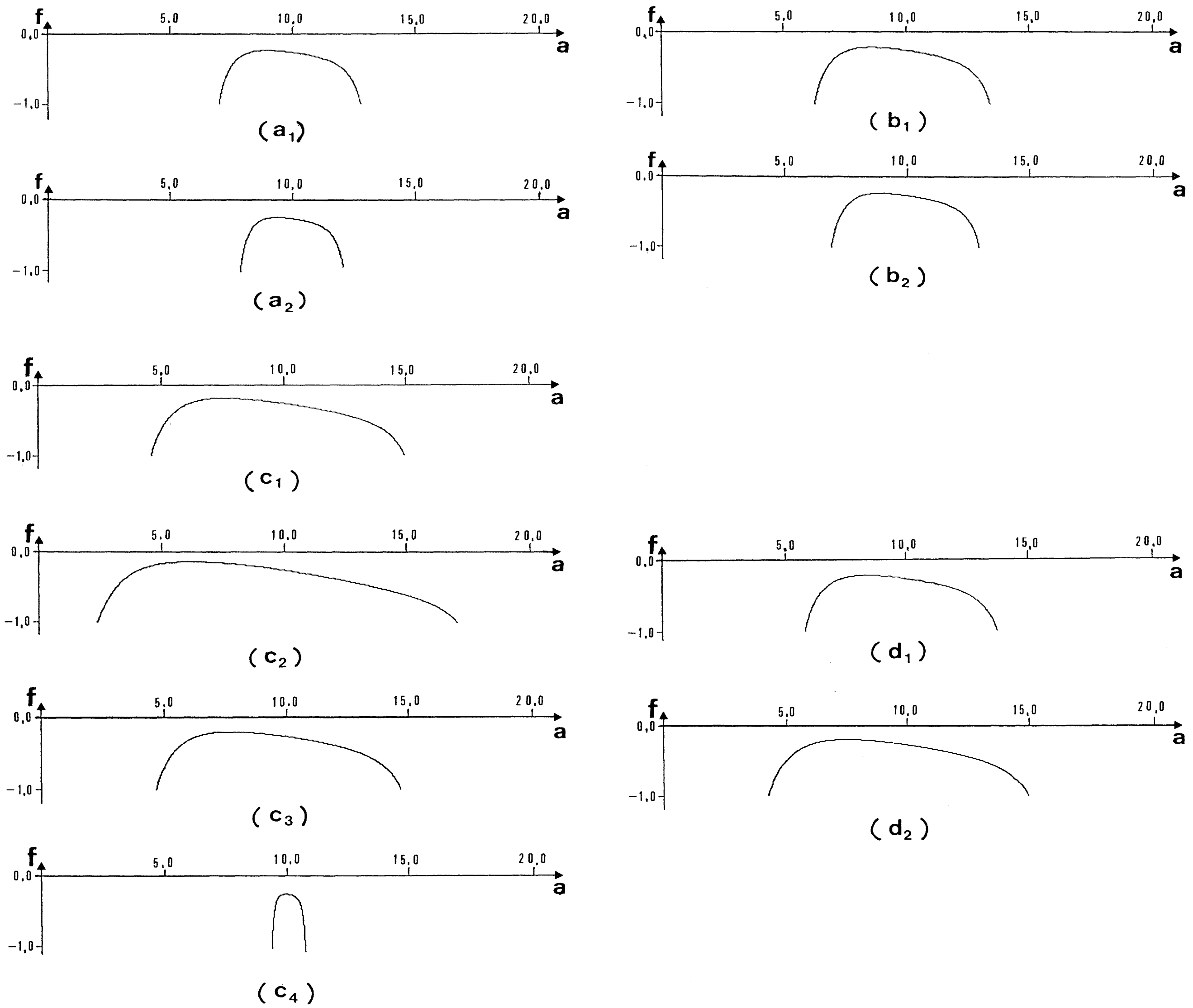

FIG. 3. B-type wormholes, which were denoted by the triangles in Figs. 1(a)–1(c). (a) For scalar at $(\beta,\bar{r}_1)=(a_1)$ $(-1.0,2.0)$ and (a_2) $(-2.0,7.0)$; (b) for the spinor at $(\beta,\bar{r}_2)=(b_1)$ $(-1.5,1.0)$ and (b_2) $(-2.0,2.0)$; (c) for the vector at $(\beta,\bar{r}_3)=(c_1)$ $(1.0,0.5)$, (c_2) $(-1.0,0.5)$, (c_3) $(-1.0,0.125)$, and (c_4) $(1.664,0.375)$; and (d) for higher-derivative gravity at $\beta=(d_1)-1.0$ and $(d_2)-2.0$.

to arrive at an accurate value of f at $a=0$. Another difficulty of the accurate calculation is not to forget the initial condition in spite of the accumulation of error during the process of computer work.

In any case several types of wormhole solutions are really found in the $1/N$ expansion scheme. The region where the wormhole exists will extend to a wider region than the one given in Fig. 1 if the solutions were searched for by using a more extended initial condition. If we further add the distribution of C-type solutions in Fig. 1, then the map will be filled by wormhole solutions of various types. Then we can say that the wormhole configurations must be taken into account in any calculation in the $1/N$ expansion scheme even if the axion field were not included in the theory.

V. VALUE OF S_E

The next problem is to estimate the value of the Euclidean action $S_E(g^{cl}_{\mu\nu})$, where $g^{cl}_{\mu\nu}$ represents the wormhole configurations given above. Since $g^{cl}_{\mu\nu}$ depends on only the Euclidean time τ through $a(\tau)$, S_E can be written as

$$S_E=2\pi^2\int d\tau\, a(\tau)^3\mathcal{L}(a(\tau))$$

$$=2\pi^2\int da\frac{a(\tau)^3}{\sqrt{1+f(a)}}\mathcal{L}(a(\tau))\ , \qquad (15)$$

where $\mathcal{L}(a(\tau))$ is the Lagrangian given by Eq. (1). We can show that S_E is finite as follows.

The factor $1/\sqrt{1+f(a)}$ diverges at $a=a_m$ and a_b because $f=-1$ at these values of a. This divergence is, however, canceled by the integral measure da. In order to see this, it is determined how $f(a)$ approaches -1 by assuming the following form for $f(a)$:

$$f(a)=-1+(a-a_0)^q[A_0+A_1(a-a_0)+A_2(a-a_0)^2+\cdots] , \quad (16)$$

where a_0 denotes a_m or a_b. By substituting this form

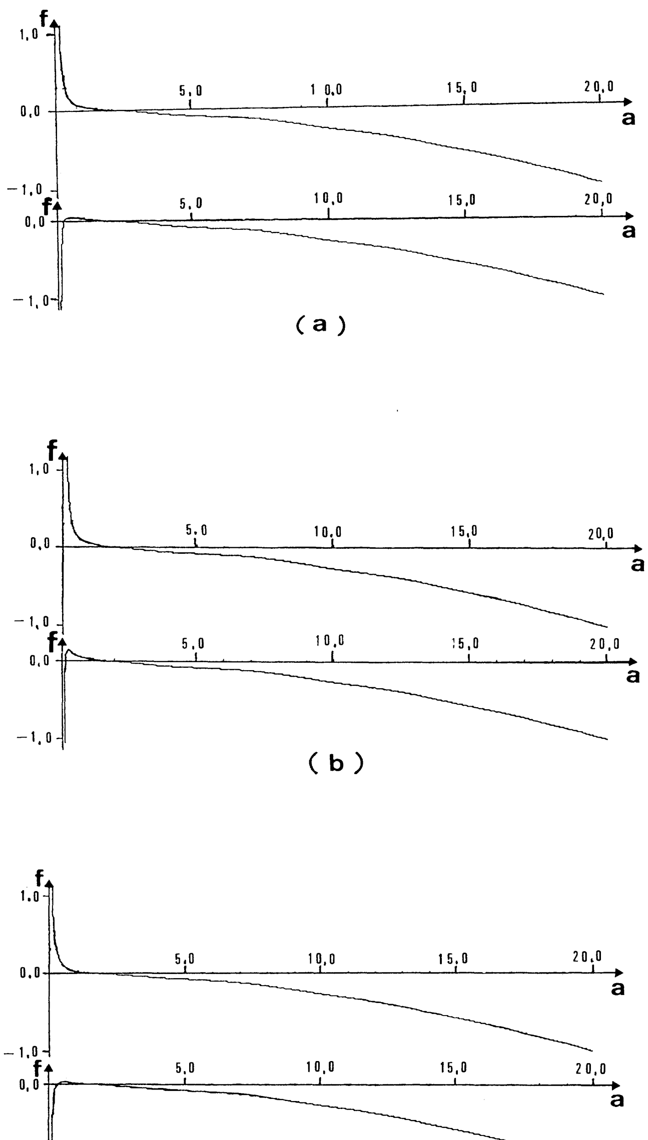

FIG. 4. Solutions with ($a_0=0.5$) or without ($a_0=0.0$) the axion are shown. The upper graphs of (a)–(c) represent the solution without the axion, and the lower ones are the C-type wormhole with the axion. (a) For scalar at $(\beta,\bar{r}_1)=(0.5,1.0)$, (b) for the spinor at $(\beta,\bar{r}_2)=(0.5,0.5)$, and (c) for the vector at $(\beta,\bar{r}_3)=(2.0,0.25)$. The initial condition and other parameters are the same as those of Fig. 1.

into Eq. (8), we can determine q, A_0, and other coefficients. It can be shown that q is equal to 1 in order to obtain a consistent solution in the form of Eq. (16). In this case we obtain, near $a \gtrsim a_b$,

$$da\frac{1}{\sqrt{1+f(a)}} \sim -2\left[\frac{a_b}{A_0}\right]^{1/2}\frac{1}{\sin^2\theta}d\theta , \quad (17)$$

where A_0 is positive and the integral variable a is changed to θ as $a=a_b/\sin^2\theta$, where $\theta \lesssim \pi/2$. This measure does not diverge at $a\sim a_b$. For $a\lesssim a_m$ we obtain, by the replacement $a=a_m\sin^2\theta$ with $\theta\lesssim\pi/2$,

$$da\frac{1}{\sqrt{1+f(a)}} \sim 2\left[\frac{a_m}{|A_0|}\right]^{1/2}\sin\theta\, d\theta , \quad (18)$$

where we can see $A_0<0$. From Eqs. (17) and (18) it can be said that the measure part is finite at $f=-1$.

Next, we see that $\mathcal{L}(a)$ is finite in the range $a_b\le a\le a_m$. The higher-derivative part $\mathcal{L}_{\rm HD}(a)$ is explicitly written as

$$a^3\mathcal{L}_{\rm HD}(a)=6\kappa^2 a\left[(f+\tfrac{1}{2}af')+\frac{3}{4}\left[\frac{r_0}{a}\right]^2(f+\tfrac{1}{2}af')^2+(Ha)^2\right] . \quad (19)$$

This is not singular for the wormhole solutions given here in the range $a_m<a<a_b$. So $\int da\,(a^3/\sqrt{1+f}\,)\mathcal{L}_{\rm HD}$ is finite. Since the second term in Eq. (1) is the loop correction of the conformal invariant matter field, it can be rewritten through the conformal transformation of the background metric (the wormhole solution), $g^{cl}_{\mu\nu}=\Omega^2\tilde{g}_{\mu\nu}$, where $\tilde{g}_{\mu\nu}$ is the metric of S^4 with a unit radius, as

$$W=-\frac{i}{2}\mathrm{Tr}\ln[(\mu\Omega)^2D(\tilde{g}_{\mu\nu})] , \quad (20)$$

where μ is an appropriate mass scale. This is estimated by separating it into two parts. One is $\mathrm{Tr}\{\ln[D(\tilde{g}_{\mu\nu})]\}$, which is regularized and can be considered as a part of the cosmological constant in $\mathcal{L}_{\rm HD}$. The other part is written as

$$-\frac{i}{2}\mathrm{Tr}\ln(\mu\Omega)^2=\pi^2\int d\eta\,\Omega^4\ln(\mu\Omega)^2 =\pi^2\int\frac{da}{\dot{a}}a^3\ln(\mu a)^2 , \quad (21)$$

where η is Euclidean time in the metric $\tilde{g}_{\mu\nu}$ and it is related to $a(\tau)$ as $\eta=\int^{\tau}d\tau'/a(\tau')$. And $\Omega(\eta)=a(\tau)$. It can be seen that Eq. (21) is also finite for an appropriately chosen scale factor μ. Then we can say that S_E is finite for the wormhole configurations. The numerical estimation of S_E for some wormhole solution obtained here is not given, but we can say that the more the wormhole solution departs from the de Sitter solution, the larger the value of S_E.

VI. COSMOLOGICAL CONSTANT

According to Coleman,[1] we can estimate the partition function under the de Sitter background S^4 with a radius

$L=\sqrt{6\kappa^2/\Lambda}$, and we obtain the weight factor Z to fix the parameters contained in the theory:

$$Z=\exp\left\{\exp\left[48\pi^2\left[\frac{\kappa^4}{\Lambda}-\frac{4}{3}\beta\right]+\frac{8}{3}\pi^2[c_0-Nc_1\ln(L\mu)]\right]\right\}, \qquad (22)$$

where we assumed that wormhole instantons exist and the Coleman mechanism works. c_1 is given by Grinstein and Hill,[2] and it is positive for all spin states if their interactions with gravity are conformally invariant. For nonzero c_1, the $\ln(\Lambda)$ term is present as a result of the N-matter-field loop corrections. But this term does not change the sharp peak of Z at $\Lambda=0$. So the statement that Λ vanishes due to the wormhole survives also in this scheme.

VII. CONCLUSION

The existence of wormhole solutions are shown here by numerical analyses for the $1/N$ expansion scheme of gravitational theory, where loop corrections of the large number N matter fields are included. These types of matter couple to gravity in a conformally invariant way. Here the scalar, spinor, and vector are considered as matter fields. For each case we found two typical solutions which are classified as A- and B-type solutions. The A-type solution is the same kind of solution as the one which has been obtained in higher-derivative gravity. The B-type solution was also found in higher-derivative theory with the parameter $\beta<0$, where the Euclidean path integral would not give a finite value of the partition function. However, this solution exists in the presence of the N vector loop correction in spite of positive β. This is because of the fact that $\beta'<0$, which corresponds to a β of higher-derivative gravity effectively, even if β is positive.

The map of the distribution of these two types of solutions are given for the three planes of parameter space to see the effect of each matter loop correction. The distributions of the A- and B-type solutions are clearly separated by the critical line $\beta'=0$, above (under) which the A-type (B-type) solutions appear. Just on this line there is no wormhole, however, and we find a de Sitter solution.

If we add the axion field to our theory, C-type wormhole solutions are found above the critical line $\beta'=0$. This is easily found at the point in the parameter space where the A-type solution was not found. This solution is considered as a modified axion-induced wormhole given by Giddings and Strominger. Then we can say that the large loop correction cannot wash out the instanton solution, which existed when the loop corrections were not included. The washing out of the instantons expected in QCD (Ref. 5) does not occur in gravitational theory for wormhole solutions.

We considered here three types of matter fields which couple to gravity in a conformal invariant way. The loop corrections of these fields give the same qualitative contribution to the propagator of the graviton.[3] However, these corrections give different coefficients for the trace anomaly, which causes a qualitative difference of the distribution of the two types of wormhole solutions. For the case of the vector, the sign of γ is different from that of the scalar and spinor, and this is the reason why B-type solutions appear in the region of $\beta>0$. On the contrary, the A-type solution appears for $\beta<0$ in the case of the scalar and spinor.

In any case it is assured that there are some kinds of wormhole solutions in the $1/N$ expansion scheme, and they should be taken into account in quantum gravity.

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