# Quark-hadron phase transition of the early Universe in the nontopological soliton model

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Based on the nontopological soliton model, we discuss the quark-hadron phase transition of the early Universe in the hot big-bang model. We extract the parameter spaces which are relevant for small supercooling of the phase transition and the inhomogeneous nucleosynthesis. In particular, we explore the possibility that the early Universe is a cold universe filled with very dense and degenerate baryons. This cold universe is then reheated by the latent heat released from the quark-hadron phase transition. Without fine-tuning any parameter in the soliton model, we find that a typical potential in the model would dilute the dense baryons to an acceptable baryon asymmetry of the Universe. We also discuss the generation of density fluctuations during the phase transition in the cold universe.

## I. INTRODUCTION

In the hot big-bang cosmology, it is presumed that a quark-hadron phase transition had occurred in the early stage of the Universe when the temperature was about 100–200 MeV. It is also expected that a quark-gluon phase may exist in the dense cores of compact objects such as neutron stars. On the other hand, quark-gluon plasma may be formed in the hot central region and in the dense fragmentation region in heavy-ion-collision experiments. The nature of the phase transition is still unknown since we know little about QCD physics. However, the transition is probably of first order, as suggested by computer simulations of lattice QCD with dynamical fermion.[1]

The quark-hadron phase transition of the early Universe has been studied in the literature. These studies can be classified into three different categories: (1) The quark-gluon plasma condenses into a gas of hadrons in a way which is in thermodynamic equilibrium as the Universe expands;[2,3] (2) the Universe is supercooled to a temperature well below the critical temperature and then undergoes an out-of-equilibrium nucleation of hadron bubbles in the quark-gluon-plasma sea;[4,5] (3) is in between the first and the second categories, in which the Universe undergoes a phase transition with small supercooling.[6–9] In this scenario coexistence between the quark-gluon and hadron phases can be established after nucleation at a coexistence temperature. This cosmic separation of phases may generate isothermal baryon-density fluctuations.[7] As a consequence, there could occur an inhomogeneous nucleosynthesis[7,9,10] with the baryonic mass density $\Omega_B=1$, thus closing the Universe with baryons. (In the standard hot big-bang model, the abundance of light elements produced in the homogeneous nucleosynthesis restricts $\Omega_B \lesssim 0.2$.[11] On the other hand, inflation predicts $\Omega=1$, altogether with the observation that the luminous matter density $\Omega_{\rm lum} \lesssim 0.02$; we need dark matter for closing the Universe.[12]) Unfortunately, it is agreed[10] that one cannot have $\Omega_B=1$ and get the correct light-element abundances. However, inhomogeneities can have an important effect on element abundances. In addition, the later cooling of the quark phases may make them contract to form stable quark nuggets.[6] If the nuggets survive the evaporation into hadrons from their surfaces in the later epoch of the Universe after their formation, they could be a promising candidate for dark matter.[6] Further studies[13] have shown that quark nuggets are very unlikely to survive hot evaporation. In the cold-universe scenario, which we shall discuss below, however, nuggets might survive more easily.

In most of the discussions of the quark-hadron phase transition in the early Universe, the thermodynamic properties of both quark and hadron phases have been considered separately and then matched at the critical temperature. In the nucleation of hadronic bubbles, it has been assumed that the nucleating action has a certain general functional form. In this paper we shall discuss the quark-hadron phase transition in the hot big-bang universe in the language of the phenomenological nontopological soliton model of quark-hadron physics.[14] Although the model is premature, it has been applied to fit the hadronic properties, and the results are encouraging.[15] We think that the description in terms of an effective potential as in the nontopological soliton model is a convenient way to investigate the evolution of the early Universe in the quark-hadron phase transition, especially when the phase transition is of first order. Here we are confined to the nontopological soliton model rather than others, just because the model is simple and adequate enough for our purposes. Other discussions of the quark-hadron phase transition in an effective-field theory, for example, based on the topological soliton model can be found elsewhere.[16]

The development of a cold universe as a nonstandard scenario for the evolution of the Universe has been explored previously.[17–19] Recently, in the context of supersymmetric model, it was proposed[20,21] that the baryon asymmetry of the Universe can be generated by the decays of the scalar lepton and quark fields in a cold universe with temperature of about 1 TeV. [In the hot big-bang model, it is presumed that the baryon asymmetry is generated in the SU(5) grand-unified-theory (GUT) model by the decays of the superheavy bosons at temperature of $10^{15}$ GeV (Ref. 22).] In this mechanism the resulting baryon-photon number-density ratio $n_B/n_\gamma$ could be as large as $10^3$ just after the decays of the scalar fields, thus leading to a cold universe filled with very dense fermions.[20] The possible dilution of this large baryon asymmetry to an acceptable value of $n_B/n_\gamma \simeq 10^{-10}$ in the later epoch of the Universe has been discussed in Ref. 23. Here, based on the nontopological soliton model, we shall consider another possibility that a cold universe filled with very dense fermions undergoes a first-order quark-hadron phase transition to a reheated universe with baryon asymmetry $n_B/n_\gamma \simeq 10^{-10}$, which is needed for the ensuing nucleosynthesis.

This paper is organized as follows. In Sec. III we shall recapitulate the nontopological soliton model. In Sec. III we shall discuss the quark-hadron phase transition in a hot big-bang universe. In Sec. IV we shall discuss the quark-hadron phase transition in a cold universe. Section V is our conclusion.

## II. NONTOPOLOGICAL SOLITON MODEL

The nontopological soliton model was invented a decade ago by Lee and Feinberg.[14] It is a phenomenological model of QCD physics. The starting point of this model is from the fact that quarks are confined in hadrons or no color singlet has been found. Phenomenologically, this fact can be described by introducing a parameter called color-diaelectric constant $\kappa$, which is near one inside a hadron and equal to zero outside. It means that a color field is absent except inside a hadron. Furthermore, we can introduce a scalar field $\sigma$ with an effective potential

$$U(\sigma)=\frac{a}{2!}\sigma^2-\frac{b}{3!}\sigma^3+\frac{c}{4!}\sigma^4+B \ , \tag{2.1}$$

where $a$, $b$, and $c$ are positive parameters. $B$ is called the bag constant. The potential $U(\sigma)$ has a local minimum at $\sigma=0$ and an absolute minimum at $\sigma=\sigma_{\text{vac}}$ with $U(\sigma_{\text{vac}})=0$. Then the field $\sigma$ is related to the diaelectric constant $\kappa$ by

$$\frac{\sigma}{\sigma_{\text{vac}}}=1-\kappa \ . \tag{2.2}$$

Therefore, inside a hadron is a perturbative vacuum with energy density $B$, and outside is a normal vacuum.

The simplest nontopological soliton model is given by the Lagrangian

$$\mathcal{L}=i\sum_{i=1}^{n_F}\bar{\psi}_i\gamma^\mu\partial_\mu\psi_i+\tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma)-\sum_{i=1}^{n_F}f\sigma\bar{\psi}_i\psi_i \ , \tag{2.3}$$

where $\psi_i$ represents the quark field and $n_F$ is the number

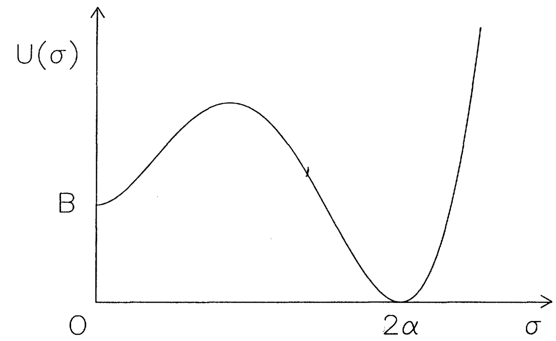


FIG. 1. Typical potential $U(\sigma)$ in the nontopological soliton model.

of flavors. Note that we have introduced Yukawa coupling of $\sigma$ to $\psi_i$ with a common coupling constant $f$. Because of the nonlinearity of the potential $U(\sigma)$, the Lagrangian (2.3) carries nontopological soliton solutions which could be identified with hadrons; inside each is a perturbative vacuum decorated with localized quark fields. The model based on Lagrangian (2.3) and its modified complications has been extensively studied and applied to fit the hadronic properties.[15] In the following our discussions will be based on Lagrangian (2.3). Here, for our purposes, we reparametrize the potential $U(\sigma)$ as

$$U(\sigma)=B\left[\frac{\beta}{\alpha^4}\sigma^2(2\alpha-\sigma)^2+\frac{\sigma^2}{2\alpha^2}\left[\frac{\sigma}{2\alpha}-\frac{3}{2}\right]+1\right] . \tag{2.4}$$

The parameters $\alpha,\beta$ are related to $a$, $b$, and $c$ in the potential (2.1) by

$$\begin{aligned}\frac{8B}{\alpha^2}\left[\beta-\frac{3}{16}\right]&=a \ ,\\ \frac{24B}{\alpha^3}\left[\beta-\frac{1}{16}\right]&=b \ ,\\ \frac{24B\beta}{\alpha^4}&=c \ .\end{aligned} \tag{2.5}$$

Note that $\beta$ must be greater than $\frac{3}{16}$ and $\alpha$ must be positive. We have sketched a typical potential $U(\sigma)$ in Fig. 1.

## III. HOT UNIVERSE

With respect to QCD physics, we can imagine that the present Universe is located at the absolute minimum of the potential $U(\sigma)$ in the nontopological soliton model where $\sigma=\sigma_{\text{vac}}$. Of course, the Universe is filled with stable and massive nontopological solitons (protons, neutrons, and so on); inside each is a perturbative vacuum decorated with free quarks.

When we discuss the quark-hadron phase transition in the hot early Universe, we have to include the temperature corrections to the potential $U(\sigma)$. Since $U(\sigma)$ is a phenomenological potential, we will not include any loop corrections to $U(\sigma)$ at temperature $T=0$. At finite temperature $T$, we again will not consider any $\sigma$ loop corrections because $\sigma$ is a phenomenological field. (For more about the nontopological soliton model and $\sigma$ loop corrections at finite temperature, the reader should refer to Ref. 24. However, it is found that our results will not change qualitatively when the $\sigma$ loop corrections at finite temperature are taken into account.) In the one-fermion-loop expansion at finite temperature $T$, it has been found[25] that the potential is corrected to

$$U(\sigma,T)=U(\sigma)+\frac{n_F}{12}f^2T^2\sigma^2 , \tag{3.1}$$

where $n_F$ is the light-quark degrees of freedom. Here we could define a critical temperature $T_c$ at which both the minima of the potential become degenerate. $T_c$ is found to be given by

$$T_c^2=\frac{3}{n_F f^2}\frac{B}{\alpha^2}\left[1+\frac{1}{16\beta}\right] . \tag{3.2}$$

When $T>T_c$, the perturbative vacuum ($\sigma=0$) becomes the lowest-energy state. The soliton solutions then disappear and the quarks are no longer confined. This signifies a phase transition from the hadron phase to the quark phase.

In the early Universe with $T>T_c$, the Universe is in the perturbative vacuum, and quarks are free to move around. The total energy density of the Universe is given by

$$\rho=\left[g_B+\frac{7}{8}g_F\right]\frac{\pi^2}{30}T^4+B , \tag{3.3}$$

where $g_B$ ($g_F$) is the total number of bosonic (fermionic) degrees of freedom at temperature $T$, $B$ is the energy density of the perturbative vacuum, and the energy density due to the quarks and gluons is included.

As the Universe cools down to $T\lesssim T_c$, the perturbative vacuum becomes metastable and nucleation of hadronic bubbles through thermal fluctuations begins. The nucleation rate per unit time per unit volume is given by[26]

$$\lambda(T)=CT_c^4\exp(-S_3/T) , \tag{3.4}$$

where $C$ is a coefficient of order unity and $S_3$ is the three-dimensional action corresponding to the O(3)-symmetric bubble. The fraction of the Universe which is unaffected by the nucleation during this period is

$$f(t)=\exp\left[-\int_{t_c}^{t}dt'\lambda(t')\frac{4\pi}{3}V_s^3(t-t')^3\right] , \tag{3.5}$$

where $t_c$ is the time when $T=T_c$ and $V_s\simeq 1/\sqrt{3}$ is the speed of the shock wave which expands into the quark phase. The total number density of nucleation sites is

$$N_n=\int_{t_c}^{\infty}dt\, f(t)\lambda(T) . \tag{3.6}$$

If $T_f$ is the temperature at which $f(t_f)\simeq 0$, and $T_{\rm re}$ is the reheating temperature of the Universe just after the completion of the phase transition, then the baryon asymmetry would be diminished by a factor $(T_{\rm re}/T_f)^3$. We expect that $T_{\rm re}$ should be of order $T_c$. Therefore, $T_f$ should be close to $T_c$ if we assume that $n_B/n_\gamma\simeq 10^{-10}$ before the quark-hadron phase transition.

From $U(\sigma,T)$ in Eq. (3.1), it is easily found that the energy difference between the metastable vacuum and the normal vacuum when $T\lesssim T_c$ is given by

$$\epsilon\simeq L\eta , \tag{3.7}$$

where

$$L\equiv 2B\left[1-\frac{1}{16\beta}\right]^2\left[1+\frac{1}{16\beta}\right] ,$$
$$\eta\equiv\frac{T_c-T}{T_c} . \tag{3.8}$$

Since $\eta_f=(T_c-T_f)/T_c\ll 1$, $S_3$ in Eq. (3.4) is well approximated by the thin-wall approximation and is given by[26]

$$S_3\simeq\frac{16\pi S_1^3}{3\epsilon^2} . \tag{3.9}$$

In Eq. (3.8), $S_1$ is the surface energy of the bubble wall given by

$$S_1=\int_0^{\sigma_0}\{2[U(\sigma,T_c)-B]\}^{1/2}d\sigma$$
$$=\frac{4}{3}\alpha(2\beta B)^{1/2}\left[1-\frac{1}{16\beta}\right]^3 , \tag{3.10}$$

where $\sigma_0=2\alpha(1-1/16\beta)$ is such that $U(\sigma_0,T_c)-B=0$.

From Einstein's equation, the age of the Universe $t$ and the temperature $T$ are related by

$$t=\left[\frac{9}{164\pi^3G}\right]^{1/2}\frac{1}{T^2} , \tag{3.11}$$

where $G$ is Newton's constant.

By using Eqs. (3.4), (3.5), and (3.11), when $\eta_f\ll 1$, it has been found[9] that

$$\eta_f^{-12}\exp\left[\frac{16\pi S_1^3}{3T_cL^2\eta_f^2}\right]=\frac{3^8}{2^{17}41^2\pi^9}\frac{CL^8V_s^3}{S_1^{12}G^2} . \tag{3.12}$$

[Note that we have found a factor of $2^{17}$ in the denominator of Eq. (3.12). In Ref. 9, instead, they obtained a factor of $2^{12}$.] As mentioned above, the cosmic separation of phases during the phase transition with small supercooling could generate isothermal baryon-density fluctuations. The length scale of the fluctuations is characterized by the mean separation $l$ per nucleation site. It has been found[9] from Eqs. (3.4)–(3.6) and (3.11) that

$$l\equiv N_n^{-1/3}$$
$$=\left[\frac{9}{164\pi^3G}\right]^{1/2}\frac{1}{T_c^2\pi^{2/3}}\left[\frac{3V_sT_cL^2\eta_f^3}{8S_1^3}\right] , \tag{3.13}$$

when $\eta_f\ll 1$. Significant effects on nucleosynthesis re-

TABLE I. Critical temperature $T_c$, degree of supercooling $\eta_f$, and length scale of density fluctuations $l$ from different sets of values of the model parameters $a$ (fm$^{-2}$), $b$ (fm$^{-1}$), $c$, and $f$. The values are taken from fits to the hadronic mass spectrum. We have assumed $n_F=2$.

| $a$ | $b$ | $c$ | $f$ | $T_c$ (MeV) | $\eta_f$ | $l$ (m) |
|---|---|---|---|---|---|---|
| 51.6 | 799.9 | 4000 | 14.8 | 30.3 | 0.264 | 321.28 |
| 1.6 | 69 | 500 | 9.57 | 44.8 | $2.05\times10^{-2}$ | 12.33 |
| 0 | 58 | 500 | 9.16 | 55.9 | $8.46\times10^{-3}$ | 3.38 |
| 44.6 | 1194 | 10 000 | 11.7 | 49.7 | $6.84\times10^{-2}$ | 32.48 |
| 11.6 | 834 | 10 000 | 10.96 | 106.1 | $5.01\times10^{-3}$ | 0.57 |
| 0 | 700 | 10 000 | 10.98 | 125.8 | $2.14\times10^{-3}$ | 0.18 |
| 28.25 | 672 | 5000 | 11.28 | 41.3 | $9.32\times10^{-2}$ | 63.36 |
| 7.51 | 474 | 5000 | 10.09 | 92.6 | $6.77\times10^{-3}$ | 1.01 |
| 0 | 399 | 5000 | 10.01 | 111.2 | $2.87\times10^{-3}$ | 0.30 |

quires $l > 10$ m.

In Eq. (3.12) the degree of supercooling is governed by $S_1$, $L$, $T_c$, and $G$, and depends only weakly on $C$ and $V_s$. Hence we simply take $C\simeq1$ and $V_s\simeq1/\sqrt{3}$. Note that $T_c$, $L$, and $S_1$ are related to the model parameters $\alpha$, $\beta$, $B$, and $f$ by Eqs. (3.2), (3.8), and (3.10), respectively. Therefore, Eq. (3.12) allows us to find $\eta_f$ in terms of $\alpha$, $\beta$, $B$, and $f$. The result has been plotted in Fig. 2, where we have defined a new variable $\delta\equiv T_c/B^{1/4}$ replacing $n_F$ and $f$. Also, we have found that the curve is highly insensitive to the changes of $B$. In particular, we use values of these model parameters from fits to the hadronic mass spectrum[15] and $n_F=2$ in Eqs. (3.2) and (3.8)–(3.13) to evaluate $T_c$, $\eta_f$, and $l$. These results are tabulated in Table I.

However, it is useful to find an approximate solution for $\eta_F$ in Eq. (3.12). Since $\beta>\frac{3}{16}$, $L\simeq2B$ from Eq. (3.8). Also, we adjust the parameter $f$ in Eq. (3.2) such that $T_c\simeq B^{1/4}$, which is a reasonable approximation. (In Ref. 9 they found $L\simeq4B$ and $T_c\simeq0.72B^{1/4}$ by studying the thermodynamics of both quark and hadron phases, where $L$ is the latent heat of the phase transition. Here $L$ is expected to be different from the latent heat and is just the energy difference between two different vacua.) Hence a good approximation solution of Eq. (3.12) is

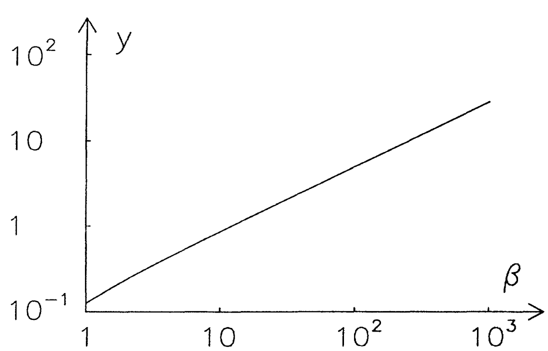


FIG. 2. Curve shows the relations between the degree of supercooling $\eta_f$ and the model parameters $\alpha$, $\beta$, $B$, and $\delta\equiv T_c/B^{1/4}$. We have defined $y\equiv\delta^{1/2}\eta_f/[(2\alpha)^4/B]^{3/8}$.

$$\eta_f\simeq0.3\frac{S_1^{3/2}}{T_c^{1/2}L}\ll1 , \qquad (3.14)$$

provided that

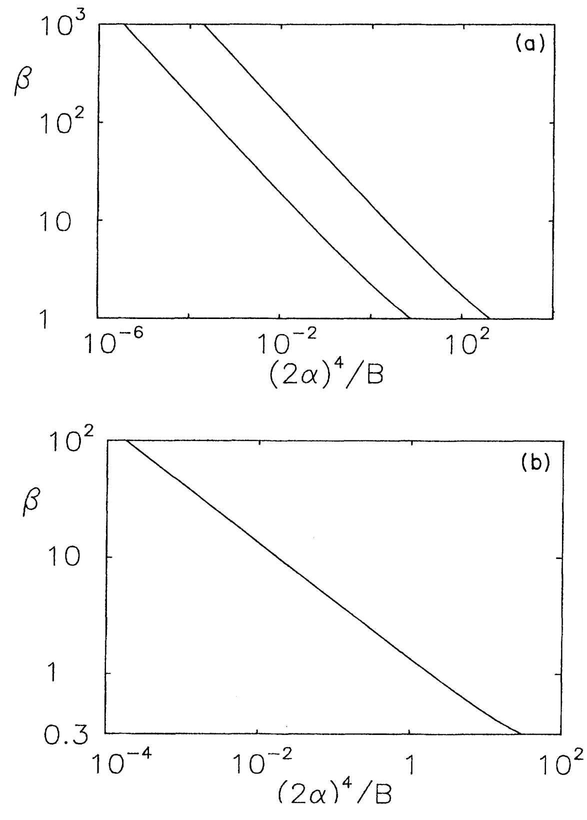


FIG. 3. (a) Upper curve is drawn from Eq. (3.17). Below the curve is the parameter space which is for not diminishing the baryon asymmetry of the Universe and for small supercooling of the quark-hadron phase transition. The lower curve is drawn from Eq. (3.18) with $B=56$ MeV fm$^{-3}$. Above the curve is the parameter space which is relevant for inhomogeneous nucleosynthesis. (b) Curve is the equality of Eq. (4.19), which gives the required parameters in a cold universe.

$$|\ln(B/\text{MeV}^4)| \ll 186.6 \,, \tag{3.15a}$$

$$|\tfrac{3}{2}\ln(S_1^4/B^3)-22.8| \ll 186.6. \tag{3.15b}$$

Substituting Eq. (3.14) in Eq. (3.13), we thus obtain

$$l \simeq (1.4\times 10^5 \text{ m})(S_1/\text{MeV}^3)^{3/2}(T_c/\text{MeV})^{-13/2} \,. \tag{3.16}$$

Then, from Eq. (3.14), we obtain a constraint on the parameters $\alpha$, $\beta$, and $B$ as

$$\left[\frac{(2\alpha)^4}{B}\right]^{3/8}(2\beta)^{3/4}\left[1-\frac{1}{16\beta}\right]^{9/2} \ll 12.2 \,, \tag{3.17}$$

for not diminishing the baryon asymmetry of the Universe during the quark-hadron phase transition. Meanwhile, the condition (3.17) is required for a phase transition with small supercooling. We have plotted the curve with the left-hand side of the inequality (3.17) equal to 12.2 in Fig. 3(a). Also, from Eq. (3.10) and $T_c \simeq B^{1/4}$, the condition $l > 10$ m is translated into

$$\left[\frac{(2\alpha)^4}{B}\right]^{3/8}(2\beta)^{3/4}\left[1-\frac{1}{16\beta}\right]^{9/2} > 1.3\times 10^{-4}(B^{1/2}/\text{MeV}^2) \,. \tag{3.18}$$

We have plotted this condition in Fig. 3(a).

As we have shown above, the inequalities (3.17) and (3.18) are valid only when (3.15a) and (3.15b) are satisfied. The condition (3.15a) is generally satisfied and (3.15b) can be rewritten in terms of $\alpha$ and $\beta$ as

$$4.2\times 10^{-18} \ll \left[\frac{(2\alpha)^4}{B}\right]^{3/8}(2\beta)^{3/4}\left[1-\frac{1}{16\beta}\right]^{9/2} \ll 4.2\times 10^{60} \,. \tag{3.19}$$

We see that the results (3.17) and (3.18) are within very good approximation.

## IV. COLD UNIVERSE

Let us suppose, contrary to the usual belief, that initially the Universe was cold with the baryon asymmetry $n_B/n_\gamma \gg 1$. This condition, indeed, can be obtained in an Affleck-Dine mechanism for baryogenesis as mentioned above. In the Affleck-Dine mechanism, decays of scalar fermion fields could result in a universe with baryon asymmetry $n_B/n_\gamma \simeq 10^3$ at a temperature of order 1 TeV.

In the context of the nontopological soliton model, when we consider the quark-hadron phase transition at finite fermion density, we have to include the finite-density effects to the potential $U(\sigma)$. It has been found[27] that, at one-fermion-loop quantum corrections, the corrected potential is given by

$$U(\sigma,\mu) = U(\sigma) + \frac{n_F}{4\pi^2}f^2\mu^2\sigma^2 \,, \tag{4.1}$$

for $\mu/T \gg 1$, where $\mu^2 \equiv (1/n_F)\sum_{i=1}^{n_F}\mu_i^2$ and $\mu_i$ is the chemical potential for the $i$th fermionic species. When comparing the potential in Eq. (4.1) with that in Eq. (3.1), we find that the finite-density effects are similar to the finite-temperature effects. At finite fermion density $\mu > \mu_c = (\pi/\sqrt{3})T_c$, where $T_c$ is given by Eq. (3.2), the ground state is the perturbative vacuum with $\sigma = 0$. When $\mu \lesssim \mu_c$, the perturbative vacuum becomes metastable, and then a phase transition to the normal vacuum follows.

In a cold universe with $n_B/n_\gamma \gg 1$, the universe consists of degenerate massless quarks with chemical potential $\mu \gg \mu_c$, which are moving freely in the perturbative vacuum with vacuum energy density $B$. For each light fermionic species, $\mu_i = (3\pi^2)^{1/3}n_i^{1/3}$, where $n_i$ is its number density. We expect that all $n_i$ are equal. Hence the chemical potential $\mu$ is related to the baryon density $n_B$ [$n_B \equiv \frac{1}{3}n_F n$, where $n = (1/n_F)\sum_{i=1}^{n_F} n_i$] by

$$\mu = (3\pi^2)^{1/3}n^{1/3} \,. \tag{4.2}$$

Thus the critical number density $n_c$ is given by

$$n_c = \frac{\pi}{9\sqrt{3}}T_c^3 \,. \tag{4.3}$$

At this stage the total energy of the universe is given by

$$\rho = \tfrac{3}{4}(3\pi^2)^{1/3}n_F n^{4/3} + B \,. \tag{4.4}$$

Therefore, as the universe expands, it will become dominated by the perturbative vacuum energy density when $n$ has decreased to

$$n_0 = \left[\frac{64}{81\pi^2}\right]^{1/4} n_F^{-3/4}B^{3/4} \,, \tag{4.5}$$

which is comparable to $n_c$ when $B \simeq T_c^4$. As a result, the universe expands exponentially with

$$R(t) = e^{H(t-t_0)} \,, \tag{4.6}$$

where $H$ is the Hubble parameter given by $H^2 = 8\pi GB/3$. Note that we have set $n = n_0$ and $R = 1$ when $t = t_0$. Meanwhile, the perturbative vacuum becomes metastable and the universe starts to nucleate hadronic bubbles.

The bubble nucleation in the cold universe is mainly through quantum tunneling. The nucleation rate per unit volume per unit time is given by

$$\lambda(\mu) = CB\exp(-S_4) \,, \tag{4.7}$$

where $C$ is a coefficient of order unity and $S_4$ is the four-dimensional action corresponding to the O(4)-symmetric bubble. It has been found[28] that $S_4$ is given by

$$S_4 = 2\pi^2\int_0^\infty r^3 dr\left[\frac{1}{2}\left[\frac{d\sigma}{dr}\right]^2 + U(\sigma,\mu) - B\right] \,, \tag{4.8}$$

where $\sigma = \sigma(r)$ is the solution of the differential equation

$$\frac{d^2\sigma}{dr^2} + \frac{3}{r}\frac{d\sigma}{dr} = U'(\sigma,\mu) \,, \tag{4.9}$$

with the boundary conditions

$$\frac{d\sigma}{dr} = 0 \quad \text{when } r = 0 \,, \tag{4.10a}$$

$$\sigma=0 \quad \text{when } r\to\infty \ . \tag{4.10b}$$

The prime in Eq. (4.9) represents partial differentiation with respect to $\sigma$.

The probability that an arbitrary point remains in the quark phase at time $t$ is given by[29]

$$p(t)=\exp\left[-\int_{t_0}^{t}dt'\lambda(t')R^3(t')V(t,t')\right] ,$$
$$V(t,t')=\frac{4\pi}{3}\left[\int_{t'}^{t}\frac{dt''}{R(t'')}\right]^3 , \tag{4.11}$$

where we have assumed that the bubbles are expanding at near the speed of light. Since baryon number is conserved at this epoch, we have $nR^3=\text{const}$. Then, Eqs. (4.2) and (4.6) lead to

$$\dot{\mu}=-H\mu \ , \tag{4.12}$$

when $t>t_0$, where the dot represents time differentiation. Substituting Eqs. (4.7) and (4.12) in Eq. (4.11), we obtain

$$p(\mu)=\exp\left[-b\int_{\mu}^{\mu_0}d\mu'\frac{\exp[-S_4(\mu')]}{\mu'^4}(\mu'-\mu)^3\right] , \tag{4.13}$$

where $b=\frac{4}{3}\pi B/H^4$.

Let $t_*$ be the time at which $p(\mu)$ decreases rapidly to zero. At this time the universe is reheated by the radiation released from the perturbative vacuum (see below) to a temperature $T_*$, which is estimated as

$$\frac{\pi^2}{15}T_*^4=B \ . \tag{4.14}$$

In order to have the right baryon asymmetry for ensuing nucleosynthesis and present observational limit, we require $n_*/n_\gamma\simeq10^{-10}$, where $n_\gamma=0.244T_*^3=0.334B^{3/4}$. This gives

$$\mu_*=(3\pi^2)^{1/3}n_*^{1/3}\simeq0.996\times10^{-3}B^{1/4} \ . \tag{4.15}$$

With this chemical potential $\mu_*$, the coefficient $a_*/2$ of the $\sigma^2$ term in the potential $U(\sigma,\mu)$ is given by

$$\frac{a_*}{2}=\frac{a}{2}+\frac{n_F}{4\pi^2}f^2\mu_*^2 \ . \tag{4.16}$$

If we assume that $a=51.6\ \text{fm}^{-2}$, $b=799.9\ \text{fm}^{-1}$, and $c=4000$ in the potential $U(\sigma)$ in Eq. (2.1) (where $B=0.142\ \text{fm}^{-4}$), and $f=14.8$, which is satisfactory for fitting a part of the static properties of the hadrons,[15,24] then $a_*\simeq a(1+10^{-7})$. Therefore, unless there exists fine-tuning in the present model, we will assume that $a_*\simeq a$ in general for naturalness.

Since $\mu_*\ll\mu_0$, where $\mu_0\simeq2.51B^{1/4}$ from Eqs. (4.2) and (4.5), $p(\mu_*)$ in Eq. (4.13) can be approximated by

$$p(\mu_*)\simeq\exp\left[-b\exp(-S_0)\ln\left[\frac{\mu_0}{\mu_*}\right]\right] , \tag{4.17}$$

where $S_0\equiv S_4(\mu=0)$. Therefore. at $t=t_*$, when the universe is already filled with hadronic bubbles and the baryon asymmetry is $n_*/n_\gamma\simeq10^{-10}$, we should have $p(\mu_*)\lesssim e^{-1}$. This gives

$$\ln b\gtrsim S_0 \ . \tag{4.18}$$

For $B=56\ \text{MeV fm}^{-3}$, we find $b=3.07\times10^{78}$. Hence Eq. (4.18) approximately gives

$$180.7\gtrsim S_0 \ . \tag{4.19}$$

If $\beta$ in Eq. (2.4) is much greater than 1, the action $S_0$ can be approximated by the thin-wall approximation[28] as

$$S_0=\frac{(2\alpha)^4}{B}\frac{27\pi^2}{2}\bar{S}_1^4 , \tag{4.20}$$

where $\bar{S}_1$ is given by

$$\bar{S}_1=\int_0^1(2V_0)^{1/2}d\bar{\sigma}=\tfrac{2}{3}\sqrt{2\beta} , \tag{4.21}$$

with $V_0=16\beta\bar{\sigma}^2(1-\bar{\sigma})^2$. Otherwise, we numerically solve Eqs. (4.9) and (4.10) with $\mu=0$ to obtain $S_0$ from Eq. (4.8). For example, with $a=51.6\ \text{fm}^{-2}$, $b=799.9\ \text{fm}^{-1}$, and $c=4000$ in the potential $U(\sigma)$, we find numerically that $S_0\simeq96$. If $a=52\ \text{fm}^{-2}$ with others remaining the same, we find that $S_0\simeq200$. We thus see that the condition (4.19) can be easily satisfied without fine-tuning any parameter in the model. We have plotted the condition (4.19) in Fig. 3(b), which constraints the parameter space spanned by $\alpha$, $\beta$, and $B$.

In the thin-wall approximation, all the energy released by converting the perturbative vacuum to the normal vacuum is transferred initially to the walls of the bubbles.[28] This energy can be thermalized when the bubble walls collide with one another many times. Outside the thin-wall approximation, the bubble nucleation leaves behind a homogeneous classical field $\sigma'$ with mass $m_{\sigma'}^2=U''(\sigma_{\text{vac}})$ (here $\sigma'$ is the shifted field with $\sigma=\sigma'+\sigma_{\text{vac}}$). The energy stored in the classical field is given by $E_{\sigma'}\simeq\frac{1}{2}m_{\sigma'}^2\sigma_0'^2$, which should be of order $B$, where $\sigma_0'$ is the initial amplitude of $\sigma'$. This energy can be interpreted as particles of mass $m_{\sigma'}$ at rest with number density $\frac{1}{2}m_{\sigma'}\sigma_0'^2$.[30] In the normal vacuum with $\sigma=\sigma_{\text{vac}}$, the quarks obtain an effective mass $m_q=f\sigma_{\text{vac}}$. Because of the absence of free quarks, we expect that $2m_q>m_{\sigma'}$. Indeed, this condition is satisfied with the above set of values for $a$, $b$, $c$, and $f$. Therefore, the thermalization of the energy $E_{\sigma'}$ must at least go through a one-fermion-loop process $\sigma'\to2\gamma$, as shown in Fig. 4. The decay rate $\Gamma(\sigma'\to2\gamma)$ has been calculated as[31]

$$\Gamma(\sigma'\to2\gamma)=\frac{n_Ff^2}{4\pi}\left[\frac{e^2}{4\pi}\right]^2\frac{m_{\sigma'}}{4\pi^2}\left[\frac{m_{\sigma'}}{m_q}\right]^2|I|^2 , \tag{4.22}$$

where $e$ is the electron charge and $I=I(m_q^2/m_{\sigma'}^2)$. When $m_q^2/m_{\sigma'}^2\to\infty$, $I=\frac{1}{3}$. On the other hand, the expansion rate of the universe at this epoch is characterized by the Hubble parameter $H\simeq(8\pi B/3M_P^2)^{1/2}$ $(M_P\equiv1/\sqrt{G})$. Since $H$ is suppressed by the Planck mass, we have in general $\Gamma\gg H$. For example, if we input $a=51.6\ \text{fm}^{-2}$, $b=799.9\ \text{fm}^{-1}$, $c=4000$, and $f=14.8$, we will get $\Gamma\simeq6.8\times10^{-5}\ \text{fm}^{-1}$ (for $n_F=2$) from Eq. (4.22) and $H\simeq1.8\times10^{-20}\ \text{fm}^{-1}$. This means that the classical field $\sigma'$ would undergo out-of-equilibrium decay into radia-

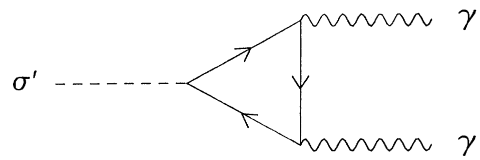


FIG. 4. Feynman diagram for the one-fermion-loop decay process $\sigma' \to 2\gamma$.

tion, with a decay rate very fast compared to the expansion time of the universe.

After we have discussed the phase transition to the hadronic phase from a cold universe of free quarks, we are now to estimate the density fluctuations generated during the phase transition. The magnitude of the density fluctuations is characterized by the mass scale contained in one hadronic bubble. From above, we know that the size of a typical bubble at time $t = t_*$ is given by

$$V = R^3(t_*)\frac{4\pi}{3}\left[\int_{t_0}^{t_*}\frac{dt}{R(t)}\right]^3 = \frac{4\pi}{3}\frac{1}{H^3}(e^{H(t_*-t_0)}-1)^3 , \tag{4.23}$$

where we have used Eq. (4.6) for $R(t)$. From the conservation of baryon number, we have

$$\frac{n_0}{n_*} = e^{3H(t_*-t_0)} . \tag{4.24}$$

Hence, inserting Eq. (4.24) into (4.23) and knowing that $n_0 \gg n_*$, we find that

$$V \simeq \frac{4\pi}{3}\frac{1}{H^3}\frac{n_0}{n_*} . \tag{4.25}$$

Then the total mass contained in a bubble after the phase transition is calculated as

$$M \simeq \tfrac{1}{3} n_F n_* m_N V , \tag{4.26}$$

where $m_N \simeq 940$ MeV is the mass of a nucleon. Using Eqs. (4.5), (4.25), (4.26), and $H = (8\pi B/3M_P^2)^{1/2}$, we find that

$$M \simeq \frac{4\pi}{3} m_N \frac{n_F n_0}{3H^3} \simeq 31 M_\odot (56\ \text{MeV fm}^{-3}/B)^{3/4} , \tag{4.27}$$

for $n_F = 2$, where $M_\odot$ is the solar mass. This is the characteristic mass scale in density fluctuations generated during the phase transition. However, this mass scale is mostly below the Jeans mass in the later evolution of the universe and is probably damped out during the acoustic phase.[32]

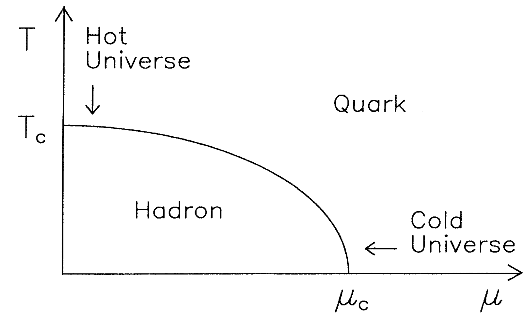


FIG. 5. $T$-$\mu$ phase diagram of the quark-hadron phase transition at finite temperature and density.

On the contrary, if the density fluctuations associated with bubbles are isothermal with the entropy density $n_\gamma/n_B$ remaining small even after the phase transition, they will grow during the unstable phase. As a consequence, they could form massive stars, which could conceivably play a role as seeds for explosive galaxy-formation models, and their remnants may form the dark matter in galactic halos and clusters.[19]

## V. CONCLUSION

Based on the nontopological soliton model, we have discussed the quark-hadron phase transition of the early Universe by using the phenomenological potential $U(\sigma)$ for hadron physics. We have discussed the phase transition in a hot universe and a cold universe, respectively. The quantum corrections at finite temperature and finite density to the potential $U(\sigma)$ are reflected in the $T$-$\mu$ phase diagram shown in Fig. 5. We have probed the cosmological implications to the phase transition at two extreme conditions (high $T$ and large $\mu$). It has been shown that a cold universe, filled with very dense fermions and then reheated by the latent heat of the quark-hadron phase transition, could be an interesting nonstandard scenario for the evolution of the Universe. Some of our results are summarized in Figs. 3(a) and 3(b), where we have given the parameter spaces which are relevant for cosmology.

## ACKNOWLEDGMENTS

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