PHYSICAL REVIEW D VOLUME 43, NUMBER 12 15 JUNE 1991

ARTICLES

Einstein equivalence principle and the polarization of radio galaxies

Sean M. Carroll and George B. Field
Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138
(Received 26 December 1990)

The theoretical foundations of metric theories of gravitation rest largely on the Einstein equivalence principle (EEP). Schiff has conjectured that no consistent Lorentz-invariant theory can obey the weak equivalence principle and not obey the EEP. However, a counterexample has been proposed by Ni in a theoretical framework for studying electromagnetism coupled to gravity. Under the assumption that gravitational fields vary smoothly over a Hubble time, we show that Ni's counterexample would lead to a rotation in the plane of polarization of radiation from distant radio sources. As such rotation is not observed, we may put stringent limits on the magnitude of the parameter which violates the EEP, of less than about 10^{-40} GeV.

I. INTRODUCTION

Modern gravitation theory is founded on the principle of equivalence. This principle comes in three forms: the weak, Einstein, and strong equivalence principles (WEP, EEP, and SEP). The WEP states that the laws of motion of freely falling bodies are the same in a gravitational field as in a uniformly accelerating frame; this implies that the inertial mass of a body is equal to its gravitational mass, regardless of composition. The EEP extends the WEP to say that the outcome of all experiments involving exclusively nongravitational forces is the same in a gravitational field as in a uniformly accelerated frame. The SEP further extends the EEP to include all experiments, gravitational and otherwise. Each principle is taken to apply in small enough regions of spacetime.

The physical consequence of the EEP is that gravitation may be described by a "metric theory," i.e., a formalism in which gravitation manifests itself as the curvature of the spacetime manifold and test particles move along geodesics of a symmetric metric $g_{\mu\nu}$ defined on this manifold.[1,2] The dynamics of the metric are not specified; indeed, the EEP permits the existence of any number of "gravitational fields" which enter the field equations for $g_{\mu\nu}$, such as the Brans-Dicke scalar, vector fields, background metric tensors, and so on. These fields are distinguished as "gravitational" since they interact with the metric directly, rather than through their energy-momentum tensor exclusively; however, since the EEP demands that the metric alone determines the motion of test bodies, they cannot couple directly to matter fields. In the process of generalizing a nongravitational physical theory from flat to curved spacetime, the EEP implies the minimal coupling prescription, in which the Minkowski metric $\eta_{\mu\nu}$ is replaced by $g_{\mu\nu}$ and partial derivatives ∂_μ are replaced by covariant derivatives ∇_μ.

In contrast with the EEP, the more restrictive SEP disallows any gravitational field other than the metric. Theories consistent with the SEP are either general relativity (GR), or extensions of GR with more complex Lagrangians, such as those which contain derivatives of the metric of order higher than two. The less restrictive WEP makes no statements about what fields or interactions are allowed, but requires that the dynamics of the theory conspire to move test bodies of arbitrary composition on identical trajectories through spacetime.

The WEP is experimentally tested by Eötvös-Dicke-Braginsky experiments.[1] It is possible to put extremely precise limits on violations of the WEP by comparing the relative accelerations of objects with different compositions. The EEP, on the other hand, is tested by gravitational redshift experiments; any metric theory of gravity will predict the same redshift. The best precision obtainable in these experiments is typically of order $\sim 10^{-4}$, while the WEP limits are as good as $\sim 10^{-12}$.

Schiff has conjectured[2,3] that any consistent Lorentz-invariant theory of gravity which obeys the WEP would necessarily obey the EEP. If true, this conjecture would allow the high-precision tests of the WEP also to constrain violations of the EEP, which would dramatically increase our experimental confidence in all metric theories of gravity. While Schiff's conjecture has not been proven in full generality, it has been demonstrated to hold under certain conditions. Lightman and Lee,[4] for example, were able to show that Schiff's conjecture would hold for electromagnetically interacting systems in a static, spherically symmetric gravitational field.

In a more general framework, however, Ni[5] was able to find a unique counterexample to Schiff's conjecture. Ni found the most general interaction Lagrangian for electromagnetic systems in a gravitational field, subject to the following restrictions: (i) uncharged particles follow geodesics of a Riemannian metric; (ii) electromagnetic gauge invariance holds; (iii) interactions with derivatives of the gravitational fields are excluded; and (iv) the Lagrangian is quadratic in derivatives of the electromagnetic potential A_μ, yielding linear equations of motion. Ni found that an interaction defined by the Lagrange density

$$\mathcal{L}_N = -\frac{1}{16\pi}\sqrt{g}\,\phi\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \tag{1}$$

led to violation of the EEP while obeying the WEP. Here g is minus the determinant of the metric $g=-\det(g_{\mu\nu})$, ϕ is a dimensionless scalar function of the gravitational fields (that is, $g_{\mu\nu}$ and any other scalar, vector, and other fields in the theory of gravity under consideration), $F_{\mu\nu}$ is the Maxwell tensor $F_{\mu\nu}=\nabla_\mu A_\nu-\nabla_\nu A_\mu=\partial_\mu A_\nu-\partial_\nu A_\mu$, and ϵ is the Levi-Civita tensor $\epsilon^{\mu\nu\rho\sigma}=g^{-1/2}e^{\mu\nu\rho\sigma}$. In this expression $e^{\mu\nu\rho\sigma}$ is the alternating symbol normalized to $e^{0123}=+1$. Our conventions are those of Misner, Thorne, and Wheeler,[6] and we set $c=\hbar=1$.

Ni's Lagrangian is intriguing as a counterexample to Schiff's conjecture. If the WEP, but not the EEP, were a symmetry of nature, we would have every reason to expect that a coupling such as Eq. (1) would contribute to the interaction between electromagnetism and gravitation. In this paper we show that, should a term such as (1) be significant in a cosmological context, the plane of polarization of radiation emitted by astrophysical sources would be rotated as the radiation propagates to Earth. As such a rotation is not observed, we can put limits on the parameter ϕ and, hence, conclude that nature chooses not to violate the EEP in this unique fashion.

II. POLARIZATION EFFECTS

It is helpful to write $\mathcal{L}_N$ in a different form. Defining the dual Maxwell tensor as

$$\tilde{F}^{\mu\nu}\equiv\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}\ , \tag{2}$$

and noting that both F and $\tilde{F}$ are antisymmetric, we can integrate by parts to write Eq. (1) in the form

$$\begin{aligned}\mathcal{L}_N &= -\frac{1}{8\pi}\sqrt{g}\,\phi F_{\mu\nu}\tilde{F}^{\mu\nu}\\ &= -\frac{1}{4\pi}\sqrt{g}\,\phi\tilde{F}^{\mu\nu}\nabla_\mu A_\nu\\ &= \frac{1}{4\pi}\sqrt{g}\,[A_\nu\nabla_\mu(\phi\tilde{F}^{\mu\nu})-\nabla_\mu(\phi\tilde{F}^{\mu\nu}A_\nu)]\ ,\end{aligned} \tag{3}$$

where ∇_μ denotes the covariant derivative relative to $g_{\mu\nu}$. An exact differential does not affect the classical action and, hence, not the field equations; so we can ignore the divergence in Eq. (3). Recalling that the homogeneous Maxwell equations are

$$\nabla_\mu\tilde{F}^{\mu\nu}=0\ , \tag{4}$$

we see that, to within a divergence,

$$\mathcal{L}_N=\frac{1}{4\pi}\sqrt{g}\,(\nabla_\mu\phi)A_\nu\tilde{F}^{\mu\nu}\ . \tag{5}$$

It is clear from Eq. (5) that only the gradient of ϕ, not ϕ itself, enters the field equations. This is consistent with the well-known fact that a Lagrangian of the form $\text{const}\times(\sqrt{g}F\tilde{F})$ has no effect on the classical field equations. Note that, while (5) is not manifestly invariant under gauge transformations for electromagnetism, a gauge transformation $A_\mu\to A_\mu+\partial_\mu\lambda$ changes $\mathcal{L}_N$ by an irrelevant divergence.

Equation (5) is similar to the Chern-Simons Lagrangian discussed by Carroll, Field, and Jackiw[7] (CFJ):

$$\mathcal{L}_{\rm CS}=-\frac{1}{8\pi}\sqrt{g}\,p_\mu A_\nu\tilde{F}^{\mu\nu}\ . \tag{6}$$

CFJ studied $\mathcal{L}_{\rm CS}$ in flat spacetime as an interaction term added to the free Maxwell Lagrangian $\mathcal{L}_{\rm Max}=-\sqrt{g}F_{\mu\nu}F^{\mu\nu}/16\pi$. The external vector p_μ was taken to be a constant vector field which violates Lorentz invariance by defining a preferred reference frame. CFJ noted that if $p_\mu=\nabla_\mu\theta$, where θ is a dynamical scalar field, so that

$$\mathcal{L}_{\rm CS}=-\frac{1}{8\pi}\sqrt{g}\,(\nabla_\mu\theta)A_\nu\tilde{F}^{\mu\nu}\ , \tag{7}$$

gauge and Lorentz invariance are preserved. Comparing with Eq. (5), we see that $\mathcal{L}_N=\mathcal{L}_{\rm CS}$ if

$$\theta=-2\phi \tag{8}$$

is a function of the gravitational fields. Proceeding as CFJ did, but taking into account the effects of curvature, we find that the field equations corresponding to $\mathcal{L}=\mathcal{L}_{\rm Max}+\mathcal{L}_N$ are

$$\nabla_\mu F^{\mu\nu}=-2(\nabla_\mu\phi)\tilde{F}^{\mu\nu}\ , \tag{9}$$

with the Bianchi identities

$$\nabla_\mu\tilde{F}^{\mu\nu}=0 \tag{10}$$

unaltered.

We use these equations to describe propagation of electromagnetic waves in the Universe. To accomplish this we take spacetime to possess the geometry of a Riemannian manifold with symmetric metric and the usual Levi-Civita connection. That we can do this is not immediately obvious, since the physical principle which allows us to make these assumptions is the EEP, and Ni has shown that a theory of gravity which includes the term (1) does not obey the EEP. However, we are in the special situation of considering a theory which does obey the EEP except for the unique addition of Ni's Lagrangian. Therefore, absent this term, the gravitational fields and spacetime geometry are that of a metric theory, with the usual covariant derivative structure. The addition of the term (1) does not add any new geometrical objects to the theory or alter any of the existing ones; rather, it merely introduces a new interaction between the existing gravitational fields and the electromagnetic field. Therefore, treating the theory as "almost a metric theory" is justified. However, it is important to note that we can (if we wish) rewrite the formalism in terms of torsion, in which case the geometry itself is altered. This formulation has been carried out by Ni in Ref. 8.

Since spacetime is Riemannian, the metric for a spatially flat, homogeneous, and isotropic universe can be written in the Roberson-Walker form

$$ds^2=R^2(\eta)(-d\eta^2+d\mathbf{x}^2) , \tag{11}$$

where $d\eta=dt/R$ is conformal time, R is the scale factor, and $\mathbf{x}$ represents Cartesian coordinates in three-space. We see that, for this metric,

$$g=R^8 . \tag{12}$$

To solve Eqs. (9) and (10) we write them in terms of $\mathbf{E}$ and $\mathbf{B}$ as defined by Turner and Widrow:[9]

$$F^{\mu\nu}=R^{-2}\begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix} . \tag{13}$$

The dual tensor $\tilde{F}^{\mu\nu}$ is obtained from $F^{\mu\nu}$ by letting $\mathbf{E}\to\mathbf{B}$ and $\mathbf{B}\to-\mathbf{E}$.

In Ni's Lagrangian, ϕ depends only on the gravitational fields, which in a Robertson-Walker cosmology depend only on η. If we drop terms in $\nabla\phi$, Eq. (9) becomes

$$\frac{\partial}{\partial\eta}(R^2\mathbf{E})-\nabla\times(R^2\mathbf{B})=-2\phi' R^2\mathbf{B} \tag{14}$$

and

$$\nabla\cdot\mathbf{E}=0 , \tag{15}$$

where $\phi'=d\phi/d\eta$, and $\nabla\times$ and $\nabla\cdot$ represent the usual differential operators in Cartesian three-space. Equation (10) becomes

$$\frac{\partial}{\partial\eta}(R^2\mathbf{B})+\nabla\times(R^2\mathbf{E})=0 \tag{16}$$

and

$$\nabla\cdot\mathbf{B}=0 . \tag{17}$$

If we differentiate Eq. (16) by η and use Eqs. (14) and (17), we obtain the wave equation

$$\frac{\partial^2}{\partial\eta^2}(R^2\mathbf{B})-\nabla^2(R^2\mathbf{B})=2\phi'\nabla\times(R^2\mathbf{B}) . \tag{18}$$

Equation (18) has solutions of the form

$$R^2\mathbf{B}(\mathbf{x},\eta)=e^{-i\mathbf{k}\cdot\mathbf{x}}R^2\mathbf{B}(\eta) , \tag{19}$$

so that $\nabla\to-i\mathbf{k}$. If we take the x axis to point along $\mathbf{k}$, so that $\mathbf{k}\cdot\mathbf{x}=kx$, Eq. (18) implies two coupled equations for $B_y(\eta)$ and $B_z(\eta)$, which are simplified by defining

$$F_\pm(\eta)=R^2B_\pm(\eta)=R^2(B_y\pm iB_z) , \tag{20}$$

in terms of which Eq. (18) can be written

$$\frac{d^2F_\pm}{d\eta^2}+(k^2\mp 2k\phi')F_\pm=0 . \tag{21}$$

The next step depends on the behavior of $\phi(\eta)$. In this section we assume that ϕ changes very little over the period of the wave, given by the solution to Eq. (21) with $\phi'\simeq 0$ as $2\pi/k$. Then we can use a WKB solution to Eq. (21):

$$F_\pm(\eta)=\exp\left[ik\int\left[1\mp 2\frac{\phi'}{k}\right]^{1/2}d\eta\right] , \tag{22}$$

where we have chosen the positive root for definiteness. Since the change in ϕ over one period,

$$\Delta\phi=\phi'\Delta\eta=\phi'\Delta x=\frac{2\pi\phi'}{k} , \tag{23}$$

is very small by assumption (where we have used $\Delta\eta=\Delta x$ along the light ray), we can expand the radical in Eq. (22), and then, upon restoring the common factor e^{-ikx}, we find that

$$B_\pm(x,\eta)=e^{-ikx}R^{-2}F_\pm(\eta)=R^{-2}e^{i\sigma_\pm} , \tag{24}$$

where the phase $\sigma_\pm$ is

$$\sigma_\pm=k(\eta-x)\mp\phi(\eta)-\frac{1}{2k}\int(\phi')^2d\eta+O(k^{-2}) . \tag{25}$$

From this we see that both positive and negative modes are waves propagating in the x direction, but with different phase shifts that depend on $\phi(\eta)$. The energy density, proportional to $|B|^2$, goes as R^{-4}, as expected. From Eq. (20) we see that F_+ (F_-) represents a wave of positive (negative) helicity.

The frequency measured by a comoving observer ($\mathbf{x}=$const) is

$$\begin{aligned}\omega_\pm=\left[\frac{\partial\sigma}{\partial t}\right]_x&=\frac{1}{R}\left[\frac{\partial\sigma}{\partial\eta}\right]_x\\ &=\frac{1}{R}\left[k\mp\phi'-\frac{1}{2k}(\phi')^2+O(k^{-2})\right]\\ &=\hat{k}\mp\frac{\phi'}{R}-\frac{1}{2\hat{k}}\left[\frac{\phi'}{R}\right]^2+O(\hat{k}^{-2}) ,\end{aligned} \tag{26}$$

where

$$\hat{k}=\frac{k}{R} \tag{27}$$

is the wave number modified by the redshift effect.

III. LIMITS ON EEP VIOLATION

On the basis of Eq. (26) we can verify that the theory satisfies the weak equivalence principle (WEP), but not the Einstein equivalence principle (EEP). The latter is easier to see: By measuring the relative phase of waves of opposite helicity, experiments in a freely falling frame will depend on $\phi'(\eta)$ and, hence, on the gravitational fields on which it depends, contrary to the EEP. We exploit this fact later in this paper.

According to the WEP, test bodies follow trajectories that are independent of their internal structure and composition. For radiation we interpret this to mean that the group velocity v_g associated with wave packets should be independent of their helicity. The group velocity for the two helicities is given by

$$v_{g\pm}=\frac{\partial\omega_{\pm}}{\partial\hat{k}}=1+\frac{1}{2\hat{k}^2}\left[\frac{\phi'}{R}\right]^2+O(\hat{k}^{-3})$$
$$=1+\frac{1}{2}\left[\frac{\phi'}{k}\right]^2+O(k^{-3}) \ . \qquad (28)$$

As usual, a "test body" is defined to be small enough in size that couplings to inhomogeneities in the gravitational fields may be neglected.[2,6] For radiation this definition implies that couplings of order $(\phi'/k)^2$ and higher may be neglected, since the size of a wave packet is $\lambda\sim 1/k$. Since Eq. (28) tells us that, to this order, the group velocity of a wave packet is $v_g=1$ independent of helicity, the WEP is satisfied.

The degree to which the EEP is violated (hence the effective degree of violation of Schiff's conjecture within this framework) can be determined by observing the change $\Delta\chi$ in position angle χ of the plane of polarized radiation from distant synchrotron sources at redshift z due to the phase shift between the modes of opposite helicity. According to CFJ,[7]

$$\Delta\chi=\tfrac{1}{2}(\sigma_+-\sigma_-)=-\delta\phi \ , \qquad (29)$$

where

$$\delta\phi=\phi(0)-\phi(z) \qquad (30)$$

is the change in ϕ since the epoch of redshift z. CFJ showed that for sources with $\langle z\rangle=0.9$, observations limit $|\Delta\chi|$ to <0.1, whence $|\delta\phi|<0.1$ with 95% confidence.

In Ni's formulation, ϕ is a function of the fundamental gravitational fields (but not their derivatives), but is otherwise arbitrary. It is therefore difficult to limit rigorously $\dot{\phi}$, as the dependence of ϕ on cosmic time is unknown. Nevertheless, in the cosmological context under consideration, it is reasonable to assume that any function of the gravitational fields is currently varying smoothly on a time scale comparable to the Hubble time $H_0^{-1}\sim 10^{17}$ sec. That is,

$$\dot{\phi}_0\sim H_0\delta\phi \ , \qquad (31)$$

which implies the limit

$$|\dot{\phi}_0|\lesssim 10^{-18}\,\text{sec}^{-1}\approx 10^{-41}\ \text{GeV} \ . \qquad (32)$$

Evidently, the fact that no rotation in polarization is observed over cosmological scales places a stringent bound on $|\dot{\phi}_0|$.

We may compare the strength of this upper bound with what might be obtained from hypothetical solar system tests.[8] To make this comparison, consider a theory with a Brans-Dicke scalar coupled to electromagnetism through Ni's Lagrangian.[10] That is, we imagine a theory of gravitation and electromagnetism defined by a Lagrangian

$$\mathcal{L}=\mathcal{L}_{\text{BD}}+\mathcal{L}_{\text{Max}}-\frac{\alpha}{8\pi}\sqrt{g}\,\Phi F_{\mu\nu}\tilde{F}^{\mu\nu} \ , \qquad (33)$$

where α is some coupling constant and

$$\mathcal{L}_{\text{BD}}=\frac{1}{16\pi}\sqrt{g}\left[\Phi\mathcal{R}-\omega\frac{\partial_\mu\Phi\partial^\mu\Phi}{\Phi}\right] \qquad (34)$$

is the Brans-Dicke Lagrangian. Here $\mathcal{R}$ is the Ricci scalar, ω is a dimensionless constant, and Φ is a scalar field whose value today is related to Newton's constant by

$$\Phi_0=\frac{1}{G} \ ; \qquad (35)$$

it is this field which we take to couple to $F\tilde{F}$ through Ni's Lagrangian (1) with $\phi\equiv\alpha\Phi$. Weinberg[11] has solved the Brans-Dicke field equations for a spatially flat Robertson-Walker universe to obtain

$$\Phi=\Phi_0\left[\frac{t}{t_0}\right]^{2/(3\omega+4)}=\Phi_0(1+z)^{-1/(\omega+1)} \ , \qquad (36)$$

where t_0 is the present age of the Universe, $t_0\sim H_0^{-1}\sim 10^{17}$ sec. Since radar-echo delay experiments in the solar system[12] have set the limit on the Brans-Dicke parameter $\omega>500$, we may expand this for large ω to find the change in Φ between a source at redshift z and today:

$$\Delta\Phi(z)\approx\frac{\ln(1+z)}{\omega+1}\Phi_0 \ . \qquad (37)$$

Our limit $|\delta\phi|=|\alpha\Delta\Phi|<0.1$ for $\langle z\rangle=0.9$ then yields

$$|\alpha|\lesssim 0.2(\omega+1)\Phi_0^{-1} \ . \qquad (38)$$

We argued in Eq. (5) that only the gradient of ϕ affects electromagnetism through Ni's term; therefore, $\dot{\phi}_0$ is the quantity which enters present-day experiments. From Eqs. (36) and (38) this is limited by

$$|\dot{\phi}_0|=|\alpha\dot{\Phi}_0|\lesssim 0.4\frac{\omega+1}{3\omega+4}t_0^{-1} \ . \qquad (39)$$

Since $\omega\gg 1$, $|\alpha\dot{\Phi}_0|\lesssim 10^{-18}$ sec^{-1}, in agreement with Eq. (32).

For comparison, consider the solution for the Brans-Dicke scalar in the solar system:[11]

$$\Phi=\frac{M_\odot}{r(\omega+2)}+\text{const} \ , \qquad (40)$$

where $M_\odot$ is the mass of the Sun. The greatest change in Φ we may hope to measure is the difference in Φ between the surface of the Sun and a point at infinity. Using $M_\odot/r_\odot=2\times 10^{-6}G^{-1}$, this is

$$\Delta\Phi(r_\odot-r_\infty)=2\times 10^{-6}(\omega+2)^{-1}\Phi_0 \ . \qquad (41)$$

Therefore, if we are sensitive to a rotation in polarization angle of $\delta\chi$, and the rotation due to the Ni effect is approximately equal to the change in ϕ, then we can potentially place the limit

$$|\alpha|\lesssim 5\times 10^5(\omega+2)\Phi_0^{-1}\delta\chi \ . \qquad (42)$$

Comparing this result to Eq. (38), we see that experiments sensitive to rotations of polarization angle of 10^{-6} rad are necessary to duplicate the precision of the cosmological test. While this comparison was performed for the specific case of "Brans-Dicke-Ni" theory, it helps to confirm our suspicion that the cosmological test is likely to be more constraining than all but the most precise solar system experiments.

IV. DISCUSSION

We have considered the observational consequences of coupling gravitation to electromagnetism through the Ni Lagrangian

$$\mathcal{L}_N = -\frac{1}{8\pi}\sqrt{g}\,\phi F_{\mu\nu}\tilde{F}^{\mu\nu} . \tag{43}$$

The scalar ϕ is taken to be a function of the gravitational fields present in a theory of gravity (which otherwise obeys the EEP). The addition of this Lagrangian leads to a rotation in the plane of polarization of radiation. Noting that a ϕ that varies smoothly with time in an homogeneous and isotropic universe would affect the correlation between polarization angle and position angle of radio galaxies, we used the nonexistence of such an effect to place the limit given by Eq. (32) on the magnitude of this parameter. Our limit applies only to functions which vary monotonically, rather than oscillating or behaving in a more complex fashion; nevertheless, we believe this to be a reasonable assumption in the cosmological context under consideration.

Ni's Lagrangian is unique in that, under a certain general set of assumptions, it is the only possible way to violate the EEP without violating the WEP. It is remarkable that Schiff's conjecture possesses only one counterexample within this framework: As noted above, such an interaction might be expected if the WEP, but not the EEP, were a symmetry of nature. In this paper we have argued that Ni's Lagrangian has negligible effects in the contemporary Universe, if it exists at all. (Our test does not address the possibility that ϕ may be unimportant today but significant in the early Universe.) Within the assumptions of Ni's framework, this result implies that tests of the WEP (such as Eötvös experiments) provide strong support for the EEP and, hence, for all metric theories of gravity.[13] Possible future work would loosen the assumptions of Ni's framework to include couplings to derivatives of the gravitational fields and extension of the formalism to spontaneously broken gauge theories.[5]

Recent progress in theories such as supergravity and superstrings has renewed interest in alternatives to general relativity, including scalar gravitational fields.[14] It is conceivable that a term such as Ni's could arise in the low-energy effective theory from a unified scheme. We believe that cosmological tests such as the one presented in this paper may prove useful in constraining models unifying gravitation with particle physics.

Note added: After this paper was submitted, we became aware of related work by Wolf.[15] Wolf explored observational consequences of the Lagrangian (1), but did not consider the test proposed in this paper.

ACKNOWLEDGMENTS

We would like to thank R. Jackiw and C. Will for helpful conversations. This work was supported in part by NASA under Grant No. NAGW-931 and the National Science Foundation.

[1] C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, England, 1981); in *300 Years of Gravitation,* edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1987).

[2] K. S. Thorne, D. L. Lee, and A. P. Lightman, Phys. Rev. D **7**, 3563 (1973).

[3] L. I. Schiff, Am. J. Phys. **28**, 340 (1960).

[4] A. P. Lightman and D. L. Lee, Phys. Rev. D **8**, 364 (1973).

[5] W.-T. Ni, Phys. Rev. Lett. **38**, 301 (1977). Ni's analysis of electromagnetism was extended to non-Abelian gauge theories (such as QCD) by W.-T. Ni, Phys. Lett. A **120**, 174 (1987).

[6] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[7] S. M. Carroll, G. B. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990). Cited in text as CFJ.

[8] W.-T. Ni, in *Precision Measurement and Fundamental Constants II,* Natl. Bur. Stand. (U.S.) Spec. Publ. No. 617, edited by B. N. Taylor and W. D. Phillips (U.S. GPO, Washington, D.C., 1984).

[9] M. S. Turner and L. M. Widrow, Phys. Rev. D **37**, 3428 (1988).

[10] This theory is the simplest possible example with nonzero Ni coupling. In general relativity there is no nontrivial ϕ which may be constructed; the only gravitational field is the metric $g_{\mu\nu}$, and at any point we may choose coordinates in which the metric takes the Minkowski form $g_{\mu\nu}=\mathrm{diag}(-1,1,1,1)$; so any scalar constructed from $g_{\mu\nu}$ must be constant across spacetime.

[11] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).

[12] R. D. Reasenberg, I. I. Shapiro, P. E. MacNeil, R. B. Goldstein, J. C. Breidenthal, J. P. Brenkle, D. L. Cain, T. M. Kaufman, T. A. Komarek, and A. I. Zygielbaum, Astrophys. J. Lett. **234**, L219 (1979).

[13] There is a loophole in this argument: Since Eötvös experiments are performed on normal matter (electrons, protons, and neutrons), a nonmetric theory could have strong effects on the motion of more exotic particles and still escape detection. This property has been exploited in the context of cosmological dark matter by T. Damour, G. W. Gibbons, and C. Gundlach, Phys. Rev. Lett. **64**, 123 (1990); and in the context of cosmions which have settled in stars by J. W. Moffat, Phys. Rev. D **39**, 474 (1989).

[14] See, for example, J. P. Derendinger, L. E. Ibañez, and H. P. Nilles, Nucl. Phys. **B267**, 365 (1986).

[15] C. Wolf, Phys. Lett. A **132**, 151 (1988); **145**, 413 (1990).