# Remark on the Yukawa sector of the standard model

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The Goldstone-fermion interaction in the standard model is examined. We show that massless fermions still couple to Goldstone particles. An application in the technicolor model shows that electronlike quarks should couple to pions.

In the standard model,[1] mass generation remains a mystery. Because the Yukawa sector is not a gauge interaction, its couplings are totally arbitrary. However, the Yukawa sector provides a Higgs-fermion coupling to preserve the unitarity for $\bar{F}F \to V_L V_L$ ($V$ can be either $W$ or $Z$; $F$ denotes quarks and leptons). But with massless fermions, the Higgs-fermion vertex is not needed and one can simply set the corresponding Yukawa coupling to zero. Furthermore, the Yukawa sector also generates Goldstone-fermion couplings, and we may wonder if this vertex disappears for massless fermions like the Higgs-fermion vertex. In this Brief Report we will elaborate on this last point and discuss only massless fermions with $\mathrm{SU}_c(3)\times \mathrm{U}_{\mathrm{cm}}(1)$ gauge interactions. We show that they must continue to couple to the Goldstone particles.

Let us consider a world where the $\mathrm{SU}_L(2)\times \mathrm{U}_Y(1)$ gauge interaction is turned off. There is $[\mathrm{SU}_L(2)\times \mathrm{U}_Y(1)]^{\mathrm{global}}\times[\mathrm{SU}_c(3)]^{\mathrm{gauge}}$ symmetry. For simplicity, we shall also restrict ourselves to one generation. The Lagrangian here is

$$\mathcal{L}=\mathcal{L}_1+\mathcal{L}_2+\mathcal{L}_3+\mathcal{L}_4 \,, \tag{1}$$

where

$$\mathcal{L}_1=-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} \,,$$

$$\begin{aligned}\mathcal{L}_2=&\overline{\begin{bmatrix}\nu_e\\ e\end{bmatrix}}_L i\gamma^\mu\partial_\mu \begin{bmatrix}\nu_e\\ e\end{bmatrix}_L+\bar{e}_R i\gamma^\mu\partial_\mu e_R\\ &+\overline{\begin{bmatrix}u\\ d\end{bmatrix}}_L i\gamma^\mu D_\mu \begin{bmatrix}u\\ d\end{bmatrix}_L\\ &+\bar{u}_R i\gamma^\mu D_\mu u_R+\bar{d}_R i\gamma^\mu D_\mu d_R \,,\end{aligned}$$

$$\mathcal{L}_3=\partial_\mu\Phi^\dagger\partial^\mu\Phi-V(\Phi) \,,$$

$$\mathcal{L}_4=f^e\overline{\begin{bmatrix}\nu_e\\ e\end{bmatrix}}_L\Phi e_R+f^u\overline{\begin{bmatrix}u\\ d\end{bmatrix}}_L\tilde{\Phi}u_R+f^d\overline{\begin{bmatrix}u\\ d\end{bmatrix}}_L\Phi d_R+\mathrm{H.c.} \,,$$

with

$$\tilde{\Phi}=i\tau_2\Phi^* \,,$$

$$D_\mu=\partial_\mu-ig_c\frac{\lambda^i}{2}G^i_\mu \,,$$

$$V(\Phi)=\frac{\lambda}{4}\left[\Phi^+\Phi-\frac{V^2}{2}\right]^2 .$$

As $\Phi$ develops a vacuum expectation value due to $V(\Phi)$,

$$[\mathrm{SU}_L(2)\times \mathrm{U}_Y(1)]^{\mathrm{global}}\times[\mathrm{SU}_c(3)]^{\mathrm{gauge}}$$

is broken down to

$$[\mathrm{U}_{\mathrm{em}}(1)]^{\mathrm{global}}\times[\mathrm{SU}_c(3)]^{\mathrm{gauge}} .$$

As a result, fermions get masses and three Goldstone particles appear. One can see that there are Goldstone-fermion vertices from the Yukawa sector. To see it clearly let us parametrize $\Phi$ as

$$\Phi=e^{-i\xi\cdot\tau/V}\begin{bmatrix}0\\ \dfrac{V+\eta}{\sqrt{2}}\end{bmatrix}\simeq\frac{1}{\sqrt{2}}\begin{bmatrix}-\xi^2-i\xi^1\\ V+\eta+i\xi^0\end{bmatrix} , \tag{2}$$

so that $\langle|\eta|\rangle=\langle|\xi^i|\rangle=0$, with $\eta$ being the Higgs particle and $\xi^i$ being the Goldstone particles. From $\mathcal{L}_4$ one has

$$\begin{aligned}\mathcal{L}_4=&\frac{f^e}{\sqrt{2}}V\bar{e}_L e_R+\frac{f^e}{\sqrt{2}}\bar{e}_L e_R\eta+\frac{f^u}{\sqrt{2}}V\bar{u}_L u_R+\frac{f^u}{\sqrt{2}}\bar{u}_L u_R\eta\\ &+\frac{f^d}{\sqrt{2}}V\bar{d}_L d_R+\frac{f^d}{\sqrt{2}}\bar{d}_L d_R\eta+\frac{f^e}{\sqrt{2}}i\bar{e}_L e_R\xi^0\\ &+\frac{f^d}{\sqrt{2}}i\bar{d}_L d_R\xi^0-\frac{f^u}{\sqrt{2}}i\bar{u}_L u_R\xi^0+\mathrm{H.c.}+\cdots .\end{aligned} \tag{3}$$

For discussion let us calculate the amplitude $\xi^0\to\mathrm{gluon}+\mathrm{gluon}$ for on-shell gluons and Goldstones. As shown in Fig. 1, the calculation is finite and gives a nonvanishing result, which for each fermion in the triangle is

$$A(\xi^0\to gg)\sim\frac{f^i}{m_i}\frac{g_c^2}{\sqrt{2}4\pi^2}\epsilon^{\mu\nu\sigma\rho}k_{1\mu}k_{2\nu}\epsilon^1_\sigma\epsilon^2_\rho . \tag{4}$$

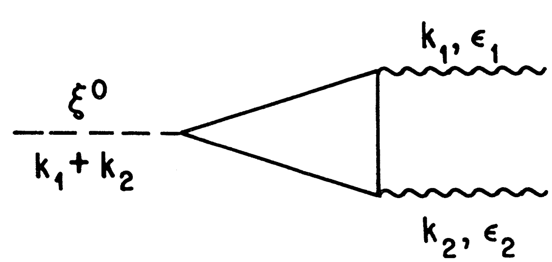


FIG. 1. Effective coupling of $\xi^0$ to two gluons or photons.

 

Here $f^i$ is the Yukawa coupling and $m_i$ is the corresponding fermion mass. If $f^u$ and $f^d$ are not zero, these triangles will involve $u$ and $d$ quarks, which contribute with opposite signs (see the Lagrangian above), and so

$$A(\xi^0 \to gg) = 0 \ . \tag{5}$$

However, imagine that $m_d \neq 0$, but $m_u = 0$ (this is one way proposed to solve the strong *CP* problem[2]), and set $f^u = 0$ at the beginning, so that there is no vertex such as $\bar{u}\gamma_5 u \xi^0$. The triangle now gives a nonvanishing result:

$$A(\xi^0 \to gg) \neq 0 \ . \tag{6}$$

Note, however, that (4) has a fixed value in the limit $f^u \to 0$ with $f^u V/\sqrt{2} \equiv m_u$. In this limit, $A(\xi^0 \to gg) = 0$, which is the same answer as for the massive $u$ quark. We summarize the results in Table I.

One can argue that $A(\xi^0 \to gg) = 0$ is a physical solution in two ways. First, $A(\xi \to gg) \neq 0$ means that the global currents coupling to the Goldstone fields have an anomaly (see below), resulting in an inconsistency of the theory; second, $A(\xi^0 \to gg) = 0$ is a smooth solution (i.e., there should not be a difference between the Yukawa coupling being very small and being set to zero.) By these considerations one can conclude that effectively the massless $u$ quark should still couple to Goldstone particles, which is different from what happens in the Higgs-fermion vertex.

TABLE I. Amplitude $\xi^0 \to$ gluon + gluon.

| | $A(\xi^0 \to gg)$ |
|---|---|
| $f^u \neq 0,\ f^d \neq 0$ | 0 |
| $f^u = 0,\ f^d \neq 0$ | $\neq 0$ |
| $f^u \to 0,\ f^d \neq 0$ | 0 |

Let me try to explain the first point above in detail because the second point is obvious. First, we will show that if $A(\xi^0 \to gg) \neq 0$, $m^2_{\xi^0} \neq 0$, which results in an inconsistency of the theory. Equivalently, one can get $A(\xi^0 \to gg) \neq 0$ by using an effective Lagrangian (see below)

$$\mathcal{L} = \frac{\xi^0}{V} \frac{g_c^2}{32\pi^2} G^a_{\mu\nu} G^{\tilde{a}\mu\nu} \ . \tag{7}$$

Similar to the discussions of axions,[2] one can argue that a mass term for the Goldstone $\xi^0$ is induced:

$$m^2_{\xi^0} \sim \frac{1}{V^2} \int d^4x \left\langle 0 \left| T \left[ \frac{g_c^2}{32\pi^2} G^a_{\mu\nu} G^{\tilde{a}\mu\nu}(x),\ \frac{g_c^2}{32\pi^2} G^a_{\mu\nu} G^{\tilde{a}\mu\nu}(0) \right] \right| 0 \right\rangle \ . \tag{8}$$

What will happen with a massive $\xi^0$ if we try to gauge the theory? The Higgs mechanism suggests that $\xi^0$ should be absorbed by the gauge field, so that $Z^0$ can get a mass. However, the mixing term in $\mathcal{L}_3$, when one gauges the theory,

$$\mathcal{L}_{\text{mix}} \sim g_z V Z_\mu \partial^\mu \xi^0 \ , \tag{9}$$

will generate a gauge-boson mass at zero momentum through Fig. 2, if and only if $m^2_{\xi^0} = 0$. Otherwise, the pole of the Goldstone is shifted, resulting in $m_Z^2 = 0$.

This should not be a problem of the standard model with a massless $u$ quark. To avoid it one has to be a bit careful and note that really the $u$ quark still couples to $\xi^0$, so that $m^2_{\xi^0} = 0$. One can also understand this point as follows. Examining in detail the Lagrangian (3), one can see that there are two chiral U(1) curents, which can be written as

$$\begin{aligned} \mathrm{U}^{\xi^0}(1):\ & d_L \to e^{-i\beta} d_L \ , \\ & d_R \to d_R \ , \\ & \xi^0 \to \xi^0 + V\beta \ ; \\ \mathrm{U}(1):\ & u_L \to e^{i\gamma} u_L \ , \\ & u_R \to u_R \ , \end{aligned} \tag{10}$$

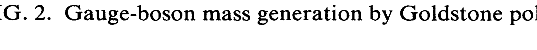

FIG. 2. Gauge-boson mass generation by Goldstone pole.

where $\beta, \gamma$ are free parameters. Having two anomalous U(1) currents, it is possible to construct an anomaly-free current, which one can use to gauge.

To discuss this concretely, consider making a local transformation on the $d$ quark, to remove the $\xi^0$ field from the Yukawa interactions:

$$\begin{aligned} & d_L \to e^{i\xi^0/V} d_L \simeq \left[ 1 + i\frac{\xi^0}{V} \right] d_L \ , \\ & d_R \to d_R \ . \end{aligned} \tag{11}$$

This transformation will generate a derivative interaction of the $\xi^0$ arising from the fermion kinetic energy terms:

$$\bar{d}_L i\gamma^\mu \partial_\mu d_L \to \bar{d}_L i\gamma^\mu \partial_\mu d_L - \bar{d}_L \gamma^\mu d_L \partial_\mu \frac{\xi^0}{V} \ . \tag{12}$$

However, in making the chiral transformation (11), one generates also an "anomalous" nonderivative interaction of $\xi^0$ to the $[\mathrm{SU}_c(3)]^{\text{gauge}}$ fields:[3]

$$\mathcal{L}_{\text{anomaly}} \sim \frac{\xi^0}{V} \frac{g_c^2}{32\pi^2} G^a_{\mu\nu} G^{\tilde{a}\mu\nu} \ , \tag{13}$$

which gives the same amplitude as (4). As remarked above, one can combine the two U(1)'s into an anomaly-free current. To see how a similar term to (13) emerges to cancel (13), consider making a transformation on the $u$ quark,

$$\begin{aligned} & u_L \to e^{i\alpha\xi^0/V} u_L \simeq \left[ 1 + i\alpha\frac{\xi^0}{V} \right] u_L \ , \\ & u_R \to u_R \ , \end{aligned} \tag{14}$$

with $\alpha$ being a free parameter. By this transformation one generates

$$\mathcal{L}_{\text{anomaly}} \sim \alpha \frac{\xi^0}{V} \frac{g_c^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} \ . \tag{15a}$$

To cancel explicitly (13), $\alpha$ must be fixed to be $-1$. [If $\alpha$ is not fixed to be $-1$, $\xi^0$ would be an axion. When $\alpha$ is $-1$, the transformations (11) and (14) are part of an $SU_L(2)$ transformation.] However, $\xi^0$ is a local field, and so the transformation (14) also generates a derivative interaction of $\xi^0$ from the $u$-quark kinetic energy term

$$\mathcal{L}_{\bar{u}u\xi^0} \sim \bar{u}_L \gamma^\mu u_L \partial_\mu \frac{\xi^0}{V} \ . \tag{15b}$$

This is the additional $\bar{u}u\xi^0$ vertex which is needed with massless $u$ quark to guarantee consistency. Now the whole Lagrangian for the quark sector is

$$\mathcal{L} \sim (\bar{u}_L \gamma^\mu u_L - \bar{d}_L \gamma^\mu d_L) \partial_\mu \frac{\xi^0}{V} + \frac{f^d}{\sqrt{2}} \bar{d}_L (V+\eta) d_R + \text{H.c.} + SU_c(3) - \text{gauge interactions} \ . \tag{16}$$

One can see that no coupling such as $\bar{u}u\eta$ exists with the massless $u$ quark, but there is a vertex $\bar{u}u\xi^0$ with derivative interactions. It is true that $\mathcal{L}_4$ can be recovered by an integration by parts. However, no anomalous term such as (7) can be generated because the $\xi^0$ couples now to current $J^\mu = (\bar{u}_L \gamma^\mu u_L - \bar{d}_L \gamma^\mu u_L)$, which is anomaly-free. The key point here is that to have a consistent treatment we have to fix the gauge of the $u$-quark field.

The expression for the Lagrangian $\mathcal{L}_4$ in (16) serves to remind us that, irrespective of the mass of the quarks, there is really no anomalous term. In this formulation, however, the $u$ quark is in fact derivatively coupled to $\xi^0$. Apparently, the advantage of Lagrangian (16) over (3) is that Yukawa terms can be treated freely, with the same results as the smooth solution. If the Higgs field decouples, (16) becomes the effective Lagrangian, discussed in Ref. 4, which is the basis for nonlinear realization of the broken symmetry, where Goldstone fields couple to full theory.

Let us now extend the discussion to the lepton sector. Because $U_{\text{em}}(1)$ is an unbroken symmetry, one can formally gauge it (the photon field is a combination of $T^3$ and Y gauge fields). In this case a similar consideration about the triangle in Fig. 1 with the replacement of gluons by photons requires that also the massless electron couple derivatively to the Goldstone particle. A similar consideration can be applied to a dynamical electroweak symmetry-breaking theory. Let us take the technicolor model as an example. Consider, again, the standard model, but this time without the elementary Higgs scalar, and for simplicity restrict the discussion again to one family of fermions. Without the Higgs sector fermions and gauge bosons should be massless. But strong chiral-symmetry breaking can generate gauge-boson masses. The basic idea is well known. When turning off electroweak interactions, the quark sector has an $SU_L(2)\times SU_R(2)$ chiral symmetry. As $\langle \bar{u}u + \bar{d}d \rangle \neq 0$, driven by strong interaction, the chiral symmetry is broken down to $SU_{L+R}(2)$. As a result, three Goldstone bosons $\pi^{\pm,0}$ appear. And it is well known that pions couple to the quark with a derivative coupling through the nonlinear realization of broken symmetry. (Here we use the chiral quark model, where the quark should be a constituent quark.[5] In general, the effective Lagrangian will include pions and a nucleon.[6])

Gauge bosons get masses in the technicolor model by absorbing these pions. However, there appears to be a problem due to the $\pi^0 \to \gamma\gamma$ anomaly. However, as we discuss above, in this unrealistic world, $A(\pi^0 \to \gamma\gamma)$ actually vanishes. The reason is the following. As chiral symmetry is broken, quarks get dynamical masses, which are of the order of chiral-symmetry-breaking scale. But in this renormalizable theory the electron cannot get mass from the quark condensate. However, the massless electron provides another freedom to cancel the anomaly, resulting in a derivative coupling of $\pi^0$ to the electron:

$$\mathcal{L} \sim -\bar{e}\gamma^\mu \gamma_5 e \partial_\mu \frac{\pi^0}{2f_\pi} \ . \tag{17}$$

[In the real world the electron gets mass from the Higgs sector. Furthermore, the physical pions are massive and couple dominantly to the hadronic current, and so $A(\pi^{\text{phys}} \to \gamma\gamma)$ does not vanish. However, mixing between $\xi^0$ and $\pi^0$ also generates a term such as (17), but much suppressed by a factor of $(f_\pi/V)^2$.]

In conclusion, by focusing on anomalies we have shown that massless fermions do not decouple from Goldstone fields. This is in contrast with the Higgs-fermion vertex, which vanishes as $m_f = 0$. These derivative couplings of the massless fermions to the Goldstone fields are necessary to guarantee the absence of unphysical anomalous couplings of the Goldstone fields.

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