BRIEF REPORTS

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QCD corrections to $t \to W^+ + b$

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The $O(\alpha_s)$ QCD corrections to the process $t \to W^+ b$ are calculated in the standard model for a heavy top quark. For $m_W/M_t < 0.8$, the corrections reduce the partial width by 10% relative to the tree-level result. This reduction of the top-quark width enhances the signal for $t\bar{t}$ bound-state resonances near threshold in e^+e^- collisions, studied recently by Strassler and Peskin.

I. INTRODUCTION

The experimental lower bound for the standard-model top-quark mass is now up to 89 GeV according to recent Fermilab Collider Detector Facility data.[1] If there is no exotic physics (supersymmetry, technicolor, extended Higgs sector, etc.) below the few-hundred-GeV scale, the dominant decay mode of such a heavy standard-model top quark is $t \to W^+ b$, since other modes are suppressed by the small mixing angles. The total width of the heavy top quark is then approximately the same as the partial width for $t \to W^+ b$. For $m_t \gg m_W$ this width is broad, since it is proportional to m_t^3. Intuitively, this broad width will smear out any structure near the $t\bar{t}$ production threshold. Recently, following the ideas of Fadin and Khoze,[2] Strassler and Peskin[3] have shown that despite its broad width, for a sufficiently heavy top quark ($m_t > 140$ GeV), $t\bar{t}$ threshold production in e^+e^- collisions can still exhibit a resonance peak due to QCD nonrelativistic bound-state effects and/or Higgs-boson-exchange effects. The threshold production behavior of $t\bar{t}$ states in e^+e^- collisions for $m_t < 140$ GeV, where a spectroscopy of toponium states presumably exists, has also been studied recently by Kwong.[4] The shape of the resonance depends crucially on the total width of the top quark, while its peak value and location are sensitive to α_s.[3] Using these effects, the authors of Refs. 2 and 3 argue that one can make precise measurements of the strong coupling constant α_s and the width and the mass of the top quark in the future e^+e^- machines beyond the CERN collider LEP II.

While all the $O(\alpha_s)$ effects in $t\bar{t}$ production near threshold remain to be analyzed, we present here the calculation of one important contribution, namely, the $O(\alpha_s)$ QCD corrections to the tree-level width for $t \to W^+ b$. These $O(\alpha_s)$ corrections to the top-quark width should be next in importance to the static one-gluon-exchange correction in $t\bar{t}$ production near threshold studied in Refs. 2–4. Since the top quark is quite heavy, the bottom-quark mass is negligible and we shall take the bare bottom-quark mass to be zero in our calculation. This approach is slightly different from the earlier calculations[5,6] where m_b was retained and allowed to vanish only in the final results. These earlier calculations also emphasized the QCD corrections to the lepton spectrum of the semileptonic decay mode of $t \to W^{+*} b \to l^+ \nu_l b$, where W^{+*} is virtual. For the W^+ on shell, our results agree. In our approach, neglecting the bare bottom-quark mass from the outset not only simplifies the calculations but also clarifies some misleading statements in the literature[7] concerning the massless renormalization scheme. In Sec. II, we present the virtual- and real-gluon corrections to the tree-level rate for $t \to W^+ b$. In Sec. III, we discuss the implication of our results for the threshold effects in the production process $e^+e^- \to t\bar{t}$ recently analyzed by Strassler and Peskin.[3]

II. $O(\alpha_s)$ QCD CORRECTIONS TO $t \to W^+ b$

A. Tree-level decay width

For completeness, we recall[8] the partial decay width for the process $t \to W^+ b$, which is given by the diagram shown in Fig. 1(a):

$$\Gamma_0 = \frac{G_F m_t^3}{8\sqrt{2}\pi} |V_{tb}|^2 \lambda^{1/2}\left(1, \frac{m_b}{m_t}, \frac{m_W}{m_t}\right) \times \left[\left(1 - \frac{m_b^2}{m_t^2}\right)^2 + \left(1 + \frac{m_b^2}{m_t^2}\right)\frac{m_W^2}{m_t^2} - 2\frac{m_W^4}{m_t^4}\right]. \quad (1)$$

Here $\lambda(x,y,z) = (x^2 - y^2 - z^2)^2 - 4y^2z^2$. In the limit $m_b \to 0$, the tree-level rate simplifies considerably, becoming

$$\Gamma_0 = \frac{G_F m_t^3}{8\sqrt{2}\pi} |V_{tb}|^2 \beta_W^4 (3 - 2\beta_W^2), \quad (2)$$

where we have defined $\beta_W=(1-m_W^2/m_t^2)^{1/2}$, the velocity of the W^+ in the top-quark rest frame. Clearly, for a very heavy top quark the width becomes very broad, increasing as m_t^3.

B. QCD virtual corrections

We will use dimensional regularization to regulate all the ultraviolet divergences in the virtual one-loop corrections. To regulate the infrared divergences associated with soft and collinear gluon emission, we introduce a gluon mass parameter λ in the gluon propagator, which is justified since the non-Abelian nature of QCD does not enter at this order in α_s. We also adopt the on-shell mass-renormalization scheme. Calculating the self-energy correction for the top quark in Fig. 1(b), one finds its wave-function renormalization constant to be

$$Z_2^t-1=\frac{C_F\alpha_s}{4\pi}\left[-\overline{\gamma}_\epsilon-2\ln\left[\frac{\lambda^2}{m_t^2}\right]-\ln\left[\frac{\mu^2}{m_t^2}\right]-4\right] . \quad (3)$$

Similarly, from Fig. 1(c), one finds the wave-function renormalization constant for the (massless) bottom quark to be

$$Z_2^b-1=\frac{C_F\alpha_s}{4\pi}\left[-\overline{\gamma}_\epsilon+\ln\left[\frac{\lambda^2}{m_t^2}\right]-\ln\left[\frac{\mu^2}{m_t^2}\right]+\frac{1}{2}\right] . \quad (4)$$

Here, $\overline{\gamma}_\epsilon=1/\epsilon+\ln 4\pi-\gamma_E$, where $D=4-2\epsilon$ is the space-time dimension and γ_E is Euler's constant. The color factor $C_F=\frac{4}{3}$ for SU(3) and μ is the 't Hooft mass parameter in the dimensional-regularization scheme. From the calculation of the vertex correction in Fig. 1(d), we can extract the renormalization constant Z_1 associated with the vertex:

$$1-Z_1=\frac{C_F\alpha_s}{4\pi}\left[\overline{\gamma}_\epsilon-\frac{1}{2}\ln^2\left[\frac{\lambda^2}{m_t^2}\right]+2\ln\beta_W^2\ln\left[\frac{\lambda^2}{m_t^2}\right]-2\ln\left[\frac{\lambda^2}{m_t^2}\right]+\ln\left[\frac{\mu^2}{m_t^2}\right]-2\ln^2\beta_W^2+3\ln\beta_W^2-2\mathrm{Li}_2(1-\beta_W^2)-2-\frac{\pi^2}{2}\right] . \quad (5)$$

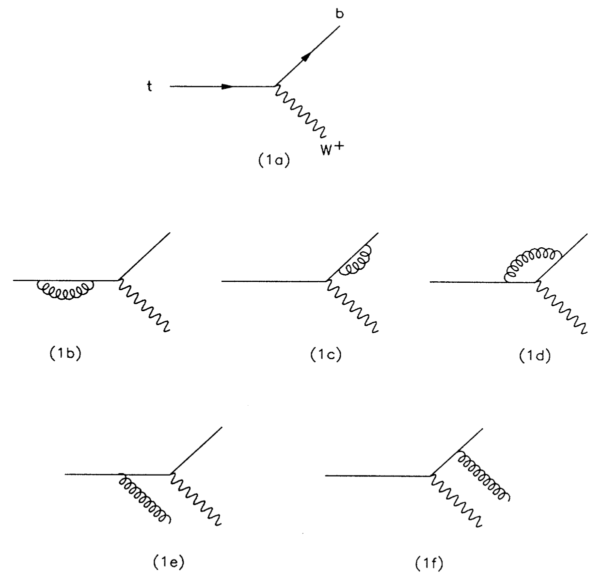

FIG. 1. Tree-level diagram for $t\rightarrow W^+b$ and the $O(\alpha_s)$ corrections. (a) tree; (b)–(d) virtual corrections, (e) and (f) real-gluon emmission. Counterterms are not shown.

Here $\mathrm{Li}_2(z)=-\int_0^z dt\,\ln(1-t)/t$ is the Spence function.

Using Lorentz invariance, one can write the transition amplitude for the process $t\rightarrow W^+b$ in the general form

$$i\mathcal{M}=\overline{u}(b)\frac{ig}{\sqrt{2}}V_{tb}\times\left[\gamma_\mu LF_1(k^2)+\frac{C_F\alpha_s}{2\pi}F_2(k^2)i\sigma_{\mu\nu}k^\nu m_t R\right]\times u(t)\epsilon^\mu(k,\lambda) , \quad (6)$$

where $k^\nu=(t-b)^\nu$, $\sigma_{\mu\nu}=i[\sigma_\mu,\sigma_\nu]/2, L,R=(1\mp\gamma_5)/2$, and $F_{1,2}(k^2)$ are form factors. To $O(\alpha_s)$, F_1 is given on shell by the sum of Figs. 1(a)–1(d):

$$F_1=1+\frac{1}{2}(Z_2^t-1)+\frac{1}{2}(Z_2^b-1)+(1-Z_1)$$
$$=1+\frac{C_F\alpha_s}{4\pi}\left\{\ln\left[\frac{\lambda^2}{m_t^2}\right]\left[-\frac{5}{2}+2\ln\beta_W^2-\frac{1}{2}\ln\left[\frac{\lambda^2}{m_t^2}\right]\right]-2\ln^2\beta_W^2+3\ln\beta_W^2-2\mathrm{Li}_2(1-\beta_W^2)-\frac{15}{4}-\frac{\pi^2}{2}\right\} ; \quad (7)$$

F_2, on shell, comes from the Pauli term of Fig. 1(d):

$$F_2=\frac{1}{2m_W^2}\ln\beta_W^2 . \quad (8)$$

Using these results it is straightforward to calculate the renormalized decay rate to order α_s. One finds

$$\Gamma_{\rm ren}=\Gamma_0\left[1+\frac{C_F\alpha_s}{2\pi}\left\{\ln\left[\frac{\lambda^2}{m_t^2}\right]\left[-\frac{5}{2}+2\ln\beta_W^2-\frac{1}{2}\ln\left[\frac{\lambda^2}{m_t^2}\right]\right]\right.\right.$$
$$\left.\left.-2\ln^2\beta_W^2+\left[\frac{6(1-\beta_W^2)}{(3-2\beta_W^2)}\right]\ln\beta_W^2-2\mathrm{Li}_2(1-\beta_W^2)-\frac{15}{4}-\frac{\pi^2}{2}\right\}\right]. \tag{9}$$

All the ultraviolet divergences and the μ-dependent terms have canceled in $\Gamma_{\rm ren}$, as they must, but the infrared-divergent part is still present. We note that this infrared-divergent piece in $\Gamma_{\rm ren}$ has the same form as in the virtual QCD corrections to $t\to H^+b$,[9] as well as in the process $H^+\to t\bar{b}$.[10] This is due to the fact that these infrared divergences are associated with the soft-gluon contribution and factorization of the amplitude is known to occur in this kinematical region.

C. Real Bremsstrahlung corrections

In order to cancel the infrared divergences in Eq. (9) we must consider the bremsstrahlung of soft real gluons. The relevant diagrams are shown in Figs. 1(e) and 1(f). The matrix element for hard-gluon emission has recently been analyzed numerically by several groups[11] and effects as large as 80% are obtained for $m_t > 200$ GeV. Here, we analytically calculate the total rate, including both soft- and hard-gluon emission. We regulate the infrared divergences associated with the soft and collinear real-gluon emission using the same gluon mass parameter λ. After a straightforward but tedious calculation, one obtains the rate for the decay accompanied by a real gluon, including both soft- and hard-gluon emission:

$$\Gamma_{\rm real}=\Gamma_0\frac{C_F\alpha_s}{2\pi}\left[\ln\left[\frac{\lambda^2}{m_t^2}\right]\left[\frac{5}{2}-2\ln\beta_W^2+\frac{1}{2}\ln\left[\frac{\lambda^2}{m_t^2}\right]\right]\right.$$
$$+2\ln^2\beta_W^2-5\ln\beta_W^2+2\left[\frac{(1-\beta_W^2)(\beta_W^2-2)(2\beta_W^2-1)}{\beta_W^4(3-2\beta_w^2)}\right]\ln(1-\beta_W^2)$$
$$\left.+2\,\mathrm{Li}_2\beta_W^2+\frac{4-3\beta_W^2}{\beta_W^2(3-2\beta_W^2)}+\frac{21}{4}-\frac{\pi^2}{2}\right]. \tag{10}$$

As for the virtual-gluon corrections, the infrared-divergent piece in the rate including real-gluon emission has the same form as in the processes $t\to H^+bg$ (Ref. 9) and $H^+\to t\bar{b}g$ (Ref. 10), which can be understood as a consequence of factorization in the soft-gluon kinematical region. However, the remaining piece in Eq. (10), which is free of infrared divergences, contains both soft- and hard-gluon contributions and does not have the same form as in the corresponding charged-Higgs-boson cases.[9,10]

D. Physical decay width

The physical decay width to order α_s after renormalization and inclusion of real-gluon emission is $\Gamma=\Gamma_{\rm ren}+\Gamma_{\rm real}$, that is, simply the sum of Eqs. (9) and (10):

$$\Gamma=\Gamma_0\left\{1+\frac{C_F\alpha_s}{2\pi}\left[2\left[\frac{(1-\beta_W^2)(2\beta_W^2-1)(\beta_W^2-2)}{\beta_W^4(3-2\beta_W^2)}\right]\ln(1-\beta_W^2)-\frac{9-4\beta_W^2}{3-2\beta_W^2}\ln\beta_W^2\right.\right.$$
$$\left.\left.+2\mathrm{Li}_2\beta_W^2-2\mathrm{Li}_2(1-\beta_W^2)-\frac{6\beta_W^4-3\beta_W^2-8}{2\beta_W^2(3-2\beta_W^2)}-\pi^2\right]\right\}. \tag{11}$$

All the infrared divergences have canceled in Eq. (11), as they must, in accord with the Kinoshita-Lee-Nauenberg theorem.[12]

In the very-massive-top-quark limit $m_W/m_t\to 0$ $(\beta_W\to 1)$, Eq. (11) reduces to

$$\Gamma=\Gamma_0\left[1-\frac{C_F\alpha_s}{2\pi}\left[\frac{2\pi^2}{3}-\frac{5}{2}\right]\right]. \tag{12}$$

Note that there is no large logarithmic term such as $\ln(m_W/m_t)$ in this limit. In the opposite limit $m_W/m_t\to 1$ $(\beta_W\to 0)$, one finds

$$\Gamma=\Gamma_0\left[1-\frac{C_F\alpha_s}{2\pi}\left[3\ln\beta_W^2+\frac{4\pi^2}{3}-\frac{9}{2}\right]\right]. \tag{13}$$

As $\beta_W\to 0$, there is a $\ln\beta_W$ singularity, but this limit should not be taken literally since we have neglected the bottom-quark mass altogether. In any event, the top quark is expected to be rather massive. We note that Eqs. (11)–(13) agree with Ref. 6.

III. CONCLUSIONS

In Fig. 2, we plot Γ/Γ_0 versus m_W/m_t [Eq. (11)]. For $m_W/m_t < 0.8$, the QCD corrections are essentially constant and are accurately given by Eq. (12). This implies a 10% reduction of the decay rate relative to the tree-level result. Such a reduction of the top-quark width could be quite significant in the search for new phenomena in future high-energy experiments. One example was recently analyzed by Strassler and Peskin.[3] They show that for $m_t > 140$ GeV the shape of a bound-state resonance of the $t\bar{t}$ system is quite sensitive to the width of the toponium state, which is, to a very good approximation, simply twice that of the top quark. The sensitivity to the top-quark width of the $e^+e^- \to t\bar{t}$ production cross section near threshold as a function of energy is displayed in Figs. (15) and (16) of Ref. 3 for $m_t = 150$ and 210 GeV, respectively. These graphs clearly show that a reduction of the top-quark decay width significantly enhances the signal for a $t\bar{t}$ resonance. Our calculation of the $O(\alpha_s)$ QCD corrections to the tree-level decay rate for $t \to W^+ b$ shows that QCD effects do indeed reduce the width. Other effects due to electroweak corrections in the standard model are expected to be much smaller. We also note that Fig. 2 indicates that the large effects due to hard-gluon bremsstrahlung found in Ref. 11 are canceled if the complete $O(\alpha_s)$ corrections are taken into account.

We conclude that the QCD effects calculated here, which reduce the top-quark width, can be significant and should be included in considering new phenomena associated with a heavy top quark in the standard model.

Note added. While this manuscript was being written, we received a paper[13] in which the same QCD corrections are calculated using slightly different techniques. Our results agree. After this paper was submitted for publication, we became aware of another publication by Czarnecki,[14] who had obtained the same result using strictly dimensional regularization to regulate both the ultraviolet and infrared divergencies.

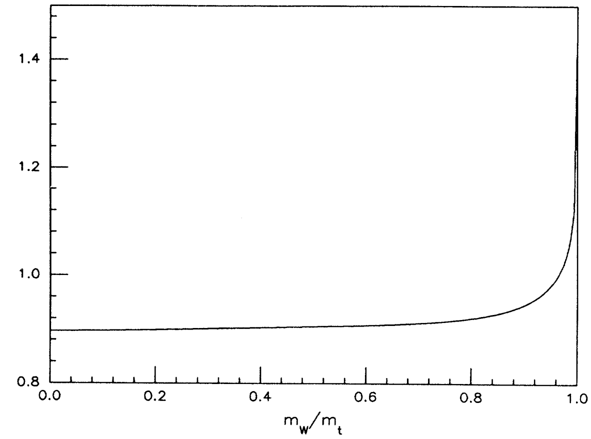

FIG. 2. Plot of Γ/Γ_0 versus m_W/m_t for $\alpha_s = 0.12$.

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