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Nonequilibrium fluctuations in cosmic vacuum decay

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With the help of the Landau-Lifshitz theory for nonequilibrium fluctuations four recent phenomenological models for the decay of the effective cosmological constant are analyzed. The model of Ozer and Taha as well as the one of Chen and Wu successfully pass our test.

I. INTRODUCTION

One of the main puzzles concerning our current understanding of the physical world is the small value of the effective cosmological constant $\Lambda < 10^{-120}M_P^2$, or, what is the same, the tiny value of the vacuum energy density $\rho_v = \Lambda(8\pi G)^{-1}$, as witnessed by cosmic observation. The status of this problem has recently been reviewed by Weinberg.[1]

Several authors[2-5] have tried independently to account for the current small value of Λ by assuming that the cosmological term is not really constant but its value continually decreases as the Universe expands. The rationale behind this is that the energy density of the vacuum should spontaneously decay into massive and massless particles, hence reducing Λ to a value compatible with astronomical constraints. This view has gained some support from Gasperini[4] who argues that according to an earlier work by Gibbons and Hawking[6] the cosmological term can also be interpreted as a measure of the temperature of the cosmic vacuum. Hence, if at some time close to the Planck era the vacuum were in thermal contact with radiation, the Friedmann expansion of the Universe would not only decrease the radiation temperature but the vacuum temperature as well.

So, there exist at the moment different phenomenological laws relating the energy density of the vacuum to cosmic time, or, what is the same, to the scale factor $R(t)$ of the Universe. These are the following.

(i) The law of Ozer and Taha:

$$\Lambda = \frac{3}{\chi R^2} , \tag{1}$$

with $\chi \equiv 8\pi G$ and R given by $R^2 = R_0^2 + t^2$.

(ii) The law of Freese *et al.*:

$$\rho_v = \frac{3x}{4\chi(1-x)^2 t^2} , \quad 0 \le x < 1 , \tag{2}$$

$$R \sim t^{1/2(1-x)} , \tag{3}$$

x being a phenomenological constant parameter.

(iii) The law of Gasperini:

$$\Lambda = \frac{12\pi^2 b^2}{R^{2n}}, \quad \tfrac{9}{2} < n < 2 , \tag{4}$$

where b is a constant to be determined and n a phenomenological constant parameter.

(iv) The law of Chen and Wu:

$$\Lambda = \frac{\gamma}{R^2} , \tag{5}$$

where γ is a positive-semidefinite constant of order unity.

Note that the dependence of Λ on R is the same as in the model of Ozer and Taha, Eq. (1), and in the model of Chen and Wu, Eq. (5). However, both models are actually very different from each other as they differ on their assumptions and lead to different conclusions.

All of them make use of the Friedmann-Robertson-Walker scenario with the equations

$$\left[\frac{\dot{R}}{R}\right]^2 + \frac{k}{R^2} = \tfrac{1}{3}(\chi\rho + \Lambda) \quad (k = 1, 0, -1) , \tag{6}$$

$$\dot{\rho} + 3(\rho + P)\frac{\dot{R}}{R} = -\frac{\dot{\Lambda}}{\chi} . \tag{7}$$

Here and throughout, ρ and P stand for the energy density and pressure of matter plus radiation, respectively. The overdot means derivation with respect to cosmic time.

However, since these equations do not determine Λ but only constrain its behavior we need to introduce additional assumptions which, although reasonable in principle, depend to some extent on the personal taste of their authors.

Therefore, assuming the idea of a varying cosmic vacuum energy density happens to be correct, one is naturally led to ask which, if any, of these laws is the correct one. Of course, we do not have the answer to this. However, the laws mentioned, (1)–(5), can be analyzed from the thermodynamic point of view. This analysis tests the likelihood of these laws. Effectively, the flux of energy $d\rho_v/dt$ from the vacuum to radiation and/or matter must fluctuate around its mean value. As we will see later the knowledge of these fluctuations will rule out some of the above laws. Effectively, it turns out that the fluctuations predicted by two of the models (those by Freese *et al.* and by Gasperini) are at variance with cosmic observation.

Our purpose in this paper is to study the fluctuations of the fluxes mentioned around their average values and draw some conclusions from the behavior of these fluctuations. To this end we shall employ the well-known Landau-Lifshitz (LL) fluctuation hydrodynamic theory[7] which applies in equilibrium and nonequilibrium classical statistical theory.[8]

According to Landau and Lifshitz, if the flux $\dot{y}_i$ of a given thermodynamic quantity, which evolves in a general dissipative process, is governed by

$$\dot{y}_i = \sum_j \Gamma_{ij} Y_j + \delta \dot{y}_i \tag{8}$$

and the entropy rate by

$$\dot{S} = \sum_i (\pm Y_i \dot{y}_i) , \tag{9}$$

then the second moments in the fluctuations of the fluxes obey

$$\langle \delta \dot{y}_i \delta \dot{y}_j \rangle = k_B (\Gamma_{ij} + \Gamma_{ji}) \delta_{ij} \delta(t_i - t_j) . \tag{10}$$

Here and throughout angular brackets denote a statistical average with respect to the steady state, and the fluctuations $\delta \dot{y}_i$ are considered spontaneous deviations from the steady-state value $\langle \dot{y}_i \rangle$, so $\langle \delta \dot{y}_i \rangle$ vanishes. The quantities Γ_{ij} and Y_i stand for the phenomenological transport coefficients and the thermodynamic force conjugate to the flux $\dot{y}_i$, respectively. In (9) the minus sign must be taken when the product $Y_i \dot{y}_i$ is negative, otherwise the plus sign should be considered.

This theory, as mentioned earlier, is only valid for linear steady-state fluxes. However, at first sight the fluxes

$$\dot{\Lambda} \approx -\Lambda \frac{\dot{R}}{R} \tag{11}$$

following from expressions (1)–(5) are neither linear nor steady. So it may seem inconsistent to apply the LL theory to the problem we wish to study. Nevertheless, for cosmic times much greater than unity, or what is the same, for large-scale factors, both the right-hand sides of Eqs. (1)–(3) and the Hubble term $H \equiv \dot{R}/R$ are quite small and nearly constant. Hence, we can say that at sufficiently large cosmic time, $t \gg t_P$, $\dot{\Lambda}$ becomes steady for all practical purposes. By this we mean that the change of $\dot{\Lambda}$ in the interval of time in which a measure of it is performed produces a negligible result. After these remarks, it becomes clear that it is valid to apply the LL theory to our problem provided that we restrict ourselves to cosmic times larger than, say, $10^3 t_P$.

It should be noted that due to the absence of a reliable theory accounting for the decay of the vacuum these fluctuations cannot be analyzed from first principles. So, if such an analysis is desired one has to rely on some phenomenological method and, to our knowledge, the LL one is more suitable for the problem under consideration.

Before going any further it is worth noting that if the cosmological term were constant the entropy would remain fixed, as long as we deal with a perfect cosmic fluid. But, if Λ is allowed to vary the Universe expansion will give rise to an entropy production[2]

$$T\dot{S} = -\frac{V}{\chi} \dot{\Lambda} ,$$

where T denotes the temperature of the fluid.

In what follows we study the fluctuations of the fluxes following expressions (1)–(5) in turn. We consider only the possibility of the vacuum decaying into radiation inasmuch as the possibility of the vacuum decaying into nonrelativistic particles has been dismissed by Freese *et al.*[3] on observational grounds.

II. THE APPROACH OF OZER AND TAHA

One of the main ingredients in the model of Ozer and Taha[2] is the assumption that the energy density always coincides with the critical value $3H^2\chi$. This, together with Eqs. (6) and (7) and the principle of entropy increase, yields $k=1$ for the spatial curvature and leads to Eq. (1). The latter gives the dependence of the cosmological term on the scale factor. Furthermore, if the cosmic fluid consists of pure radiation,

$$\rho = 3P = \frac{\pi^2}{30} g_{\text{eff}} T^4 , \tag{12}$$

the dependence of the energy density on R takes the form

$$\rho = \Lambda[1 - (R_0/R)^2] , \tag{13}$$

where g_{eff} denotes the effective number of relativistic degrees of freedom.

This model has the advantage of being free of the main cosmological problems that beset many other scenarios.

The flux associated with Eq. (1),

$$\dot{\Lambda} = -\frac{6}{\chi R^2} H , \tag{14}$$

becomes quasisteady and almost linear in H for sufficiently large R whence it is permissible to apply the LL theory sketched above. To this end we add to the right-hand side of the last equation a stochastic term $\delta\dot{\Lambda}$:

$$\dot{\Lambda} = -\frac{6}{\chi R^2} H + \delta\dot{\Lambda} . \tag{15}$$

So, we have an expression for the flux $\dot{\Lambda}$ that parallels Eq. (8). Obviously its average value $\langle \dot{\Lambda} \rangle$ is given by the first term on the right-hand side of (15).

The production of entropy reads

$$\dot{S} = \frac{12\pi^2}{\chi T^2} , \tag{16}$$

where T stands for the radiation temperature

$$T = (90/\chi\pi^2 g_{\text{eff}})^{1/4} (t/R)^{1/2} .$$

If we compare (15) and (16), respectively, with (8) and (9) we get a sort of phenomenological transport coefficient $\Gamma = (3T/\pi^2 R^5)H$. Thus the second moments in the fluctuations of $\dot{\Lambda}$ simply read

$$\langle \delta\dot{\Lambda}(t_i) \delta\dot{\Lambda}(t_j) \rangle = \frac{6T}{\pi^2 R^5} H \delta(t_i - t_j) , \tag{17}$$

or, in terms of t,

$$\langle \delta\dot{\Lambda}(t_i) \delta\dot{\Lambda}(t_j) \rangle = B \frac{t^{3/2}}{(R_0^2 + t^2)^{15/4}} \delta(t_i - t_j) \tag{18}$$

with $B \equiv (6/\pi^2)(90/\pi^2 \chi g_{\rm eff})^{1/4}$. This shows that, as expected, the second moments decrease with time as the expansion proceeds.

III. THE APPROACH OF FREESE *et al.*

This model is built on the assumption that the energy densities of the vacuum and radiation redshift at the same rate. As a consequence, for large t,

$$\frac{\rho_v}{\rho_{\rm rad}+\rho_v} \to x \ , \tag{19}$$

where x is a nonvanishing phenomenological constant parameter whose value lies somewhere between zero and unity. Using this condition, in addition to Eqs. (6) and (7), with ρ and P being those of pure radiation [Eq. (12)] and $k=0$, one obtains expressions (2) and (3) for the radiation-dominated era. The radiation temperature reads

$$T=[\tfrac{16}{45}\pi^3 g_{\rm eff}(1-x)]^{-1/4} t^{-1/2} \ . \tag{20}$$

The flux associated with Eq. (2) is

$$\dot{\rho}_v = -4(1-x)\rho_v H \ , \tag{21}$$

whereas the production of entropy adopts the form

$$\dot{S}=\frac{R^3}{T}4(1-x)\rho_v H \ , \tag{22}$$

where we have taken $V=R^3$. For large cosmic time $\dot{\rho}_v$ becomes quasisteady and linear in H. Hence, the LL theory can be applied. It yields, for the second moments,

$$\langle \delta\dot{\rho}_v(t_i)\delta\dot{\rho}_v(t_j)\rangle = \frac{8(1-x)}{(3\pi)^{1/2}}\left[\frac{\rho_v^{1/2}}{R}\right]^3 H\delta(t_i-t_j) \ . \tag{23}$$

IV. GASPERINI APPROACH

This model rests on two key assumptions. The first one refers to the thermal interpretation of the cosmological term. According to it this quantity can be thought as a parameter measuring the intrinsic temperature of the cosmic vacuum:[6]

$$\Lambda = 12\pi^2 T_v^2 \ . \tag{24}$$

So, it might be said that Λ represents the contribution of the vacuum temperature to the geometry. Accordingly it seems natural to think that at some early time, close to the Planck era, the vacuum and radiation were in thermal contact: $T_v \cong T_{\rm rad}$. As the Universe expands both temperatures should decrease. This simple picture is intended to offer an explanation of the current very small value of Λ. Recently, we have shown[9] that if such an equilibrium ever occurred the maximum value of Λ at that time should be lower than $2.8M_P^2$.

The second assumption refers to the dependence of the vacuum temperature on the scale factor

$$T_v = \frac{b}{R^n(t)} \ , \tag{25}$$

which is contrived to satisfy the requirement of Freese *et al.* that the vacuum energy density redshifts at the same rate as the radiation does—see expression (19) above.

Equation (4) automatically follows from the combination of (24) and (25). Inserting the former into the energy balance equation (7) and assuming that the adiabatic relation for the energy density of matter $d(\rho_{\rm mat}R^3)/dt=0$ remains valid one has, for large R,

$$\rho = \frac{n\Lambda}{\chi(n-2)} \ . \tag{26}$$

Here ρ refers to the radiation energy density. Since this quantity has to be positive the restriction $n<2$ immediately follows. The restriction $n<\frac{9}{5}$ comes from observational limits on nucleosynthesis and entropy generation.

The flux associated with Eq. (4) is

$$\dot{\Lambda} = -\frac{24\pi^2 b^2}{R^{2n}} H \ , \tag{27}$$

with $H=[8\pi/(2-n)]bR^{-n}$, and the entropy production

$$\dot{S}=\frac{2nR\Lambda^3}{T} H \ , \tag{28}$$

where

$$T=\left[\frac{30}{\chi\pi^2 g_{\rm eff}}\left[\frac{n\Lambda}{2-n}\right]\right]^{1/4} \tag{29}$$

denotes the radiation temperature.

As in the previous cases the flux (27) is quasisteady for large R; hence, it makes sense to apply the LL theory. We obtain

$$\langle \delta\dot{\Lambda}(t_i)\delta\dot{\Lambda}(t_j)\rangle = \frac{C}{R^m}\delta(t_i-t_j) \ , \tag{30}$$

$$C \equiv 384\pi\left[\frac{360}{\chi g_{\rm eff}}\left[\frac{n}{2-n}\right]^5 b^2\right]^{1/4}, \quad m \equiv 3(n+1)+\frac{n}{2} \ .$$

V. THE APPROACH OF CHEN AND WU

The main argument of Chen and Wu[5] leading to Eq. (5) runs as follows. Assuming from the start that Λ must vary as a power of the scale factor, the most natural expression for Λ should be of the form

$$\Lambda \propto M_P^4 (L_P/R)^n \quad (\hbar = c = 1) \ . \tag{31}$$

If one tries to recover the Planck constant $\hbar$ one is led to the conclusion that the choice $n=2$ is the only one compatible with the nonappearance of $\hbar$ in the right-hand side of the Einstein classical equation once (31) is substituted into. Obviously, it would be quite disturbing that such an appearance would occur for times larger than the Planck time. From observational considerations related to the flatness of the Universe it follows that γ cannot be negative.

This model, apart from being compatible with the present observational limit on Λ and not conflicting with the predictions of the radiation-dominated epoch of the standard model, has some interesting consequences for the matter-dominated epoch which make it easier to reconcile observation with the inflationary scenario.

The flux associated with Eq. (5) and the production of entropy read

$$\dot{\Lambda} = -2\Lambda H \ , \tag{32}$$

$$\dot{S} = 2\gamma \frac{RH}{T} \ , \tag{33}$$

respectively, where

$$T = \left[\frac{30}{\pi^2 g_{\text{eff}}} \right]^{1/4} \left[\frac{A}{R^4} + \frac{\gamma}{\chi R^2} \right]^{1/4} \tag{34}$$

denotes the radiation temperature and A is a positive constant. The Hubble parameter can be obtained from the combination of equations (7) and (9) in Ref. 5:

$$H = \left[\frac{\chi A}{3R^4} + \left[\frac{2\gamma}{3} - k \right] \frac{1}{R^2} \right]^{1/2} . \tag{35}$$

It is clear that $\dot{\Lambda}$ becomes quasisteady for large R whence it is admissible to apply the LL theory. It yields, for the second moments of the fluctuations of the flux,

$$\langle \delta\dot{\Lambda}(t_i)\delta\dot{\Lambda}(t_j) \rangle = \frac{2\chi\gamma T}{R^5} H \delta(t_i - t_j) \ . \tag{36}$$

VI. CONCLUDING REMARKS

As expected, the strength of the fluctuations decreases in all the cases [Eqs. (17), (18), (23), (30), and (36)] as the Universe expands. However, there is an important aspect in which the fluctuations corresponding to the models of Ozer and Taha[2] and of Chen and Wu[5] differ from those of the two other models. If we calculate the ratio

$$\zeta \equiv \left. \frac{\langle \delta\dot{\Lambda}(t_i)\delta\dot{\Lambda}(t_j) \rangle}{\langle \dot{\Lambda}(t_i) \rangle \langle \dot{\Lambda}(t_j) \rangle} \right|_{t_i = t_j}$$

for the different models we find

$$\begin{gathered} \zeta_{\text{OT}} \sim t^{-1/2}, \quad \zeta_{\text{F}} \sim R^{(2-5x)}, \\ \zeta_{\text{G}} \sim R^{(5n-6)/2}, \quad \zeta_{\text{CH}} \sim R^{-1/2} \ , \end{gathered} \tag{37}$$

where the ζ's refer to models of the Ozer and Taha, Freese *et al.*, Gasperini and Chen and Wu, respectively. Relations (37) show that whereas ζ_{OT} and ζ_{CH} decrease with cosmic expansion ζ_{F} and ζ_{G} increase. (Bear in mind that the lower phenomenological bounds on x and n are $\frac{1}{4}$ and $\frac{9}{5}$, respectively.) This means that sooner or later the fluctuations of the fluxes in the two latter models will become greater than the average value of the corresponding flux. That is to say, these models predict for a large-scale factor an erratic behavior of the fluxes. In contrast, the models of Refs. 2 and 5 are free of such anomalous behavior since the LL theory applies only for times greater than unity.

In this light we can say that expressions (1) and (5) appear to be much more reliable than (2) and (4). This point of view is reinforced when one realizes that an erratic behavior for cosmic times greater than, say, $10^3 t_P$, would distort the microwave background away from the Planck spectrum well beyond the limits allowed by cosmic observation—see Sec. 3.2 in Ref. 3. Effectively, even if the vacuum decays with a Planckian distribution there is no grounds for which the corresponding fluctuations $\langle (\delta\dot{\rho}_v)^2 \rangle^{1/2}$ should keep such a spectrum.

This outcome shows that those models based on Eqs. (2)–(4), respectively, should be dismissed as incompatible with observation. Furthermore, it suggests that if the effective cosmological constant indeed diminishes with cosmic expansion its dependence on the scale factor should take the form $\Lambda \propto R^{-2}$.

In view of this result one could think that the root of the failure of the models given by Eqs. (2)–(4) lies in their common hypothesis, which seems rather reasonable, that the vacuum energy density and the radiation energy density should redshift at the same rate. It is to say that the ratio $\rho_v/(\rho_v + \rho_r)$ approaches, for large R, a nonvanishing constant value between zero and unity. Nevertheless, it is not so since in the model of Ozer and Taha this ratio approaches $(2\chi)^{-1}$ whereas in the one of Chen and Wu it approaches $\frac{1}{2}$. In both cases this feature does not constitute a hypothesis of these models; on the contrary it follows from them as a by-product.

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