Massive and massless Majorana particles of arbitrary spin: Covariant gauge couplings and production properties

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It is shown that massive particles that are their own antiparticles can possess only one specific type of multipolar moments—the so-called toroidal moments. Unlike the familiar charge and magnetic moments, these moments do not directly interact with the (external) static electromagnetic fields but only with the external current, and hence lead to contact interactions. On the other hand, massless Majorana particles, with the exception of those with spin $\frac{1}{2}$, do not have *any* electromagnetic form factor. It is also found that regardless of the spin of the Majorana particle the differential cross section for the production of a Majorana pair by a spin-1 particle at a high-energy e^+e^- machine has a unique angular distribution.

INTRODUCTION

Majorana particles are very special in the sense that they are identical to their antiparticles. Usually the term Majorana has been used only for fermions but we will generalize it to include real bosons too, which coincide with their antiparticles. Majorana particles are predicted by various versions of grand-unified theories and are unavoidable in supersymmetric gauge theories. They usually represent gauge fields. Moreover, since they can only live in spaces with restricted dimensionality[1] some theories cannot be formulated in or extended to arbitrary higher dimensions. Many investigations have been devoted to the study of the electromagnetic properties of the spin-$\frac{1}{2}$ Majorana particle.[2] These studies successively appealed to *C*, *CP*, and *CPT* transformations[3] of the Majorana field with the result that regardless of whether or not the particle is massive it can only have one *P*-violating moment, called the anapole,[4] in striking difference with a Dirac particle, which can have charge, magnetic, as well as electric dipole moments. The peculiarity of the anapole moment is that it does not interact with the external electromagnetic field, but only with the external current. This gives rise to a contact interaction. The study, by one of the authors, of the electromagnetic properties of the Z^0,[5-7] which is a spin-1 Majorana particle, in the context of the SU(2)×U(1) model, has shown that unlike the W^+ and W^- the Z^0 has only one moment which is of the anapole type. Radescu[8] has demonstrated that Majorana fermions of higher spins can also have only an anapole and higher multipoles of it. In a recent Letter[9] we generalize this result to a Majorana particle of any spin. These derivations were derived by carrying a multipole expansion of the electromagnetic current and by relying on the *CPT* properties of the Majorana particles.

In this paper we rederive these results for the case of a Majorana (M) particle with arbitrary spin without the use of the somewhat old-fashioned formalism of the multipole expansion of the current. But we will, instead, present a manifestly covariant formulation. This seems to us more suited for the modern formulation of field theory and, as we will see, can greatly facilitate the calculation as well as the analysis of the electromagnetic couplings. For instance many of these calculations in specific models or theories involve lengthy loop diagrams. Our results can then help in easily isolating the specific contributing terms and at least check the Lorentz structure of the final answer.

Another reason for avoiding the formalism of the multipole expansion adapted to the Majorana particles is that in this case the approach heavily relies on the *CPT* transformation of the field and implicitly assumes the Hermiticity of the electromagnetic current, i.e., the reality of the various form factors $F_i(k^2)$. However, it is well known that this is only true if there are no absorptive parts and that we are below certain thresholds. Our approach does not impose any condition on the reality of the various $F_i(k^2)$. A case in point is the $ZZ\gamma$ coupling in the standard model which is generated via a fermion triangle. The form factor does develop an imaginary part since (apart from the top contribution) $m_{\text{fermion}} < M_Z/2$.[5,6]

Our principal idea is based on the following. We consider the channel $\gamma \to MM$ and require the corresponding matrix element, under the interchange of the two identical Majorana particles, to be symmetric for integer-spin Majorana particles (bosons) and antisymmetric for the half-integer-spin case (fermions).[10] In both cases, we impose current conservation. It is essential to keep terms proportional to the invariant mass of the photon ("k^2 terms"). These, in fact, are the ones which generate the anapole contact term contribution:

$$k^2 S_\mu - k\cdot S\, k_\mu \ ,$$

where k is the photon four-momentum (μ characterizing its polarization) and where S is some functional in the Majorana fields and momenta.

One needs to write down the general covariant forms for the current, which might seem a rather laborious task for higher-spin fields; however, the identical-particle symmetry argument together with the properties of the wave function of a spin-s field make it possible to generate the form of the current of any Majorana particle from a knowledge of the much-studied spin-$\frac{1}{2}$ and spin-1 cases.

The use of fields for spin-s particles does not necessarily mean that our approach only deals with elementary relativistic particles. For the case of composite objects these fields should be regarded as extrapolating fields.

The covariant derivation of the form factors of massive bosonic Majorana particle is carried in Sec. I, where we indeed find that only anapole structures are possible. For a particle of spin s there are $2s$ C-violating terms, half of which are P violating and the rest are CP violating. The parity-violating terms exhibit also a P-wave behavior; that is, if produced through a photon or a spin-1 particle, the cross section is proportional to $\beta^{3/2}$ (β being the velocity of the Majorana particle). Massive fermionic Majorana particles are treated in Sec. II with the same conclusions as in the bosonic case: $(2s+1)/2$ P-violating, with the P-wave behavior form factors and $(2s-1)/2$ T-violating ones.

Massless Majorana particles are treated separately due to the difficulties inherent to massless higher-spin ($s>1$) fields.[11] It also has been known for some time[12] that the massless limit is generally ambiguous if not ill defined.[13] Part of the problem is that fields describing massless particles are subject to quite constraining conditions. To circumvent these problems, which arise in a manifestly covariant formulation, we propose in Sec. IV an approach based on the helicity formalism. We find that the *only* massless Majorana which has a single photon coupling is the spin-$\frac{1}{2}$ Majorana particle. As a byproduct of the approach we point out that the number of invariants (form factors) describing the coupling of a vector boson to a massive particle-antiparticle pair (not a Majorana particle) of spin s is $6s+1$ and then also confirming that only $2s$ terms remain if the particle is a Majorana particle.

In Sec. V we still exploit the approach based on the helicity formalism adapted to the case of a Majorana particle to study the symmetry properties of the amplitude and the cross section for producing a Majorana particle. In the most interesting case, that of a production through a vector boson, at an e^+e^- machine, say, we find for instance that regardless of the spin of the Majorana particle, the differential cross section always has a $1+\cos^2(\theta)$ distribution, where θ is the scattering angle. In Sec. VI we give some applications of our theorems and an analysis to current models and theories in particle physics (standard model, supersymmetry, supergravity, etc.) and show how the results of some previous calculations could have easily been guessed or arrived at.

In the appendixes we have collected some nontrivial manipulations, tricks, and identities which we have used in the text and which, on their own right, can be a good exercise of getting accustomed to the algebra of higher-spin fields.

I. MASSIVE MAJORANA PARTICLE WITH INTEGER SPIN

A boson of spin s, mass m, and momentum k is defined by a tensor of ranks s, $A_{\alpha_1\cdots\alpha_i\cdots\alpha_s}(k)$ with the properties that it is[14] completely symmetric,

$$\epsilon^{\alpha_i\alpha_j\mu\nu}A_{\alpha_1\cdots\alpha_i\cdots\alpha_j\cdots\alpha_s}=0 \ , \tag{1.1}$$

with vanishing traces,

$$g^{\alpha_i\alpha_j}A_{\alpha_1\cdots\alpha_i\cdots\alpha_j\cdots\alpha_s}=0 \ , \tag{1.2}$$

divergence-free,

$$k^{\alpha_i}A_{\alpha_1\cdots\alpha_i\cdots\alpha_s}(k)=0 \ , \tag{1.3}$$

and satisfying the wave equation

$$(k^2-m^2)A_{\alpha_1\cdots\alpha_i\cdots\alpha_s}(k)=0 \ . \tag{1.4}$$

The last two conditions are on-shell conditions.

Consider the transition

$$\gamma(k,\mu)\to M(k_1,\alpha)M(k_2,\beta) \ . \tag{1.5}$$

In the following, α and β will stand for the sequences of the indices $\alpha_1\cdots\alpha_s$ and $\beta_1\cdots\beta_s$.

The matrix element can be written as

$$M_\mu=\langle M_1M_2J_\mu(0)|0\rangle=T_{\mu;\alpha_1\cdots\alpha_s,\beta_1\cdots\beta_s}(k_1,k_2)A^{\alpha_1\cdots\alpha_s}(k_1)A^{\beta_1\cdots\beta_s}(k_2) \tag{1.6}$$

where J_μ is the electromagnetic current. Current conservation imposes the transervality condition

$$k_\mu M^\mu=0 \ , \tag{1.7}$$

whereas Bose symmetry requires that under the interchange 1 and 2 the tensor $T_{\mu;\alpha_1\cdots\alpha_s,\beta_1\cdots\beta_s}(k_1,k_2)$ be symmetric in any

$$\alpha_i\leftrightarrow\beta_j \text{ and } k_1\leftrightarrow k_2 \ . \tag{1.8}$$

It is more important to trade the two independent variables k_1 and k_2 for

$$k=k_1+k_2, \quad P=k_1-k_2 \ . \tag{1.9}$$

Since the tensor A is symmetric, with vanishing traces and divergenceless the most general form of $T_{\mu;\alpha_1\cdots\alpha_s,\beta_1\cdots\beta_s}$ we may write, as suggested by conditions (1.8) and (1.9), is

$$T_{\mu;\alpha_1\cdots\alpha_s,\beta_1\cdots\beta_s}=T_\mu S_{\alpha_1\cdots\alpha_s,\beta_1\cdots\beta_s}+T_{\mu;\alpha_1\beta_1}S_{\alpha_2\cdots\alpha_s,\beta_2\cdots\beta_s} \ , \tag{1.10}$$

where the tensors S and T are both symmetric under the transformation in (1.8). Decomposition (1.10) reflects the fact that the index μ can be carried either by (a) a four-vector T_μ, (b) the metric tensor $g_{\mu}\ldots$, or (c) by the Levi-Civita tensor $\epsilon_\mu\ldots$. These two last possibilities are collected in $T_\mu;\alpha_1\beta_1$. One then imposes current conservation on the tensors T. T_μ is the spin-zero case. This four-vector can only be a combination of k_μ and P_μ. Transversality of the current imposes the combination

$$k_\mu P\cdot k - P_\mu k^2 \ . \tag{1.11}$$

But this is not allowed under Bose symmetry since it contains an odd number of P. To obtain an even number of P one can multiply the previous equation by $P\cdot k$; however $P\cdot k=0$. Therefore we recover the well-known result that a scalar particle does not have any electromagnetic form factor.

$T_{\mu\alpha_1\beta_1}$ is the spin-1 case whose most general form is

$$T_{\mu\alpha_1\beta_1}=F_1(k^2)(k_\mu\epsilon_{\alpha_1\beta_1\rho\lambda}P^\rho k^\lambda - k^2\epsilon_{\alpha_1\beta_1\rho\mu}P^\rho)+F_2(k^2)\left[k_\mu k_{\alpha_1}k_{\beta_1}-\frac{k^2}{2}(g_{\mu\alpha_1}k_{\beta_1}+g_{\mu\beta_1}k_{\alpha_1})\right] . \tag{1.12}$$

This has been derived by starting with the most general $V^+V^-V^0$ where the V's are vector bosons as can be found in Hagiwara *et al.*[15] for W^+W^-V. This contains seven independent invariants. Requiring Bose symmetry and the transversality of the current only the two previous invariants survive.

F_1 is parity violating but CP conserving, while F_2 is CP violating. The most general symmetric $S_{\alpha\beta;\alpha_1\beta_1}$, taking into account the fact that the tensor representing the wave function has vanishing traces (hence no $g_{\alpha_i\alpha_j}$) symmetric (no $\epsilon_{\alpha_i\alpha_j}\ldots$) and that P_α is equivalent to k_α in view of (1.3), is

$$S_{\alpha\beta;\alpha_1\beta_1}=\begin{cases} g_{\alpha_2\beta_2}\cdots g_{\alpha_s\beta_s} \ , \\ k_{\alpha_2}k_{\beta_2}g_{\alpha_3\beta_3}\cdots g_{\alpha_s\beta_s} \ , \\ \cdots \ , \\ k_{\alpha_2}k_{\beta_2}k_{\alpha_3}k_{\beta_3}\cdots k_{\alpha_s}k_{\beta_s} \ . \end{cases} \tag{1.13}$$

Equations (1.10), (1.12), and (1.13) are not manifestly symmetric under the permutation among the indices α's and among the indices β's. The symmetrization will be taken care of by contracting T and S with the polarization tensors $A_{\alpha_1\cdots\alpha_s}(k_1)$ and $A_{\alpha_1\cdots\alpha_s}(k_2)$. There are s terms in Eq. (9), the last one defining the highest multipole. Note that substituting the "pair" $k_{\alpha_i}k_{\beta_i}$ or $g_{\alpha_i\beta_i}$ by the only remaining symmetric "pair" $\epsilon_{\alpha_i\beta_i\lambda\rho}P^\lambda k^\rho$ does not introduce any new form factor. To wit, we can show that (see Appendix A)

$$\left[k_\mu k_{\alpha_1}k_{\beta_1}-\frac{k^2}{2}(g_{\mu\alpha_1}k_{\beta_1}+g_{\mu\beta_1}k_{\alpha_1})\right]\epsilon_{\alpha_2\beta_2\lambda\rho}P^\lambda k^\rho=(k_\mu\epsilon_{\alpha_1\beta_1\lambda\rho}P^\lambda k^\rho-k^2\epsilon_{\alpha_1\beta_1\lambda\mu}P^\lambda)\left[k_{\alpha_2}k_{\beta_2}-\frac{k^2}{2}g_{\alpha_2\beta_2}\right] \tag{1.14}$$

and

$$(k_\mu\epsilon_{\alpha_1\beta_1\lambda\rho}P^\lambda k^\rho-k^2\epsilon_{\alpha_1\beta_1\lambda\mu}P^\lambda)\epsilon_{\alpha_2\beta_2\lambda\rho}P^\lambda k^\rho=2\left[k_\mu k_{\alpha_1}k_{\beta_1}-\frac{k^2}{2}(g_{\mu\alpha_1}k_{\beta_1}+g_{\mu\beta_1}k_{\alpha_1})\right][g_{\alpha_2\beta_2}(k^2-4m^2)-2k_{\alpha_2}k_{\beta_2}] \ . \tag{1.15}$$

This relation is readily derived by contracting the antisymmetric tensors. m is the mass of the Majorana particle.

Equations (1.14) and (1.15) are contained in (1.10) as a combination of (1.12). Therefore for integral spin-s there are $2s$ form factors, half of which are P violating and the rest are T violating. If C is conserved all form factors vanish. Note that, in fact, we have generated the electromagnetic form factors of a general Majorana boson from those of the basic spin-1 case. They are all of the anapole type; i.e., they give rise to a contact interaction. This is best seen by considering, for instance, $e^+e^-\to MM$ (via a photon exchange). One sees from (1.12) that the k_μ part vanishes because of current conservation whereas the k^2 part drops against the k^2 from the photon propagator, hence leading effectively to a contact interaction e^+e^-MM.

Another important property concerns the parity-violating but CP-conserving terms. One should note that they involve the "odd" momentum P with $P^2=k^2-4m^2$. This is a P-wave (although rather amusing, its unfortunate that all these properties, P violating, P momentum, and P wave, bear the same initial P) factor which means that if the Majorana pair is produced via a single photon the cross section is suppressed at threshold being proportional to $\beta^{3/2}$ (β being the velocity of any one of the Majorana in their center of mass).

II. MASSIVE MAJORANA FERMIONS

A fermion of spin $s+\frac{1}{2}$, mass m, and momentum k may be described by a generalized Rarita-Schwinger spinor,[14] $u_{\alpha_1\ldots\alpha_s}$ which has zero traces, symmetric in the indices α, and which satisfies Eqs. (1.1)–(1.3) and

$$\gamma^{\alpha_i}u_{\alpha_1\cdots\alpha_i\cdots\alpha_s}(k)=0\ , \tag{2.1}$$

$$(\not k-m)u_{\alpha_1\ldots\alpha_s}(k)=0 \tag{2.2}$$

with $\not k=k_\mu\gamma^\mu$. The analog of (1.6) is

$$M_\mu=\bar u_\alpha(k_2)T_{\mu\alpha\beta}(k,p)v_\beta(k_1)\ . \tag{2.3}$$

But now interchanging particles 1 and 2, we get

$$\begin{aligned}M'_\mu&=\bar u_\beta(k_1)T_{\mu\beta\alpha}(k,-p)v_\alpha(k_2)\\&=v^t_\alpha(k_2)T^t_{\mu\beta\alpha}(k,-p)u^t_\beta(k_1)\ ,\end{aligned} \tag{2.4}$$

where t denotes the transpose matrix. ($T_{\mu\alpha\beta}$ may contain γ matrices.) Introducing the charge-conjugation operator $C(C^\dagger=C^{-1},\ C^t=-C)$ so that

$$v^t_\alpha(k)=-\bar u_\alpha(k)C\ , \tag{2.5}$$

we may write

$$M'_\mu=-\bar u_\alpha(k_2)CT^t_{\mu\beta\alpha}(k,-p)C^{-1}v_\beta(k_1)\ . \tag{2.6}$$

Since Fermi statistics requires

$$M'_\mu=-M_\mu \tag{2.7}$$

we must have

$$CT^t_{\mu\beta\alpha}(k,-p)C^{-1}=T_{\mu\beta\alpha}(k,p)\ . \tag{2.8}$$

Again we may write $T_{\mu\alpha\beta}$ in exactly the same form as (1.10), where due to (2.1) the S terms are C numbers and have the form (1.13). As compared to the bosonic case the only complication is the (possible) presence of the Dirac γ matrices in the T tensors. This time, in decomposition (1.10), T_μ is nonzero and is, in fact, derived as in the spin-$\frac{1}{2}$ case,[10]

$$T_\mu=F_0(k^2)(k_\mu\not k-k^2\gamma_\mu)\gamma_5\ , \tag{2.9}$$

whereas $T_{\mu\alpha_1\beta_1}$ is an adaptation of the previous spin-1 case (here it represents the spin-$\frac{3}{2}$ case):

$$T_{\mu\alpha_1\beta_1}=\left[k_\mu k_{\alpha_1}k_{\beta_1}-\frac{k^2}{2}(g_{\mu\alpha_1}k_{\beta_1}+g_{\mu\beta_1}k_{\alpha_1})\right][F_1(k^2)\gamma_5+F_2(k^2)]\ . \tag{2.10}$$

Note that F_0 is parity violating but CP conserving. Hence the possible form factors are

$$\begin{aligned}M_\mu=\bar u^{\,\alpha}(k_2)\Bigg[&[F_0(k^2)(k_\mu\not k-k^2\gamma_\mu)\gamma_5]S_{\alpha\beta}\\&+\left[k_\mu k_{\alpha_1}k_{\beta_1}-\frac{k^2}{2}(g_{\mu\alpha_1}k_{\beta_1}+g_{\mu\beta_1}k_{\alpha_1})\right][F_1(k^2)\gamma_5+F_2(k^2)]S_{\alpha\beta;\alpha_1\beta_1}\Bigg]v^\beta(k_1)\ .\end{aligned} \tag{2.11}$$

Among all the different $S^{\alpha\beta}$ of the form (1.13) which multiply $F_0(k^2)$ only

$$S_{\alpha\beta}=g_{\alpha_1\beta_1}g_{\alpha_2\beta_2}\cdots g_{\alpha_s\beta_s} \tag{2.12}$$

gives an independent form factor. Indeed, we can show (Appendix B) that

$$\begin{aligned}\bar u_\alpha(k_2)(k_\mu\not k-\gamma_\mu k^2)\gamma_5v_\beta(k_1)k_{\alpha_i}k_{\beta_j}=&\,2m\bar u_\alpha(k_2)\gamma_5v_\beta(k_1)\left[k_\mu k_{\alpha_i}k_{\beta_j}-\frac{k^2}{2}(k_{\alpha_i}g_{\mu\beta_j}+k_{\beta_j}g_{\mu\alpha_i})\right]\\&-\frac{k^2}{2}g_{\alpha_i\beta_j}\bar u_\alpha(k_2)(k_\mu\not k-\gamma_\mu k^2)\gamma_5v_\beta(k_1)\ ,\end{aligned} \tag{2.13}$$

which is a combination of F_0, with $S_{\alpha\beta}$ given by (2.12), and $F_2(k^2)$. Therefore for Majorana fermions of spin s there are $(2s+1)/2$ P-violating moments and $(2s-1)/2$ T-violating moments, which are all of the anapole type (see the previous section).

It is even possible to rewrite the F_1, P-violating terms so that they have a structure very similar to the integer-spin case. From Appendix D [(see Eq. (D6)] the F_1 terms may be cast in the form

$$\begin{aligned}&\left[k^\mu k^{\alpha_1}k^{\beta_1}-\frac{k^2}{2}(g^{\mu\alpha_1}k^{\beta_1}+g^{\mu\beta_1}k^{\alpha_1})\right]\bar u_\alpha(k_2)\gamma_5u_\beta(k_1)\\&\quad=\frac{i}{2}(k^\mu\epsilon^{\alpha_1\beta_1\rho\sigma}k_\rho P_\sigma-k^2\epsilon^{\alpha_1\beta_1\mu\sigma}P_\sigma)\bar u_\alpha(k_2)v_\beta(k_1)+mg^{\alpha_1\beta_1}\bar u_\alpha(k_2)[(k^\mu\not k-k^2\gamma^\mu)\gamma_5]v_\beta(k_1)\ .\end{aligned} \tag{2.14}$$

While the last term is in fact the spin-$\frac{1}{2}$ case and goes into F_0, the first term has the same structure as in the spin-1 case.

Moreover, the F_0 term can be rewritten as

$$\bar u_\alpha(k_2)[(k_\mu\not k-\gamma_\mu k^2)\gamma_5]v_\beta(k_1)=ik^\rho P^\sigma\epsilon_{\mu\lambda\rho\sigma}\bar u_\alpha(k_2)\gamma^\lambda v_\beta(k_1)\ . \tag{2.15}$$

Therefore this again shows that the P-violating terms have the P-wave factor.

III. THE ANAPOLE OR TOROIDAL MOMENT[16]

As can be clearly seen from the covariant decomposition of the electromagnetic current of a Majorana particle and as has been noted in Sec. II, all the form factors may be cast in the following structure:

$$J_\mu^{\text{anapole}} = k_\mu k\cdot S - k^2 S_\mu \;, \tag{3.1}$$

where S represents some vector characterizing the Majorana particle and which can be expressed as a functional of the Majorana field and momenta.

The above structure leads to a contact interaction as the piece k_μ does not contribute due to current conservation whereas k^2 drops against the k^2 from the photon propagator. One way of seeing what is so peculiar about this type of moment is to notice that it does not interact with the external fields (i.e., $F^{\mu\nu}$ or $\tilde{F}^{\mu\nu}$) but rather only with the external current. The Lagrangian representing the interaction of the "anapole current" with the electromagnetic field may be written as:

$$\mathcal{L}_A^{\text{eff}} = J_\mu^{\text{anapole}} A^\mu \equiv S^\mu \partial^\nu F_{\mu\nu} \;. \tag{3.2}$$

However from Maxwell equations one has

$$\mathcal{L}_A^{\text{eff}} = S\cdot \mathcal{J}^{\text{ext}} \;, \tag{3.3}$$

where $\mathcal{J}^{\text{ext}}$ is the external current. There are some similarities with the charge radius. However a truly neutral particle such as a Majorana particle does not have any charge distribution or charge radius associated with it. It is also easy to see that by going to the Breit frame,[17] which is the natural frame when studying form factors in the static limit, the timelike component of the four-current, i.e., the charge, is zero.[6]

For instance, take the current contained in (1.12),

$$j_\mu = k_\mu \epsilon_{\alpha_1\beta_1\rho\lambda} P^\rho k^\lambda - k^2 \epsilon_{\alpha_1\beta_1\rho\mu} P^\rho \;, \tag{3.4}$$

and consider its time component (charge), i.e., $\mu=0$, in the Breit frame. Since in this frame k has no time component, $k_0=0$, whereas P has only a time component, $P_i=0$, it is clear that $j_0=0$.

We have seen that the anapole current breaks some fundamental discrete symmetries, either P or T and always C invariance, and leads to a contact interaction. These are the main reasons one hardly finds any discussion about this moment in textbooks. However to see that such moments can arise, consider (part of the following argument is borrowed from Khriplovich[18]) the expansion of the (three-)vector potential $\mathbf{A}(R)$ generated by a localized current density $\mathbf{j}$ of a stationary system at some large distance R from the source:

$$\begin{aligned}\mathbf{A}(R) &= \int d^3r \frac{\mathbf{j}(r)}{|\mathbf{R}-\mathbf{r}|} \\ &= \frac{1}{R}\int d^3r\, \mathbf{j}(r) + \partial_k \frac{1}{R}\int d^3r\, r_k \mathbf{j}(r) \\ &\quad + \frac{1}{2}\partial_k\partial_l \frac{1}{R}\int d^3r\, r_k r_l \mathbf{j}(r) \;. \end{aligned} \tag{3.5}$$

It is well known that the first term vanishes due to the continuity condition while the second gives rise to a magnetic moment. The last term induces both a quadrupole magnetic moment and an anapole moment. The latter comes from the vector contribution which arises from the case $k=l$ and leads to the anapole-induced vector potential.

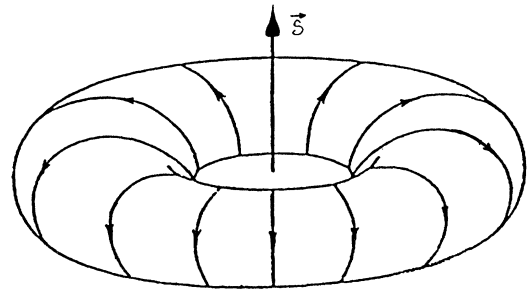

FIG. 1. A solenoid bent around a torus will generate an anapole moment directed along the axis of the torus. The flow of the current is indicated by the arrow. (From Ref. 16.)

$$\mathbf{A}_{\text{anapole}} = \frac{1}{2}\nabla^2 \frac{1}{R}\int d^3r\, r^2 \mathbf{j}(r) = \mathbf{T}\delta(R) \tag{3.6}$$

with

$$\mathbf{T} = -2\pi \int d^3r\, r^2 \mathbf{j}(r) \;. \tag{3.7}$$

The presence of the delta function $\delta(R)$ again is a manifestation of a contact interaction.

As first pointed out by Zeldovich[4] a classical model of the anapole can be pictorially visualized by a current flowing through a wire helix wound around a torus (Fig. 1). The anapole moment is a toroidal moment directed along the axis of the torus (i.e., the axial symmetry of the system). The charge density ρ for this system is zero as is the magnetic dipole moment. The interaction with a test particle can only take place, however, if the particle penetrates the torus.

The anapole *current* is a polar vector, but for a particle at rest one only has the spin S, which is an axial vector. This reflects the fact that C is violated. This violation is responsible for a depolarization via the spin precession leading to a helical spin structure in analogy with the current around the torus.

IV. MASSLESS CASE

If the Majorana is massless, its representing wave function will be subject to subsidary conditions and it is not all obvious how we can take the massless limit in Eqs. (1.12) and (2.11), a limit which may after all not exist. We have already alluded to this difficulty in the Introduction, inherent to formulating a consistent covariant formulation of high-spin fields (spin-$s>1$). Apart from this difficulty one should expect that since the electromagnetic Majorana couplings are C violating that the currents which generate these couplings are not conserved. This is especially true with P-violating couplings which are not conserved or most often anomalous at the quantum level.

It is well known that the massless limit cannot be taken. This occurs even for the spin-1 case. For instance the study of $ZZ\gamma$ which gives rise to an anapole shows[6,7] that in $\gamma\to ZZ$ one of the Z is longitudinal while the other is transverse. Now if the Z were massless it would not have a longitudinal mode and we would expect $\gamma\to ZZ$ not to exist in this limit. However, if one tries to take the limit $M_Z\to 0$ one finds a singularity which reflects the fact that the Z is coupled to a current which is *not* conserved and hence the limit is not smooth. Therefore, the massless case needs a separate treatment. There is a general theorem by Weinberg and Witten[19] stating that a massless but charged particle cannot acquire a nonzero expectation value of the electromagnetic current if its spin is higher than $\frac{1}{2}$. Here we extend the theorem to neutral particles.

A. How many form factors for a non-Majorana particle?

However, before doing so and since the formalism we will follow is well adapted, we would like to show that the maximum number of form factors, without imposing any C, P, or T invariance, for any particle (not a Majorana) of spin s is $6s+1$. This only follows from angular momentum conservation and should greatly help when writing all the possible form factors in a phenomenological approach. For instance it was first believed that the number of independent invariants for the W^+W^-Z or the $W^+W^-\gamma$ vertex was equal to 9.[20] However a subsequent careful analysis[15] showed that two of these invariants were redundant by use of nontrivial relations. Recently,[21] there has been a study devoted to counting the number of form factors associated with a conserved current based on the multipole expansion approach. We will see below how the helicity formalism can very easily and directly give us this number.

Consider two particles in their center-of-mass (c.m.) frame with total energy W, helicity λ_1, and λ_2, their relative momentum being specified by the angles θ,ϕ. Using the Jacob-Wick[22] helicity formalism, we develop such a state with a definite total angular momentum J, with the result

$$|W\theta\phi,\lambda_1\lambda_2\rangle=\sum_{J\geq|\lambda|}\sum_{M=-J}^{+J}\left[\frac{2J+1}{4\pi}\right]^{1/2}e^{-iM\phi}d^J_{M,\lambda}(\theta)|WJM;\lambda_1\lambda_2\rangle\ . \tag{4.1}$$

$d^J_{M,\lambda}(\theta)$ are the usual rotation functions and

$$\lambda=\lambda_1-\lambda_2\ . \tag{4.2}$$

We first notice that the number of form factors or invariants representing the coupling of a vector boson to a particle-antiparticle pair is restricted by the values of the components of the independent $d^{J=1}_{m,m'}$ functions. We only take the $J=1$ as, for the photon, the scalar part $J=0$ does not contribute in the c.m. frame due to current conservation, while for a general massive vector boson the counting only concerns the vector part which is the case of the Z in e^+e^-, for instance, since its scalar part breaks chirality and hence is negligible. Then since

$$|\lambda|\leq 1 \tag{4.3}$$

we have either (a) $\lambda_1=\lambda_2$, which occurs $2s+1$ times ($\lambda=0$), or (b) $\lambda_1=\lambda_2\pm 1$, with each possibility occurring $2s$ times ($\lambda=\pm 1$). Hence there can be at most $6s+1$ form factors. This allows at most 4 independent form factors for spin $\frac{1}{2}$, as is well known, and 7 for spin 1, as has been recently discovered.[15] If the scalar part is important, for example if the spin-1 boson is not on shell or if it is not coupled to a conserved current, then one just looks at the $J=0$ part. In this case only $\lambda_1=\lambda_2$ is allowed giving $2s+1$ possible form factors. Then all in all there can be at most $(6s+1)+(2s+1)=2(4s+1)$.

B. Number of form factors for a massive Majorana

In the case where the two particles are identical with spin s, one has to properly symmetrize (PS) the state,[23]

$$|W\theta\phi\lambda_1\lambda_2\rangle_{\rm PS}=\frac{1}{\sqrt{2}}[|W\theta\phi\lambda_1\lambda_2\rangle+P_{12}(-)^{2s}|W\theta\phi\lambda_1\lambda_2\rangle] \tag{4.4}$$

(P_{12} is the permutation operator[22]) with the result

$$|W\theta\phi,\lambda_1\lambda_2\rangle_{\rm PS}=\frac{1}{\sqrt{2}}\sum_{J\geq|\lambda|}\sum_{M=-J}^{+J}\left[\frac{2J+1}{4\pi}\right]^{1/2}e^{-iM\phi}d^J_{M,\lambda}(\theta)[|WJM,\lambda_1\lambda_2\rangle+(-)^J|WJM,\lambda_2\lambda_1\rangle]\ . \tag{4.5}$$

One of the consequences of (4.5) is that two identical Majorana particles with the same helicity cannot be in a state with odd angular momentum. For a spin-zero Majorana, $\lambda_1=\lambda_2=0$, and hence no coupling with the photon is allowed whether or not they are massless.

Therefore since the case $\lambda_1=\lambda_2$ does not occur when coupling to a spin-1, and since the previous equation shows that there is a $\lambda_1\leftrightarrow\lambda_2$ symmetry, the maximum number of form factors in the case of identical Majorana reduces to just $2s$, which again confirms the results of our covariant decomposition as well as the results of the multipole expansion which we carried out in a previous paper.[9] For the spin-1 which is not coupled to a conserved current, taking into account the $J=0$ part tells us that now $\lambda_1=\lambda_2$ is allowed giving an additional $2s+1$ form factors.

C. Massless Majoranas

For a general massless Majorana of spin s, only two helicity states may exist

$$\lambda_1=\pm s \; ; \tag{4.6}$$

hence, either

$$\lambda_1=\lambda_2=\pm s \tag{4.7}$$

or

$$\lambda_1=-\lambda_2=\pm s \; . \tag{4.8}$$

For J odd, (4.7) is ruled out whereas the $d^J_{M,\lambda}$ functions in (4.5) impose that opposite helicities can form a spin J only if

$$|\lambda|=2s\le J \; . \tag{4.9}$$

Therefore a vector particle cannot decay into two massless Majorana particles unless they have spin $\frac{1}{2}$. This theorem embodies Yang's theorem[24] which states that a vector particle cannot decay into two photons.

Now consider more specifically the case of the electromagnetic current: $\langle MM|J_\mu(0)|0\rangle$. We take the Majorana particles in their c.m. so that $k_\mu=(k_0,0)$. From current conservation we see that the spin-0 part of the current, J_0, has a vanishing matrix element

$$k_\mu J^\mu=k_0J_0=0 \tag{4.10}$$

and since we have just shown that the spin-1 part does not couple to a massless Majorana particle if its spin is different from $\frac{1}{2}$, then the only massless Majorana particle which may have an electromagnetic structure has spin $\frac{1}{2}$.

For a nonconserved current, $J=0$ contributes and one has to take (4.7) which allows two more form factors.

V. PRODUCTION OF A MAJORANA PAIR

A. Production of two particles

Consider the production of an arbitrary spin-s Majorana pair with helicity λ_1 and λ_2 by two particles a and b with helicity λ_a,λ_b. Denoting by W the c.m. total energy, by θ the scattering angle, by ϕ the azimuthal angle, and by T the transition matrix, the amplitude for the process is

$$f_{\lambda_1\lambda_2;\lambda_a\lambda_b}(\theta,\phi)={}_{\rm PS}\langle W\theta\phi\lambda_1\lambda_2|T|W00\lambda_a\lambda_b\rangle=\sum_{J\ge\min(|\lambda|,|\lambda'|)}\frac{2J+1}{4\pi}e^{i\lambda\phi}d^{*J}_{\lambda',\lambda}(\theta)T^J_{\lambda_1\lambda_2,\lambda_a\lambda_b} \tag{5.1}$$

with

$$\lambda=\lambda_1-\lambda_2,\quad \lambda'=\lambda_a-\lambda_b \tag{5.2}$$

and where according to (4.5) we must have

$$T^J_{\lambda_1\lambda_2,\lambda_a\lambda_b}=(-)^JT^J_{\lambda_2\lambda_1,\lambda_a\lambda_b} \; . \tag{5.3}$$

This combined with

$$d^J_{\lambda',-\lambda}(\pi-\theta)=(-)^{J-\lambda'}d^J_{\lambda,\lambda}(\theta) \tag{5.4}$$

shows that

$$f_{\lambda_1\lambda_2,\lambda_a\lambda_b}(\pi-\theta,\phi+\pi)=f_{\lambda_1\lambda_2,\lambda_a\lambda_b}(\theta,\phi) \tag{5.5}$$

and that the differential cross section summing over the final helicities has the property

$$\sigma_{\lambda_a\lambda_b}(\theta,\phi)=\sigma_{\lambda_a\lambda_b}(\pi-\theta,\phi+\pi) \; . \tag{5.6}$$

B. Production via a vector boson V in e^+e^-

In this special case, we are exchanging a particle with $J=1$. In the high-energy limit chirality is conversed at e^+e^-V (V can be $\gamma,Z,\ldots$, and e^+e^- generically stands for an $f\bar{f}$ pair) and hence, using the previous notation only

$$\lambda_a=-\lambda_b=\pm\tfrac{1}{2} \tag{5.7}$$

occur giving $\lambda'=\pm1$. Also in (5.1) only $J=1$ contributes. Without loss of generality we may set $\phi=0$. Summing over final helicities we get

$$\left[\frac{4\pi}{3}\right]^2\sigma_{\lambda_a\lambda_b}(\theta)=\sum_{\lambda_1\lambda_2}|d^1_{\lambda'\lambda}(\theta)|^2T^{J=1}_{\lambda_1\lambda_2,\lambda_a\lambda_b}T^{J=1*}_{\lambda_1\lambda_2\lambda_a\lambda_b}$$

$$=\sum_{\lambda_1}|d^1_{\lambda',1}(\theta)|^2T_{\lambda_1(\lambda_1-1),\lambda_a\lambda_b}T^*_{\lambda_1(\lambda_1-1),\lambda_a\lambda_b}+|d^1_{\lambda',1}(\theta)|^2T_{(\lambda_1-1)\lambda_1,\lambda_a\lambda_b}T^*_{(\lambda_1-1)\lambda_1,\lambda_a\lambda_b} \tag{5.8}$$

($\lambda_1=\lambda_2$ does not occur). Using (5.3) with $J=1$ and the explicit form of the $d^1_{\lambda',1}$ for $\lambda'=\pm1$ gives

$$\left[\frac{4\pi}{3}\right]^2\sigma_{\lambda_a,\lambda_b=-\lambda_a}(\theta)=[|d^1_{\lambda',1}(\theta)|^2+|d^1_{\lambda',-1}(\theta)|^2]\sum_\mu|T_{\mu,\mu-1;\lambda_a,\lambda_b}|^2=\frac{1+\cos^2\theta}{2}\sum_\mu|T_{\mu,\mu-1;\lambda_a\lambda_b}|^2 \; . \tag{5.9}$$

This shows that $e^+e^- \to V \to MM$ has the unique angular distribution $1+\cos^2\theta$ regardless of the spin of the Majorana particle. This result has been explicitly verified for $s=\frac{1}{2}$ (Ref. 24) and $s=1$.[6,7]

VI. APPLICATIONS FROM GAUGE FIELD THEORIES: FORM FACTORS OF ELEMENTARY MAJORANA PARTICLES

The preceding analysis has shown that a γ^*MM vertex where M is a Majorana particle of any spin leads to a contact interaction. In other words, since M carries no electric charge this coupling cannot be generated by a minimal gauge prescription nor can it be induced through the use of the electromagnetic field tensor $F^{\mu\nu}$. Therefore in any renormalizable field theory if a γ^*MM is to exist, it will only be induced at the loop level and hence may be considered of a purely quantum origin. Moreover, since T violation is only extremely weakly broken, the anapole which one might encounter in current field theories will most certainly be parity violating. This already tells us that if one wants to calculate the induced γ^*MM one must look for P-violating (or C-violating) contributions.

A. Spin 0

Neutral spin-0 particles, either scalar particles such as the minimal standard model Higgs boson or pseudoscalar such as the neutral pion, do not have any single-photon electromagnetic coupling at any order of the loop expansion in any theory regardless of whether or not C is violated.

B. Spin $\frac{1}{2}$

This is the first most interesting case which has received wide attention. In this most simple case one may think of M as being a gluino ($\tilde{g}$), for instance in a supersymmetric theory. Parity violation in such a theory may not only come from the weak sector as in the standard model through the $SU(2)_{weak}$ gauge couplings but could also occur if the two opposite-chirality squarks corresponding to the same partner quark have different masses. For $\gamma\tilde{g}\tilde{g}$ only the parity-violating part of the triangle diagram involving quarks and squarks contributes, the total contribution being proportional to the squark mass splitting. This is confirmed by explicit calculations.[25]

One can extend some of these conclusions to the case $Z \to \tilde{g}\tilde{g}$. Here, however there is also parity violation from the axial-vector coupling of the Z to the quarks. The case of the vector coupling is analogous to the photon case and its contribution vanishes for no mass splitting between the squarks. This process has been considered in Ref. 26. In both cases, when considering for instance $e^+e^- \to \tilde{g}\tilde{g}$ there is a P-wave factor and the distribution is indeed verified to be $(1+\cos^2\theta)$.

C. Spin 1

One typical example is that of the standard Z^0 particle. The $ZZ\gamma$ is generated at one-loop order via a fermion triangle.[27,6,7] One can also draw loops involving the charged W but since the trilinear and quadrilinear vector-boson couplings in $SU(2)\times U(1)$ are C conserving this particular contribution must be zero. In the fermion triangle only one axial coupling (only one γ_5 at a time) contributes to the trace so that one gets a C-violating contribution. These calculations have been considered by one of us,[6] confirming our general previous results. It is also interesting to note that one can in principle generate a $\gamma\gamma\gamma$ vertex, but in the minimal standard model this will only be induced at the two-loop order. Moreover our theorem about massless Majorana particles tells us that at least two of the photons will have to be off shell.

D. Spin $\frac{3}{2}$

Here also one needs a massive particle. One obvious candidate is the gravitino in a theory which breaks C invariance. The theory which comes to mind is $N=1$ broken supergravity. However even with a maximal breaking of supersymmetry we have recently shown[28] that at one loop no gravitino form factor can be generated, at least for on-shell gravitinos. This may be partly due to the very constraining conditions the on-shell gravitino field is subject to.

E. Spin 2

One can think of the graviton but since it is massless any attempt to calculate its single-photon interaction will lead to a null result.

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APPENDIX A

We want to show that

$$\left[k_\mu k_{\alpha_1} k_{\beta_1} - \frac{k^2}{2}(g_{\mu\alpha_1} k_{\beta_1} + g_{\mu\beta_1} k_{\alpha_1})\right] \epsilon_{\alpha_2\beta_2\lambda\rho} P^\lambda k^\rho = (k_\mu \epsilon_{\alpha_1\beta_1\lambda\rho} P^\lambda k^\rho - k^2 \epsilon_{\alpha_1\beta_1\lambda\mu} P^\lambda)\left[k_{\alpha_2} k_{\beta_2} - \frac{k^2}{2} g_{\alpha_2\beta_2}\right] . \tag{A1}$$

This derivation relies heavily on the use of Schouten's identity:

$$g_{\alpha\lambda}\epsilon_{\mu\nu\rho\sigma}+g_{\alpha\mu}\epsilon_{\nu\rho\sigma\lambda}+g_{\alpha\nu}\epsilon_{\rho\sigma\lambda\mu}+g_{\alpha\rho}\epsilon_{\sigma\lambda\mu\nu}+g_{\alpha\sigma}\epsilon_{\lambda\mu\nu\rho}=0 \ . \tag{A2}$$

We only need to transform the term in k^2 of the left-hand side (LHS) of (A1). To this end, use of the previous relation leads to

$$-g_{\mu\alpha_1}\epsilon_{\alpha_2\beta_2\lambda\rho}P^\lambda k^\rho=g_{\alpha_1\alpha_2}\epsilon_{\beta_2\lambda\rho\mu}P^\lambda k^\rho+g_{\alpha_1\beta_2}\epsilon_{\lambda\rho\mu\alpha_2}P^\lambda k^\rho+P_{\alpha_1}\epsilon_{\rho\mu\alpha_2\beta_2}k^\rho+k_{\alpha_1}\epsilon_{\mu\alpha_2\beta_2\lambda}P^\lambda \ . \tag{A3}$$

One should note that the first term on the RHS of (A3) does not contribute to the trace condition (1.2), in the second term one can interchange α_1 and α_2 due to the symmetry of the wave function, and in the third term one may replace [in view of (1.3)] P_{α_1} by k_{α_1}.

We can then write

$$(g_{\mu\alpha_1}k_{\beta_1}+g_{\mu\beta_1}k_{\alpha_1})\epsilon_{\alpha_2\beta_2\lambda\rho}P^\lambda k^\rho=-2k_{\alpha_1}k_{\beta_1}\epsilon_{\mu\alpha_2\beta_2\lambda}P^\lambda+g_{\alpha_2\beta_2}(k_{\beta_1}\epsilon_{\lambda\rho\mu\alpha_1}P^\lambda k^\rho-k_{\alpha_1}\epsilon_{\lambda\rho\mu\beta_1}P^\lambda k^\rho) \ . \tag{A4}$$

Another use of Schouten's identity permits one to transform the last term in (A4) into

$$-k_{\alpha_1}\epsilon_{\lambda\rho\mu\beta_1}=k\cdot P\epsilon_{\rho\mu\beta_1\alpha_1}+k^2\epsilon_{\mu\beta_1\alpha_1\lambda}P^\lambda+k_\mu\epsilon_{\beta_1\alpha_1\lambda\rho}P^\lambda k^\rho+k_{\beta_1}\epsilon_{\alpha_1\lambda\rho\mu}P^\lambda k^\rho \ . \tag{A5}$$

The first term vanishes ($k\cdot P=0$) while the last term cancels against the one in the previous equation. Replacing in (A4) lead to (A1).

APPENDIX B

We want to show that

$$k^\alpha k^\beta \bar{u}_\alpha(k_2)(k_\mu \not{k}-\gamma_\mu k^2)\gamma_5 v_\beta(k_1) \tag{B1}$$

can be written as a combination of F_0 and F_2. For this we write the second term as

$$k^\alpha k^\beta \bar{u}_\alpha(k_2)\gamma_\mu\gamma_5 v_\beta(k_1)=k^\beta k^\nu \bar{u}_\alpha(k_2)g^\alpha_\nu\gamma_\mu\gamma_5 v_\beta(k_1)$$
$$=\frac{k^\beta k^\nu}{2}\bar{u}_\alpha(k_2)(\gamma^\alpha\gamma_\nu+\gamma_\nu\gamma^\alpha)\gamma_\mu\gamma_5 v_\beta(k_1)=\frac{k^\beta k^\nu}{2}\bar{u}_\alpha(k_2)(\gamma_\nu\gamma^\alpha\gamma_\mu)\gamma_5 v_\beta(k_1) \tag{B2}$$

(where we used $\bar{u}_\alpha(k_2)\gamma^\alpha=0$). Now we use the identity

$$\gamma_\nu\gamma_\alpha\gamma_\mu=\gamma_\nu g_{\alpha\mu}-\gamma_\alpha g_{\mu\nu}+\gamma_\mu g_{\alpha\nu}+i\gamma_5\epsilon_{\nu\alpha\mu\rho}\gamma^\rho \tag{B3}$$

to write

$$k^\alpha k^\beta \bar{u}_\alpha(k_2)\gamma_\mu\gamma_5 v_\beta(k_1)=k_\beta g^\alpha_\mu \bar{u}_\alpha(k_2)\not{k}\gamma_5 v_\beta(k_1)-ik^\nu k^\beta\epsilon^\alpha_{\mu\nu\rho}\bar{u}_\alpha(k_2)\gamma^\rho v_\beta(k_1) \ . \tag{B4}$$

One can write a similar relation by transforming k^β instead of k^α. Adding the two gives

$$k^\alpha k^\beta \bar{u}_\alpha(k_2)\gamma_\mu\gamma_5 v_\beta(k_1)=\tfrac{1}{2}\{[\bar{u}_\alpha(k_2)\not{k}\gamma_5 v_\beta(k_1)](k^\alpha g^\beta_\mu+k^\beta g^\alpha_\mu)+i\bar{u}_\alpha(k_2)\gamma_\rho v_\beta(k_1)(k^\beta k^\nu\epsilon^{\rho\alpha}_{\mu\nu}-k^\alpha k^\nu\epsilon^{\rho\beta}_{\mu\nu})\} \ . \tag{B5}$$

The last terms containing the Levi-Civita tensors can be rewritten using Schouten's identity. For instance,

$$-k^\alpha k^\lambda\epsilon_{\mu\beta\lambda\rho}=k^\mu k^\lambda\epsilon_{\beta\lambda\rho\alpha}+k^\beta k^\lambda\epsilon_{\lambda\rho\alpha\mu}+k^2\epsilon_{\rho\alpha\mu\beta}+k^\rho k^\lambda\epsilon_{\alpha\mu\beta\lambda} \ . \tag{B6}$$

Noting that the last term in ($B6$) does not contribute because of current conservation one has

$$\bar{u}_\alpha(k_2)\gamma_\mu\gamma_5 v_\beta(k_1)k^\alpha k^\beta=\tfrac{1}{2}\{[\bar{u}_\alpha(k_2)\not{k}\gamma_5 v_\beta(k_1)](k^\alpha g^\mu_\beta+k^\beta g^\alpha_\mu)+i\bar{u}_\alpha(k_2)\gamma_\rho v_\beta(k_1)(k_\mu\epsilon^{\alpha\beta\rho\nu}k^\nu-k^2\epsilon^{\alpha\beta\rho}_\mu)\} \ . \tag{B7}$$

This enables us to write

$$k^\alpha k^\beta \bar{u}_\alpha(k_2)k_\mu[(\not{k}-\gamma_\mu k^2)\gamma_5]v_\beta(k_1)=\bar{u}_\alpha(k_2)\not{k}\gamma_5 v_\beta(k_1)\left[k_\mu k^\alpha k^\beta-\frac{k^2}{2}(k^\alpha g^\beta_\mu+k^\beta g^\alpha_\mu)\right]$$
$$+i\frac{k^2}{2}\bar{u}_\alpha(k_2)\gamma_\rho v_\beta(k_1)(k_\mu\epsilon^{\alpha\beta\rho\nu}k_\nu-k^2\epsilon^{\alpha\beta\rho}_\mu) \ . \tag{B8}$$

The last trick we use is to transform the Levi-Civita tensor in (B8) to a γ_5. For this we use this relation combined with (B3),

$$\bar{u}_\alpha(k_2)\gamma^\alpha\gamma^\rho\gamma^\beta v_\beta(k_1)=\bar{u}_\alpha(k_2)(\gamma^\alpha g^{\rho\beta}-\gamma^\rho g^{\alpha\beta}+\gamma^\beta g^{\rho\alpha}+i\gamma_5\epsilon^{\alpha\rho\beta\lambda}\gamma_\lambda)\gamma_5 v_\beta(k_1) \ , \tag{B9}$$

to be able to write

$$g^{\alpha\beta}\bar{u}_\alpha(k_2)\gamma^\lambda\gamma_5 u_\beta(k_1)=i\epsilon^{\alpha\beta\lambda\rho}\bar{u}_\alpha(k_2)\gamma_\rho u_\beta(k_1) \tag{B10}$$

[where we used $\bar{u}_\alpha(k_2)\gamma^\alpha=\gamma^\beta u_\beta(k_1)=0$] so that, at last,

$$k^\alpha k^\beta \bar{u}_\alpha(k_2)[(k^\mu \not{k}-k^2\gamma^\mu)\gamma_5]v_\beta(k_1)=2m\bar{u}_\alpha(k_2)\gamma_5 v_\beta(k_1)\left[k^\mu k^\alpha k^\beta-\frac{k^2}{2}(k^\alpha g^{\mu\beta}+k^\beta g^{\mu\alpha})\right] -\frac{k^2}{2}g^{\alpha\beta}\bar{u}_\alpha(k_2)(k^\mu\not{k}-k^2\gamma^\mu)\gamma_5 v_\beta(k_1)\ . \qquad \text{(B11)}$$

The first term is of the F_2 type while the last one is of the F_0 type.

APPENDIX C

We want to show that the first anapole term of lowest order, i.e., the one that would appear in the spin-$\frac{1}{2}$ case, can be reexpressed in a form similar to that of the spin-1 case, i.e., in terms of the Levi-Civita tensor.

We start by using

$$\bar{u}_\alpha(k_2)k_\mu\not{k}\gamma_5 v_\beta(k_1)=2mk_\mu\bar{u}_\alpha(k_2)\gamma_5 v_\beta(k_1)\ . \qquad \text{(C1)}$$

We next use identity (B3) to transform the product $\not{k}_1\gamma_\mu\not{k}_2$ which permits us to write

$$\bar{u}_\alpha(k_2)\gamma_\mu\gamma_5\left[\frac{k^2}{2}-m^2\right]v_\beta(k_1) =\bar{u}_\alpha(k_2)[m(k_\mu-m\gamma_\mu)\gamma_5+ik_1^\rho k_2^\sigma\epsilon_{\rho\sigma\mu\lambda}\gamma^\lambda]v_\beta(k_1)\ . \qquad \text{(C2)}$$

Therefore, this leads to the desired relation

$$\bar{u}_\alpha(k_2)[(k_\mu\not{k}-\gamma_\mu k^2)\gamma_5]v_\beta(k_1) =ik^\rho P^\sigma\epsilon_{\mu\lambda\rho\sigma}\bar{u}_\alpha(k_2)\gamma^\lambda v_\beta(k_1)\ . \qquad \text{(C3)}$$

APPENDIX D

Consider the F_0 term in the fermion case (in the following the indices α and β which appear everywhere but as indices of the spinors stand for any α_i and β_i):

$$F_0=g^{\alpha\beta}\bar{u}_\alpha(k_2)[(k^\mu\not{k}-k^2\gamma^\mu)\gamma_5]v_\beta(k_1)\ . \qquad \text{(D1)}$$

We then use identity ($B10$) together with the Gordon identity

$$\bar{u}_\alpha(k_2)\gamma^\rho(u_\beta(k_1)=\frac{1}{2m}\bar{u}_\alpha(k_2)(P^\rho+i\sigma^{\rho\nu}k_\nu)v_\beta(k_1) \qquad \text{(D2)}$$

giving

$$F_0=\frac{i}{2m}(k_\mu\epsilon^{\alpha\beta\rho\sigma}k_\rho P_\sigma-k^2\epsilon^{\alpha\beta\mu\sigma}P_\sigma)\bar{u}_\alpha(k_2)v_\beta(k_1) -\frac{1}{2m}(k^\mu\epsilon^{\alpha\beta\rho\sigma}k_\rho P_\sigma-k^2\epsilon^{\alpha\beta\mu\rho})\bar{u}_\alpha(k_2)\sigma_{\rho\nu}k^\nu v_\beta(k_1)\ . \qquad \text{(D3)}$$

The first term is obviously the anapole as is the spin-1 case. So we just concentrate on the second term

$$F'=(k^\mu\epsilon^{\alpha\beta\rho\sigma}k_\rho P_\sigma-k^2\epsilon^{\alpha\beta\mu\rho})\bar{u}_\alpha(k_2)\sigma_{\rho\nu}k^\nu v_\beta(k_1) =(k^\alpha\epsilon^{\beta\mu\rho\sigma}k_\sigma-k^\beta\epsilon^{\alpha\mu\rho\sigma}k_\sigma)\bar{u}_\alpha(k_2)\sigma_{\rho\nu}k^\nu v_\beta(k_1)\ , \qquad \text{(D4)}$$

where Schouten's identity has been applied as in Appendix A.

Next we introduce a new Levi-Civita tensor via the identity

$$\sigma^{\rho\nu}=\gamma_5\frac{i}{2}\epsilon^{\rho\nu\lambda\tau}\sigma_{\lambda\tau}$$

and replace it in the previous equation to be able to contract the antisymmetric tensors. The σ Dirac matrices which appear have at least an α or a β index which we expand in order to use condition $\gamma^\beta u_\beta(k_1)=\bar{u}_\alpha(k_2)\gamma^\alpha=0$, so that

$$F'=-2\left[k^\mu k^\alpha k^\beta-\frac{k^2}{2}(g^{\mu\alpha}k^\beta+g^{\mu\beta}k^\alpha)\right]\bar{u}_\alpha(k_2)\gamma_5 v_\beta(k_1). \qquad \text{(D5)}$$

Therefore

$$F_0=g^{\alpha\beta}\bar{u}_\alpha(k_2)[(k^\mu\not{k}-k^2\gamma^\mu)\gamma_5]v_\beta(k_1) =\frac{i}{2m}(k^\mu\epsilon^{\alpha\beta\rho\sigma}k_\rho P_\sigma-k^2\epsilon^{\alpha\beta\mu\sigma}P_\sigma)\bar{u}_\alpha(k_2)v_\beta(k_1) +\frac{1}{m}\left[k^\mu k^\alpha k^\beta-\frac{k^2}{2}(g^{\mu\alpha}k^\beta+g^{\mu\beta}k^\alpha)\right] \times\bar{u}_\alpha(k_2)\gamma_5 v_\beta(k_1)\ . \qquad \text{(D6)}$$

So indeed the equivalent term of the spin-1 case is a combination of the F_0 and F_1 form factors.

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