Light Higgs bosons in three-generation Calabi-Yau superstring theory

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(Received 30 October 1990)

An analysis of the Higgs sector is given in three-generation superstring models which arise via Calabi-Yau compactifications where the flux breaking at the compactification scale reduces the E_6 grand unified group to $[SU(3)]^3$ and a further spontaneous breaking at an intermediate scale M_I reduces this symmetry down to the standard-model gauge group symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$. The analysis is carried out within the framework of matter parity invariance which is needed for proton stability and the assumption that spontaneous breaking at the intermediate scale is triggered by supersymmetry breaking. Conditions for the existence of a pair of light Higgs doublets needed for the further breaking of the $SU(2)_L \times U(1)_Y$ electroweak symmetry are obtained. These conditions act as constraints on the moduli space of the theory in order that the $(27)^3$ interactions accommodate light Higgs bosons below the intermediate mass scale.

I. INTRODUCTION

One of the important ingredients necessary for an acceptable supersymmetric low-energy theory of particle interactions is the existence of a pair of light Higgs doublets which are needed for the breaking of $SU(2)_L \times U(1)_Y$ electroweak symmetry. We shall investigate this problem here within the framework of three-generation models which arise from nonsimply connected Calabi-Yau compactifications.[1] Three examples of such manifolds are the $CP^3 \times CP^3/Z_3$ model,[2] $CP^3 \times CP^2/Z_3 \times Z'_3$ model,[3,4] and the $CP^2 \times CP^2/Z_3 \times Z'_3 \times Z''_3$ model.[2,5,6] However, the analysis we shall carry out will not be specific to any particular three-generation model.[2] Rather, the analysis we carry out will be applicable to a whole class of three-generation models within Calabi-Yau compactifications under the following phenomenologically desirable restrictions. (i) We shall assume that on the nonsimply connected manifold the group E_6 breaks to $[SU(3)]^3$. In general, flux breaking[7] allows for the additional possibilities such as $SU(6) \times U(1)$, but such alternate breakings do not appear to be physically viable; i.e., it does not seem feasible to reduce this symmetry down to the standard-model gauge group. (ii) We shall also assume in our analysis that matter parity invariance holds. This is needed for proton stability.[8,9] (iii) We shall make the further technical assumption that all the light generations of quarks and leptons have a single matter parity signature[10] (i.e., odd). We will see then that phenomenologically acceptable Higgs bosons have the other matter parity signature (i.e., even). (More complicated matter parity assignments may be analyzed, but we shall not do so here.)

We review briefly some basic ingredients in the reduction of the theory to the standard models. After compactification, the symmetry of the four-dimensional (4D) theory is $E_6 \times (N=1$ supergravity). The massless particles at this level are either E_6 singlets of $H^1(\text{End}\,T)$ or are nonsinglet states belonging to the **27** and $\overline{\mathbf{27}}$ representations of E_6.[1] On the nonsimply connected manifold, E_6 has been assumed to break to $[SU(3)]^3 \equiv SU(3)_C \times SU(3)_L \times SU(3)_R$, where C=color and L,R=left,right. The particle decomposition of the multiplets is given in Appendix A. To achieve the standard model, the $[SU(3)]^3$ symmetry must be broken down further to $SU(3)_C \times SU(2)_L \times U(1)_Y$. This breaking can be brought about by N and ν_i^c vacuum-expectation-value (VEV) growth[11–13] at an intermediate scale M_I below and most likely close to the compactification scale, i.e., $M_I \lesssim M_c$. The symmetry breaking at M_I is governed by the following.

(a) A soft supersymmetric- (SUSY-) breaking term in the potential of the form

$$-\sum m_i^2(N_iN_i^\dagger+\nu_i^c\nu_i^{c\dagger}+H_i^2H_i^{2\dagger}+H'_{2i}H'^{\dagger}_{2i}) - \sum \overline{m}\,_i^2(\overline{N}_i\overline{N}\,_i^\dagger+\overline{\nu}\,_i^c\overline{\nu}\,_i^{c\dagger}+\overline{H}\,_i^2\overline{H}\,_i^{2\dagger}+\overline{H}\,'_{2i}\overline{H}\,'^{\dagger}_{2i}) , \qquad (1.1a)$$

where the soft-supersymmetry-breaking masses $m_i,\overline{m}_i \lesssim 10^3$ GeV.[14]

(b) An F part of the potential that arises from contributions from both the renormalizable and nonrenormalizable interactions of the following form in the superpotential:

$$W=(27)^3+(\overline{27})^3+\sum \lambda_i(27_i\times\overline{27}_i)^n/M_c^{2n-3} . \qquad (1.1b)$$

(c) A D part of the potential which is governed by the gauge transformations under $SU(3)_L \times SU(3)_R$ and is given by

$$D=\tfrac{1}{8}\sum (D_L^\alpha D_L^{\alpha\dagger}+D_R^\alpha D_R^{\alpha\dagger}) , \qquad (1.1c)$$

where

$$D_L^\alpha=\tfrac{1}{2}g_L \sum (L_{ir}^{l\dagger}L_{ir}^{l'}-\bar{L}\,_{ir}^{l}\bar{L}\,_{ir}^{l'\dagger})(t^\alpha)^l{}_{l'}\ , \qquad (1.1\text{d})$$

$$D_R^\alpha=\tfrac{1}{2}g_R \sum (L_{ir}^{l\dagger}L_{ir'}^{l}-\bar{L}\,_{ir}^{l}\bar{L}\,_{ir'}^{l\dagger})(t^\alpha)^{r'}{}_{r}\ . \qquad (1.1\text{e})$$

Here t^α are the Gell-Mann matrices and $g_{L,R}$ are the $SU(3)_{L,R}$ gauge coupling constants.

It was shown in Ref. 13 that the lowest-lying vacuum solutions of the potential of Eq. (1.1) that preserve $SU(2)_L\times U(1)_Y$ invariance are those that preserve matter parity. One has then $\langle H_i\rangle=0=\langle H_i'\rangle$, and one can always relabel the generations so that only $\langle N_1\rangle$ and $\langle \nu_2^c\rangle$ are nonzero. One finds[15]

$$\langle N_1\rangle \cong \left[\frac{\Sigma_1^2 M_c^{4n-6}}{2n^2(2n-1)\lambda_1^4}\right]^{-1/(4n-4)}\ ,$$
$$\langle \nu_2^c\rangle \cong \left[\frac{\Sigma_2^2 M_c^{4n-6}}{2n^2(2n-1)\lambda_2^4}\right]\ , \qquad (1.2\text{a})$$

where $\Sigma_i^2\equiv m_i^2+\bar{m}\,_i^2>0$. Similar results hold for the mirror generations. $\langle N_1\rangle^2$, $\langle \bar{N}_1\rangle^2$ and $\langle \nu_2^c\rangle^2,\langle \bar{\nu}_2^c\rangle^2$ differ by soft-SUSY-breaking corrections arising from the D terms:

$$\langle \bar{N}_1\rangle^2=\langle N_1\rangle^2+O(\Delta_i^2)\ ,$$
$$\langle \bar{\nu}_2^c\rangle^2=\langle \nu_2^c\rangle+O(\Delta_i^2)\ , \qquad (1.2\text{b})$$

where $\Delta_i^2\equiv(m_i^2-\bar{m}\,_i^2)$. The estimates of the VEV's show that $\langle N_1\rangle\geq 10^{15}$ GeV for $n\geq 3,4$. The value $n=2$ is actually excluded by proton stability.[9] This result makes an important statement regarding the allowed complex structure of the moduli space of the three generation Calabi-Yau models. We shall see in Secs. II and III that the ratio of the VEV's of Eq. (1.2a) appears importantly in the analysis of the Higgs boson. Thus we define a parameter ϵ so that

$$\epsilon=\frac{\langle \nu_2^c\rangle}{\langle N_1\rangle}=\left[\left|\frac{\Sigma_2}{\Sigma_1}\right|^{1/2}\frac{\lambda_1}{\lambda_2}\right]^{1/(n-1)}\ ,\quad n\geq 3,4\ . \qquad (1.3)$$

From the general size of parameters in Eq. (1.3), one expects ϵ to lie in the range $30\lesssim\epsilon\lesssim 0.30$ if we assume $10^{-3}\lesssim\lambda_1/\lambda_2\lesssim 10^3$ (i.e., λ_1 and λ_2 are of comparable size).

A remarkable result of the spontaneous breaking of Eq. (1.2) is that it leads to the existence of four new light $SU(3)_C\times SU(2)_L\times U(1)_Y$ neutral chiral supermultiplets in addition to the normal standard-model states. These new states are[16]

$$n_1=(N_1+\bar{N}_1)/\sqrt{2},\quad \bar{\nu}_2^c=(\nu_2^c+\bar{\nu}_2^c)/\sqrt{2}\ ,$$
$$n_2=\cos\theta\, N_2+\sin\theta\,\bar{\nu}_1^c\ , \qquad (1.4)$$
$$\bar{n}_2=\cos\theta\,\bar{N}_2+\sin\theta\,\nu_1^c\ ,$$

where $\tan\theta=\epsilon=\langle \nu_2^c\rangle/\langle N_1\rangle$. These states mix with neutrino states after electroweak symmetry breaking and contribute to neutrino masses.[17,18] However, since these light states are $SU(3)_C\times SU(2)_L\times U(1)_Y$ singlets, they do not affect the analysis of the nonsinglet Higgs fields.

The plan of this paper is as follows. In Sec. II we compute the Higgs-boson mass matrix below the intermediate mass scale and exhibit the constraint equations that must be satisfied to allow for a pair of light Higgs bosons. In Sec. III we solve the constraint equations and exhibit the conditions that the $(27)^3$ Yukawa couplings must satisfy to generate light Higgs bosons. In Sec. IV we discuss in detail the phenomenologically acceptable possibility, which can generate a pair of light Higgs doublets. In Sec. V we exhibit the low-energy effective interactions for this case and discuss the phenomenological viability of this case. In Sec. VI we illustrate how this possibility narrows down the available possibilities for spontaneous breaking at the intermediate scale.

II. HIGGS-BOSON MASS MATRIX BELOW THE INTERMEDIATE MASS SCALE

Since matter parity is preserved by spontaneous breaking, the mass matrix containing the fields in the lepton nonets factorizes into a block-diagonal form with an M_2-odd and an M_2-even part. The M_2-odd part contains mass terms for fields involving C-even leptons, C-odd Higgs bosons, and their mirrors. Diagonalization of this part of the mass matrix gives three massless generations of leptons. This is guaranteed by the rectangular nature of the mass matrix in this sector. The M_2-even part of the mass matrix contains C-even Higgs bosons, C-odd leptons, and their mirrors. Here the matrix is square and there is no kinematical restriction (such as the rectangular form of the mass matrix) which forces the existence of light Higgs bosons. Thus whether or not light Higgs doublets exist in the theory is a much more detailed question. In this section we determine the form of the Higgs-boson mass matrix to investigate this question.

In the notation $n=(1,m)$ for the C-even generations and $r=(2,s)$ for the C-odd generations (see Appendix A for the definition of C), the Higgs-boson mass matrix will involve the following fields: $\Phi_a=(\bar{H}_1,\bar{H}_m,H_1',H_m',\bar{l}_2,\bar{l}_s)$ and $\Phi_b=(H_1,H_m,\bar{H}_1',\bar{H}_m',l_2,l_s)$. The sources of mass terms in the theory below the intermediate scale are the D and F terms, where we neglect mass terms $O(m_i)$ from soft supersymmetry breaking. Now we state an important lemma regarding contributions of the D terms to the M_2-even mass matrix.

Lemma. There are no contributions to the M_2-even matrix from the D terms after spontaneous breaking below the intermediate scale when spontaneous breaking at the intermediate scale preserves matter parity.

We shall give the proof of the lemma by actual construction. The analysis of Goldstone and Higgs analysis after spontaneous breaking at the intermediate scale shows that the following combination of $SU(2)_L$ doublets is absorbed:[19]

$$\frac{\cos\theta}{\sqrt{2}}(l_1+\bar{l}\,_1^\dagger)-\frac{\sin\theta}{\sqrt{2}}(H_2'+\bar{H}\,_2'^\dagger)\ , \qquad (2.1)$$

where $\tan\theta=\epsilon=\langle \nu_2^c\rangle/\langle N_1\rangle$. The following combination of $SU(2)_L$ doublets becomes superheavy by gaining mass through the D term:

$$\frac{\cos\theta}{\sqrt{2}}(l_1-\bar{l}\,_1^\dagger)-\frac{\sin\theta}{\sqrt{2}}(H_2'-\bar{H}\,_2'^\dagger)\ . \qquad (2.2)$$

We note that the lepton doublets $(l_1, \bar{l}_1^\dagger)$ and Higgs doublets $(H'_2, \bar{H}'^\dagger_2)$ that enter in Eqs. (2.1) and (2.2) are all M_2 odd. Thus the absorption and the D-mass growth does not involve M_2-even Higgs bosons and leptons. Thus there is no D-mass growth for particles that enter in the M_2-even mass matrix. All of the M_2-even mass matrices thus arise from the F terms. From the renormalizable $(27)^3$ interactions one has, for the lepton mass terms,

$$W_{\rm ren} = -\lambda^3_{ijk}(H_i H'_j \langle N_k\rangle + H_i \langle \nu^c_j \rangle l_k) \ , \tag{2.3a}$$

and a similar relation holds for $(\overline{27})^3$ interactions. From the nonrenormalizable terms, one has

$$W_{\rm NR} = \sum \lambda_{ijkl}(\langle 27_i \overline{27}_j\rangle^{n-1}/M_C^{2n-3}) L_k \bar{L}_l \ . \tag{2.3b}$$

The M_2-even mass matrix now takes the form

$$\begin{array}{c} \\ l_2 \\ l_s \\ \bar{H}_1 \\ \bar{H}_m \\ H'_m \\ H'_1 \end{array} \begin{array}{cccccc} \bar{l}_2 & \bar{l}_{s'} & \bar{H}'_1 & \bar{H}'_{m'} & H_{m'} & H_1 \\ \end{array}$$

$$\begin{array}{c} l_2 \\ l_s \\ \bar{H}_1 \\ \bar{H}_m \\ H'_m \\ H'_1 \end{array} \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & M^3_{2m'} & M^3_{21} \\ 0 & M^1_{ss'} & 0 & 0 & M^3_{sm'} & M^3_{s1} \\ \bar{M}^3_{12} & \bar{M}^3_{1s'} & \bar{M}^2_{11} & \bar{M}^2_{1m'} & 0 & 0 \\ \bar{M}^3_{m2} & \bar{M}^3_{ms'} & \bar{M}^2_{m1} & \bar{M}^2_{mm'} & M^1_{mm'} & 0 \\ 0 & 0 & 0 & M^1_{mm'} & M^2_{mm'} & M^2_{m1} \\ 0 & 0 & 0 & 0 & M^2_{1m'} & M^1_{11} \end{array} \right] , \tag{2.4a}$$

where M_{ij} arises from $W_{\rm NR}$ (which always pairs a particle with a mirror particle), while the remaining mass terms arise from the $W_{\rm ren}$, so that

$$M^2_{ij} = -\lambda^3_{ij1}\langle N_1\rangle, \quad M^3_{ij} = -\lambda^3_{ij2}\langle \nu^c_2\rangle \ . \tag{2.4b}$$

Identical formulas hold for $\bar{M}^2$ and $\bar{M}^3$ in terms of $\bar{\lambda}^3$, $\bar{N}_1$, and $\bar{\nu}^c_2$. As will be seen in Sec. III, all the Higgs bosons become superheavy for arbitrary entries in Eq. (2.4). Detailed analyses verify that the low-energy theory without any constraints on the mass matrix of Eq. (2.4a) leads to phenomenologically disastrous results, in that no light Higgs bosons exist.[20] This phenomenological disaster can be averted, however, if we assume that the Higgs-boson mass matrix obeys certain constraints.

We shall deduce the necessary constraints on the Higgs-boson mass matrix by first assuming that the diagonalization of Eq. (2.4) produces a pair of light doublets H and H' and seeing what constraints this implies. We may expand the fields that enter the mass matrix of Eq. (2.4) as follows: The light Higgs doublet H' appears in the expansion of Φ'_a such that

$$\begin{aligned} &\bar{H}_n = \bar{V}_{nH} H' + \bar{V}_{n\alpha}\eta_\alpha, \quad H'_n = V'_{nH} H' + V'_{n\alpha}\eta_\alpha \ , \\ &l_r = V_{rH} H' + V'_{r\alpha}\eta_\alpha \ , \end{aligned} \tag{2.5}$$

where n_α are the superheavy eigenstates in the eigenmode expansion of Φ'_a. The condition that Eq. (2.4) possess a left massless eigenvector (H') reads

$$\bar{V}_{1H}\bar{M}^3_{12} + \bar{V}_{mH}\bar{M}^3_{m2} = 0 \ , \tag{2.6a}$$

$$V_{sH} M^1_{ss'} + \bar{V}_{1H}\bar{M}^3_{1s'} + \bar{V}_{mH}\bar{M}^3_{ms'} = 0 \ , \tag{2.6b}$$

$$\bar{V}_{1H}\bar{M}^2_{11} + \bar{V}_{mH}\bar{M}^2_{m1} = 0 \ , \tag{2.6c}$$

$$\bar{V}_{1H}\bar{M}^2_{1m'} + \bar{V}_{mH}\bar{M}^2_{mm'} + V'_{mH} M^1_{mm'} = 0 \ , \tag{2.6d}$$

$$\begin{aligned} &V_{2H} M^3_{2m'} + V_{sH} M^3_{sm'} + \bar{V}_{mH} M^1_{mm'} \\ &\qquad + V_{mH} M^2_{mm'} + V'_{1H} M^2_{1m'} = 0 \ , \end{aligned} \tag{2.6e}$$

$$V_{2H} M^3_{21} + V_{sH} M^3_{s1} + V'_{mH} M^2_{m1} + V'_{1H} M^2_{11} = 0 \ . \tag{2.6f}$$

Similarly, the light Higgs doublet H arises in the eigenmode expansion of Φ_a:

$$\begin{aligned} &H_n = V_{nH} H + V_{n\alpha}\omega_\alpha, \quad \bar{H}'_n = \bar{V}'_{nH} H + \bar{V}'_{n\alpha}\omega_\alpha \ , \\ &\bar{l}_r = \bar{V}_{rH} H' + \bar{V}_{r\alpha}\omega_\alpha \ , \end{aligned} \tag{2.7}$$

where ω_α are superheavy states in the eigenmode expansion. The conditions that Eq. (2.4) has a massless right eigenvector are

$$M^3_{21} V_{1H} + M^3_{2m} V_{mH} = 0 \ , \tag{2.8a}$$

$$M^3_{s1} V_{1H} + M^3_{sm} V_{mH} + M^1_{ss'} \bar{V}_{s'H} = 0 \ , \tag{2.8b}$$

$$\bar{M}^2_{1m} \bar{V}'_{mH} + \bar{M}^2_{11} V'_{1H} + \bar{M}^3_{1s} \bar{V}_{sH} + \bar{M}^3_{12} \bar{V}_{2H} = 0 \ , \tag{2.8c}$$

$$\begin{aligned} &M^1_{mm'} V_{m'H} + M^2_{mm'} \bar{V}_{m'H} + \bar{M}^2_{m1} \bar{V}'_{1H} + \bar{M}^3_{ms} \bar{V}_{sH} \\ &\qquad + \bar{M}^3_{m2} \bar{V}_{2H} = 0 \ , \end{aligned} \tag{2.8d}$$

$$M^2_{m1} V_{1H} + M^2_{mm'} V_{m'H} + M^1_{mm'} \bar{V}'_{m'H} = 0 \ , \tag{2.8e}$$

$$M^2_{11} V_{1H} + M^2_{1m'} V_{m'H} = 0 \ . \tag{2.8f}$$

In Sec. III we will derive the conditions that the Yukawa couplings must satisfy because of the constraints of Eqs. (2.6) and (2.8) in order that one has one pair of light Higgs doublets which is needed for an $N=1$ supersymmetric low-energy theory.[21]

III. CONDITIONS ON YUKAWA COUPLINGS FOR LIGHT HIGGS DOUBLETS

The condition for the existence of one light Higgs doublet H' is that there be a nonzero solution of Eq. (2.6). Of course, once one Higgs doublet H' becomes massless, the mass matrix of Eq. (2.4) is forced to develop a second light Higgs doublet of type H; that is, there will be a nonzero solution of Eq. (2.8) for $v_a \equiv \{V_{nH}, \bar{V}'_{nH}, \bar{V}_{rH}\}$. We begin by analyzing Eqs. (2.6). From Eqs. (2.6a) and

(2.6b) we have

$$\bar{V}_{1H}=0=\bar{V}_{mH}\ , \tag{3.1a}$$

provided

$$\bar{M}^{\,3}_{12}\bar{M}^{\,2}_{m1}-\bar{M}^{\,3}_{m2}\bar{M}^{\,2}_{11}\neq 0\ . \tag{3.1b}$$

Equation (2.6b) gives

$$V_{sH}=0\ , \tag{3.2a}$$

provided

$$\det(M^1_{ss'})\neq 0\ . \tag{3.2b}$$

We can recast the remaining constraint equations into a more transparent form. Here we combine Eqs. (2.6e) and (2.6f) into a single equation:

$$V_{rH}M^3_{n'r}+V'_{nH}M^2_{n'n}+\bar{V}_{mH}M^1_{mn'}=0\ , \tag{3.3}$$

where we have used the fact that $M^1_{m1}\equiv 0$. Next, following the analysis of Eqs. (3.1) and (3.2), we insert in the results of Eqs. (3.1a) and (3.2a) in Eq. (3.3), which gives

$$V_{2H}M^3_{n'2}+V'_{nH}M^2_{n'n}=0\ . \tag{3.4}$$

Finally, the remaining Eq. (2.6d) reads

$$V'_{mH}M^1_{mm'}=0\ . \tag{3.5}$$

We can combine Eqs. (3.4) and (3.5) into a single equation, which reads

$$M_{pq}\Psi_q=0\ , \tag{3.6a}$$

where

$$M_{pq}=\begin{bmatrix} M^2_{n'n} & M^3_{n'2} \\ M^1_{nm'} & 0 \end{bmatrix},\quad \Psi_q\equiv(V'_{nH},\bar{V}_{2H})\ . \tag{3.6b}$$

Thus we have

$$\Psi_q=0\ , \tag{3.7a}$$

provided

$$\det(M_{pq})\neq 0\ . \tag{3.7b}$$

Thus the constraints of Eqs. (3.1b), (3.2b), and (3.7b), which we choose to be the independent set, imply no light Higgs bosons and violation of one of these constraints will lead to a pair of light Higgs bosons.

Of the three possible constraints of Eqs. (3.1b), (3.2b), and (3.7b) that can be violated to generate a pair of light Higgs bosons, only the violation of the Eq. (3.7b) constraint is phenomenologically viable. Thus, for example, if Eq. (3.1b) holds and M_{pq} is nonsingular, then the light Higgs boson H' lies entirely in the $\bar{H}_n$ sector, while the violation of Eq. (3.2b) implies that H' lies entirely in the $\bar{l}_s$ sector. Thus, if M_{pq} is nonsingular, the light Higgs doublet H' lies in the $\overline{27}$ (i.e., $\bar{H}_n,\bar{l}_s$). However, since all the u quarks lie in the 27, there will be no mass growth for the u quarks after $SU(2)_L\times U(1)_Y$ breaking if the light Higgs boson H' lies in the $\overline{27}$. Further, since violation of more than one constraint will lead to the existence of more than one pair of Higgs doublets, which generally lead to flavor-changing neutral-current problems, one is led uniquely to the possibility that on phenomenological grounds one must discard violation of Eq. (3.1b) or (3.2b) and require that the light pair of Higgs bosons arise from violation of Eq. (3.7b), i.e.,

$$\det(M_{pq})=0\quad(\text{one null vector})\ . \tag{3.8}$$

$\det(M_{pq})$ in Eq. (3.8) depends on three types of masses, $M^1_{nm'}$, $M^2_{n'n}$, and $M^3_{n'2}$, which are all dynamically different. The satisfaction of Eq. (3.8) as a consequence of cooperative effects among the three types of masses will require a miracle (although string theory is known to exhibit miracles). We adopt here the approach that the satisfaction of Eq. (3.8) comes about by individual properties of M^1, M^2, or M^3. We have then the following three possibilities:

$$M^1_{mm'}=0\ , \tag{3.9a}$$

$$M^2_{m2}=0\quad(\lambda_{22n}=0)\ , \tag{3.9b}$$

$$M^3_{n1}=0\quad(\lambda_{11n}=0)\ . \tag{3.9c}$$

Condition Eq. (3.9a) is not viable on phenomenological grounds. Thus, using Eq. (3.9a) in Eqs. (2.8e) and (2.8f), one finds

$$V_{nH}=0,\quad \bar{V}'_{nH}\neq 0\ , \tag{3.10a}$$

while the remainder of Eqs. (2.8) implies the additional constraints

$$\bar{V}_{sH}=0,\quad \bar{V}_{2H}\neq 0\ . \tag{3.10b}$$

Thus we find that the H-type Higgs boson lies completely in the $(\bar{l}_2,\bar{H}'_n)$ sector of the mirror generations $\overline{27}$, which again makes all the u quarks massless. Thus case (3.9a) is not acceptable.

The analyses of cases of Eqs. (3.9b) and (3.9c) are more involved to check their phenomenological viability. For Eq. (3.9b) one finds $H'=l_2$, and H is a linear combination of the fields H_n, $\bar{H}'_n$, and $\bar{l}_r$. A detailed analysis shows that while the current neutrino mass limits can be accommodated here, it will lead to d quarks heavier than u quarks. This case is analyzed in greater detail in Appendix B. Case Eq. (3.9c) is phenomenologically the only acceptable one, and we discuss this case in detail in Sec. IV.

IV. ANALYSIS OF THE LIGHT-HIGGS-BOSON CONDITION $\lambda_{11n}=0$

The condition $\lambda_{11n}=0$ gives[22] $M^2_{1n}=M^2_{n1}=0$. From Eq. (2.4) this implies that H'_1 decouples from the mass matrix and is the light Higgs boson and, hence, $H'=H'_1$. Next, to find the H-type Higgs boson, one may invert Eq. (2.7) and write

$$H=\sum_n V_{nH}H_n+\sum_n \bar{V}'_{nH}\bar{H}'_n+\sum_r \bar{V}_{rH}\bar{l}_r\ , \tag{4.1a}$$

$$\sum_n (V_{nH})^2+\sum_r (\bar{V}'_{nH})^2+\sum_r (\bar{V}_{rH})^2=1\ . \tag{4.1b}$$

As discussed in Refs. 17 and 18, dangerously large neutrino mass terms arise from $(27)^3$ Yukawa interactions unless λ^3_{12r} is suppressed: i.e.,

$$\lambda^3_{12r}=\delta^2\tilde{\lambda}^3_{12r}\ , \tag{4.2}$$

where $\tilde{\lambda}^3_{12r}=O(\lambda^3_{m2r})$ and δ^2 is small. Thus we have here two parameters of expansion, i.e., $\epsilon\equiv\tan\theta$ and δ. We can use Eqs. (2.8) and (4.1b) to estimate the relative size of V_{nH}, $\bar{V}'_{nH}$, and $\bar{V}_{rH}$. We find

$$V_{1H}\sim 1,\quad V_{mH}=O(\delta^2)\ , \tag{4.3a}$$

$$\bar{V}'_H\sim\delta^2,\quad \bar{V}'_{mH}=O(\delta^2)\ , \tag{4.3b}$$

$$\bar{V}_{2H}\sim\delta^2/\epsilon,\quad \bar{V}_{sH}=O(\delta^2\epsilon)\ . \tag{4.3c}$$

Analysis of the neutrino masses depends crucially on the size of ϵ and δ in Eq. (4.3). Consistency with the current data requires[17,18]

$$\epsilon\lesssim 0.03-0.05,\quad \delta^2\approx\epsilon^3\ . \tag{4.4}$$

With the input of Eqs. (4.3) and (4.4), one sees from Eq. (4.1) that the light Higgs doublet H is mostly H_1 with a small admixture of $\bar{l}_2$ and much smaller components of other fields.

V. LOW-ENERGY INTERACTIONS OF LIGHT HIGGS BOSONS

An important constraint on the choice of alternatives for the light Higgs bosons H,H' is that they generate acceptable low-energy interactions. This means that the Higgs interaction structure with quarks and leptons should be what one expects in the (SUSY) standard model. Specifically, the Higgs structure should allow for the quark/lepton mass growth and allow for a quark/lepton mass hierarchy after $SU(2)_L\times U(1)_Y$ electroweak symmetry breaking. That this will turn out to be the case is nontrivial. In this section we analyze the phenomenological viability of case (3.9c).

To test the phenomenological viability of case (3.9c), we need to obtain the interaction structure of the light Higgs boson with the light-quark and light-lepton fields. The leptonic spectrum arises from diagonalization of the M_2-odd mass matrix in the fermionic sector, i.e., the terms of $\xi_a M_{ab}\xi'_b$, $a=1,\dots,m_a$, $b=1,\dots,m_b$, where $m_a=m_b+3$, $\xi_a=(\lambda_L^{(-)},l_n,\bar{H}_r,H'_r)$, and $\xi'_b=(\lambda_L^{(+)},\bar{l}_n,H_r,\bar{H}'_r)$. Here $\lambda_L^{(\pm)}=\pm(\lambda^L_{4\pm},\lambda^L_{b\pm})$ with $\lambda_{4\pm}=(\lambda_4\pm i\lambda_5)$, 2, etc., where λ^L_α are the $SU(3)_L$ gaugino fields. After N_1 and ν_2^c VEV growth, the leptonic mass matrix receives contributions from two sources: from the F and D parts. The D part in the fermionic sector consists of gaugino interactions

$$\mathcal{L}_{\text{gaugino}}=-ig_L\lambda_{4-}\gamma^0[L^3_{ir}(L_i^\dagger)^2_r-\bar{L}^2_{ir}(\bar{L}^\dagger)^3_r]$$
$$-ig_L\lambda_{6-}\gamma^0[L^3_{ir}(L_i^\dagger)^2_r-\bar{L}^2_{ir}(\bar{L}^\dagger)^3_r]\ . \tag{5.1}$$

In Eq. (5.1) fields with a dagger are Bose fields and those without are Weyl spinors. After spontaneous breaking at the intermediate, the relevant part of Eq. (5.1) that connects to the M_2-odd leptonic mass matrix is

$$g_L\lambda_L^{(-)\lambda}\gamma^0(\nu_1^c\tilde{H}^{\lambda'\dagger}_1+N_1 l_1^{\lambda\dagger})+\text{H.c.}\ , \tag{5.2}$$

where $\tilde{H}_1$ is the Higgsino field. Goldstone analysis of spontaneous breaking shows that the combination $(\sin\theta l_1-\cos\theta H'_1)$ is the fictitious Goldstone field, which in the unitary gauge can be set to zero. It is then only the orthogonal combination that remains in Eq. (5.2). The mass term from Eq. (5.2) after spontaneous breaking can be simulated by a mass term in the superpotential. The total lepton mass matrix in the M_2-odd sector, including also the mass growth from the F terms, is given by

$$\begin{aligned}\xi_a M^{(0)}_{ab}\xi'_b=&\lambda_L^{(-)}M_{45}(\cos\theta\bar{l}_1+\sin\theta\bar{H}'_1)+(l_1\cos\theta+H'_1\sin\theta)M_{45}\lambda_L^{(+)}\\&+l_m(M^1_{mm'}\bar{l}_{m'}+M^3_{m2}H_2+M^3_{ms'}H_{s'})+\bar{H}_2(\bar{M}^3_{21}\bar{l}_1+\bar{M}^3_{2m'}\bar{l}_{m'}+\bar{M}^2_{22}\bar{H}'_2+\bar{M}^2_{2s'}\bar{H}'_{s'})\\&+\bar{H}_s(\bar{M}^3_{s1}\bar{l}_1+\bar{M}^3_{sm'}\bar{l}_{m'}+M^1_{ss'}H_{s'}+\bar{M}^2_{s2}\bar{H}'_2+\bar{M}^2_{ss'}\bar{H}'_{s'})+H'_2(M^2_{22}H_2+M^2_{s'2}H_{s'})\\&+H'_s(M^2_{2s}H_2+M^2_{ss'}H_{s'}+M^1_{ss'}\bar{H}'_{s'})\ ,\end{aligned} \tag{5.3a}$$

where

$$M_{45}=g_L(N_1^2+\nu_2^{c2})^{1/2}\ . \tag{5.3b}$$

Diagonalization of Eq. (5.3) yields three massless eigenstates l_p ($p=1,2,3$), which are three linear combinations of the ξ_a. The remaining m_b components of ξ_a and all of the ξ'_b become supermassive. We expand ξ_a in terms of the light and heavy modes:

$$\begin{aligned}&l_n=l_pU^\dagger_{pn}+\chi_\alpha U^\dagger_{\alpha n},\quad H'_r=l_pU^\dagger_{pr}+\chi_\alpha U^\dagger_{\alpha r}\ ,\\&\bar{H}_r=l_p\bar{U}^\dagger_{pr}+\chi_\alpha\bar{U}^\dagger_{\alpha r},\quad \lambda_L^{(-)}=l_pU^\dagger_{p(-)}+\chi_\alpha U^\dagger_{\alpha(-)}\ ,\end{aligned} \tag{5.4}$$

where χ_α ($\alpha=1,\dots,m_b$) are the supermassive modes in the expansion, and $U^\dagger_{pn}$, etc., are the projection on to the light states l_p. One can carry out a δ,ϵ expansion[18,19] of $U^\dagger_{pn}$, etc., as in the Higgs case discussed in Sec. IV. The results of this analysis are summarized in Table I.

One can carry out a diagonalization for the remaining fields also. We have

$$\begin{aligned} e_n^c &= e_p^c U_{pn}^{1\dagger} + \chi_\alpha^e U_{\alpha n}^{1\dagger} , \\ q_n &= q_p U_{pn}^{2\dagger} + \chi_\alpha^q U_{\alpha n}^{2\dagger}, \quad u_n^c = u_p^c U_{pn}^{2\dagger} + \chi_\alpha^u U_{\alpha n}^{2\dagger} , \\ d_n^c &= d_p^c U_{pn}^{3\dagger} + \chi_\alpha^d U_{\alpha n}^{3\dagger} , \\ D_r^c &= d_p^c U_{pr}^{3\dagger} + \chi_\alpha^D U_{\alpha n}^{3\dagger}, \quad \bar{D}_r = d_p^c \bar{U}_{pr}^{3\dagger} + \chi_\alpha^{\bar{D}} \bar{U}_{\alpha n}^{3\dagger} , \end{aligned} \tag{5.5}$$

where e_p^c, q_p, u_p^c, d_p^c are the light fields and χ_α^e, etc., are supermassive fields.

The low-energy theory below the intermediate mass scale is obtained by integrating out the heavy fields. The low-energy interactions consist of two parts:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_W^{\text{eff}} + \mathcal{L}_{\text{gaugino}}^{\text{eff}} , \tag{5.6a}$$

where $\mathcal{L}_W^{\text{eff}}$ arises from a W_{eff} which has the form

$$\begin{aligned} W_{\text{eff}} = (\lambda_{pp'}^{(l)} H' e_p^c l_{p'} + \lambda_{pp'}^{(u)} H q_p u_p^c + \lambda_{pp'}^{(d)} H' q_p d_{p'}^c) + [(\lambda_p H l_p n_2 + \bar{\lambda}_p H l_p \bar{n}_2) + (m_1 n_2 \bar{n}_2 + \tfrac{1}{2} m_2 n_2 n_2 + \tfrac{1}{2} m_3 \bar{n}_2 \bar{n}_2) \\ + (m_4 n_1 \hat{\nu}_2^c + \tfrac{1}{2} m_5 n_1 n_1 + m_6 \hat{\nu}_2^c \hat{\nu}_2^c)] + W_{\text{seesaw}} \end{aligned} \tag{5.6b}$$

and (see Appendix A for notation)

$$\begin{aligned} \lambda_{pp'}^l &= -\lambda_{1mm'}^3 U_{pm}^{1\dagger} U_{p'm'}^\dagger , \\ \lambda_p &= -(c\lambda_{n2r}^3 U_{pr}^\dagger V_{nH} + s\bar{\lambda}_{1rr'}^3 U_{pr}^\dagger \bar{V}_{r'H}) , \\ \bar{\lambda}_p &= -(s\lambda_{1mm'}^3 U_{pm}^\dagger V_{m'H} + c\bar{\lambda}_{2rn} \bar{U}_{pr}^\dagger V'_{nH}) , \\ \lambda_{pp'}^{(u)} &= \lambda_{mnk}^4 U_{pm}^{2\dagger} V_{nH} U_{p'k}^{2\dagger} , \\ \lambda_{pp'}^{(d)} &= \lambda_{mnk}^4 U_{pm}^{2\dagger} V'_{nH} U_{p'n}^{3\dagger} . \end{aligned} \tag{5.6c}$$

In Eq. (5.6b), m_i are the soft-SUSY-breaking masses ~ 1 TeV and W_{seesaw} contributions arise from

$$\begin{aligned} &(\lambda_{ps}\langle H\rangle \nu_p N_s + \bar{\lambda}_{ps}\langle H\rangle \nu_p \bar{N}_s \\ &+ \lambda_{pm}\langle H\rangle \nu_p \nu_m^c + \bar{\lambda}_{pm}\langle H\rangle \nu_p \bar{\nu}_m^c) \\ &+ \tfrac{1}{2}[M_s N_s^2 + \bar{M}_s \bar{N}_s^2 + M_m (\nu_m^c)^2 + \bar{M}_m (\bar{\nu}_m^c)^2] , \end{aligned} \tag{5.7}$$

where λ_{ps}, $\bar{\lambda}_{ps}$, etc., can be related to the fundamental couplings in $(\mathbf{27})^3$, $(\overline{\mathbf{27}})^3$, and $(\mathbf{27}\,\overline{\mathbf{27}})^n$, etc., and M_s, $\bar{M}_s$, etc., are superheavy masses $\geq 10^{15}$ GeV. We may integrate out the superheavy fields N_s, $\bar{N}_s$, ν_m^c, and $\bar{\nu}_m^c$, and obtain a seesaw mass[23] for the neutrinos:

$$W_{\text{seeaw}} = \tfrac{1}{2}\nu_p \mu_{pp'} \nu_{p'} , \tag{5.8a}$$

where

$$\mu_{pp'} = -\lambda_{pp} \frac{1}{M_s} \lambda_{p's} \langle H\rangle^2 + \cdots . \tag{5.8b}$$

Comparison with the charged-lepton masses shows that

$$\mu_{pp'} \cong \frac{m_\lambda^2}{M_s} \cong 10^{-6}\ \text{eV}, \quad m_l \cong m_\tau . \tag{5.9}$$

The $\mathcal{L}_{\text{gaugino}}^{\text{eff}}$ in Eqs. (5.6a) arises from the light leptonic content of the $\lambda_L^{(-)}$ gaugino interactions (see Table I) and is

$$\begin{aligned} \mathcal{L}_{\text{gaugino}}^{\text{eff}} = g_L U_{p(-)}^\dagger l_p \gamma^0 [(U_{p'm}^{1\dagger} V_{mH} e_{p'}^c H^\dagger + s\bar{n}_2 H_2'^\dagger) \\ + \tfrac{1}{2}(c\bar{\nu}_2^c l_{p(-)}^\dagger + s n_1 l_{p(-)}^\dagger) \\ - (s\bar{V}'_{1H} H n_2^\dagger + c\bar{V}_{2H} H n_2^\dagger)] . \end{aligned} \tag{5.10}$$

In Eq. (5.10) the Hermitian conjugates on fields stand for Bose components of the corresponding chiral multiplet.

A detailed analysis of the lepton and quark mass spectra arising from Eq. (5.6) is presented elsewhere.[18] In the neutrino sector one finds that the ν_p $(p=1,2,3)$ mix with n_2 and $\bar{n}_2$, and one gets a 5×5 neutrino mass matrix. Diagonalization of this mass matrix gives two neutral states with masses ~ 1 TeV, one very light neutrino with mass $O(\mu_{pp'}) \sim 10^{-6}$ eV and two neutrinos with masses ~ 1 eV. In the charged-lepton sector, one finds that the experimental constraint[24] $m_\nu < 18$ eV leads to $m_e/m_\tau \cong 10^{-3}$. Interesting neutrino oscillations are predicted by the theory and are analyzed in detail in Ref. 18.

VI. DISCUSSION AND CONCLUSION

The analysis of Secs. III–V has shown that only the case Eq. (3.9c) has the possibility of generating a pair of Higgs doublets which can be in confirmity with the low-energy phenomenology. This means that the key condition that must be obeyed in any three-generation Calabi-Yau model to generate a pair of light Higgs doublets is[22]

TABLE I. Light-lepton content of ξ_a in orders of ϵ and δ.

p	$U_{p1}^\dagger$	$U_{pm}^\dagger$	$U_{p2}^\dagger$	$U_{ps}^\dagger$	$\bar{U}_{pr}^\dagger$	$U_{p(-)}^\dagger$
1	ϵ	δ^2/ϵ	1	δ^2	δ^2	$\delta^2\epsilon$
2,3	0	1	0	ϵ	ϵ	ϵ^2

$$\lambda_{11n}=0 \quad (\text{all } n) . \tag{6.1}$$

Equation (6.1) is a condition which significantly narrows down the available possibilities for a light pair of Higgs bosons. We illustrate the application of Eq. (6.1) by considering the case of the symmetric $CP^3\times CP^3/Z_3$ Tian-Yau three-generation model. Here, in the notation of Ref. 11, the C-even lepton states are $\lambda_{1+},\lambda_{3+},\lambda_5,\lambda_7,\lambda_{8+}$, while the C-odd states are $\lambda_{1-},\lambda_{3-},\lambda_6,\lambda_{8+}$, where $\lambda_{1\pm}=(\lambda_1\pm\lambda_2)/\sqrt{2}$. The $(\mathbf{27})^3$ interaction in the symmetric Tian-Yau model is given by[12]

$$\begin{aligned}W_{(27)^3}=&[-\tfrac{1}{2}(\lambda_{1+})^3-\tfrac{1}{2}(\lambda_{3+})^3+\tfrac{1}{2}(\lambda_{8+})^3+\tfrac{3}{2}\lambda_{1+}\lambda_{3+}\lambda_{8+}+16\lambda_7^2\lambda_5+2\lambda_{8+}^2\lambda_5]\\&+(2\lambda_{1-}\lambda_{3-}\lambda_5+2\lambda_{1-}\lambda_{3+}\lambda_6+2\lambda_{1+}\lambda_{3-}\lambda_6\\&\quad-\tfrac{3}{2}\lambda_{1-}\lambda_{3-}\lambda_{8+}-\tfrac{3}{2}\lambda_{1-}\lambda_{3+}\lambda_{8-}+\tfrac{3}{2}\lambda_{1+}\lambda_{3-}\lambda_{8-}-3\lambda_6^2\lambda_5-2\lambda_{8-}^2\lambda_5)\ .\end{aligned}\tag{6.2}$$

Now the N_1 VEV can arise only from the C-even states, i.e., $\lambda_{1+},\lambda_{3+},\lambda_5,\lambda_7,\lambda_{8+}$ (while the ν_2^c VEV can arise from $\lambda_{1-},\lambda_{3-},\lambda_6,\lambda_{8-}$). From Eq. (6.2) we find that the L_1 generation which contains the N_1 VEV cannot be a linear combination which involves λ_{1+}, λ_{3+}, λ_{8+}, or λ_7 since that will violate Eq. (6.1). Thus L_1 is uniquely determined to be λ_5. This means that it is only when spontaneous symmetry breaking arises from N VEV growth of λ_5 that below the intermediate scale we will have a light pair of doublets. Thus, for the symmetric Tian-Yau model, we find that Eq. (6.1) can be satisfied and a light pair of Higgs doublets exist.

Now the fact that the light Higgs boson H' is identified to be H_5 has important implications. From Eq. (6.2) we find that the λ_5 couplings with all the C-odd channels are either exactly vanishing (e.g., the coupling of $\lambda_5\lambda_{1-}\lambda_{8-}$) or are of order unity. It can then be shown that the first possibility leads to a massless electron (since m_e is δ^2/ϵ and $\delta=0$ in this case), while the second possibility leads to $\delta\sim 1$ in contradiction with Eq. (4.4), which are in contradiction with the current existing upper limits on the neutrino masses. Thus the light-Higgs-boson analysis we have presented here leads to the fact that the symmetric Tian-Yau model is not viable phenomenologically.

A similar analysis of the Higgs structure can be carried out for the Gepner model.[4] Here the lepton multiplet content of the Gepner model consists of seven C-even lepton generations (L_1–L_7) and two C-odd lepton generations (L_8,L_9). The coupling structure of the theory is[25]

$$\begin{aligned}W_{(27)^3}=&\mu L_3L_4L_5+\mu^3L_3L_7L_7+\tfrac{1}{3}(L_1L_5L_6+L_1L_4L_6)\\&+\frac{\mu}{3}(L_2L_4L_6+L_2L_5L_6)\\&+\mu^2L_2L_6L_7+\frac{\lambda^2}{3}L_8L_9L_2\ ,\end{aligned}\tag{6.3}$$

where $\mu^3=0.556$ and $\lambda^3=1.15$. The analysis of the Higgs structure using Eq. (6.1) shows that the light Higgs boson H' must be linear combinations of one of the following sets:

$$\begin{aligned}&(H_1',H_2',H_3'),\ (H_1',H_7'),\ (H_3',H_6')\ ,\\&(H_4',H_7'),\ (H_5',H_7')\ .\end{aligned}\tag{6.4}$$

We note that since the only C-even–C-odd–C-odd coupling is $L_2L_8L_9$ in Eq. (6.3), one is presented in Eq. (6.4) with the following two possibilities. (I) Here H' does not contain a component of H_2'. In this case, $\delta=0$. Now, as stated already, the electron mass can be shown to obey[18] $m_e\sim\delta^2/\epsilon$, so that $m_e=0$ in this case, and further this result is not modified at the loop level. (II) Here H' does contain a component of H_2', but then from Eq. (6.3) one finds that $\delta\sim 1$ and, hence, is in violation of Eq. (4.4). Thus we find that the Gepner model is also not phenomenologically viable; i.e., it either leads to a massless electron or a large neutrino mass, in violation of the current upper bound on neutrino masses (a more detailed analysis is presented elsewhere). These flaws can be overcome by moving away from the symmetric point in the moduli space and considering the more general moduli-dependent manifolds. For the more general models, Eq. (6.1) is still expected to be a strong constraint on their complex structure.

ACKNOWLEDGMENTS

This research was supported in part under National Science Foundation Grants Nos. PHY-8706873 and PHY-8907887 and the Texas Advanced Research Program No. 3043.

APPENDIX A: NOTATION AND RENORMALIZABLE INTERACTION STRUCTURE

The $(\mathbf{27})^3$ undergoes the following decomposition under $[\mathrm{SU}(3)]^3$:

$$(\mathbf{27})_i=L_{r_i}^{l}(1,3,\bar{3})\oplus Q_{l_i}^{a}(3,\bar{3},1)\oplus(Q^c)_{a_i}^{r}(\bar{3},1,3)\ , \tag{A1}$$

where L,Q,Q^c are the lepton, quark, and antiquark nonets, $a,l,r=1,2,3$ are the $\mathrm{SU}(3)_{C,L,R}$-triplet indices, and i is the generation index. The conventional particle content of these nonets is

$$\begin{aligned}&L_i=[l=(\nu_e,e)_i;\ e_i^c;\ H_i=(\Phi_i^\dagger,\Phi_i^0)\ ;\\&\qquad H_i'=(\Phi_i^{0'},\Phi_i^{-'});\ \nu_i^c,N_i]\ ,\\&Q_i=[q_i^a=(u^a,d^a)_i;\ H_3\equiv D_i^a]\ ,\\&\qquad Q_i^c=(u_{ai}^c;\ d_{ai}^c;\ H_{3i}'\equiv D_{ai}^c)\ ,\end{aligned}\tag{A2}$$

where q_i, l_i, H_i, and H_i' are the quark, lepton, and the usual SUSY Higgs doublets and D^a, D_a^c are the usual color Higgs triplets of SU(5) SUSY grand unified theory. Further, N_i and ν_i^c are SU(5) singlets, while N_i is also an SO(10) singlet.

In the $[\mathrm{SU}(3)]^3$ phase, i.e., before the intermediate mass scale breaking, the renormalizable $(\mathbf{27})^3$ superpotential consists of the following parts.

(i) A baryon-number-violating interaction

$$\lambda_{ijk}^1 \epsilon_{aa'a''} d_i^a u_j^{a'} D_k^{a''} + \lambda_{ijk}^2 \epsilon^{aa'a''} u_{ai}^c d_{a'j}^c D_{a''k}^c \ , \tag{A3a}$$

which enters in proton decay.

(ii) An interaction among the lepton multiplets of the form

$$\lambda_{ijk}^3 (-H_i^\lambda H'_{\lambda j} N_k - H_i^\lambda \nu_j^c l_{\lambda k} + H'_{\lambda i} e_j^c l_k^\lambda) \ , \tag{A3b}$$

which enters in the analysis of the mass spectrum of the Higgs boson and the leptons.

(iii) An interaction which generates masses for the quarks

$$-\lambda_{ijk}^4 (D_i^a N_j D_{ak}^c + D_i^a \nu_j^c d_{ak}^c - q_{\lambda i}^a H_j^\lambda u_{ak}^c - q_i^{a\lambda} H'_{\lambda j} d_{ak}^c) \ . \tag{A3c}$$

(iv) A lepton-number-violating interaction

$$\lambda_{ijk}^4 (D_i^a e_j^c u_{ak}^c - q_i^{a\lambda} l_{\lambda j} D_{ak}^c) \ . \tag{A3d}$$

A similar decomposition holds for the $(\overline{\mathbf{27}})^3$ interactions of the mirror generations.

The interaction structure of Eq. (A3) holds for a wide class of three-generation Calabi-Yau models in the $[\mathrm{SU}(3)]^3$ phase after flux breaking. The details of the coupling structures λ_{ijk}^{1-4} will depend on the specific three-generation Calabi-Yau model, i.e., whether it is $CP^3 \times CP^3/Z_3$, $CP^3 \times CP^2/Z_3 \times Z_3'$, etc., as well as on the complex structure of the model.

As discussed in the Introduction, a necessary ingredient for proton stability is matter parity invariance of the interaction structure of the theory, both renormalizable as well as nonrenormalizable. We define matter parity M_2 by $M_2 = CU_z$, where U_z is an element of $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$, which reverses the sign of $\mathrm{SU}(2)_{L',R}$ doublets, leaving others unchanged, and $C^2 = 1$. It is useful to obtain combinations of generations which are C odd or C even. We shall use the notation $(r;n)$ to define (C odd; C even) generations.

APPENDIX B: ANALYSIS OF THE LIGHT-HIGGS BOSON CONDITION $\lambda_{22n} = 0$

We analyze here the light-Higgs-boson condition

$$\lambda_{22n}^3 = 0, \quad M_{n2}^3 = 0 = M_{22}^2 \ . \tag{B1}$$

Here, from Eq. (2.6), we find that

$$\bar{V}_{nH} = 0 = V'_{nH}, \quad V_{sH} = 0 \ , \tag{B2}$$

$$V_{2H} \neq 0 \ . \tag{B3}$$

Thus we find that the only light component of H' comes from l_2, i.e.,

$$H' = l_2 \ . \tag{B4}$$

Next, we turn to the H sector. Here we can utilize Eq. (2.8) to determine all of the objects V_{nH}, $\bar{V}'_{nH}$, and $\bar{V}_{sH}$ in terms of $\bar{V}_{2H}$. Thus we have

$$V_{n'H} = -\bar{V}_{m'H} M_{nm'}^1 M_{nn'}^{2-1} \ , \tag{B5}$$

$$\bar{V}_{s'H} = \bar{V}_{m'H} M_{nm'}^1 M_{nn'}^{2-1} M_{n's}^3 M_{ss'}^{1-1} \ , \tag{B6}$$

$$\bar{V}'_{m'H} = \bar{V}_{2H} (\bar{M}^2_{11} \bar{M}^3_{m2} - \bar{M}^3_{12} \bar{M}^2_{m1}) \times (Q_{1m'} \bar{M}^2_{m1} - Q_{2m'} \bar{M}^2_{11})^{-1} \ , \tag{B7a}$$

where

$$\bar{V}'_{1H} = -\bar{V}_{2H} (Q_{1m'} \bar{M}^3_{m2} - Q_{sm'} \bar{M}^3_{12}) \times (Q_{1m'} \bar{M}^2_{m1} - Q_{2m'} \bar{M}^2_{11}) \ , \tag{B7b}$$

where

$$Q_{1m'} \equiv \bar{M}^2_{am'} + M_{nm'}^1 M_{nn'}^{2-1} M_{n's}^3 M_{ss'}^{1-1} \bar{M}^3_{1s'} \ , \tag{B8}$$

$$Q_{2m'} \equiv \bar{M}^2_{\bar{m}m'} - M_{nm'}^1 M_{nm'}^{2-1} M_{mm'}^1 + M_{nm'}^1 M_{nn'}^{2-1} M_{n's}^3 M_{ss'}^{1-1} \bar{M}^3_{ms'} \ . \tag{B9}$$

The light Higgs boson H is then given by

$$H = \sum_n V_{nH} H_n + \sum_n \bar{V}'_{nH} \bar{H}'_n + \sum_r \bar{V}_{rH} \bar{l}_r \ , \tag{B10a}$$

$$\sum_n (V_{nH})^2 + \sum_n (\bar{V}'_{nH})^2 + \sum_r (\bar{V}_{rH})^2 = 1 \ . \tag{B10b}$$

Now it is easily established from Eqs. (B6)–(B10) that

$$\begin{aligned} &\bar{V}_{2H} \sim 1, \quad \bar{V}_{sH} = O(\epsilon^2) \ , \\ &V_{nH} = O(\epsilon), \quad \bar{V}'_{nH} = O(\epsilon) \ . \end{aligned} \tag{B11}$$

Equations (B11) imply that the light Higgs boson H lies mostly in the $\bar{l}_2$ sector.

The mass spectrum in the lepton sector below the intermediate mass scale under the constraint of Eq. (B1) can also be analyzed. The analysis gives

$$\begin{aligned} &U_{p1}^\dagger = O(\epsilon^2), \quad U_{pm}^\dagger \sim 1 \ , \\ &\bar{U}_{pr}^\dagger = O(\epsilon), \quad U_{pr}^\dagger = O(\epsilon) \ , \\ &U_{p(-)}^\dagger = O(\epsilon^2) \ . \end{aligned} \tag{B12}$$

Using Eqs. (B11) and (B12), one finds an interaction $g_L \epsilon^3 l_p \gamma^0 \bar{n}_2 H'^\dagger$, so that the effective $\mu \to e\gamma$ interaction that results from Eq. (5.6) is

$$\mathcal{L}_{\text{eff}} \cong e \frac{1}{4m_\mu} F^{\mu\nu} \bar{\mu} \sigma_{\mu\nu} (a_R P_R + a_L P_L) e + \text{H.c.} \ , \tag{B13}$$

where

$$a_R \approx \frac{\alpha}{8\pi^2} \frac{\epsilon^4}{\sin^2\theta_W} \left[\frac{m_e}{m_\tau} \right] \frac{m_\mu^2}{M_{\tilde{\nu}}^2} L \left[\frac{\tilde{m}_e^2}{M_{\tilde{\nu}}^2} \right] \ ,$$

$$a_L \approx \frac{\alpha}{8\pi^2} \frac{\epsilon^4}{\sin^2\theta_W} \left[\frac{m_e}{m_\tau} \right] \frac{m_e m_\mu}{M_{\tilde{\nu}}^2} L \left[\frac{\tilde{m}_e^2}{M_{\tilde{\nu}}^2} \right] \ ,$$

and $L(x)$ is a loop function, which is given by

$$L(x)=(x-1)^{-4}(\tfrac{1}{3}+\tfrac{1}{2}x-x^2+\tfrac{1}{6}x^3+x\ln x)\ . \qquad \text{(B14)}$$

The current experimental limit on a is[26] $a \lesssim 2.4\times10^{-13}$, which gives $\epsilon \lesssim 1.0$ However, the basic problem with this alternative is that it is difficult to get a heavy top quark in this case. This is so because $m_t \approx \Sigma_n \lambda^4 V_{nH}\langle H\rangle$, and since from Eq. (B11) one has $V_{nH}=O(\epsilon)$, an abnormally large value of λ^4, i.e., $(\lambda^4)^2/4\pi \gtrsim 50$, is needed to get an $m_t \sim 100$ GeV.

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