# Importance of vector mesons in nucleon form factors

M. T. Jeong
*Department of Physics, Dongshin University, Naju 520-180, Korea*

Il-T. Cheon
*Department of Physics, Yonsei University, Seoul 120-749, Korea*
(Received 5 April 1990; revised manuscript received 21 December 1990)

The effects of vector mesons on the nucleon form factors have been investigated in the cloudy bag model. Vector mesons improve remarkably the theoretical curves of nucleon form factors as well as the values of nucleon magnetic moments.

## I. INTRODUCTION

Nucleon form factors provide very precise information on the nucleon structure and, therefore, play a key role to examine the quality of models and for sensitive tests of various subnuclear theories.

Tegen and Weise[1] analyzed the nucleon form factors in the quark-confining potential model, but failed to reproduce the neutron electric form factor. In addition, their calculation violates gauge invariance and, as a result, happens to contain undesirable parameters to preserve the nucleon charge. Although similar calculations were carried out by Barik and Dash,[2] the results were also unsuccessful. Recent calculations[3,4] in the cloudy bag model[5] (CBM) show somewhat reasonable results for the nucleon form factor in the wide momentum-transfer region $q^2 \leq 20\ \mathrm{fm}^{-2}$. However, the nucleon magnetic form factors were evaluated independently of the electric form factors; i.e., charge conservation was ignored in the calculation of magnetic form factors. Therefore, these calculations suffer from lack of consistency. Alberto *et al.*[6] tried to evaluate the nucleon electromagnetic and axial form factors using the Lagrangian of the linear chiral soliton model. They obtained good agreement only for $q^2 \leq 10\ \mathrm{fm}^{-2}$. However, vector mesons were ignored in their calculations. Although they improved recently the chiral soliton model by including vector mesons[7] in the massive Yang-Mills approach, the Wess-Zumino term was still ignored, which could make significant contributions to the magnetic moments as is shown in the present work. Along similar lines, Meissner *et al.*[8] made extensive calculations of the nucleon form factors in a nonlinear chiral meson theory with vector mesons and obtained remarkable results. The nucleon form factors are well reproduced except for the neutron electric form factor, while the nucleon charge and magnetic radii are yet far from the experimental values. Their results show an irreconcilability between the form factors and quantities such as rms radii, magnetic moments, and axial coupling constant; i.e., better results for the form factors induce less satisfactory results for other quantities.

Since there is no complete calculation of these quantities on the basis of the chiral bag model with vector mesons, it may be very interesting to explore the importance of roles vector mesons play in the calculation of the nucleon form factors and magnetic moments within this model with explicit valence quark degrees of freedom.

## II. DERIVATION OF THE LAGRANGIAN WITH VECTOR MESONS

Let us start by writing down the Lagrangian density of the CBM (Ref. 9) as

$$\begin{aligned}\mathcal{L}(x)=&[\bar{q}(x)i\gamma^{\mu}\partial_{\mu}q(x)-B]\theta_V\\ &-\tfrac{1}{2}\bar{q}(x)\exp[i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)/f_{\pi}]q(x)\Delta_S\\ &+\tfrac{1}{2}[D_{\mu}\boldsymbol{\pi}(x)]^2+\tfrac{1}{2}m_{\pi}^2\boldsymbol{\pi}^2(x)\ ,\end{aligned} \tag{1}$$

where $q(x)$ and $\pi(x)$ are quark and pion fields, respectively, $B$ is the vacuum energy constant, $D_\mu$ is the covariant derivative, and $f_\pi$ is the pion decay constant. $\theta_V$ and $\Delta_S$ denote the volume and surface $\delta$ functions.

The Lagrangian-containing vector mesons can be derived by a local gauge transformation of the CBM Lagrangian (1); i.e., $\rho$ mesons are coupled to SU(2) isospin and $\omega$ mesons are coupled to U(1) baryon number. Therefore, the minimal coupling

$$\partial_{\mu}\rightarrow\partial_{\mu}-\frac{i}{2}g(\boldsymbol{\tau}\cdot\boldsymbol{\rho}_{\mu}+\omega_{\mu}) \tag{2}$$

induces the effective Lagrangian

$$\begin{aligned}\tilde{\mathcal{L}}(x)=&\bar{q}(x)i\gamma^{\mu}\left[\partial_{\mu}-\frac{i}{2}g\boldsymbol{\tau}\cdot\boldsymbol{\rho}_{\mu}(x)-\frac{i}{2}g\omega_{\mu}(x)\right]q(x)\theta_V-B\theta_V\\ &-\tfrac{1}{2}\bar{q}(x)\exp[i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)/f_{\pi}]q(x)\Delta_S+\tfrac{1}{2}[\tilde{D}_{\mu}\boldsymbol{\pi}(x)]^2+\tfrac{1}{2}m_{\pi}\boldsymbol{\pi}(x)^2\\ &-\tfrac{1}{4}\boldsymbol{\rho}_{\mu\nu}(x)\cdot\boldsymbol{\rho}^{\mu\nu}(x)-\tfrac{1}{4}\omega_{\mu\nu}(x)\omega^{\mu\nu}(x)+\tfrac{1}{2}m_V^2[\boldsymbol{\rho}_{\mu}(x)\cdot\boldsymbol{\rho}^{\mu}(x)+\omega_{\mu}(x)\omega^{\mu}(x)]-\frac{3g^2}{8\pi^2 f_{\pi}}\epsilon_{\mu\nu\alpha\beta}\partial^{\mu}\omega^{\nu}(x)\boldsymbol{\rho}^{\alpha}(x)\cdot\partial^{\beta}\boldsymbol{\pi}(x)\ ,\end{aligned} \tag{3}$$

 

where

$$\boldsymbol{\rho}_{\mu\nu}=\partial_\mu\boldsymbol{\rho}_\nu-\partial_\nu\boldsymbol{\rho}_\mu+g\boldsymbol{\rho}_\mu\times\boldsymbol{\rho}_\nu\ , \tag{4}$$

$$\omega_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu\ , \tag{5}$$

$$\tilde{D}_\mu\boldsymbol{\pi}=D_\mu\boldsymbol{\pi}+g\boldsymbol{\rho}_\mu\times\boldsymbol{\pi}\ . \tag{6}$$

The last term in Eq. (3) is the so-called Wess-Zumino term, which describes a certain anomaly process containing $\pi$, $\rho$, and $\omega$.[10]

## III. INTERACTION HAMILTONIAN AND QUANTIZED MESON FIELDS

With this effective Lagrangian (3), one can derive the interaction Hamiltonian of the vector mesons and quarks as

$$H_{\rm int}^V=\frac{g}{2}\bar{q}(x)\gamma^\mu(\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu+\omega_\mu)q(x)\theta_V\ . \tag{7}$$

Quantization of the vector-meson fields is actually not the same as that of the pion field. In order that the condition $\partial_\mu\rho^\mu=0$ be satisfied, the time component of the canonical conjugate momentum of the vector mesons should be eliminated in favor of the space component. This can be done along the lines given in Refs. 7 and 11. Thus Fourier transformations of quantized $\rho$ and $\omega$ fields are expressed as

$$\rho_\lambda^\mu(x)=\int d^3k_V N_V\epsilon^\mu(k_V)\times[\exp(ik_Vx)c_\lambda(k_V)+\exp(-ik_Vx)c_\lambda^\dagger(k_V)]\ , \tag{8}$$

$$\omega^\mu(x)=\int d^3k_V N_V\epsilon^\mu(k_V)\times[\exp(ik_Vx)d(k_V)+\exp(-ik_Vx)d^\dagger(k_V)]\ , \tag{9}$$

and yield the quantized interaction Hamiltonians for $\rho qq$ and $\omega qq$,

$$H_{\rm int}^\rho=i\sum_\lambda\int d^3k_V[W_\lambda(k_V)c_\lambda(k_V)+W_\lambda^\dagger(k_V)c_\lambda^\dagger(k_V)]\ , \tag{10}$$

$$H_{\rm int}^\omega=i\int d^3k_V[Z(k_V)d(k_V)+Z^\dagger(k_V)d^\dagger(k_V)]\ , \tag{11}$$

where vertex functions are defined as

$$W_\lambda(k_V)=-igN_V\sum_{j=1}^{3}b_j^\dagger\tau_\lambda[S_0(k_V)\epsilon^0(k_V)+S_1(k_V)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(k_V)\times\mathbf{k}_V]b_j\ , \tag{12}$$

$$Z(k_V)=-igN_V\sum_{j=1}^{3}b_j^\dagger[S_0(k_V)\epsilon^0(k_V)+S_1(k_V)\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(k_V)\times\mathbf{k}_V]b_j\ , \tag{13}$$

with the bag spatial form factors

$$S_0(k_V)=\frac{N^2}{2}\int_0^R dr\, r^2[j_0^2(\omega_0 r/R)+j_1^2(\omega_0 r/R)]j_0(k_V r)\ , \tag{14}$$

$$S_1(k_V)=N^2\int_0^R dr\, r^2 j_0(\omega_0 r/R)j_1(\omega_0 r/R)j_1(k_V r)\ . \tag{15}$$

The gauge coupling constant $g$ in Eq. (7) can be determined by the $\rho\to\pi\pi$ decay process and satisfy the well-known Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation $\sqrt{2}f_\pi g=m_\omega=m_\rho$.[12] In Eqs. (12) and (13), $N_V=[(2\pi)^3 2\omega_V]^{-1/2}$, $b_j^\dagger(b_j)$ is a quark creation (annihilation) operator, $\epsilon^\mu(k_V)$ is the polarization vector, $c_\lambda^\dagger(c_\lambda)$ and $d^\dagger(d)$ are $\rho$- and $\omega$-meson creation (annihilation) operators, respectively, and the energy and momentum of the vector meson are given by $\omega_V$ and $k_V$. $\lambda$ denotes a charge index. In Eqs. (14) and (15), $N$ is the normalization constant, $j_l$ is the spherical Bessel function, and $R$ is the bag radius.

## IV. NUCLEON ELECTROMAGNETIC FORM FACTORS

### A. Derivation of the form factors

As was done in the previous calculations,[3-5] the baryon electromagnetic current $j^\mu(x)$ can be derived by requiring the local U(1) gauge invariance after introducing the photon field. Evaluating the Feynman diagrams shown in Fig. 1, one can find the nucleon electromagnetic form factors as a sum of contributions from each diagram,

$$G_A^N(q^2)=G_{Aa}^N(q^2)+G_{Ab}^N(q^2)+G_{Ac}^N(q^2)+G_{Ad}^N(q^2)\ , \tag{16}$$

where the symbol $A$ denotes $E$ or $M$ for the electric or magnetic form factor. Since the nucleon charge is preserved by the gauge invariance, we should have $G_E^p(0)=1$ and $G_E^n(0)=0$. As is shown in the diagram (c1) of Fig. 1, the $\gamma\pi\pi$ vertex is treated in the vector-dominance model. The form factors associated with each diagram are given as

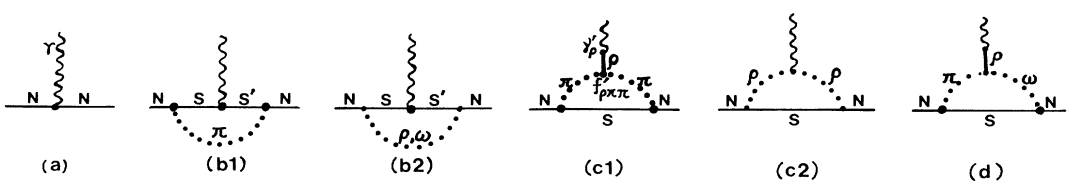


FIG. 1. Feynman diagrams for the nucleon electromagnetic form factors.

$$G_{Ea}^{N}(q^2)=Z_2 A Q_0(q) , \tag{17}$$

$$G_{Eb}^{N}(q^2)=\frac{Q_0(q)}{36\pi^2}\sum_{SS'}\left[\left[\frac{f_Q}{m_\pi}\right]^2\int_0^\infty dk_\pi \frac{B_{SS'}k_\pi^4 U_{NS}(k_\pi R)U_{NS'}(k_\pi R)}{\omega_\pi(\omega_{NS}+\omega_\pi)(\omega_{NS'}+\omega_\pi)} +2g^2\int_0^\infty dk_V\left[\frac{C_{SS'}k_V^2 S_1^2(k_V)}{\omega_V(\omega_{NS}+\omega_V)(\omega_{NS'}+\omega_V)}+\frac{D_{NN}k_V^4 S_0^2(k_V)}{m_V^2\omega_V^3}\right]\right] , \tag{18}$$

$$G_{Ec}^{N}(q^2)=\frac{1}{72\pi^3}\sum_{S}\left[\frac{\gamma_\rho' f_{\rho\pi\pi}'}{em_\rho^2}\left[\frac{m_\rho^2}{m_\rho^2+q^2}\right]\left[\frac{f_Q}{m_\pi}\right]^2\int_0^\infty d^3k_\pi \frac{E_S k_\pi^2(\hat{\mathbf{k}}_\pi\cdot\hat{\mathbf{k}}'_\pi)U_{NS}(k_\pi R)U_{NS}(k'_\pi R)}{(\omega_\pi+\omega'_\pi)(\omega_{NS}+\omega_\pi)(\omega_{NS}+\omega'_\pi)} +2g^2\int_0^\infty d^3k_V\frac{E_S(\hat{\mathbf{k}}_V\cdot\hat{\mathbf{k}}'_V)S_1(k_V)S_1(k'_V)}{(\omega_V+\omega'_V)(\omega_{NS}+\omega_V)(\omega_{NS}+\omega'_V)}\right] , \tag{19}$$

$$G_{Ed}^{N}(q^2)=\frac{1}{72\pi^3}\left[\frac{q^2}{8\pi^2 f_\pi}\right]\left[\frac{f_Q}{m_\pi}\right]\left[\frac{m_\rho^2}{m_\rho^2+q^2}\right]\sum_S\int_0^\infty d^3k_\pi k_\pi k'_\pi(\hat{\mathbf{k}}_\pi\times\hat{\mathbf{q}})^2 U_{NS}(k_\pi R) \times\left[\frac{F_S k_\pi(\omega_\pi+\omega'_V+\omega_{NS})S_i(k'_V)}{\omega_\pi\omega'_V(\omega_\pi+\omega'_V)(\omega_\pi+\omega_{NS})(\omega'_V+\omega_{NS})}+\frac{G_N S_0(k'_V)}{\omega_\pi\omega'_V(\omega_\pi+\omega'_V)}\right] , \tag{20}$$

$$G_{Ma}^{N}(q^2)=Z_2 A m_N Q_1(q) , \tag{21}$$

$$G_{Mb}^{N}(q^2)=\frac{m_N}{81\pi^2}Q_1(q)\sum_{SS'}\left[\left[\frac{f_Q}{m_\pi}\right]^2\int_0^\infty dk_\pi\frac{B_{SS'}k_\pi^4 U_{NS}(k_\pi R)U_{NS'}(k_\pi R)}{\omega_\pi(\omega_{NS}+\omega_\pi)(\omega_{NS'}+\omega\pi)} +2g^2\int_0^\infty dk_V\left[\frac{C_{SS'}k_V^2 S_1^2(k_V)}{\omega_V(\omega_{NS}+\omega_V)(\omega_{NS'}+\omega_V)}+\frac{D_{NN}k_V^4 S_0^2(k_V)}{m_V^2\omega_V^3}\right]\right] , \tag{22}$$

$$G_{Mc}^{N}(q^2)=\frac{m_N}{144\pi^3}\sum_S\left[\left[\frac{\gamma_\rho' f_{\rho\pi\pi}'}{em_\rho^2}\right]\left[\frac{m_\rho^2}{m_\rho^2+q^2}\right]\left[\frac{f_Q}{m_\pi}\right]^2 \times\int_0^\infty d^3k_\pi\frac{E_s k_\pi^2(\hat{\mathbf{k}}_\pi\times\hat{\mathbf{q}})^2 U_{NS}(k_\pi R)U_{NS}(k'_\pi R)(\omega_\pi+\omega'_\pi+\omega_{NS})}{\omega_\pi\omega'_\pi(\omega_\pi+\omega'_\pi)(\omega_{NS}+\omega_\pi)(\omega_{NS}+\omega'_\pi)} +2g^2\int_0^\infty d^3k_V\frac{E_S(\hat{\mathbf{k}}_V\times\hat{\mathbf{q}})^2 S_1(k_V)S_1(k'_V)(\omega_V+\omega'_V+\omega_{NS})}{\omega_V\omega'_V(\omega_V+\omega'_V)(\omega_{NS}+\omega_V)(\omega_{NS}+\omega'_V)}\right] , \tag{23}$$

$$G_{Md}^{N}(q^2)=\frac{m_N}{144\pi^3}\left[\frac{g^2}{8\pi^2 f_\pi}\right]\left[\frac{f_Q}{m_\pi}\right]\left[\frac{m_\rho^2}{m_\rho^2+q^2}\right] \times\sum_S\int_0^\infty d^3k_\pi k_\pi U_{NS}(k_\pi R)(\hat{\mathbf{k}}_\pi\cdot\hat{\mathbf{k}}_V)\left[\frac{F_S S_1(k'_V)}{(\omega_\pi+\omega'_V)(\omega_\pi+\omega_{NS})(\omega'_V+\omega_{NS})}+\frac{G_N k'_V S_0(k'_V)}{\omega_\pi^2\omega_V'^2}\right] , \tag{24}$$

with $\mathbf{k}'_{V,\pi}=\mathbf{k}_{V,\pi}+\mathbf{q}$ and $\omega_{NS}=m_S-m_N$. The suffixes $S$ and $S'$ denote $N$ or $\Delta$. We have also introduced vertex functions with the photon momentum transfer $q$ in the forms

$$Q_0(q)=N^2\int_0^R dr\, r^2[j_0^2(\omega_0 r/R)+j_1^2(\omega_0 r/R)]j_0(qr) , \tag{25}$$

$$Q_1(q)=N^2\int_0^R dr\, r^3 j_0(\omega_0 r/R)j_1(\omega_0 r/R)j_1(qr)/(qr) . \tag{26}$$

$U_{NS}(k_\pi R)$ is defined with the $\pi NS$ form factor[3] as

$$U_{NS}(k_\pi R)=[j_0(k_\pi R)+j_2(k_\pi R)]g_{\pi NS}(k_\pi) . \tag{27}$$

Coefficients in Eqs. (17)–(24) are given by $A=1(0)$, $B_{NN}=25(50)$, $C_{NN}=34(50)$, $B_{\Delta\Delta}=C_{\Delta\Delta}=128(-32)$, $D_{NN}=45(9)$, $E_N=50(-50)$, $E_\Delta=32(-32)$, $F_N=160(70)$, $F_\Delta=32(32)$, and $G_N=240(-240)$ for the proton (neutron) electric form factor and $A=4(-8/3)$, $B_{NN}=25(-100)$, $C_{NN}=-2(-82)$, $B_{N\Delta}=640(-640)$, $B_{\Delta\Delta}=C_{\Delta\Delta}=640(-160)$, $D_{NN}=351(-189)$, $E_N=100(-100)$, $E_\Delta=32(-32)$, $F_N=160(40)$, $F_\Delta=-64(-64)$, and $G_N=240(-120)$ for the proton (neutron) magnetic form factor. The renormalization function $Z_2$ is given in the following form when the vector mesons are involved:

$$Z_2=\left\{1+\frac{1}{12\pi^2}\sum_S\left[\left[\frac{f_Q}{m_\pi}\right]^2\int_0^\infty dk_\pi\frac{a_{NS}k_\pi^4U_{NS}^2(k_\pi R)}{\omega_\pi(\omega_\pi+\omega_{NS})^2}+2g^2\int_0^\infty dk_V\left[\frac{b_{NS}k_V^2S_1^2(k_V)}{\omega_V(\omega_V+\omega_{NS})^2}+\frac{c_{NN}k_V^4S_0^2(k_V)}{m_V^2\omega_V^3}\right]\right]\right\}^{-1}, \tag{28}$$

where $a_{NN}=25$, $a_{N\Delta}=32$, $b_{NN}=28$, $b_{N\Delta}=32$, and $c_{NN}=18$.

Figure 1(d) denotes the Wess-Zumino term, and the coupling of $\gamma$ to the $\pi\rho\omega$ vertex has been treated by the vector-meson-dominance procedure. The $\pi qq$ coupling constant $f_Q=0.483$ and the effective $Vqq$ coupling constant $q=4.1$ should be used for numerical calculations. Furthermore, the coupling constant $(\gamma'_\rho f'_{\rho\pi\pi}/em_\rho^2)=0.80$ is determined in a context of the gauge invariance.

### B. Results of numerical calculations

The results of numerical calculations with $R=0.95$ fm are shown in Figs. 2 and 3. The dotted curves are contributions from the quark, i.e., Figs. 1(a) and 1(b), and the dot-dashed and dot-dot-dashed curves show contributions from the pion and vector mesons, respectively. The contribution of the diagram 1(d) is included into the dot-dot-dashed curve. It is obvious that the vector-meson contribution is larger than the pion contribution in larger region of momentum transfer. The solid curve is the result in which all contributions are included, and the dashed curve is obtained without vector mesons. The data of the neutron electric form factor can usually be extracted by using the wave function of deuteron. The Hamada-Johnson wave function[15] gives the open circles, while the McGee wave function[16] gives the solid circles. Agreement of our result with the data is excellent. The importance of the vector meson is obvious. On the other hand, the nucleon magnetic form factors are shown in Fig. 3. Although the neutron magnetic form factor can well be reproduced, the proton magnetic form factor shows less agreement. One can see that the disagreement is reduced by introducing the vector mesons in the processes.

The nucleon magnetic moments can be obtained from the values of the magnetic form factors at $q^2=0$. Our results are 2.26 and $-1.59$ for the proton and neutron, respectively. Of course, these values are not in agreement with the data, $\mu_p=2.7928$ and $\mu_n=-1.9130$. As was pointed out,[4,5,7] the c.m. correction plays rather an important role to determine the nucleon magnetic moments in the CBM bag model. This correction is estimated as $\mu_{\text{c.m.}}=\mu_Q(1+\langle p^2\rangle/2m^2)$, where $\mu_Q$ is the quark contribution and $m=5.9/R$.[4,15] There are two ways to esti-

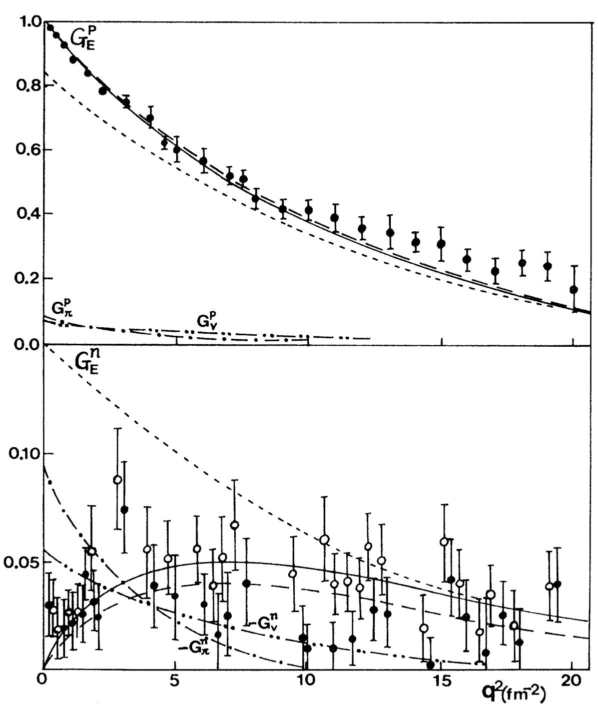


FIG. 2. Nucleon charge form factors obtained with $R=0.95$ fm. The solid and dashed curves show the results with and without vector-meson contribution, respectively. The dotted curve is the quark contribution. The dot-dashed and dot-dot-dashed curves are the pion and vector-meson contributions, respectively. Experimental data were taken from Ref. 3.

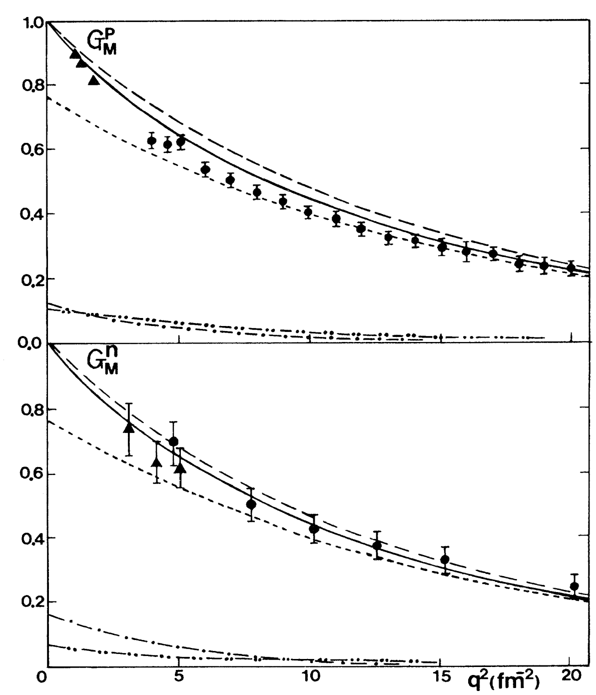


FIG. 3. Nucleon magnetic form factors normalized to unity at $q^2=0$. Explanations of each curve are same as those given Fig. 2. Experimental data were taken form Refs. 3 and 14.

TABLE I. Nucleon magnetic moments. The values given in parentheses are for the neutron.[a]

| | Diagrams | Bag Radius $R$ (fm) 0.90 | 0.95 | 1.00 |
|---|---|---|---|---|
| $\mu_Q$ | (1a),(1b) | 1.68(−1.16) | 1.77(−1.22) | 1.85(−1.28) |
| $\mu_\pi$ | (1c) | 0.30(−0.30) | 0.27(−0.27) | 0.25(−0.25) |
| $\mu_\rho$ | (1c) | 0.04(−0.04) | 0.04(−0.04) | 0.04(−0.04) |
| $\mu_{\rm WZ}$ | (1d) | 0.21(−0.09) | 0.18(−0.08) | 0.16(−0.07) |
| $\mu_{\rm c.m.}$ | method I | 0.43(−0.30) | 0.45(−0.31) | 0.48(−0.33) |
| | method II | 0.30(−0.21) | 0.32(−0.22) | 0.34(−0.23) |
| $\mu$ | method I | 2.66(−1.89) | 2.71(−1.92) | 2.78(−1.96) |
| | method II | 2.53(−1.80) | 2.58(−1.83) | 2.64(−1.86) |

[a] $\mu_p^{\rm expt}=2.792\,845\,6(11)$, $\mu_n^{\rm expt}=-1.913\,042\,11(88)$.

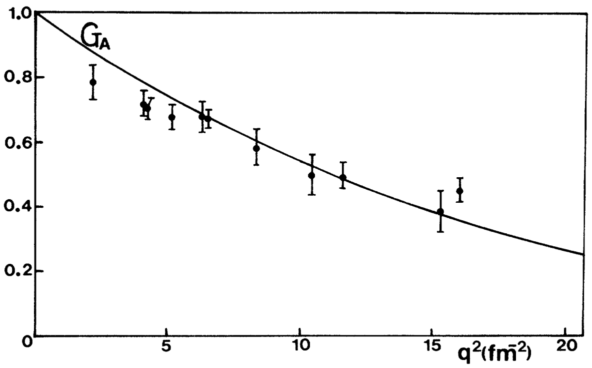


FIG. 4. Axial form factor normalized to unity at $q^2=0$. Experimental data were taken from Ref. 19.

mate the mean momentum spread in the bag. One is to use the formula[4] $\langle p^2\rangle=17.85/R^2$, which yields $\langle p^2\rangle=0.51m^2$ and the other is $\langle p^2\rangle=0.36m^2$ in Ref. 8. We refer to the first one as method I and the second as method II. The results are given in Table I.

The nucleon radii have also been calculated with the c.m. correction,[8] $\langle r^2\rangle=\langle r^2\rangle_Q(1+\langle p^2\rangle/3m^2)$. The results are listed in Table II.

## V. DERIVATION OF THE AXIAL FORM FACTOR

In the present framework, we are able to evaluate the axial form factor which also provides sensitive informations of the baryon structure. The axial form factor $G_A(q)$ is generally given in the following integral:

$$\int d^3x\langle f|A_\mu(x)|i\rangle\exp(iq\cdot x)=\bar{u}_f\frac{\tau}{2}[\gamma^\mu\gamma_5 G_A(q^2)+q^\mu\gamma_5 G_P(q^2)]u_i\ , \tag{29}$$

where $u$ is the spinor, $G_P(q^2)$ is the pseudoscalar form factor, and the axial-vector current is derived from the Lagrangian (3) as

$$A_\mu=\bar{q}(x)\gamma_\mu\gamma_5\frac{\tau}{2}q(x)\theta_V+f_\pi\partial_\mu\pi(x)\ . \tag{30}$$

The second term which plays an important role to satisfy partial conservation of axial-vector current (PCAC) vanishes in the matrix element when the pion field exists inside the bag.[20] As there is no contribution from the diagrams (1c) and (1d), we find the axial form factor as a sum of two terms coming from the diagrams (1a) and (1b):

$$G_A(q^2)=G_{Aa}(q^2)+G_{Ab}(q^2)\ , \tag{31}$$

$$G_{Aa}(q^2)=\tfrac{5}{3}Z_2Q_2(q)\ , \tag{32}$$

$$G_{Ab}(q^2)=\frac{Q_2(q)}{324\pi^2}\sum_{SS'}\left[\left[\frac{f_Q}{m_\pi}\right]^2\int_0^\infty dk_\pi\frac{I_{SS'}k_\pi^4U_{NS}(k_\pi R)U_{NS'}(k_\pi R)}{\omega_\pi(\omega_{NS}+\omega_\pi)(\omega_{NS'}+\omega_\pi)}+2g^2\int_0^\infty dk_V\left[\frac{J_{SS'}k_V^2S_1^2(k_V)}{\omega_V(\omega_{NS}+\omega_V)(\omega_{NS'}+\omega_V)}+\frac{K_{NN}k_V^4S_0^2(k_V)}{m_V^2\omega_V^3}\right]\right]\ , \tag{33}$$

where

$$Q_2(q)=N^2\int_0^R dr\,r^2\{[j_0^2(\omega_0r/R)-j_1^2(\omega_0r/R)]j_0(qr)+2j_1^2(\omega_0r/R)j_1(qr)(qr)^{-1}\}\ , \tag{34}$$

TABLE II. Nucleon properties for $R=0.95$ fm. The rms radii are given in unit of fm. Experimental values were taken Refs. 8 and 19. Symbols I and II denote the methods used to evaluate the c.m. correction.

| c.m. | $r_E^p$ | $(r_E^n)^2$ | $r_M^p$ | $r_M^n$ | $r_A$ | $\mu_p$ | $\mu_n$ | $g_A$ |
|---|---|---|---|---|---|---|---|---|
| no | 0.78 | −0.180 | 0.77 | 0.80 | 0.59 | 2.26 | −1.59 | 1.11 |
| I | 0.84 | −0.126 | 0.82 | 0.85 | 0.64 | 2.71 | −1.92 | 1.30 |
| II | 0.82 | −0.143 | 0.81 | 0.84 | 0.62 | 2.58 | −1.83 | 1.24 |
| Expt. | 0.86±0.01 | −0.119±0.004 | 0.86±0.06 | 0.88±0.07 | 0.68±0.02 | 2.79±0.0 | −1.91±0.0 | 1.262±0.005 |

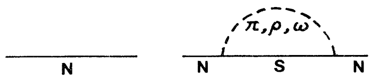


FIG. 5. Diagrams for the bag energy.

and $I_{NN}=125$, $I_{N\Delta}=1280$, $I_{\Delta\Delta}=800$, $J_{NN}=80$, $J_{N\Delta}=1280$, $J_{\Delta\Delta}=800$, and $K_{NN}=135$. The result of numerical calculation is shown in Fig. 4. Agreement is excellent. We can evaluate the axial-vector coupling constant at $q^2=0$, i.e.,

$$g^A \equiv G_A(0)=1.11 , \tag{35}$$

which is slightly smaller than the experimental value $g_A^{\text{expt}}=1.262\pm0.005$.[21]

The c.m. correction improves it by a factor of $(1+\langle p^2\rangle/3m^2)$ and results in 1.34 and 1.28 with methods I and II, respectively. The nucleon axial radius can be obtained by

$$\langle r_A^2\rangle = -\frac{6}{g_A}\frac{d}{dq^2}G_A(q^2)\bigg|_{q^2=0} ,$$

which yields $\langle r_A^2\rangle^{1/2}=0.59$ fm. When the c.m. correction is considered, we obtain 0.64 and 0.62 fm, respectively with methods I and II. The experimental value[19] is $0.68\pm0.02$ fm. The bag radius $R=0.95$ fm has been used in this calculation.

## VI. BAG ENERGY

Since the vector mesons are taken into account in the present model, it is very interesting to explore the effects of vector mesons on the bag energy.

In the limit of a static, spherical cavity, the energy of the MIT bag is expressed as[22]

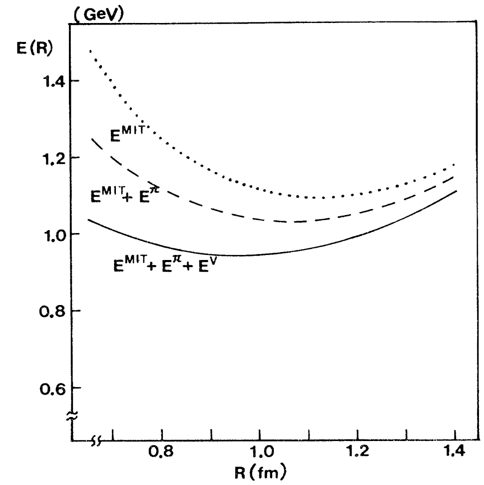


FIG. 6. Bag energy vs the bag radius. The dotted curve shows the result of the MIT bag. The dashed and solid curves are obtained with the pion alone and pion plus vector mesons, respectively.

$$E^{\text{MIT}}(R)=\frac{3\omega_0}{R}+\frac{4\pi B}{3}R^3-\frac{Z_0}{R}-\frac{\alpha_S}{R}$$
$$=\frac{3\omega_0-Z'}{R}+\frac{4\pi B}{3}R^3 . \tag{36}$$

When the pion and vector mesons $\rho$ and $\omega$ are taken into account, we can obtain the bag energy in the form

$$E(R)=E^{\text{MIT}}(R)+E^{\pi}(R)+E^{V}(R) , \tag{37}$$

by evaluating the diagram Fig. 5, where

$$E^{\pi}(R)=\frac{-1}{12\pi^2}\left[\frac{f_Q}{m_\pi}\right]^2\sum_S\int_0^\infty dk_\pi\frac{a_{NS}k_\pi^4U_{NS}^2(k_\pi R)}{\omega_\pi(\omega_{NS}+\omega_\pi)} , \tag{38}$$

$$E^{V}(R)=\frac{-2g^2}{12\pi^2}\sum_S\int_0^\infty dk_V\left[\frac{b_{NS}k_V^2S_1^2(k_V)}{\omega_V(\omega_{NS}+\omega_V)}+\frac{c_{NS}k_V^4S_0^2(k_V)}{m_V^2\omega_V(\omega_{NS}+\omega_V)}\right] . \tag{39}$$

The values of $a_{NS}$, $b_{NS}$, and $c_{NS}$ are given below [Eq. (28)]. Other functions are defined in Secs. III and IV.

The results are shown in Fig. 6. In this calculation we have used $Z'=1.42$ and $B^{1/4}=136$ MeV. It is interesting to see that the minimum radius goes down when the mesons are involved. This arises from the mesons pressure around the bag. We find the minimum nucleon mass $m_N=940$ MeV around the bag radius $R=0.95$ fm.

## VII. CONCLUSION

The present model is successful in reproducing the experimental results for the nucleon electromagnetic form factors, axial form factor, magnetic moments, axial-vector coupling constant, and the rms radii. It should be stressed that vector mesons are very important for improving the theoretical results; particularly the Wess-Zumino term makes significant contribution to the proton magnetic moment.

In the present calculations, we have taken into account the c.m. corrections only for the quantities determined at the vanishing momentum transfer $q^2=0$. The c.m. correction seems generally to be important in the low-momentum-transfer region, and it may make appreciable contributions to the electromagnetic form factors in the small-momentum-transfer region, probably $q^2\le 3$ fm.[2] However, no reliable method is available so far to evaluate the c.m. correction in the region of $q \neq 0$; effects of the center-of-mass motion have been ignored in the present calculations of the form factors, as was done in most works published up to date. The stability of the bag is well restored around $R = 0.95$ fm in our model.

## ACKNOWLEDGMENTS

This work was supported by the Korea Science and Engineering Foundation.

[1] R. Tegen and W. Weise, Z. Phys. A **314**, 357 (1984).
[2] N. Barik and B. K. Dash, Phys. Rev. D **34**, 2092 (1986).
[3] Y. J. Oh, K. J. Kong, and Il-T. Cheon, Phys. Rev. D **37**, 2570 (1988).
[4] S. Lee, K. J. Kong, and Il-T. Cheon, Phys. Lett. B **202**, 21 (1988).
[5] S. Theberge and A. W. Thomas, Nucl. Phys. **A393**, 252 (1983); S. Theberge, A. W. Thomas, and G. A. Miller, Can. J. Phys. **60**, 59 (1982).
[6] P. Alberto, E. Ruiz Arriola, M. Fiolhais, F. Grummer, J. N. Urbano, and K. Goeke, Phys. Lett. B **208**, 75 (1988).
[7] E. Ruiz Arriola, P. Alberto, K. Goeke, and J. N. Urbano, Phys. Lett. B **236**, 381 (1990).
[8] U.-G. Meissner, N. Kaiser, and W. Weise, Nucl. Phys. **A466**, 685 (1987); U.-G. Meissner, N. Kaiser, H. Weigel, and J. Schechter, Phys. Rev. D **39**, 1956 (1989).
[9] G. A. Miller, A. W. Thomas, and S. Theberge, Phys. Lett. **91B**, 192 (1980); A. W. Thomas, S. Theberge, and G. A. Miller, Phys. Rev. D **24**, 216 (1981).
[10] U.-G. Meissner and N. Kaiser, Z. Phys. A **325**, 267 (1986).
[11] T. D. Lee, *Particle Physics and Introduction to Field Theory* (Harwood Academic, New York, 1981), Chap. 4.
[12] K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. **16**, 225 (1966); Riazuddin and Fayyazuddin, Phys. Rev. **158**, 1447 (1967); V. Bernard, B. Hiller, and W. Weise, Phys. Lett. B **205**, 16 (1988).
[13] T. Janssens, R. Hofstadter, E. B. Hughes, and M. R. Yearians, Phys. Rev. **142**, 922 (1966); D. T. Drickey and L. N. Hand, Phys. Rev. Lett. **9**, 1327 (1966); D. Benakas, D. Drickey, and D. Frerejacque, Phys. Rev. **148**, 1327 (1966).
[14] E. B. Hughes, T. A. Griffy, M. R. Yearian, and R. Hofstadter, Phys. Rev. **139**, B458 (1965).
[15] T. Hamada and J. D. Johnson, Nucl. Phys. **34**, 382 (1962).
[16] I. McGee, Phys. Rev. **151**, 772 (1966).
[17] V. Vento, Phys. Lett. **107B**, 5 (1981); C. W. Wong, Phys. Rev. D **24**, 1416 (1981).
[18] J. F. Donoghue and K. Johnson, Phys. Rev. D **21**, 1975 (1980).
[19] A. del Guerra *et al.*, Nucl. Phys. **B107**, 65 (1976); W. A. Mann *et al.*, Phys. Rev. Lett. **31**, 844 (1973).
[20] W. Weise, *Quarks and Nuclei* (World Scientific, Singapore, 1984).
[21] B. Bopp, D. Dubbers, L. Hornig, E. Klemt, J. Last, and H. Schutze, Phys. Rev. Lett. **56**, 919 (1986).
[22] P. J. Mulders and A. W. Thomas, J. Phys. G **9**, 1159 (1983).