Model dependence of the cosmological upper bound on the Higgs-boson mass

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We consider an electroweak scenario for baryon asymmetry generation in an extended model with two Higgs doublets. The upper bound on the mass of the lightest Higgs boson is obtained. It is shown to coincide with that of the theory with one Higgs doublet, namely, $m_H < m_{cr} \simeq 45$ GeV, if the masses of all Higgs bosons do not exceed the mass of the W boson substantially. Otherwise the upper bound is shown still to exist but is greater than m_{cr} and depends on the actual values of the scalar-field self-coupling constants.

I. INTRODUCTION

Observation of the Higgs boson and the top quark, the necessary ingredients of the standard electroweak theory, is a challenging problem of high-energy physics nowadays. To summarize the recent progress in the Collider Detector at Fermilab[1] (CDF) and CERN LEP-I (Ref. 2) one has the following lower bounds:

$$m_t > 77 \text{ GeV } (95\% \text{ C.L.}) ,$$

$$m_H > 24 \text{ GeV } (95\% \text{ C.L.}) .$$

At the same time a theoretical cosmological upper bound on the Higgs-boson mass M_H (Ref. 3) within the standard electroweak theory has been reported recently:

$$m_H < m_{cr} \simeq 45 \text{ GeV} . \tag{1}$$

The first inequality comes from the requirement that the baryon asymmetry of the universe[4] (BAU) which might be created in an electroweak first-order phase transition[3] must not be washed out by the anomalous reactions with B nonconservation[5] (for a review see Ref. 6). Equation (1) implies also an upper bound on the top-quark mass[7] $m_t < m_t^{cr}$. The value m_t^{cr} comes from the vacuum stability bounds[8] which give in the one-loop approximation[8] $m_t^{cr} \simeq 80$ GeV. The more detailed treatment of the full renormalization-group-improved Higgs potential involving two-loop effects gives $m_t^{cr} \simeq 120$ GeV.[9] According to Sher,[10] for a top quark heavier than

$$m_t^{cr} = 95 \text{ GeV} + 0.60\, m_H \tag{2}$$

the effective potential becomes unbounded from below. The statement holds true for a Higgs-boson mass smaller than 240 GeV, just as in our case.

Suppose that in the near future the Higgs boson and top quark with these properties will not be found. What will be the meaning of this fact for the problem of B nonconservation and BAU generation?

Let us first take the standard electroweak theory. With the mass out of the range (1) electroweak baryogenesis seems impossible. Therefore, one should invoke some other mechanism for B nonconservation, say a grand unified theory (GUT). In this case we should require also $B-L$ nonconservation, because only the $B-L$ part of the asymmetry survives[5,11] in the source of universe expansion due to the equilibrium character of anomalous reactions. $B-L$ nonconservation implies, in turn, unusual proton decay modes, neutron and neutrino oscillations.

The other possibility is to consider somehow an extended electroweak theory, where (1) may not be valid. In particular, extension of the Higgs sector may have its impact on both the experimental and theoretical constraints mentioned above. The reason is the appearance of a variety of new decay modes which should be taken into account in the experimental searches. As far as theoretical bounds are concerned the additional scalar fields in general imply new self-coupling constants, while the relations between the mass spectrum of the theory and Lagrangian parameters are modified.

In the present paper we consider the two-Higgs-doublet version of the electroweak theory in order to explore the model dependence of the cosmological bound on the Higgs-boson mass. The question of the bound on the top-quark mass mass due to vacuum stability in extended models was discussed recently by Sher,[10] and the bound (2) is weakened significantly if the scalars (or at least some scalars) have masses greater than m_W.

An extended Higgs sector involving for instance two doublets of the scalar fields is of particular interest since it is a typical feature of supersymmetric versions of the electroweak model.[12,13]

The paper is organized as follows. In the first section we present the general framework of the derivation of the cosmological upper bound based on the electroweak scenario of the BAU production.[3]

In Sec. II we review the general properties of the model with two doublets.

The third section is devoted to the case of light Higgs bosons. We find that in order for the BAU to survive after the electroweak phase transition the mass of the lightest neutral Higgs boson must be bounded from above by the same magnitude as in the case of one Higgs doublet.

In the last section we consider theory with Higgs-boson masses comparable to m_W and larger. We show that the mass of one of the Higgs bosons still should be cosmologically bounded from above but this bound depends crucially on the scalar self-coupling constants and may be relatively large.

II. THE ORIGIN OF THE COSMOLOGICAL UPPER BOUND ON THE HIGGS-BOSON MASS

Consider the standard electroweak theory with one Higgs doublet following Refs. 3 and 7. Suppose that at the moment of electroweak phase transition a nonzero baryon asymmetry $\Delta=\Delta_{\rm in}$ is formed with $\Delta\equiv n_B/n_\gamma$, here n_B is a density of baryonic charge and n_γ is a density of relic photons. The source of the baryon excess is not important here in essence.

Immediately after the phase transition at temperature T_c the rate of anomalous fermion number nonconservation is still appreciable, if the value of the scalar field condensate $v(T)$ is relatively small: $v(T_c)\ll v_0\simeq 250$ GeV. This leads to the dilution of real baryons. In the range of temperatures

$$m_W(T)<T<\frac{m_W(T)}{\alpha_W}\ , \tag{3}$$

where $m_W(T)=g_W v(T)/2$, the ratio Γ of anomalous electroweak fermion number nonconservation may be evaluated semiclassically[5,14] with the use of the sphaleron configuration.[15] In this approximation the leading Boltzmann exponent and the preexponential factor given by the determinant of small fluctuations around the sphaleron may be represented in the form

$$\Gamma=A(T)\exp[-E^{\rm sph}(T)/T]\ , \tag{4}$$

where $E^{\rm sph}(T)$ is the temperature-dependent effective sphaleron mass incorporating those terms in the preexponential factor which are singular in the high-temperature limit, $A(T)$ is the rest of the determinant, which was estimated in Ref. 14. According to Refs. 15 and 5 one has

$$E^{\rm sph}(T)=\frac{4\pi v(T)}{g_W}B\ , \tag{5}$$

where $B=B(\lambda/g_W^2)$ is a numerical coefficient: $B(0)=1.56$, $B(\infty)=2.72$. The rate Γ enters the kinetic equation for the dilution of baryonic charge. The integration of kinetic equation gives the suppression factor S in the form[3,7]

$$S\equiv\Delta_{\rm final}/\Delta_{\rm in},\quad S=\exp[-\Omega\zeta^6\exp(-\zeta)]\ , \tag{6}$$

where

$$\zeta\equiv E^{\rm sph}(T_c)/T_c \tag{7}$$

and the term Ω is given by

$$\Omega=\frac{2\times10^{-5}}{B^7}\frac{M_{\rm Pl}}{T_c} \tag{8}$$

with $M_{\rm Pl}$ being the Planck mass. The final value of the baryon asymmetry should coincide with the observations $\Delta_{\rm final}=\Delta_{\rm obs}=10^{-8}-10^{-10}$. At the same time the largest value of $\Delta_{\rm in}$ which may be generated within the standard electroweak theory is $\Delta_{\rm in}=10^{-5}-10^{-6}$.[3,16] Therefore, we require $S_{(m_H,m_t)}\geq 10^{-5}$. The last inequality was found[3,7] to imply

$$\zeta(m_H,m_t)\geq 45 \tag{9}$$

with rather high accuracy due to the exponential behavior of S in (6). Equation (9) is the origin of the upper bound on the Higgs-boson mass versus top-quark mass.

To obtain the function $\zeta(m_H,m_t)$ one should fix first the temperature dependence of the scalar field condensate $v(T)$. The effective potential $V_T(\phi)$ of the scalar field at high temperatures $T\gg g\phi$, $\sqrt{\lambda}\,\phi$ looks like

$$V_T(\phi)=-\tfrac{1}{2}\mu^2(T)\phi^2-\delta T\phi^3+\tfrac{1}{8}\lambda\phi^4+C\phi^4\ln\frac{\phi^2}{AT^2}\ , \tag{10}$$

$$\mu^2(T)=\mu^2-\omega T^2,\quad \omega\equiv\tfrac{1}{4}(2m_W^2+m_Z^2+2m_t^2)\ , \tag{11}$$

$$\delta\equiv\frac{1}{4\pi}(2m_W^3+m_Z^3)\ , \tag{12}$$

$$C\equiv\frac{1}{64\pi^2}(6m_W^4+3m_Z^4-12m_t^4)\ , \tag{13}$$

where all the particle masses are expressed in the units of v_0 and A is some number of order 1, $\phi^2\equiv2\langle\varphi^\dagger\varphi\rangle_{\rm vac}$. The expressions (11)–(13) are valid in the case of relatively light Higgs boson: $m_H\ll m_W$, $\lambda\ll g_W^2$.

The phase transition from the unbroken phase with $v=0$ to the broken one with $v\neq0$ occurs when the barrier separating those two phases disappears. It implies the vanishing curvature of the effective potential at the origin, $\mu^2(T_c)=0$. The critical temperature is given by

$$T_c^2=4\mu^2/(2m_W^2+m_Z^2+2m_t^2)\ . \tag{14}$$

The corresponding value of the scalar field condensate is

$$v(T_c)=6\frac{\delta}{\lambda}T_c\ . \tag{15}$$

It follows from Eq. (15) that the light Higgs boson implies strongly first-order phase transition while the values $\lambda\geq g_W^2$ would imply the weak first-order or the second-order phase transition.[17] Then in the region of coupling constants,

$$g_W^2>\lambda>\frac{g_W^4}{16\pi^2}\ , \tag{16}$$

the critical temperature satisfies (3), so the sphaleron estimates are applicable. The last step is the combining of expression (15) for temperature-dependent sphaleron mass (5) with inequality (9).

Note, that the obtained magnitude of the bound on the Higgs-boson mass is insensitive to the value of the top-quark mass[7] and, moreover it does not alter after the calculating of the renormalization-group corrections to the scalar potential. The reason is that the expansion parameter $\alpha_W^2(m_t/m_W)^4\ln(\phi/v_0)$ is small in the domain of $\varphi \simeq v_0$.

To summarize, in order to get a cosmological upper bound on the Higgs-boson mass we should find a sphaleron mass just after the first-order phase transition. Consider electroweak theory with two SU(2) doublets of complex scalar fields φ_1 and φ_2 with U(1) hypercharge $Y=1$. The most general scalar potential which spontaneously breaks $SU(2)_L \times U(1)$ down to $U(1)_{EM}$ looks like

$$\begin{aligned} V = &-\mu_1^2\varphi_1^\dagger\varphi_1 - \mu_2^2\varphi_2^\dagger\varphi_2 + \mu_3^2\varphi_1^\dagger\varphi_2 + \mu_4^2\varphi_2^\dagger\varphi_1 \\ &+ \tfrac{1}{2}\lambda_1(\varphi_1^\dagger\varphi_1)^2 + \tfrac{1}{2}\lambda_2(\varphi_2^\dagger\varphi_2)^2 + h_1(\varphi_1^\dagger\varphi_2)(\varphi_2^\dagger\varphi_1) \\ &+ h_2(\varphi_1^\dagger\varphi_1)(\varphi_2^\dagger\varphi_2) + \tfrac{1}{2}h_3[(\varphi_1^\dagger\varphi_2)^2 + (\varphi_2^\dagger\varphi_1)^2] \ . \end{aligned} \tag{17}$$

In order to eliminate Higgs-mediated flavor-changing neutral currents it is convenient to impose on the Lagrangian one of the discrete symmetries

$$\varphi_2 \to -\varphi_2, \quad u_R^i \to -u_R^i, \quad i=1,\ldots,n_f \ , \tag{18}$$

where u_R^i stands for the up quarks and n_f is the number of generations, or

$$\varphi_2 \to -\varphi_2 \ .$$

The first symmetry implies that the φ_1 field gives masses to down quarks and leptons while φ_2 is coupled only to up quarks. The second means that all fermions couple to the same doublet and none couple to the other. In fact, the choice of discrete symmetry is irrelevant in our discussion. The coupling constants in (17) are real due to the Hermiticity of the potential. Note that discrete symmetry requires $\mu_3^2=\mu_4^2=0$. The relevant minimum of the effective potential is of the form (see general discussion of the extrema in Ref. 10)

$$\langle\varphi_1\rangle_{\rm vac} = \begin{bmatrix} 0 \\ v_1/\sqrt{2} \end{bmatrix}, \quad \langle\varphi_2\rangle_{\rm vac} = \begin{bmatrix} 0 \\ v_2/\sqrt{2} \end{bmatrix} \ . \tag{19}$$

The following conditions guarantee that the vacuum (19) with v_1 and v_2 being real and positive is stable:

$$\begin{aligned} &\lambda_1 > 0, \quad \lambda_2 > 0 \ , \\ &h \equiv h_1 + h_2 + h_3 > -\sqrt{\lambda_1\lambda_2} \ , \\ &h_2 + h_3 < 0, \quad h_3 < 0 \ . \end{aligned} \tag{20}$$

The explicit expressions for v_1 and v_2 are

$$\begin{aligned} &v_1^2 = 2(\lambda_2\mu_1^2 - h\mu_2^2)/D \ , \\ &v_2^2 = 2(\lambda_1\mu_2^2 - h\mu_1^2)/D \ , \\ &D = \lambda_1\lambda_2 - h^2 > 0 \ , \end{aligned} \tag{21}$$

and $v_0^2 \equiv v_1^2 + v_2^2 = (250\ \text{GeV})^2$.

The spectrum of the physical-Higgs-boson excitations around the vacuum (19) contains a charged particle H^+ and three neutral bosons $H_{1,2,3}^0$ with masses m_+ and $m_{1,2,3}$, respectively,

$$\begin{aligned} &m_{1,2}^2 = \tfrac{1}{2}\{\lambda_1 v_1^2 + \lambda_2 v_2^2 \pm [(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + 4v_1^2 v_2^2 D]^{1/2}\} \ , \\ &m_3^2 = -2h_3 v^2, \quad m_+^2 = -(h_1+h_3)v^2 \ . \end{aligned} \tag{22}$$

It is easy to prove the following inequality which will be used later in Sec. IV:

$$\min\{m_{1,2}^2\} \le \min\{\lambda_1,\lambda_2\}v^2 \ . \tag{23}$$

The theory under consideration should have a sphaleron solution. The upper bound on the sphaleron mass may be derived in the following way. Let us construct a noncontractible loop in the configuration space connecting vacua with different topological numbers for the case of two doublets with the use of one-doublet ansatz (see, e.g., Ref. 18). Namely, we take the gauge field to be the same as in the standard model, while $\varphi_1 = (v_1/v)f$, $\varphi_2 = (v_2/v)f$, where f is the configuration for one-doublet scheme. The contribution of the scalar self-interaction to the energy of gauge-Higgs configuration is determined by

$$V = \frac{v^4}{8} f^2(f^2-2)(\lambda_1 c^4 + \lambda_2 s^4 + 2hc^2s^2) \ , \tag{24}$$

where $c \equiv \cos\beta$, $s \equiv \sin\beta$, $\tan\beta \equiv v_2/v_1$. All other contributions are the same as in the one-doublet case. This means that the sphaleron energy in the case of two doublets is not bigger than that of one doublet [see Eq. (5)] with the numerical coefficient $B = B(\lambda^{\rm eff}/\alpha_W)$, where

$$\lambda^{\rm eff} = \lambda_1 c^4 + \lambda_2 s^4 + 2hc^2 s^2 \ . \tag{25}$$

In particular for $\mu_1 = \mu_2$ one has $\lambda^{\rm eff} = \gamma$:

$$\gamma \equiv \frac{D}{\lambda_1 + \lambda_2 - 2h} \ . \tag{26}$$

One can show that $\gamma > 0$ as far as $D > 0$. Note also that γ is the minimal value of $\lambda^{\rm eff}(\beta)$.

IV. PHASE TRANSITION WITH LIGHT HIGGS BOSONS

Consider first the case when all scalar particles are lighter than the W boson. Our final aim here is to find the BAU suppression factor S defined in (6). As explained in Sec. II one has to determine for this end the effective sphaleron mass right after the electroweak phase transition.

The temperature-dependent one-loop effective potential for the vacuum expectation values (VEV's) defined by (19) is of the form

$$\begin{aligned} V_T(v_1,v_2) = &-\tfrac{1}{2}\mu_1^2(T)v_1^2 - \tfrac{1}{2}\mu_2^2(T)v_2^2 - \delta T v^3 \\ &+ \tfrac{1}{8}\lambda_1 v_1^4 + \tfrac{1}{8}\lambda_2 v_2^4 + \tfrac{1}{4}h v_1^2 v_2^2 + C v^4 \ln\frac{v^2}{AT^2} \ , \end{aligned} \tag{27}$$

where $v^2 \equiv (v_1^2+v_2^2)$,

$$\begin{aligned} &\mu_i^2(T)=\mu_i^2-\omega_i T^2 , \\ &\omega_1 \equiv \tfrac{1}{4}(2m_W^2+m_Z^2) , \\ &\omega_2 \equiv \tfrac{1}{4}[2m_W^2+m_Z^2+2m_t^2 v^2(0)/v_2^2(0)] , \end{aligned} \tag{28}$$

C and δ are defined in accordance with (12) and (13) with substitution $m_t \to m_t(v_2/v)[v(0)/v_2(0)]$, where $v_i(0)$ are VEV's at zero temperature. The coefficient C is numerically unimportant and will be neglected in what follows.

At the moment of phase transition the curvature of the effective potential (27) vanishes in some direction. Without loss of generality suppose that $\mu_1^2(T_c)=0$ and $\mu_2^2(T_c)\equiv\Delta\mu^2\le 0$. Then the extrema of the effective potential satisfy the equations

$$\begin{aligned} &v_1(\lambda_1 v_1^2+hv_2^2-6\delta T_c v)=0 , \\ &v_2(\lambda_2 v_2^2+hv_1^2-6\delta T_c v-2\Delta\mu^2)=0 . \end{aligned} \tag{29}$$

There are three solutions to the system (29):

$$(v_1,0):\ v_1=6\frac{\delta T_c}{\lambda_1} , \tag{30}$$

$$(0,v_2):\ v_2=3\frac{\delta T_c}{\lambda_2}\pm\left[\left[3\frac{\delta T_c}{\lambda_2}\right]^2+\frac{2\Delta\mu^2}{\lambda_2}\right]^{1/2} ; \tag{31}$$

$$(v_1=v_c\cos\beta_c, v_2=v_c\sin\beta_c):\ \sin^2\beta_c=\frac{\lambda_1-6\delta T_c/v_c}{\lambda_1-h},\quad v_c=3\frac{\delta T_c}{\gamma}\pm\left[\left[3\frac{\delta T_c}{\gamma}\right]^2+2\Delta\mu^2\frac{\lambda_1-h}{D}\right]^{1/2} . \tag{32}$$

The second and third extrema exist provided the following constraints are satisfied:

$$\frac{|\Delta\mu^2|}{\mu_1^2}<\frac{9}{2}\frac{\delta^2}{\omega_1\lambda_2},\quad \frac{9}{2}\frac{\delta^2}{\gamma^2\omega_1^2}\frac{D}{\lambda_1-h} , \tag{33}$$

λ as in (16) one concludes the extrema (31) and (32) exist only for very small values of $\Delta\mu^2$:

$$\frac{|\Delta\mu^2|}{\mu_1^2}<\frac{g_W^4}{\lambda} . \tag{34}$$

For the most natural case the Lagrangian parameters μ_1 and μ_2 are different and (34) does not hold true. The only extremum at the moment of the phase transition is given by Eq. (30). Equation (30) allows one to obtain the effective sphaleron mass (5) and (7) at the moment of phase transition:

$$\zeta=\frac{4\pi}{g_W}B\frac{v_1}{T_c} . \tag{35}$$

Then the requirement (9) that the suppression of the baryon asymmetry should not be too large yields a constraint on the combination of coupling constants of the form

$$\lambda_1<\frac{\pi}{5}B\alpha_W . \tag{36}$$

We need to convert (36) into the constraint on the Higgs-boson masses. For this end we use inequality (23) which gives exactly the same upper bound on the neutral-Higgs-boson mass m_1 as in (1).

Suppose now that due to some reasons $\Delta\mu^2$ is small enough so that all three minima coexist. In what follows we shall neglect corrections of the order of $|\Delta\mu^2|/\mu_1^2 \ll 1$ so one should take $\Delta\mu^2=0$ as well. The extremum (32) is found to correspond to the global minimum. Evaluating the spectrum of small fluctuations around the minimum (32) one may easily show that this minimum exists as far as the vacuum (19) is realized at $T=0$. One has

$$v_c=6\frac{\delta}{\gamma}T_c,\quad \sin^2\beta_c=\frac{\lambda_1-h}{\lambda_1+\lambda_2-2h} , \tag{37}$$

which implies in particular that the angle β_c coincides with its zero-temperature value and $\lambda^{\rm eff}$ in (25) is $\lambda^{\rm eff}_{\rm min}=\gamma$. Therefore, instead of (36) we have

$$\gamma<\frac{\pi}{5}B\alpha_W . \tag{38}$$

Now for $\mu_1^2=\mu_2^2$ (we neglect corrections to this equality coming from the t quark) one can derive from Eq. (21): $2\mu^2=\gamma v_0^2$. In the case under consideration ($\Delta\mu^2=0$) the masses of the two neutral Higgs bosons (22) look like

$$m_{1,2}^2=(2\mu^2, 2\mu^2(1-h/\gamma)) . \tag{39}$$

So we again recover the old upper bound on the Higgs-boson mass $m_1<45$ GeV.

To summarize, in the two-doublet version of the electroweak theory with a small Higgs self-interaction we obtain an upper bound on the mass of one neutral Higgs boson which coincides numerically with that in the case of one Higgs doublet.

V. THE MODEL WITH HEAVY HIGGS BOSONS

In the case of one Higgs doublet the region of $m_H \gtrsim m_W$, $\lambda \gtrsim g_W^2$ corresponds to the parametrically small value of the scalar condensate at the moment of phase transition $v(T_c)\sim g_W T_c$ which implies the weakly first- or second-order phase transition. Then all the baryon asymmetry produced at the phase transition is washed out. In this section we shall consider the model with relatively heavy Higgs bosons. Namely, we will suppose that scalar self-couplings are not small compared with g_W^2:

$$\lambda, h \propto g_W^2 \ . \tag{40}$$

We will show, that this case provides us with the opportunity to have both the first-order phase transition which preserves BAU and a heavy enough Higgs bosons which guarantees the stability of the vacuum.

If the Higgs bosons are heavy (40) the contribution of the scalar loops to the effective potential should be taken into account. In general, it modifies the structure (27) of the one-loop effective potential at finite temperature. In particular, the term linear in the temperature becomes a function of v_1^2 and v_2^2 separately whereas it was a function of combination $(v_1^2+v_2^2)$ before. It is given by expression

$$-\frac{1}{12\pi} T\,\mathrm{Tr}[\mathbf{M}^2(\varphi)]^{3/2} \ , \tag{41}$$

where $\mathbf{M}^2$ is the mass matrix of scalar fields in the presence of scalar condensate. However, if we consider a definite direction in the space of Higgs fields $v_1 = v\cos\beta$, $v_2 = v\sin\beta$ the structure (27) of the effective potential does not alter, while the coefficients $\beta_i(\beta)$, $\delta(\beta)$, $C(\beta)$ receive additional terms proportional to the scalar self-coupling constants.

In order to have a first-order phase transition and unsuppressed baryon production, the vacuum expectation value v should be of order of T_c. Let us consider first the case when both temperature-dependent masses are equal to zero at critical temperature.

Again, we have in general three solutions of the type (30)–(32), where the third type of solution is the global minima. To get $v(T_c) \sim T_c$ we have to demand that

$$\gamma \propto g_W^3 \ , \tag{42}$$

while other combinations of coupling constants should be big enough (say, $\lambda \simeq g_W^2$) to ensure a stable Higgs potential. With this choice of parameter, solutions of the type (30) or (31) give a parametrically small value of the scalar condensate $v_c \sim (g_W^3/\lambda)T_c \sim g_W T_c$ and are, therefore, irrelevant.

To obtain the solution $v_1 = v_c\cos\beta_c$, $v_2 = v_c\sin\beta_c$, we notice that after a change of variables,

$$\begin{aligned} w_1 &= \varphi_1\cos\beta + \varphi_2\sin\beta \ , \\ w_2 &= -\varphi_2\sin\beta + \varphi_2\cos\beta \ , \end{aligned} \tag{43}$$

the zero-temperature effective potential becomes

$$\begin{aligned} V = &-\mu_1^2(\beta)w_1^\dagger w_1 - \mu_2^2(\beta)w_2^\dagger w_2 - \mu_3^2(\beta)w_1^\dagger w_2 - \mu_4^2(\beta)w_2^\dagger w_1 + \tfrac{1}{2}\lambda_1(\beta)(w_1^\dagger w_1)^2 + \tfrac{1}{2}\lambda_2(\beta)(w_2^\dagger w_2)^2 \\ &+ h_1(b)(w_1^\dagger w_2)(w_2^\dagger w_1) + h_2(\beta)(w_1^\dagger w_1)(w_2^\dagger w_2) + \tfrac{1}{2}h_3(\beta)[(w_1^\dagger w_2)^2 + (w_2^\dagger w_1)^2] \\ &+ [\tfrac{1}{2}\epsilon_1(\beta)w_1^\dagger w_1 + \epsilon_2(\beta)w_2^\dagger w_2](w_1^\dagger w_2 + w_2^\dagger w_1) \ . \end{aligned} \tag{44}$$

The evaluation of the rotation-dependent coupling constants shows that

$$\begin{aligned} &\lambda_1(\beta_0) = \lambda^{\mathrm{eff}} = \gamma, \quad \epsilon_1(\beta_0) = 0 \ , \\ &\beta_0 = \tfrac{1}{2}\arccos\left[\frac{\lambda_2 - \lambda_1}{\lambda_2 + \lambda_1 - 2h}\right] \ . \end{aligned} \tag{45}$$

Then if we choose $\beta_c = \beta_0[1 + O(g_W^3)]$, the coupling constants are

$$\lambda_1(\beta_c) = \gamma[1 + O(g_W^3)] \propto g_W^3, \quad \epsilon_1(\beta_c) \propto g_W^3 \ . \tag{46}$$

Then the minimum of the high-temperature effective potential is

$$\begin{aligned} &w_1(\beta_c) = \begin{bmatrix} 0 \\ \xi T_c \end{bmatrix} \ , \\ &w_2(\beta_c) = 0, \quad \beta_c = \beta_0(1 + \eta g_W^3) \ , \end{aligned} \tag{47}$$

where the numerical coefficient ξ and η are the functions of λ_i/g_W^2, h_i/g_W^2.

Expressed in the terms of the constants of the potential (44) the function ξ like

$$\begin{aligned} \xi &= \frac{6\kappa}{\lambda_1(\beta_c)} \ , \\ \kappa &= \delta + \frac{1}{24\pi\sqrt{2}}\{3(1+\sqrt{3})[\lambda_1(\beta_c)]^{3/2} \\ &\qquad + [h(\beta_c)]^{3/2} + 2[h_2(\beta_c)]^{3/2} \\ &\qquad + [h(\beta_c) - 2h_3(\beta_c)]^{3/2}\} \ , \end{aligned} \tag{48}$$

and

$$\begin{aligned} \cos\beta_c = \frac{1}{16\xi\pi\sqrt{2}}\{&3(1+\sqrt{3})[\lambda_1(\beta_c)]^{1/2} \\ &+ [h(\beta_c)]^{1/2} + 2[h_2(\beta_c)]^{1/2} \\ &+ [h(\beta_c) - 2h_3(\beta_c)]^{1/2}\} \ . \end{aligned} \tag{49}$$

We use the fact that $M^2(\varphi)$ becomes diagonal after the rotation (43). Now (9) implies the upper bound on the Higgs-boson mass:

$$\min\{m_1^2, m_2^2\} < \frac{8\pi B}{45}\frac{\kappa}{g_w}v_0^2 \ . \tag{50}$$

Note, however, that κ is a function of λ_i, as in (5). Thus the numerical value of the upper bound on the Higgs-boson mass depends on the choice of the coupling constants of the scalar potential. For example, the critical mass of the lightest Higgs boson in the theory with coupling constants $\lambda_1=\lambda_2=4g_W^2$, $h=-4g_W^2(1-0.25g_W)$, $h_2=5g_W^2$, $h_3=-9g_W^2$, $\mu_1=\mu_2=45$ GeV is equal to 67 GeV.

The treatment of the case when at the transition temperature only one of the masses is equal to zero is in complete analogy with the previous one. If, say, $\mu_1(T_c)=0$ then the coupling λ_1 has to be of the order g_W^3. This also implies the upper bound on the lightest Higgs boson, but it differs from (1) due to contribution of other Higgs bosons to linear in temperature term in effective potential.

We conclude that if the neutral Higgs boson will not be observed experimentally in the mass interval 24–45 GeV it may still be consistent with the cosmological upper bounds on the Higgs-boson mass necessary for electroweak BAU generation. The price for that is the extension of the standard model.

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