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CP-violating but P-preserving electromagnetic couplings of the $W^{\pm}$ and Z^0

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We show that the electroweak gauge bosons $W^{\pm}$ and Z^0 can have CP-violating (but P-preserving) couplings to the photon and Z^0 boson that are very weakly bounded by current limits on fermion electric dipole moments, and by present data from the CERN e^+e^- collider LEP. They nevertheless can be large enough to be detectable, for instance, in gauge-boson pair production at LEP 200.

I. INTRODUCTION AND SUMMARY

The advent of precision measurements on the Z^0 pole,[1] and of access to TeV energies at hadron colliders,[2] has put the standard model under increasingly searching scrutiny over the past year. Although it has survived all tests, most theorists expect it to ultimately fail as experiments improve.

In the absence of the direct production of any new unexpected particles, whatever "new physics" lies beyond the standard model is most likely to be first detected through deviations between accurately made standard-model predictions and experiment. Quantities that are forbidden or unobservably small within the standard model are very clean observables for these purposes because their unambiguous observation necessarily implies the existence of some sort of new physics. CP-violating processes are good examples of this type because although CP is broken in the standard model it is broken in a very specific way with unobservably small consequences for many quantities such as electric dipole moments[3] (EDM's) for elementary fermions, neutrons, atoms, and molecules. Indeed, given the fantastic accuracy of the bounds on such EDM's,[4] this is one of the standard model's most successful predictions.

Flavor-diagonal CP-violating effective interactions are usually thought to be quite strongly bounded by the constraint that they not contribute too large an EDM for fundamental fermions. Current limits[4,5] on the electron and neutron EDM can probe CP-violating physics up to scales of thousands[6] of TeV and so would appear to rule out its appearance in accelerator experiments for the foreseeable future.

Our purpose in this article is to present loopholes in the reasoning of the previous paragraph. Our main point is that although most CP-violating effective interactions are very strongly bounded by EDM measurements there are effective interactions that are large enough to be detected in the near future at, say, the CERN e^+e^- collider LEP but which are nevertheless naturally suppressed in their contributions to fermion EDM's. We illustrate this observation by considering effective couplings of the $W^{\pm}$ boson that preserve parity P but break charge conjugation C and hence also CP. We show that, unlike P- and CP-violating couplings, those that preserve parity contribute only very weakly to the neutron and electron EDM's or to current experiments at LEP, but could nonetheless be large enough to be eventually detected through W^+W^- pair production at LEP 200 or through WZ production at the Fermilab Tevatron.

The very exent of the present agreement between the standard model and experiment suggests that the energy scale associated with any new physics is probably quite large compared to the masses of the electroweak gauge bosons. This observation invites the application of effective-Lagrangian techniques, which exploit the small ratio between the weak and the new physics scales. It can be used not only in our present analysis, but also more generally to search for observables in which new phenomena might be most easily detectable in upcoming experiments.

We choose to organize our presentation in the following way. We start with a discussion of the general form required of any candidate gauge-boson moment by the general requirements of Lorentz and $SU_L(2)\times U_Y(1)$ gauge invariance, together with the standard-model particle content. This presentation is followed by an estimate of the implications of a representative CP-odd but P-even interaction for experiments. The estimate starts with a discussion of the domain of applicability of the effective-Lagrangian approach, as quantified by the requirement that the S matrix due to the effective interaction satisfy the unitarity bound. This sets the upper limit on the potential size of effect that may be entertained, even in principle. We then show that the interaction discussed is naturally much too small to have been detected to date, such as through its contribution to the neutron EDM or through its effects given the current Z^0 sample at LEP. It can nevertheless potentially produce detectable effects in the near future at LEP or the Tevatron or through precision measurements of neutral-current processes. We find that the most sensitive probe of these interactions may be found at LEP once collision energies rise above the threshold for W^+W^- pair production.

II. GENERAL $W^{\pm}$ AND Z^0 COUPLINGS

We start by considering the general requirements of Lorentz invariance on the form of the couplings of the weak bosons to the photon and the Z^0. The charged boson $W^{\pm}$ can *a priori* have three distinct types of *CP*-odd electromagnetic couplings (on shell). These are given in terms of the matrix elements[7,8] of the electromagnetic current $J^{\lambda}_{\rm em}$ by

$$\langle W^-|J^{\lambda}_{\rm em}|W^-\rangle \equiv -ie\varepsilon^*_{\mu}(p_1)\Gamma^{\mu\nu\lambda}_{\rm em}(p_1,p_2)\varepsilon_{\nu}(p_2)\ ,$$
$$\Gamma^{\mu\nu\lambda}_{{\rm em},T{\rm odd}} = -f_{\gamma}(q^2)\varepsilon^{\mu\nu\lambda\rho}q_{\rho} - \frac{1}{M_W^2}g_{\gamma}(q^2)p^{\lambda}\varepsilon^{\mu\nu\sigma\rho}q_{\sigma}p_{\rho} + \frac{i}{M_W^2}h_{\gamma}(q^2)[q^2(q^{\mu}\eta^{\lambda\nu}+q^{\nu}\eta^{\lambda\mu})-2q^{\mu}q^{\lambda}q^{\nu}]\ . \tag{1}$$

In this expression p_i^{ν} and $\varepsilon_{\mu}(p_i)$ are the W-boson four-momentum and polarization vector, respectively. The second equality in Eq. (1) gives only the T-odd part of this matrix element. p^{μ} and q^{μ} are the sum and difference, $p=p_1+p_2$ and $q=p_1-p_2$, of the four-momenta of the initial and final W's.

$f_{\gamma}(q^2)$ and $g_{\gamma}(q^2)$ violate both P and T and may be physically interpreted through their contributions to the electric dipole[9–12] (d_W) and magnetic quadrupole ($\tilde{Q}_W$) moments of the W boson:

$$d_W = \frac{e}{2M_W}[f_{\gamma}(0)-4g_{\gamma}(0)]\ ,$$
$$\tilde{Q}_W = -\frac{e}{M_W^2}f_{\gamma}(0)\ . \tag{2}$$

By contrast, form factor $h_{\gamma}(q^2)$ is T odd but P even and so cannot contribute to either d_W or $\tilde{Q}_W$. Since it vanishes when the photon is on shell, $q^2=0$, it also does not contribute to processes which involve real external photons.

The T-violating part of the WWZ vertex admits a similar parametrization:[7,8]

$$\langle W^-|J^{\lambda}_{\rm nc}|W^-\rangle \equiv -ie_Z\varepsilon^*_{\mu}(p_1)\Gamma^{\mu\nu\lambda}_{\rm nc}(p_1,p_2)\varepsilon_{\nu}(p_2)\ ,$$
$$\Gamma^{\mu\nu\lambda}_{{\rm nc},T{\rm odd}} = -f_Z(q^2)\varepsilon^{\mu\nu\lambda\rho}q_{\rho} - \frac{1}{M_W^2}g_Z(q^2)p^{\lambda}\varepsilon^{\mu\nu\sigma\rho}q_{\sigma}p_{\rho} + \frac{i}{M_W^2}h_Z(q^2)[q^2(q^{\mu}\eta^{\lambda\nu}+q^{\nu}\eta^{\lambda\mu})-2q^{\mu}q^{\lambda}q^{\nu}]\ . \tag{3}$$

The normalization of Eq. (3) differs from that of Eq. (1) by the replacement $e\rightarrow e_Z=e/\sin\theta_W\cos\theta_W$ where θ_W represents, as usual, the electroweak mixing angle. The implications of these types of W couplings for asymmetries in W^+W^- pair production have been examined in Ref. 8, where it is concluded that they are potentially observable given 1000 W^+W^- pairs at LEP 200 if the Z or photon form factors are $\gtrsim 0.5$ when evaluated at momentum transfers: $q^2\approx 4M_W^2$. The form factors used here are related to those of Ref. 8 by

$$f_Z(q^2)=f_6^Z(q^2)\cos^2\theta_W\ ,$$
$$g_Z(q^2)=f_7^Z(q^2)\cos^2\theta_W\ ,$$
$$(q^2/M_W^2)h_Z(q^2)=f_4^Z(q^2)\cos^2\theta_W\ . \tag{4}$$

The photon couplings are related by expressions that differ from these only through their omission of the factors of $\cos^2\theta_W$.

Similar expressions to Eqs. (1) and (3) may also be written for the general $ZZ\gamma$ and ZZZ vertex.[13] The principal difference between the $W^{\pm}$ and Z^0 form factors is the constraint that follows from the fact that the Z^0 is its own antiparticle. The form factors that are the analogues for the Z^0 particle of $f_i(q^2)$ and $g_i(q^2)$ are inconsistent with the Majorana nature of the Z^0 and so necessarily vanish for the neutral weak boson. The *CP*-violating and *P*-preserving counterpart to $h_i(q^2)$ that is of most interest for the present purposes is therefore in this case the only *CP*-violating form factor possible.

We next turn to the implications for these general couplings of $SU_L(2)\times U_Y(1)$ gauge invariance together with the knowledge that the scale for new physics is associated with momenta q^2 that are much larger than M_W^2 and M_Z^2. This information is most efficiently summarized by the requirement that the form factors of Eqs. (1) and (3) be generated by the operators with the lowest possible dimension in an $SU_L(2)\times U_Y(1)$-invariant effective Lagrangian.[14] This Lagrangian must of course embody all current constraints on deviations from the standard model such as, for example, the requirement that the ρ parameter not differ significantly from unity.

The lowest-dimension operators with a *CP*-violating vertex involving just three electroweak gauge bosons have dimension six. Any such operator that is invariant with respect to the $SU_L(2)\times U_Y(1)$ gauge group and which only involves the usual standard-model fields (including the usual Higgs doublet) may be written as a linear combination of a basis of the following two independent ones:[6,10]

$$A\frac{g_2^3}{3!}\varepsilon_{abc}\tilde{W}^{a\mu}_{\ \nu}W^{b\nu}_{\ \lambda}W^{c\lambda}_{\ \mu} + Bg_2g_1(\phi^{\dagger}\tau_a\phi)W^a_{\mu\nu}\tilde{B}^{\mu\nu}\ . \tag{5}$$

τ_a here represent the usual Pauli matrices and ε_{abc} is the completely antisymmetric symbol in the $SU_L(2)$ gauge indices a,b, and c. The gauge coupling constants for $U_Y(1)$ and $SU_L(2)$ are each denoted g_1 and g_2, and the corresponding field strengths are $W^a_{\mu\nu}$ and $B_{\mu\nu}$. The tilde represents the duality transformation: $\tilde{B}_{\mu\nu}=\frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}B^{\lambda\rho}$. ϕ is the usual Higgs doublet whose vacuum expectation value (VEV) breaks the electroweak gauge group:

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\ v\end{bmatrix}\ .$$

The constants A and B have dimensions of inverse mass squared and may be computed in terms of the underlying parameters of any given model. The form factors, cf. Eqs. (1) and (3), induced by these effective interactions do not vary appreciably with q^2 for q^2 smaller than the scale of the new physics. Their low-energy values are determined in terms of A and B by

$$f_\gamma(0)=g_2^2(\tfrac{1}{2}Bv^2-AM_W^2)\ , \tag{6}$$

$$g_\gamma(0)=-\tfrac{1}{2}g_2^2M_W^2A\ , \tag{7}$$

and

$$f_Z(0)=g_2^2(\tfrac{1}{2}Bv^2\sin^2\theta_W-AM_W^2\cos^2\theta_W)$$
$$=f_\gamma(0)\sin^2\theta_W+2g_\gamma(0)\cos2\theta_W\ , \tag{8}$$

$$g_Z(0)=-\tfrac{1}{2}g_2^2AM_W^2\cos^2\theta_W=g_\gamma(0)\cos^2\theta_W\ . \tag{9}$$

A and B are bounded[9–11] by the requirement that they do not induce too large a neutron EDM. The EDM is induced by the one-loop correction to the electromagnetic vertex in which a quark emits a virtual weak boson (i.e., a $W^\pm$ or Z^0), which in turn emits a photon *via* the *CP*-violating electromagnetic vertex due to Eq. (5). The bound obtained in this way has been computed for $B \gg A$ (the case relevant when A and B are induced at one loop, such as in left-right models) in Refs. 9 and 10, giving the limit $g_\gamma \ll f_\gamma(0) \lesssim 1\times10^{-3}$. The general case for which B and A are arbitrary is considered in Ref. 11 where the bounds

$$f_\gamma(0)\lesssim 1\times10^{-3},\quad g_\gamma(0)\lesssim 1\times10^{-4} \tag{10}$$

are derived. These bounds assume that the effects of the operators of Eq. (5) do not cancel unnaturally with those of other operators in fermion EDM's. Although the calculation of Ref. 11 omits the graph involving the virtual Z^0, this omission is unlikely to significantly alter this bound. Notice that since both the electromagnetic and Z form factors f_Z, g_Z, f_γ, and g_γ are determined by the same two operators of Eq. (5), the WWZ couplings are bounded to a size similar to that of the $WW\gamma$ couplings. Interactions this small are unobservable[8] in W^+W^- pair production at LEP 200.

Our main point is that this same conclusion does not apply to the third T-odd and P-even form factors h_Z and h_γ, which turn out to be completely unconstrained by the current bounds on particle EDM's. If *CP*-violating $W^\pm$ or Z^0 interactions should be detected at LEP (or elsewhere), they will therefore most likely be of this form.

In order to compute the size of this bound we must, as before, determine the relevant lowest-dimension effective operator in the effective Lagrangian that can contribute to h_γ (and h_Z). The operators of lowest dimension that can possibly contribute to h_γ (and to h_Z) within the $SU_c(3)\times SU_L(2)\times U_Y(1)$-invariant effective action have dimension eight. One such operator is

$$\mathcal{O}=Cg_1\phi^\dagger(D_\mu D_\nu+D_\nu D_\mu)\phi\partial^\mu\partial_\lambda B^{\lambda\nu}+\text{c.c.} \tag{11}$$

which produces the following effective $U_{em}(1)$-invariant interactions:

$$\mathcal{L}=-\frac{C(v+H)^2}{2}[g_2^2(W_\mu^\dagger W_\nu+W_\nu^\dagger W_\mu)+e_Z^2Z_\mu Z_\nu]\partial^\mu\partial_\lambda(eF-e_Z\sin^2\theta_WZ)^{\lambda\nu}$$
$$+2C(v+H)\partial_\mu\partial_\nu H\,\partial^\mu\partial_\lambda(eF-e_Z\sin^2\theta_WZ)^{\lambda\nu}+\mathcal{L}_{\text{unphys}}\ . \tag{12}$$

$F_{\mu\nu}$ and $Z_{\mu\nu}$ denote here the usual Abelian field strengths for the electromagnetic and Z^0 fields, and $\mathcal{L}_{\text{unphys}}$ denotes those terms involving the couplings of the unphysical scalars. The $WW\gamma$ and WWZ couplings of Eq. (12) give rise to the following form factors:

$$h_\gamma(0)=-\frac{Cg_2^2v^2M_W^2}{2}=-2CM_W^4=-\frac{Cg_2^4v^4}{8}\ , \tag{13}$$

and

$$h_Z(0)=+\frac{Cg_2^2v^2M_W^2}{2}\sin^2\theta_W=+2CM_W^4\sin^2\theta_W$$
$$=+\frac{Cg_2^4v^4}{8}\sin^2\theta_W\ . \tag{14}$$

III. LIMITS FROM PRINCIPLE AND PHENOMENOLOGY

We next turn to the question of how large a coefficient C can be consistently contemplated. The limits on the size of C come in two types: (a) those of principle, due to unitarity, which mark the boundaries of applicability of the effective-Lagrangian formalism, and (b) those from experiment since no evidence for nonstandard electroweak boson couplings yet exists. We consider each of these in turn, starting first with the unitarity bounds.

A. Unitarity bounds

Unitarity bounds on the effective interactions of Eqs. (1) and (3) have been considered in a slightly different context in Ref. 15. Here each of the WWZ and $WW\gamma$ form factors are bounded by requiring that the magnitude of the amplitude for gauge-boson pair production in fermion collisions be consistent with the upper bound set by unitarity. Each of the form factors is treated in isolation in this analysis, without including the relationships that follow from $SU_L(2)\times U_Y(1)$ invariance. For the purposes of an estimate we use their results here, although it should be borne in mind that gauge invariance imposes relations among these form factors that typically improve the high-energy behavior of scattering amplitudes for spin-one particles.

Following Ref. 15 we cut off the effective theory at a momentum scale Λ above which the low-energy effective-Lagrangian approximation is expected to fail.

For $\Lambda \gg M_Z$ the condition for the validity of tree-level unitarity in the gauge-boson production rate in fermion-fermion collisions may be approximated by

$$|f_4^Z| < 0.87 \left[\frac{1\ \text{TeV}}{\Lambda}\right]^2 . \quad (15)$$

This approximate expression is within 5% of the full result[15] for $\Lambda \gtrsim 1$ TeV but should be weakened by around 20% for $\Lambda \sim 500$ GeV. Using Eqs. (4) and (14) and choosing the highest-momentum transfer possible, $q^2 \sim \Lambda^2$, in order to get the strongest limit, implies a corresponding constraint on C:

$$Cv^4 \lesssim \begin{cases} 0.2 \left[\frac{1\ \text{TeV}}{\Lambda}\right]^4 & \text{for } \Lambda > 1\ \text{TeV} , \\ 4 \quad \text{for } \Lambda \approx 500\ \text{GeV} . \end{cases} \quad (16)$$

This represents an upper bound on the combination $C\Lambda^4 \lesssim 50$ which relates the potential size of the effects of any new physics to the scale above which that physics appears. It should be compared with the experimental sensitivity (determined below) with which C may be probed in current and upcoming experiments.

B. Current phenomenological bounds

There are several places in which an effective interaction of the form of Eq. (11) might contribute to precisely measured quantities. These differ according to whether they are sensitive to *CP*-violating effects and so arise linearly in the coefficient C or whether they simply probe the existence of nonstandard W couplings and so are generated proportional to C^2. We now consider these in order of decreasing sensitivity.

1. Precision electroweak experiments

Nonstandard couplings among the electroweak bosons must in general modify the W and Z propagators through their contributions to the transverse part of the gauge-boson vacuum polarization, $\Pi_{\mu\nu} = \Pi(q^2)\eta_{\mu\nu} + \ldots$. These corrections can have detectable consequences through the discrepancies they generate between detailed standard-model predictions and observations for precisely measured quantities. The contributions of heavy physics through electroweak-boson vacuum polarization may be conveniently parametrized in terms of the quantities[16]

$$\frac{\delta\Pi_{WW}(0)}{M_W^2} - \frac{\delta\Pi_{ZZ}(0)}{M_Z^2} = \alpha(M_Z)T ,$$

$$\left[\frac{\delta\Pi_{WW}(M_W^2) - \delta\Pi_{WW}(0)}{M_W^2}\right]_{\overline{\text{MS}}} = \frac{\alpha(M_Z)}{4\sin^2\theta_W} S_W , \quad (17)$$

$$\left[\frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2}\right]_{\overline{\text{MS}}} = \frac{\alpha(M_Z)}{4\sin^2\theta_W\cos^2\theta_W} S_Z ,$$

where $\overline{\text{MS}}$ denotes the modified minimal-subtraction scheme. The present experimental bounds on T and $S \equiv S_W \approx S_Z$ are found in Ref. 16 to be $|T| < 2$ and $|S| < 4$, respectively.

The parameter T is proportional to the deviation $\Delta\rho$ of the ρ parameter from unity. It is therefore constrained by the ratio of the strengths of the charged- and neutral-current interactions at low energies. Contributions to T, or to $\Delta\rho$, are naturally suppressed to the extent that the accidental "custodial" global SU(2) symmetry of the Higgs sector is unbroken. This "custodial" SU(2) symmetry is only broken by the $U_Y(1)$ gauge couplings that appear within the effective interaction of Eq. (11) through the covariant derivatives of the Higgs field. Since this part of the effective interaction does not contribute to the W and Z vacuum polarizations at $q^2 = 0$ it quite naturally does not contribute to T.

Its contribution to S need not similarly vanish, however. Indeed this effective operator is a typical example of new physics which does not give any deviation from unity in the ρ parameter but which nonetheless produces potentially detectable contributions to the q^2-dependent part of the electroweak-boson self-energies. A rough estimate of the size of its contribution would be

$$\delta\Pi_{ZZ}(q^2) \approx \frac{e_Z^2 \sin^4\theta_W}{(4\pi)^2} q^2 \int^{\Lambda} h_Z^2 \left[\frac{p^2}{M_W^2}\right]^2 \frac{d^4p}{p^4}$$

$$\approx \frac{e_Z^2 \sin^4\theta_W}{4\pi^2} q^2 (CM_W^4)(C\Lambda^4) , \quad (18)$$

from which we obtain

$$S \approx \frac{4\sin^4\theta_W}{\pi}(CM_W^4)(C\Lambda^4) . \quad (19)$$

Comparison with the bound $|S| < 4$ then gives the condition

$$Cv^4 < 4.5 \left[\frac{1\ \text{TeV}}{\Lambda}\right]^2 . \quad (20)$$

In these expressions Λ again indicates an upper momentum cutoff above which the effective-action analysis fails to apply.

2. Contributions at the Z^0 pole

Given that the effective interaction, Eq. (11), involves *CP*-violating Z^0 couplings and that it can be observable once W^+W^- pairs are produced at LEP 200, it is natural to check whether it might also be observable, and so be bounded by, current LEP experiments running at the Z^0 resonance. In this section we investigate these effects of the operator (11) and show that they are too small to be detected to date, although they could well be large enough to be detectable in the near future.

In order to address the experimental implications, we need to first compute the effective coupling between fermions and Z^0's that is induced by the interactions of Eq. (11). This effective fermion-Z^0 coupling may be computed using the Feynman graphs of Fig. 1. We have evaluated these graphs in R_ξ gauge up to terms that are at least quadratic in fermion masses. This neglect is justified because of the small masses and/or mixing angles that are necessarily involved. The neglect of additional fermion masses also allows the omission of all graphs that involve the unphysical scalars in this gauge.

These may be interpreted as contributing to the fermion "weak dipole moment:"[17]

$$\mathcal{L}_{\mathrm{ZDM}}=-\frac{i}{2}Z_f\bar{f}\gamma_5\sigma^{\mu\nu}fZ_{\mu\nu}\ , \tag{21}$$

where the contribution to $Z_f=Z_f^{(WW)}+Z_f^{(ZZ)}+Z_f^{(Z\gamma)}$ of each of the three graphs of Fig. 1 is

$$Z_f^{(WW)}=\frac{e\alpha C m_f}{4\pi\sin\theta_W\cos^3\theta_W}q^2 f\left[\frac{M_Z^2}{M_W^2}\right]\ , \tag{22}$$

$$Z_f^{(ZZ)}=\left\{Z_f^{(WW)}\sec^2\theta_W-\frac{e\alpha^2Cv^2m_f}{48\sin^5\theta_W\cos^3\theta_W}\left[\ln\left[\frac{\Lambda^2}{M_Z^2}\right]+O(\Lambda^0)\right]\right\}(1+4Q_f\sin^2\theta_W)\ , \tag{23}$$

$$Z_f^{(Z\gamma)}=-\frac{e\alpha^2Cv^2m_f\cos\theta_W}{\sin^3\theta_W}|Q_f|\left[\ln\left[\frac{\Lambda^2}{M_Z^2}\right]+O(\Lambda^0)\right]\ . \tag{24}$$

In these expressions $q^2=q_\mu q^\mu$ is the square of the four-momentum carried by the Z^0 particle. α is the usual electromagnetic fine-structure constant, while Q_f and m_f, respectively, represent the electric charge (in units of the proton charge) and the mass of fermion type f. The function $f(z)$ appearing in Eq. (22) is defined by

$$f(z)=\int_0^1 dx\frac{1-x}{1-x+x^2z/4}\ .$$

For $z=M_Z^2/M_W^2=1.3$ we have $f=0.75$. Λ is an ultraviolet momentum cutoff above which the effective-Lagrangian analysis does not apply. In all numerical estimates that follow it suffices to take the logarithm to be one.

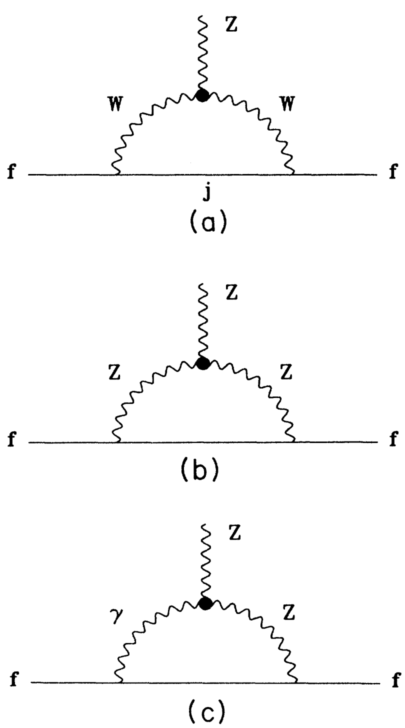

FIG. 1. The Feynman graphs through which the gauge-boson interactions of Eq. (12) contribute to the effective fermion-Z^0 vertex. Inclusion of the graph obtained by interchanging $\gamma\leftrightarrow Z$ on the internal lines of (c) is understood. Graphs not displayed here, such as those involving unphysical scalars, contribute an amount that is suppressed relative to those shown by extra factors of a small fermion mass.

For phenomenological purposes the fermions of most interest here are the τ lepton and the b quark. For both of these the contribution due to the virtual Z^0 and photon turns out to dominate, and both have roughly the same numerical value:

$$Z_b\approx Z_\tau\approx Cv^4\ln\left[\frac{\Lambda^2}{M_Z^2}\right](2\times10^{-22})\ e\ \mathrm{cm}\ . \tag{25}$$

These numbers are much too small to be detected for $Cv^4\lesssim10^4$. For purposes of comparison, the increase in the total width, $\delta\Gamma(Z\to b\bar{b})$, due to such a small coupling would be[17] $\delta\Gamma(Z\to b\bar{b})\sim|Cv^4|^2\times(10^{-9}$ MeV). For $Cv^4<10^4$ these numbers are also much smaller than the estimated accuracy[17] $\delta Z_\tau\approx6\times10^{-18}$ e cm with which Z_τ might be measured through observations of correlations in the process $Z\to\tau^+\tau^-\to\pi^+\pi^-\nu_\tau\bar{\nu}_\tau$ given the decays of 10^7 Z^0's. It is noteworthy, however, that this asymmetry furnishes a more sensitive probe to this operator than does the neutron EDM, as we see in more detail below.

An alternative place to check whether these new interactions might turn up at LEP 100 would be through decays mediated by an off-shell Higgs boson: $Z\to Hf\bar{f}$. These could proceed *via* the graph in which a virtual Z^0 boson emits a Higgs particle through a vertex due to the couplings in Eq. (12). The rate for this process is also easily seen to be much too tiny to be observable with a sample of fewer than 10^{14} Z^0's. The width for this process is

$$\Gamma(Z^0\to Hf\bar{f})=(CM_Z^4)^2\frac{\alpha M_Z}{384\pi^2\cos^2\theta_W}\frac{m_f^2}{M_Z^2}F(x)\ , \tag{26}$$

in which the function $F(x)=\frac{1}{20}(1-x^2)(1-14x^2-94x^4-14x^6+x^8)-6x^4(1+x^2)\ln x$ of the variable $x=m_H/M_Z$ satisfies the inequality $F<3\times10^{-3}$ for the mass range of current interest: $2m_H>M_Z$. This gives the branching ratio $B(Z^0\rightarrow Hb\bar{b})<(Cv^4)^2(3.5\times10^{-14})$—much too small to have been observed.

3. The neutron electric dipole moment

A good constraint might be expected to arise due to the current tight bounds on the intrinsic electric dipole moment (EDM) of the neutron or of atoms. This turns out not to give a strong limit. The analysis may be done most directly by first integrating out the $W^{\pm}$ and Z^0 particles to produce an effective operator at energy scales of the order of several GeV.

This integration procedure gives two types of contribution to the low-energy theory: (i) graphs involving an external photon line, such as in Fig. 2, generate an electric dipole moment form factor for the elementary fermions, and (ii) those involving four external fermions, such as Fig. 3, generate contact interactions. The lowest-dimension operator produced by Fig. 2 that contributes to the electromagnetic form factor is

$$\mathcal{O}_{ff\gamma}=\frac{i}{2}\sum_f D_f(\bar{f}\gamma_5\sigma^{\mu\nu}f)\Box F_{\mu\nu}\ , \tag{27}$$

with coefficient D_f given in terms of C by

$$D_f=-\frac{e\alpha C m_f}{4\pi\sin^2\theta_W\cos^2\theta_W}\times[1+\sec^2\theta_W(1+4Q_f\sin^2\theta_W)]+\text{o.t.} \tag{28}$$

All of the terms that are not written, collectively denoted "o.t." in the above, are suppressed by additional factors of a fermion mass (or, for contributions due to a virtual top quark, by small mixing angles).

The explicit derivatives that appear in Eq. (27) arise due to the overall factors of photon momentum q^2 that appear in the $WW\gamma$ vertex of Eq. (1). Since these factors just involve the momentum of the external photon they simply appear as a common factor in the final result. For this reason the operator of Eq. (27) does not contribute to the electric dipole moment of the fermion itself, however,

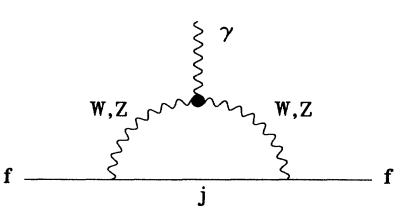

FIG. 2. The Feynman graphs that generate an effective *CP*- and *P*-violating fermion-photon interaction from the given *CP*-odd but *P*-even *W*-photon coupling. The blob represents the $WW\gamma$ (or $ZZ\gamma$) vertex with Feynman rule given by the form factor h_γ of Eq. (1). Graphs involving unphysical scalars only contribute terms involving at least two powers of a fermion mass and so may be neglected relative to those shown.

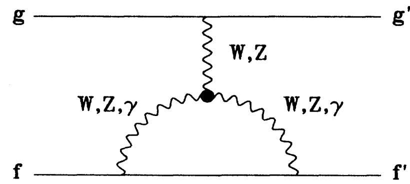

FIG. 3. Feynman graphs which generate *CP*-violating four-fermion interactions in the effective theory below the weak scale. These operators can contribute to an EDM for a bound system such as the neutron or an atom.

since the EDM is defined in terms of the value of the form factor evaluated at $q^2=0$.

Although the effective operator of Eq. (27) cannot produce an EDM for any of the fundamental fermions of the theory, it may nevertheless contribute to the EDM of a composite system, such as for a neutron or an atom through the exchange of a virtual photon between its constituents, corresponding to replacing the exchanged vector boson of Fig. 3 with a photon. Evaluation of this graph produces an effective dimension-*seven* four-fermion interaction which is equivalent to the one obtained by simplifying Eq. (27) using the electromagnetic equation of motion: $\partial_\mu F^{\mu\nu}=eJ^\nu_{\rm em}$ in which $J^\nu_{\rm em}$ is the electromagnetic current, $J^\nu_{\rm em}=\sum_\psi Q_\psi\bar{\psi}\gamma^\nu\psi$. This result becomes

$$\mathcal{O}_{4\ \rm fermion}=ie\sum_f D_f(\bar{f}\gamma_5\sigma^{\mu\nu}f)\partial_\mu J^{\rm em}_\nu\ . \tag{29}$$

Similar expressions exist for the contact terms generated by W and Z exchange. All of these 4 fermion contract interactions, due to W, Z, and photon exchange, contribute roughly the same size contributions to bound-state EDM's.

The most difficult part now remains: the estimation of the matrix elements of these four-fermion operators necessary for computing atomic and neutron EDM's. To our knowledge although matrix elements of dimension-six four-fermion couplings within a neutron or an atom have been estimated in the literature, no estimates exist for the matrix element of the dimension-seven operator of Eq. (29). Since present limits on four-fermion couplings due to the EDM's of atoms, such as for ^{199}Hg, are at best of roughly the same size[18] as those derived from the neutron EDM, we confine our attention here to the neutron.

We may form a very rough estimate of the size of the induced neutron EDM that might be generated by a representative interaction of the form of Eq. (29), by including in our figuring all factors of couplings and powers of 2π that are required by the short-distance graphs, such as those of Fig. 3, and then approximating the matrix element of the dimension-seven operator by multiplying by the power of the QCD scale, $\Lambda_{\rm QCD}\approx150$ MeV, that is required by dimensional analysis. This crudest of estimates gives

$$d_n\sim e^2D_f\Lambda^2_{\rm QCD}\approx e(Cv^4)\alpha^2G_F^2\Lambda^2_{\rm QCD}m_f\approx(Cv^4)\times(3\times10^{-32}\ e\ \text{cm})\ . \tag{30}$$

We take here a current-quark mass ≈10 MeV for m_f.

The resulting EDM is roughly seven orders of magnitude below the current experimental limit if $Cv^4 \approx 1$.

A more sophisticated estimate of the matrix element, based on an adapation of the arguments of Ref. 19, gives much the same result. By way of illustration we obtain the following result for the matrix element of the four u-quark term in Eq. (29):

$$d_n \approx -\frac{e^2}{9} M\chi D_u \langle \bar{u}u \rangle \approx (Cv^4)\times(-1.2\times 10^{-32}\ e\ \text{cm})\ , \qquad (31)$$

in which $\chi \approx 6\ \text{GeV}^{-2}$ is the "vacuum magnetic susceptibility," and

$$\langle \bar{u}u \rangle \approx -\frac{1}{(2\pi)^2}(0.55\ \text{GeV}^3)\ .$$

M is a mass that appears when a derivative is converted to a mass by using the Dirac equation for u within the matrix element of the neutron. Opinions may differ as to whether M thus produced should be a current or a constituent quark mass so, in order to be as conservative as possible, we take M to equal the neutron mass m_n. The correct choice for this mass is almost certainly smaller than this, in which case the bound arising from the neutron EDM would be correspondingly weaker, thereby strengthening our conclusion that no useful bound on C comes from the neutron EDM.

Taking the estimate of Eq. (30) implies a very weak bound on C due to the current limit on the neutron EDM of $Cv^4 < 3\times 10^6$.

C. Prospects for future detection

All of the above estimates lead to the conclusion that the effective interaction of Eq. (11) could easily be present with a strength of $Cv^4 \lesssim 5$ (if the cutoff is taken to be 500 GeV and somewhat lower for higher cutoffs) without contradicting any present experimental limits. We next argue that this is sufficient for its detection to be possible in the near future at LEP or the Tevatron.

1. LEP 200

The authors of Ref. 8 have considered the detection of the form factors of Eqs. (1) and (3) through asymmetries in W^+W^- pair production at $\sqrt{s} = 190$ GeV at LEP 200. They conclude that an anomalous WWZ form factor can be detected provided that

$$|f_4^Z(s)| \gtrsim 0.1\ . \qquad (32)$$

This translates into a lower bound for C of the form

$$Cv^4 > 0.8\ , \qquad (33)$$

which is clearly consistent with all of the above constraints for cutoffs anywhere below $\Lambda \sim 1$ TeV.

2. The Tevatron

Although the Tevatron environment is not as "clean" as that of an e^+e^- machine one can nonetheless reach higher energies than would be available even at LEP 200. It might therefore be expected in principle to set better limits on our operator. This turns out not to be the case. Various studies[20–24] have dealt with the measurements of the anomalous couplings of the weak bosons at the Tevatron. It appears that the cleanest channel in which to observe them is through $W\gamma$ production.[20] This process turns out not to bound the operator we consider here, however, since its matrix element vanishes for real photons. Hagiwara, Woodside, and Zeppenfeld[21] have presented a very careful analysis, of the hadroproduction of $W^\pm Z$ and W^+W^-, including cuts to reduce the backgrounds. However, they only consider the effect of the anomalous magnetic and the quadrupole moments of the W and do not examine CP-violating interactions. Chang and Lee,[22] on the other hand, have studied the effects of all $WW\gamma$ and WWZ anomalous couplings at the Tevatron but only present the contribution of one specific anomalous coupling at a time, allowing a deviation from their SM values as large as one. To get a limit on our operator we have reanalyzed their results while incorporating the approach of Hagiwara *et al.* to eliminate the background.

One should first note that the WW channel is more sensitive to the operator of Eq. (11) than is WZ production. This is due to the factor of q^2 that appears in its Feynman rule, representing the invariant mass of the neutral vector boson (either photon or Z) that appears in the vertex. For WW production with a total energy of 1.8 TeV, this factor corresponds to a W pair invariant mass of about 300 GeV (on average). We have assumed a center-of-mass energy of 1.8 TeV with a luminosity of 4.7 pb^{-1}. To eliminate backgrounds we choose to demand that both W's decay leptonically rather than imposing cuts on the invariant masses to keep only high-p_t events. Only 4.7% of all WW events decay in this way. Demanding at least five such events leads to the lower bound: $Cv^4 > 9$. A twofold increase in luminosity would reduce this bound to $Cv^4 > 6.5$. One of us[24] has recently obtained slightly weaker bounds by considering the ZZ production channel.

Of course the higher the energies that are available the better are the bounds that may be obtained. For instance, the recently proposed e^+e^- collider at 500 GeV is sensitive to couplings as small as $Cv^4 > 0.05$.

Clearly a CP-violating interaction strength of the type examined here can be large enough to be detected at either LEP within the next few years without having been detected to date in any experiments. It might also be detected at the Tevatron although the strength of coupling required is uncomfortably close to the unitarity bound even when the cutoff is as low as 500 GeV.

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