Supersymmetric dyons

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Using the topological part of restricted quantum chromodynamics, dyonic supermultiplets in $N=1$ supersymmetry are obtained quantum mechanically as well as in the supersymmetric version of the Georgi-Glashow model for vanishing linear momentum and in the Clifford vacuum. Constructing the Lagrangian density in such a model of restricted quantum chromodynamics, supersymmetric dyonic solutions and the classical mass of the dyon are obtained. Deriving the eigenvalue equations of bosonic and fermionic fluctuations, the corresponding one-loop corrections to the dyonic mass are calculated and it is shown that the classical mass of the dyon is not changed by one-loop quantum corrections.

I. INTRODUCTION

Supersymmetric field theories have the remarkable property that some of the perturbative effects are canceled between bosons and fermions and provide[1] a natural resolution of the gauge hierarchy problem of grand unified theories (GUT's). In most such theories, the spontaneous breaking of symmetry at mass scale M_x ($M_x \sim 10^{15}$ GeV) leads to the presence of monopoles and dyons. If supersymmetry breaks at a scale much less than M_x, the monopoles and dyons must form supermultiplets of approximately degenerate boson and fermion states. The monopoles and dyons in $N=1$ supersymmetric theories must be consistently represented by supermultiplets containing spin-0 (S monopoles) and spin-$\frac{1}{2}$ (monopolino) states. D'Hoker and Vinet[2] have presented the $N=1$ superspace formulation for the dynamical supersymmetry of the Pauli system in the presence of monopoles.

In the supersymmetric GUT's, the monopole ground state, in general, carries fractional electric charge as well as color hypercharge.[3] This is a manifestation of fermion fractionization[4] with the axial anomaly effect[5] properly taken into account. Supersymmetry provides the first realistic testing ground for the idea of fermion fractionization and induced fermionic charge on a monopole. The fractional charge of dyons arises for the Higgs-boson-mass case (and not for the Dirac-mass case) and the nontrivial topology of the background Higgs fields leads to Jackiw-Rebbi zero modes[4] even in the supersymmetric theories. It has been shown[6] that the monopole states which are connected by Jackiw-Rebbi zero-mode operators must be embedded into the fundamental multiplet of the $N=1$ supersymmetric Georgi-Glashow model, and these modes exactly coincide with the supersymmetric zero modes.[7] In general, it is very difficult to obtain the explicit form of the Jackiw-Rebbi zero modes in supersymmetric theories, and the dual nature of the fermionic zero modes in $N=2$ super Yang-Mills theories leads to several difficulties in dealing with the problem of monopoles, dyons, and dyonic supermultiplets.

Analyzing the supersymmetric generalization of monopoles in the limit of Prasad and Sommerfield[8] and Bogomolny[9] and using the supersymmetric version of the Georgi-Glashow model with vanishing potential, it has been shown[10] that the quantum corrections to the mass of a monopole are vanishing. However, some controversies have been raised[11,12] about the exact cancellation of perturbative effects between bosons and fermions, Bogomolny-bound saturation, and quantum corrections to the physical monopole mass in $N=2$ supersymmetric Yang-Mills theory. Moreover, if the Jackiw-Rebbi zero modes exist independently of supersymmetric zero modes, the dyonic states are enriched by further degeneracy with different charge and spin states, and consequently the supersymmetry becomes very much involved and cumbersome. Unfortunately, an explicit demonstration of Jackiw-Rebbi zero modes is extremely difficult. In the light of these difficulties associated with the existing supersymmetric GUT's, it becomes necessary to have an alternative approach to understand supersymmetric dyons.

Keeping these motivations in view, in this paper, dyonic supermultiplets in $N=1$ supersymmetry are obtained quantum mechanically in the topological part of restricted quantum chromodynamics.[13,14] Starting with the representation of supersymmetric algebra described by dyonic states, in the supersymmetric version of the Georgi-Glashow model, $+\frac{1}{2}$ and $-\frac{1}{2}$ spin states are constructed from the spin-0 state, and the dyonic supermultiplets are obtained for vanishing linear momentum as well as in the Clifford vacuum. Constructing the Lagrangian density in the $N=1$ supersymmetric version of restricted quantum chromodynamics (RCD) (topological part) in terms of the isotriplet gauge field and its fermionic superpartner, supersymmetric dyonic solutions are written and the classical mass of the dyon is obtained by minimizing the background potential of the theory. Separating the bosonic part of this Lagrangian in the dyonic background gauge and adding the gauge-fixing and Faddeev-Popov ghost terms to it, the eigenvalue equations of bosonic fluctuations are derived and the corresponding one-loop correction to the dyonic mass is calculated.

II. DYON SUPERMULTIPLETS

In supersymmetric theories, dyons must exist in the form of supermultiplets together with their fermionic partners. Let us demonstrate, quantum mechanically, the existence of dyonic supermultiplets in the minimal supersymmetric theory, where the monopoles and dyons form the representations of supersymmetric algebra:

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0 \,, \qquad \{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu \,. \tag{2.1}$$

Only isovector fermions are relevant in the simple supersymmetric version of the Georgi-Glashow model. Let us consider the four-component Majorana spinor ψ defined as

$$\psi = \begin{bmatrix} \psi_\alpha \\ \bar{\psi}_{\dot\alpha} \end{bmatrix} . \tag{2.2}$$

Under parity P, the component of the left-handed spinor ψ_α and its right-handed complex conjugate $\bar{\psi}_{\dot\alpha}$ transform as

$$P\psi_\alpha(x) = i\bar{\psi}^{\dot\alpha}(-x)P \quad \text{and} \quad P\bar{\psi}^{\dot\alpha}(x) = -i\psi_\alpha(-x)P \,. \tag{2.3}$$

The supersymmetric generators Q_α and $\bar{Q}_{\dot\alpha}$ also transforms under parity in an exactly similar way. The other discrete symmetry of Georgi-Glashow model is G parity. Though P parity and G parity are spontaneously broken in the Higgs vacuum, the product PG is unbroken and we have

$$(PG)Q_\alpha(PG)^{-1} = i\bar{Q}^{\dot\alpha} \quad \text{and} \quad (PG)\bar{Q}_{\dot\alpha}(PG)^{-1} = -iQ_\alpha \,. \tag{2.4}$$

Let us now introduce the states $|p,q\rangle$ characterized by the momentum and charge of the dyon. The simplest ansatz for realizing the dyons at rest in a supersymmetric theory consists of a set of supermultiplets constituted by the spin-0 states $|0,q\rangle$ for different charge labels such that

$$\bar{Q}_{\dot\alpha}|0,q\rangle = 0 \,. \tag{2.5}$$

Applying the supersymmetric operators on these states we get the spin-0 states $|\bar{0},q\rangle$ and spin-$\frac12$ states $|+\frac12,q\rangle$ and $|-\frac12,q\rangle$. States with nonzero momentum can be constructed by applying Lorentz transformations. All these states correspond to the same classical solutions. In other words, these states constitute dyonic supermultiplets. The four states $|0,q\rangle$, $|\frac12,q\rangle$, $|-\frac12,q\rangle$, and $|\bar{0},q\rangle$ are embedded in the complete supermultiplet in $N=1$ supersymmetric theory. This supermultiplet can be generalized by introducing the Clifford vacuum $|\Omega,q\rangle$ with charge $q=1$, which is annihilated by supercharge Q_i, i.e.,

$$Q_i|\Omega,q\rangle = 0 \,. \tag{2.6}$$

The other members of the multiplets are generated by the application of $\bar{Q}_i$.

The embedding of spin-$\frac12$ dyons in this manner is much more nontrivial. In order to judge whether this embedding is done by taking the spin-$\frac12$ dyons as the Clifford vacuum and constructing the supermultiplets or by incorporating them into the spin-$\frac12$ part of the fundamental multiplet in the $N=1$ supersymmetric theory, the only available way is to add a soft-supersymmetry-breaking interaction for observing the response to the main spectrum of the theory. The purpose of softly broken supersymmetry is to protect scalar mesons from quartic mass renormalization below the unification mass scale. Let us introduce the gauge-invariant perturbation $\hat{V}$ which softly breaks the supersymmetry. In the presence of this perturbation, the fermion states $|\frac12,q\rangle$ and $|-\frac12,q\rangle$ remain degenerate with mass $M_q^{(F)}$. In the basis $|0,q\rangle$ and $|\bar{0},q\rangle$, the mass matrix for spin-0 states may be written as[6,15]

$$M_q^{(B)} = \begin{bmatrix} M_q^{(F)} + \Delta_q & d_q \\ d_{q^*} & M_q^{(F)} - \Delta_q + 2\delta_q \end{bmatrix} , \tag{2.7}$$

where

$$M_q^{(F)} = \langle \tfrac12, q|\hat{H}|\tfrac12, q\rangle = \langle -\tfrac12, q|\hat{H}|\tfrac12, q\rangle \,, \tag{2.8}$$

with $\hat{H}$ as the rotationally invariant total Hamiltonian of the dyon-isovector fermion system. The mass-splitting parameters Δ_q, d_q, and δ_q may be obtained in the following form by retaining linear order in $\hat{V}$:

$$\Delta_q = \frac{1}{4M}(\bar\sigma^0)^{\dot\alpha\alpha}\langle 0,q|\{\bar{Q}_{\dot\alpha},[Q_\alpha,\hat{V}]\}|0,q\rangle \,, \tag{2.9}$$

$$d_q = -\frac{1}{4M}\langle 0,q|\{Q^\alpha,[Q_\alpha,\hat{V}]\}|0,q\rangle \,, \tag{2.10}$$

$$\delta_q = \frac{1}{32M^2}\langle 0,q|\{\bar{Q}_{\dot\alpha},[Q^{\dot\alpha},\{\bar{Q}^{\dot\alpha},[Q_\alpha,\hat{V}]\}]\}|0,q\rangle \,, \tag{2.11}$$

where averages have been taken over the states $|\frac12,q\rangle$ and $|-\frac12,q\rangle$. For deriving selection rules which restrict the form of mass splitting, we write the following transformations by using relations (2.4):

$$(PG)|0,q\rangle = |\bar{0},-q\rangle \quad \text{and} \quad (PG)|\bar{0},q\rangle = |0,-q\rangle \,, \tag{2.12}$$

showing that the electric charge is odd under the PG transformation.

If $\hat{V}$ is PG invariant, we have the following relations by applying the PG transformation to the spin-$\frac12$ states $|\frac12,q\rangle$ and $|-\frac12,q\rangle$:

$$M_q^{(F)}=M_{-q}^{(F)} , \quad (2.13a)$$

$$d_q=d_{-q^*} , \quad (2.13b)$$

and

$$\delta_q=\delta_{-q}=\tfrac{1}{2}(\Delta_q+\Delta_{-q}) , \quad (2.13c)$$

where the last relation plays a crucial role in determining the right supermultiplet structure.

Another unbroken symmetry of the Georgi-Glashow model is defined as follows in terms of R-symmetry charge:[16,17]

$$[R,Q_\alpha]=-Q_\alpha , \quad [R,\bar{Q}_{\dot{\alpha}}]=\bar{Q}_{\dot{\alpha}} . \quad (2.14)$$

Using the relations given by Eq. (2.4), we have

$$(PG)R(PG)^{-1}=-R . \quad (2.15)$$

If the soft perturbation $\hat{V}$ is chosen such that

$$\Delta R=0 ,$$

then $d_q=0$ and states $|0,q\rangle$ and $|\bar{0},q\rangle$ are not mixed by perturbation. For the perturbations which make $|\Delta R|=2$, we have

$$d_q=\delta_q=0 \quad (2.16)$$

and then the mass-squared sum rule (2.13c) is satisfied. In this case we obtain the following values of boson masses as the eigenvalues of mass matrix (2.7):

$$M_q^{(B)}=M_q^{(F)}\pm|d_q| . \quad (2.17)$$

For all other perturbations with any other values of ΔR, all the corrections to the masses vanish.

Taking various choices of perturbation, the mass splittings have been evaluated[6,15] leading to various drawbacks such as model dependence of fermion fractionization, the question of independent origin of dyon degeneracy due to fermion fractionization and supersymmetry, the origin of Jackiw-Rebbi zero modes as a consequence of hidden supersymmetry[18,19] and their explicit forms in supersymmetric theories (which involve a large number of fields), and the existence of induced color charges on GUT dyon, etc. Because of these reasons, some skepticism has been expressed[20] about these phenomena and it is therefore worthwhile to explore an alternative way to understand supersymmetric dyons. In the following section we shall make an attempt to construct the alternative supersymmetric theory of these dyons.

III. SUPERSYMMETRIC DYONS IN RESTRICTED GAUGE THEORY

Dyonic color charge and the color spin induced by fermion fractionization in Georgi-Glashow model in the presence of an isovector fermionic field can be incorporated in the restricted quantum chromodynamics (RCD),[13,14] where the unrestricted part of the gauge potential

$$\mathbf{V}_\mu=-iV_\mu^*\hat{\mathbf{m}}-\frac{1}{|q|}\hat{\mathbf{m}}\times\partial_\mu\hat{\mathbf{m}} \quad (3.1)$$

describes the dyonic flux of color isocharges and the restricted one describes the flux of topological charges. In this theory the restricted potential, containing color electric and magnetic potentials in a dual symmetric manner, has been constructed by using magnetic symmetry on global sections where color direction has been chosen by selecting color electric potential of Cartan's subgroup. The generalized field strength of gauge fields in this restricted chromodynamics describing non-Abelian dyons has been obtained as

$$\mathcal{G}_{\mu\nu}=\mathbf{G}_{\mu\nu}+|q|(\mathbf{V}_\mu\times\mathbf{V}_\nu) . \quad (3.2)$$

In Eqs. (3.1) and (3.2), the $\mathbf{V}_\mu$ vector is the isotriplet of the generalized four-vector and $\hat{\mathbf{m}}$ is isotriplet with constant length

$$\hat{\mathbf{m}}^2=\text{const}=v^2/2 . \quad (3.3)$$

In the external four-dimensional space, the multiplet $\hat{\mathbf{m}}$ behaves as a massless scalar field, components m^a of which in internal SU(2) space constitute the isotriplet $\hat{\mathbf{m}}$, where $a=1,2,3$.

The unrestricted part of the gauge potential $\mathbf{V}_\mu$, introduced by Eq (3.1), has the Abelian origin, and it has been ignored as being unnecessary in our recent work,[13] where only the restricted part of this potential has been shown responsible for quark confinement through the mechanism of dyonic condensation. The dyons appear in the restricted chromodynamic theory only through this part of the potential. As such, ignoring the unrestricted part here also, Eqs. (3.1) and (3.2) reduce to the form

$$\mathbf{V}_\mu=-\frac{1}{|q|}\hat{\mathbf{m}}\times\partial_\mu\hat{\mathbf{m}} \quad (3.4)$$

and

$$\mathcal{G}_{\mu\nu}^a=\frac{1}{|q|}(2\epsilon^{abc}\partial_\mu m_b\partial_\nu m_c+m^a\epsilon^{bcd}m_b\partial_\mu m_c\partial_\nu m_d) , \quad (3.5)$$

where

$$|q|=(e^2+g^2)^{1/2} \quad (3.6)$$

is the dimensionless coupling constant made up of electric and magnetic coupling strengths e and g, respectively. Here the massless isovector field $\mathbf{V}_\mu$ has been constructed out of the isotriplet scalar $\hat{\mathbf{m}}$ and hence the independent bosonic degree of freedom is only 1. As such, the supersymmetric generalization of RCD may be obtained by modifying the Lagrangian density into the form

$$\mathcal{L}=-\tfrac{1}{4}\mathcal{G}_{\mu\nu}^a\mathcal{G}_a^{\mu\nu}+\tfrac{1}{2}\bar{\lambda}^a\gamma^\mu D_\mu\lambda_a+\tfrac{1}{2}D_\mu m^a D^\mu m_a +\tfrac{1}{2}|q|\epsilon_{abc}m^a\bar{\lambda}^b\gamma_5\lambda^C-V(mm^*) , \quad (3.7)$$

where λ^a constitutes the isotriplet of fermionic field, γ^μ are Dirac matrices, $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$, and the covariant derivative D_μ is defined as

$$D^\mu=\partial^\mu+q\mathbf{V}^\mu\times \quad (3.8)$$

with the symbol $\times$ for cross product in internal SU(2) space. The background potential $V(mm^*)$ in Eq. (3.7) has been constructed in the form

$$V(mm^*)=|q|^2[(m^a m_a^*)^2-(m^a m_a^*)(m^b m_b^*)] \ . \tag{3.9}$$

Using Eq. (3.4) for the topological gauge potential in Eq. (3.8), we have the following expressions for the covariant derivatives of isotriplet fermionic field λ and isotriplet scalar field $\hat{\mathbf{m}}$:

$$\begin{aligned} D_\mu \lambda^a &= \partial_\mu \lambda^a + (m^a \partial_\mu m^b \lambda_b - m^b \partial_\mu m^a \lambda_b) \ , \\ D_\mu m^a &= \partial_\mu m^a + (m^a \partial_\mu m^b m_b - m^b \partial_\mu m^a m_b) \ . \end{aligned} \tag{3.10}$$

Substituting these equations along with Eq. (3.5) into Eq. (3.7), we get the Lagrangian of the topological part of the restricted gauge theory in the form

$$\begin{aligned} \mathcal{L} = &-\frac{1}{|q|^2}[\Gamma_{\mu\nu bc}(\Gamma^{\mu\nu bc}-\Gamma^{\mu\nu cb})+\tfrac{1}{2}\epsilon_{ajk}\epsilon^{bcd}m^a m_b \Gamma_{\mu\nu cd}\Gamma^{\mu\nu jk}+\tfrac{1}{2}\epsilon^{abc}\epsilon_{jkl}m_a m^j \Gamma_{\mu\nu bc}\Gamma^{\mu\nu kl}+\tfrac{1}{2}\epsilon^{bcd}\epsilon_{jkl}m_b m^j \Gamma_{\mu\nu cd}\Gamma^{\mu\nu kl}] \\ &+\tfrac{1}{2}\bar{\lambda}^a \gamma^\mu[\partial_\mu \lambda_a+(m_a \partial_\mu m_b \lambda^b - m_b \partial_\mu m_a \lambda^b)]+\tfrac{1}{2}[\partial_\mu m^a+(m^a\partial_\mu m^b m_b - m^b \partial_\mu m^a m_b)]^2 \\ &+\tfrac{1}{2}|q|\epsilon_{abc}m^a \bar{\lambda}^b \gamma_5 \lambda^c - V(mm^*) \ , \end{aligned} \tag{3.11}$$

where $\Gamma_{\mu\nu bc}$ has been written for $\partial_\mu m_b \gamma_\nu m_c$; μ, ν are the indices in the four-dimensional space and a, b, c, etc., are those in internal isospace. In this Lagrangian, λ^a are superpartners of the isotriplet gauge field V_μ^a [or in turn the superpartners of isotriplet scalar multiplet m^a through Eq. (3.4)]. Since the theories which transform as linear representations of supersymmetry must have the same number of bosonic and fermionic degrees of freedom, the Lagrangian (3.11) should be supersymmetric. In order to check the supersymmetric invariance of this Lagrangian, let us apply the following supersymmetric transformations:

$$\begin{aligned} \delta m^a &= \bar{\alpha}\gamma_5 \lambda^a \ , \\ \delta V_\mu^a &= -\frac{1}{|q|}\epsilon^{abc}\bar{\alpha}(\gamma_5 \lambda_b \partial_\mu m_c + m_b \gamma_5 \partial_\mu \lambda_c) \ , \\ \delta \lambda^a &= \frac{1}{|q|}\alpha\sigma^{\mu\nu}(2\epsilon^{abc}\Gamma_{\mu\nu bc}+m^a m_b \epsilon^{bcd}\Gamma_{\mu\nu cd}) \\ &\quad -i\alpha\gamma_5\gamma_\mu m^a m_b \gamma^\mu m^b \ . \end{aligned} \tag{3.12}$$

Under these transformations,

$$\delta\mathcal{L}=0 \tag{3.12a}$$

provided we assume the supersymmetry conditions

$$\begin{aligned} &\lambda_5(\bar{\lambda}\times\partial_\mu \hat{\mathbf{m}}+\hat{\mathbf{m}}\times\partial_\mu\bar{\lambda})=-i|q|\gamma_\mu\bar{\lambda} \ , \\ &[m_b,\bar{\alpha}]=0, \quad [m_b,\gamma_5]=0 \ , \end{aligned} \tag{3.13}$$

which give a generalization of the Majorana condition and the Weyl condition.[21] Condition (3.12a) shows that the Lagrangian density given by Eq. (3.7) is supersymmetric for the topological part of restricted gauge theory constructed in terms of magnetic symmetry.

The background potential given by Eq. (3.9) has two types of minima:

$$m^a=0 \quad \text{and} \quad m^a=\frac{v}{\sqrt{2}}x^a \ .$$

For both these cases the value of potential is zero, as required by supersymmetric theories. For the second value of m^a, the symmetry SU(2) breaks down to U(1) and the dyonic solutions occur with the following Julia-Zee[22] time-dependent solutions:

$$\begin{aligned} V^{ai} &= -\frac{1}{|q|}\frac{\epsilon^{abi}}{r}\hat{\mathbf{x}}_b[1-K(r)] \ , \\ V_0^a &= \frac{\hat{\mathbf{x}}^a}{|q|r}J(r) \ , \\ \sqrt{2}\,\mathrm{Re}\,m^a &= \frac{\hat{\mathbf{x}}^a}{|q|r}H(r) \ , \quad \mathrm{Im}\,m^a=0 \ , \end{aligned} \tag{3.14}$$

where $\hat{\mathbf{x}}^a=x^a/r$ is unit vector in the internal space. These solutions satisfy the coupled differential equations

$$\begin{aligned} r^2H'' &= 2HK^2 \ , \\ r^2K'' &= K(K^2-1)+K[H^2-J^2] \ , \\ r^2J'' &= 2JK^2 \ . \end{aligned} \tag{3.15}$$

A solution of these equations is given by

$$\begin{aligned} J(r) &= \alpha[cr\coth(cr)-1] \ , \\ H(r) &= \beta[cr\coth(cr)-1] \ , \\ K(r) &= cr/\sinh(cr) \ , \end{aligned} \tag{3.16}$$

with the condition

$$\alpha^2-\beta^2=-1 \ . \tag{3.17}$$

From the Lagrangian density (3.7) we get the following form of energy-momentum density tensor

$$T^{\mu\nu}=D^{\mu}m_a D^{\nu}m^a+\tfrac{1}{2}G_a^{\mu\lambda}G_{\lambda}^{a\nu} -g^{\mu\nu}[-\tfrac{1}{4}G_{ij}^aG_a^{ij}+\tfrac{1}{2}\bar{\lambda}^a\gamma^iD_i\lambda_a+\tfrac{1}{2}D_im^aD^im_a +\tfrac{1}{2}|q|\epsilon_{abc}m^a\bar{\lambda}^b\gamma_5\lambda^c-V(mm^*)] . \quad (3.18)$$

Setting $\mu=\nu=0$ in this relation, integrating T^{00} over three-space, and using relations (3.14) and (3.16), the classical mass of the dyon comes out to be

$$M=\frac{v}{\sqrt{2}}|q|=M_{\text{classical}} \quad (3.19)$$

showing that the dyons appear in the theory only through the restricted part of the potential (3.1), which carries the topological charges. A one-loop correction to this mass may be obtained by calculating the energies of Bose and Fermi fluctuations:

$$M_B=\tfrac{1}{2}\sum\omega_B \quad (3.20)$$

and

$$M_F=-\tfrac{1}{2}\sum\omega_F . \quad (3.21)$$

Let us choose the dyonic background gauge field defined by Eq. (3.14), where the fluctuation equation is given as the normal eigenvalue equation. This choice of gauge is necessary because the fluctuation equation takes a particular form in this gauge. Moreover, one may calculate one-loop diagrams with exact propagations of all particles in the background of a dyon.

In the dyonic background fields

$$V_{\mu D}^a=V_{\mu}^a-\delta V_{\mu}^a , \quad m_D^a=m^a-\delta m^a , \quad (3.22)$$

which satisfy Eq. (3.14), the bosonic part of the Lagrangian density (3.11) may be written as

$$\mathcal{L}_B=\mathcal{L}_D-\tfrac{1}{2}|(D_\nu\delta V_\mu^a)|^2+\tfrac{1}{2}(D_\mu\delta V_\nu)(D^\nu\delta V^\mu)^*-\tfrac{1}{4}|q|^2\,|(\delta V_{\mu b}\delta V_{\nu c})|^2-\tfrac{1}{2}\text{Re}[|q|\epsilon_{abc}G_{\mu\nu D}^aV^{*\mu b}\delta V^{*\nu c}] -\tfrac{1}{2}(D_\mu\delta m^a)(D^\mu\delta m_a)^*-\tfrac{1}{2}|q|\epsilon_{abc}(D_\mu m^a)V^{*\mu b}m^c+\tfrac{1}{2}|q|\epsilon_{abc}(D_\mu\delta m^a)\delta V^{*\mu b}\delta m^c +\tfrac{1}{2}|q|\epsilon^{abc}(D^\mu m_a)^*\delta V_{\mu b}m_c+\tfrac{1}{2}|q|^2[(\delta V_{\mu b}m^a)^2-(\delta V_{\mu b})^2(m_a)^2] , \quad (3.23)$$

where $\mathcal{L}_D$ is the dyonic background Lagrangian given by

$$\mathcal{L}_D=-\tfrac{1}{4}G_{\mu\nu D}^aG_{aD}^{*\mu\nu}+\tfrac{1}{2}(D_\mu m_D^a)(D^\mu m_{Da})-V(m_D^{*a}m_{Da}) \quad (3.24)$$

and D^μ is the covariant derivative given by Eq. (3.8) with $\mathbf{V}_\mu$ replaced by $\mathbf{V}_D^\mu$. To this Lagrangian let us add the following gauge-fixing term and Faddeev-Popov ghost term.

(i) Gauge-fixing term: Under an infinitesimal transformation θ, which keeps the background configuration $V_{\mu D}^a$ and m_D^a fixed, we may write the following variations of δV_μ^a and δm^a up to the lowest order in θ:

$$\delta V_\mu^{\theta a}=-\frac{1}{|q|}\epsilon^{abc}(\delta m_b^\theta\partial_\mu m_c+m_b\partial_\mu\delta m_c^\theta) =\delta V_\mu^a-\frac{1}{|q|}m^c\theta^a\partial_\mu m_c-\frac{1}{|q|}m_b\partial_\mu m^a\theta^b -\frac{1}{|q|}m_bm^a\partial_\mu\theta^b \quad (3.25)$$

and

$$\delta m^{\theta a}=\delta m^a+\epsilon^{abc}m_b\theta_c , \quad (3.26)$$

where the background gauge has been defined by the condition

$$f^a=(D^\mu\delta V_\mu)^a-|q|\epsilon^{abc}m_b\delta m_c \quad (3.27)$$

such that

$$\frac{\delta f^a}{\delta\theta^b}=\frac{1}{|q|}[D^\mu(m^a\partial_\mu m_b+m_b\partial_\mu m^a) +|q|^2(m^am_b-m^2\delta_b^a)] . \quad (3.28)$$

Thus the gauge-fixing term is given by

$$\mathcal{L}_{\text{gf}}=-\tfrac{1}{2}f^af_a =-\tfrac{1}{2}(D_\mu\delta V^\mu)^2+|q|\epsilon^{abc}(D_\mu\delta V^\mu)_am_b\delta m_c +|q|^2(m^am_b-m^2\delta_b^a)\delta m_a\delta m^b . \quad (3.29)$$

(ii) Faddeev-Popov ghost term: This term is given by

$$\mathcal{L}_{\text{FP}}=-\underline{c}_a^*(\delta f^a/\delta\theta^b)\underline{c}_b , \quad (3.30)$$

where Faddeev-Popov fields denoted by $\underline{c}$ and $\underline{c}^*$ are vectors in isospace. Using Eq. (3.28), we may write this term as

$$\mathcal{L}_{\text{FP}}=\underline{c}_a^*\left\{\frac{1}{|q|}[D^\mu(m^a\partial_\mu m^b+m_b\partial_\mu m^a)] +|q|^2(m^am^b-m^2\delta^{ab})\right\}\underline{c}_b . \quad (3.31)$$

Using Eqs. (3.23), (3.29), and (3.31), the total Lagrangian for the bosonic part may be written as

$$\mathcal{L}=\mathcal{L}_B+\mathcal{L}_{\text{gf}}+\mathcal{L}_{\text{FP}} . \quad (3.32)$$

In these equations for $\mathcal{L}_B$, $\mathcal{L}_{\text{gf}}$, and $\mathcal{L}_{\text{FP}}$, we may use the following matrix notation for the covariant derivative

$$D_\mu = \partial_\mu + i|q| V^D_{\mu a} T^a \tag{3.33}$$

in the dyonic background where T^a are usual generators of the internal gauge group SU(2).

Let us construct a three-vector $\mathbf{B}^D$ in the dyonic background as

$$B_j^D = \tfrac{1}{2}\epsilon_{ijk} G_b^{Dik} T^b \ , \tag{3.34}$$

and identify the spatial part $\delta\mathbf{V}_j$ and temporal part δV_j^0 of δV_j^μ as vectors in isospace. Then we may straight away get the following equations of motion from the Lagrangian density given by Eq. (3.23):

$$[D_\mu D^\mu - |q|^2 m^2 - 2|q|\sigma_k B^{Dk}]\delta\xi = 0 \ , \tag{3.35}$$

$$[D_\mu D^\mu - |q|^2 m^2]\delta V_0 = 0 \ , \tag{3.36}$$

$$[D_\mu D^\mu - |q|^2 m^2]\underline{c} = 0 \ , \tag{3.37}$$

where we have chosen

$$\delta\xi^a = \delta m^a + i\sigma^i \delta V_i^a \tag{3.38a}$$

and

$$m^2 = (T^c m_c)^2 \ . \tag{3.38b}$$

For getting the eigenvalue equations for the Bose fluctuations, let us take the following Fourier transform with respect to time t,

$$\delta\xi^a = \sum_{\omega_B} \delta\tilde{\xi}^a \exp(i\omega_B t) \ ,$$
$$\delta V_0^a = \sum_{\omega_{B_0}} \delta\tilde{V}_0^a \exp(i\omega_{B_0} t) \ , \tag{3.29}$$
$$\delta\underline{c}^a = \sum_{\omega_G} \delta\tilde{\underline{c}}^a \exp(i\omega_G t) \ .$$

Then Eqs. (3.35), (3.36), and (3.37) may be written in the following manner:

$$(D_i^2 - |q|^2 m^2 + 2|q| T_k V^{Dk})\delta\tilde{\xi} = -\omega_B^2 \delta\tilde{\xi} \ , \tag{3.40a}$$

$$(D_i^2 - |q|^2 m^2)\delta\tilde{V}_0 = -\omega_{B_0}^2 \delta\tilde{V}_0 \ , \tag{3.40b}$$

$$(D_i^2 - |q|^2 m^2)\tilde{\underline{c}} = -\omega_G^2 \tilde{\underline{c}} \ , \tag{3.40c}$$

where ω_B, ω_{B_0} are bosonic fluctuation frequencies and ω_G is the ghost term frequency. From these fluctuations, the one-loop contributions to dyonic mass can be written as

$$\Delta M_{\text{Bose}} = \Delta M_B = \sum \omega_B + \tfrac{1}{2}\sum \omega_{B_0} - \sum \omega_G \ , \tag{3.41}$$

where the negative sign for the ghost contribution denotes the anticommuting nature of ghost fields. Since $\sum\omega_B = \sum\omega_G = \sum\omega_{B_0}$, we may write Eq. (3.41) as

$$M_{\text{Bose}} = \tfrac{1}{2}\sum \omega_B \ . \tag{3.42}$$

Fermi fluctuations may be obtained from the following fermion equation of motion which follows from the fermionic part of the Lagrangian (3.7):

$$\gamma^\mu D_\mu \lambda^a + |q|\epsilon^{abc} m_b \gamma_5 \lambda_c = 0 \ , \tag{3.43}$$

where the covariant derivative has been defined by Eq. (3.33). Using Majorana representation and taking the Fourier transform of λ^a with respect to time t as

$$\lambda^a = u^a \exp(-i\omega_F t) \ , \tag{3.44}$$

Eq. (3.34) reduces to

$$(D_i^2 - |q|^2 m^2) u = -\omega_F^2 u \ , \tag{3.45}$$

where u is a vector in isospace with its components given by u^a. The contribution of this fluctuation to the dyon mass is

$$\Delta M_F = M_{\text{Fermi}} = -\tfrac{1}{2}\sum \omega_F \ . \tag{3.46}$$

Combining Eqs. (3.42) and (3.46), we get the following one-loop quantum correction to the dyonic mass:

$$\Delta M = \Delta M_B + \Delta M_F = \tfrac{1}{2}\left[\sum \omega_B - \sum \omega_F\right] . \tag{3.47}$$

In other words, the dyonic mass up to one-loop quantum correction is given by

$$M = M_{\text{classical}} + \tfrac{1}{2}\left[\sum \omega_B - \sum \omega_F\right] , \tag{3.48}$$

where the classical mass of dyon ($M_{\text{classical}}$) is given by Eq. (3.19).

Because of the similar nature of second-order differential equations (3.40a), (3.40b), and (3.45), the bosonic and fermionic fluctuations have the same spectrum of nonzero eigenvalues,

$$\sum \omega_F = \sum \omega_B \ . \tag{3.48a}$$

The equality between ω_B and ω_F leads to the result that the mass of the dyon is not changed by quantum corrections. In other words, in the supersymmetric limit, the non-Abelian theory of dyons in RCD falls apart, in the correct way, into degenerate supermultiplets.

IV. DISCUSSION

The dyonic states with spin $\pm\frac{1}{2}$ constructed from the spin-0 states have been shown to constitute the dyonic supermultiplet for vanishing linear momentum. When there are no spin degeneracies beyond those required by supersymmetry, it is not necessary to include larger angular momenta in the dyonic multiplets. Actually, the supermultiplets containing higher spins arise as bound or scattering states of dyons and other particles. Introducing a soft supersymmetry-breaking gauge invariant perturbation in $N=1$ supersymmetry, the mass matrix for spin-0 states has been obtained in Eq. (2.7) in terms of mass splitting parameters given by Eqs. (2.9), (2.10), and (2.11). Equation (2.17) gives the relation between the masses of bosons and fermions in dyonic supermultiplets. This relation shows that the embedding of spin-$\frac{1}{2}$ dyons is done by incorporating them in the spin-$\frac{1}{2}$ part of fundamental multiplet in $N=1$ supersymmetry, and hence the dyons in this theory can be consistently represented in supermultiplets containing spin-0 and spin-$\frac{1}{2}$ states. The supermultiplets of dyons in $N=2$ theory are similar to those in $N=1$ theory except that the fermionic dyons lose their electric charge and became neutral monopoles.

The Lagrangian density given by Eq. (3.7) is supersymmetric under the transformations (3.12), subject to the condition (3.13), which are a generalization of Majorana and Weyl conditions.[21] It leads to supersymmetric dyonic solutions with classical mass given by Eq. (3.19) when the symmetry SU(2) breaks down to U(1) by minimizing the potential (3.9). This value of dyonic mass agrees with that predicted by Julia and Zee.[22] This result shows that the dyons appear in the theory (RCD) only through the restricted part of the potential given by Eq. (3.1). Only this part, carrying the topological charges, is relevant in dyonic theory, while the unrestricted part of this potential, which is Abelian in nature, does not contribute anything to dyonic solutions. We have also demonstrated in our recent work[13] that it is only the restricted part of this potential which is responsible for quark confinement in RCD through the mechanism of dyonic condensation. On the other hand, the unrestricted part of the potential becomes confined as a result of condensation of topological charges. Due to these reasons the unrestricted part of the potential has been ignored in Eq. (3.4) while writing the supersymmetric Lagrangian (3.7) which carries only one bosonic degree of freedom and one fermionic degree of freedom. In case this unrestricted part of the potential is not ignored, one will have to introduce two fermionic degrees of freedom in this Lagrangian. It will not lead to any new physics and the mathematical calculations will become unnecessarily cumbersome, leading to difficulty in constructing the background potential of Eq. (3.9).

Choosing the dyonic background field defined by Eq. (3.22), the bosonic part of the Lagrangian of supersymmetric theory in the gauge restricted by magnetic symmetry has been obtained in the form given by Eq. (3.23). Equation (3.40) gives the bosonic fluctuations with the same frequency for the spatial bosonic field, temporal bosonic field, and the ghost field, which lead to total one-loop fluctuations and a correction to the dyonic mass in the form given by Eq. (3.42). The fermionic part of the Lagrangian (2.28) leads to the equation of motion (3.43), which in turn yields the fermionic fluctuation (3.44) in the one-loop approximation. Because of the similar nature of second-order differential equations (3.40a), (3.40b), and (3.44), the bosonic and fermionic fluctuations have the same spectrum of nonzero eigenvalues. Equations (3.46), (3.47), and (3.48) show that the classical mass of the dyon is not changed by quantum corrections, and hence it may be concluded that, in the supersymmetric limit, the non-Abelian theory of dyons in the restricted chromodynamics falls apart, in the correct way, into degenerate multiplets. In other words, in the supersymmetric generalization of RCD, the physical dyonic mass does not receive quantum corrections. The introduction of the supersymmetric dyonic model in this way and the vanishing of quantum corrections may be used for proving the interesting conjecture proposed by Montonen and Olive.[23] All these results of the supersymmetrized version of RCD agree with conclusions drawn by D'Adda *et al.*[10] by using dimensionally reduced supersymmetrized pure Yang-Mills theory in six dimensions. Using this method of dimensional reduction, we may get a two-dimensional theory from the four-dimensional supersymmetric theory presented in the preceding section by interpreting two of the spatial dimensions as internal degrees of freedom.

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