Double-field inflation

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We present an inflationary universe model which utilizes two coupled real scalar fields. The inflaton field ϕ experiences a first-order phase transition and its potential dominates the energy density of the Universe during the inflationary epoch. This field ϕ is initially trapped in its metastable minimum and must tunnel through a potential barrier to reach the true vacuum. The second auxiliary field ψ couples to the inflaton field and serves as a catalyst to provide an abrupt end to the inflationary epoch; i.e., the ψ field produces a time-dependent nucleation rate for bubbles of true ϕ vacuum. In this model, we find that bubbles of true vacuum can indeed percolate and we argue that thermalization of the interiors can more easily take place. The required degree of flatness (i.e., the fine-tuning) in the potential of the ψ field is comparable to that of other models which invoke slowly rolling fields. Pseudo Nambu-Goldstone bosons may naturally provide the flat potential for the rolling field.

I. INTRODUCTION

In 1981, Guth[1] proposed the inflationary universe model to solve several cosmological problems, notably the horizon problem, the flatness problem, and the monopole problem. During the inflationary epoch, the energy density of the Universe is dominated by a (nearly constant) false-vacuum energy term $\rho \simeq \rho_{\text{vac}} \approx \text{const}$, and the scale factor of the Universe expands exponentially:

$$H^2 = 8\pi G\rho/3 , \tag{1.1a}$$

$$R(t) = R(t_0)e^{\chi(t-t_0)} , \tag{1.1b}$$

where $H = \dot{R}/R$ is the Hubble parameter, R is the scale factor of the universe, $R(t_0)$ is the scale factor at the beginning of inflation, and χ is defined by

$$\chi = \sqrt{8\pi G\rho_{\text{vac}}/3} \tag{1.1c}$$

(notice that $\chi \approx H$ during the inflationary epoch). During this period of exponential expansion, a small causally connected region of the Universe inflates to a sufficiently large region to explain the observed homogeneity and isotropy of the Universe today, to "inflate away" the overdensity of monopoles to regions outside our horizon, and to predict a flat universe with $\Omega = 1$. A successful resolution to these cosmological problems requires at least 70 e-folds of inflation; i.e., the scale factor must increase by at least 10^{27} (for $\chi \approx \text{const}$). The period of exponential expansion must be followed by a period of thermalization, in which the vacuum energy density is converted to radiation.

In the original inflationary model[1] (now known as "old" inflation), the Universe supercools to a temperature $T \ll T_c$ during a first-order phase transition with critical temperature T_c. The nucleation rate for bubbles of true vacuum must be slow enough that the Universe remains in the metastable false vacuum long enough for at least 70 e-folds of inflation. Unfortunately, the old inflationary scenario has been shown to fail[2] because the interiors of expanding spherical bubbles of true vacuum fail to thermalize—the "graceful exit" problem. Hence this model does not produce a universe such as our own. The problem of ending old inflation will be discussed in greater detail in Sec. II, where we discuss modifications which can lead to percolation and thermalization.

Linde[3] and Albrecht and Stienhardt[4] proposed the "new" inflationary scenario, in which the effective potential (or free energy) of the inflaton field becomes very flat (the phase transition may now be second order or only weakly first order). As the field ψ "slowly rolls" down the potential, the evolution of the field is described by

$$\ddot{\psi} + 3H\dot{\psi} + \Gamma\dot{\psi} + \frac{dV}{d\psi} = 0 . \tag{1.2}$$

In the slowly rolling regime of growth, the energy density of the Universe is dominated by the vacuum contribution ($\rho \simeq \rho_{\text{vac}} \gg \rho_{\text{rad}}$) and the Universe expands exponentially. When the field approaches the true vacuum, it oscillates about the minimum, and the $\Gamma\dot{\psi}$ term gives rise to particle and entropy production. In this manner, a "graceful exit" to inflation is achieved. Many other proposed versions of inflation (e.g., the "chaotic" inflation model of Linde[5]) utilize a slowly rolling field.

All existing versions of inflation with rolling fields tend to overproduce density fluctuations and are thus highly constrained by isotropy measurements of the microwave background.[6] These measurements indicate that the amplitude of the density perturbations must be less than $\delta \approx 10^{-5}$. However, inflationary models predict[7,8] density fluctuations with amplitudes given by

$$\left.\frac{\delta\rho}{\rho}\right|_{\text{hor}} \simeq \frac{H^2}{\dot{\psi}} , \tag{1.3}$$

where the right-hand side is evaluated at the time when the fluctuation crossed outside the horizon during inflation and where $(\delta\rho/\rho)|_{\rm hor}$ is the amplitude of a density perturbation when it crosses back inside the horizon after inflation. In order for sufficient inflation to take place and for the density perturbations to be smaller than the observational limits, the potential of the rolling field must be very flat. This statement can be quantified[9] by defining a fine-tuning parameter λ through

$$\lambda \equiv \frac{\Delta V}{(\Delta\psi)^4} , \tag{1.4}$$

where ΔV is the change in the total potential $V(\psi)$ which affects the ψ field (including any interaction terms) and $\Delta\psi$ is the change in the field ψ during the slowly rolling portion of the inflationary epoch. The parameter λ is constrained to be small (i.e., $\lambda \lesssim 10^{-8}-10^{-11}$) for a general class of inflationary scenarios which contain a slowly rolling field.[9]

In Sec. II we discuss the "graceful exit" problem of old inflation and discuss a mechanism to circumvent this problem. In old inflation, a small nucleation rate (which is constant in time) allows for sufficient inflation, but the phase transition can never be completed. A large nucleation rate (also constant in time) would allow the phase transition to complete, but the Universe would not inflate sufficiently to solve the cosmological problems stated above. The basic feature of this present scenario is to have a *time-dependent* nucleation rate for bubbles of true vacuum in a first-order transition. This time dependence allows us to take advantage of the best features of both slow and fast nucleation rates. In our scenario, the nucleation rate is initially negligible and the Universe can inflate; subsequently, at the same time at every point (in a large enough region of space to encompass our Universe), the nucleation rate suddenly becomes extremely fast and the phase transition completes. In Sec. III, we discuss a particular model to obtain a time-dependent nucleation rate which can produce a fairly sudden end to the phase transition. In this model, the old inflationary field ϕ is coupled to a slowly rolling field ψ which evolves in a flat potential (like in new inflation). The ϕ field dominates the dynamics of the Universe and gives rise to an inflationary epoch. The purpose of the slowly rolling field is to give the ϕ field a time-dependent nucleation rate. When the slowly rolling field ψ approaches its vacuum expectation value (the minimum of the potential), the interaction between the fields catalyzes the old inflationary field ϕ to rapidly nucleate bubbles of true vacuum throughout space. However, the rolling field produces density fluctuations with the same amplitude as in new inflationary models, and bubble interactions on much smaller scales probably cannot erase these large-scale fluctuations; hence, this model suffers from a fine-tuning problem similar to that of new inflation. As discussed in Ref. 9, this fine-tuning problem is a generic feature of inflationary models with slowly rolling fields; a resolution (natural inflation with pseudo Nambu-Goldstone bosons) is suggested in Ref. 19. This present model—double-field inflation—thus provides a viable alternative scenario in which the end of the inflationary epoch occurs through the nucleation of bubbles. Although other inflationary models which use more than one scalar field have been proposed,[10] this present model is different in that it achieves successful inflation through a time-dependent nucleation rate and hence a time-dependent nucleation efficiency β [see Eq. (2.4)]. Some of the advantages and disadvantages of this model are discussed in Sec. IV. For example, cosmic strings can be formed at the end of the (first-order) phase transition by the inflaton field ϕ.

II. BASIC MECHANISM

Here we review the reasons for the failure of old inflation and discuss a possible mechanism to circumvent these problems. In order to use a simple but illustrative example, we consider a quantum field theory of a scalar field with a Lagrangian of the form

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V_1(\phi) , \tag{2.1}$$

where $V_1(\phi)$ is an asymmetric potential with metastable minimum ϕ_- and absolute minimum ϕ_+ (see Fig. 1). The energy difference between the vacua is ϵ. Bubbles of true vacuum (ϕ_+) expand into a false-vacuum (ϕ_-) background.

In the zero-temperature limit, the nucleation rate Γ_N (per unit time per unit volume) for producing bubbles of true vacuum in the sea of false vacuum through quantum tunneling can be calculated[11,12] and has the form

$$\Gamma_N(t) = A e^{-S_E} , \tag{2.2}$$

where S_E is the Euclidean action[12] corresponding to Eq. (2.1) and where A is a determinantal factor[11] which is generally of order T_c^4 (where T_c is the energy scale of the phase transition). In old inflation, this nucleation rate is taken to be approximately constant in time throughout the phase transition. Guth and Weinberg[2] have shown that the probability of a point remaining in the false-vacuum phase during the transition (which begins at t_i) is given by

$$p(t) = \exp\left[-\int_{t_i}^{t} dt' \Gamma_N(t') R^3(t') \frac{4\pi}{3} \left[\int_{t'}^{t} \frac{dt''}{R(t'')} \right]^3 \right] .$$

During the de Sitter phase of expansion, the exponent in Eq. (2.3) is approximately $-\frac{4}{3}\pi\beta\chi(t-t_i)$, where the dimensionless quantity β is defined by

$$\beta \equiv \frac{\Gamma_N}{\chi^4} . \tag{2.4}$$

The value of this *nucleation efficiency* β can be calculated from the potential and is crucial for determining the nature of the phase transition.

In the limit that β is small compared to unity (i.e., low nucleation efficiency), the phase transition proceeds slowly and the Universe can inflate through many e-foldings. This limit corresponds to the case of old inflation. However, when β is sufficiently small, the rate of filling the Universe with the true vacuum cannot keep up with the exponential expansion of the false vacuum and bubble percolation never occurs, i.e., the phase transition is never completed.[2] In addition, thermalization of individual

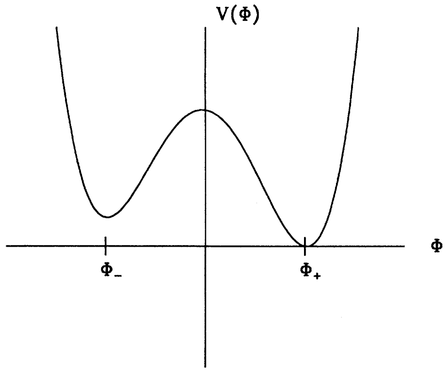

FIG. 1. Potential energy density of inflaton field ϕ as a function of field strength. The energy difference ϵ between the false vacuum (at $\phi_-=-a$) and the true vacuum (at $\phi_+=a$) provides the vacuum energy density for inflation.

bubbles or groups of bubbles never occurs. Those bubbles which nucleate early are quite large by the time later bubbles nucleate; hence a wide distribution of bubble sizes is produced. Groups of bubbles are dominated by the single largest bubble in a cluster. In any single bubble, the latent heat of the phase transition (ϵ) is entirely converted into the kinetic energy of the bubble wall[12] rather than thermalizing the interior of the bubble; in addition, collisions with much smaller bubbles cannot thermalize the interior. As a result, the (nearly) homogeneous and isotropic Universe we live in today can neither arise from a single large bubble nor from clusters of bubbles.

In the opposite limit when β is large compared to unity (i.e., high nucleation efficiency), the phase transition proceeds very rapidly. The time scale for bubbles to nucleate and percolate is small compared to the expansion time scale (which is determined by χ) for the Universe. In this limit, the phase transition is readily completed, but the Universe does not inflate sufficiently.

A critical value β_{cr} must exist,[2] such that $\beta \ge \beta_{cr}$ implies percolation (the supercritical regime) and $\beta \le \beta_{cr}$ implies no percolation (the subcritical regime). The critical value β_{cr} lies in the range (see Ref. 2).

$$0.24 \ge \beta_{cr} \ge 10^{-6} ,$$

although alternate arguments[13] have suggested $\beta_{cr} \approx 0.03$. As discussed above, β must be subcritical to allow for sufficient inflation *and* β must be supercritical to allow for percolation and hence to allow the phase transition to complete. Theories with constant β (i.e., a constant nucleation rate and a constant expansion rate χ) must clearly fail.[14]

In the present model, we consider a nucleation rate (and hence β) which can vary with time. The nucleation rate is initially small (so that $\beta<\beta_{cr}$). The Universe remains in the false vacuum and inflates for a long time. As the Universe evolves, the nucleation rate grows, and eventually β becomes supercritical. The bubbles of true vacuum can then percolate and the phase transition can be completed. As long as the time scale for β to evolve from a subcritical value to a supercritical value is long enough to allow for sufficient expansion of the Universe, a successful inflationary epoch will arise.

For definiteness, we take the potential of the inflaton field to be

$$V_1(\phi)=\tfrac{1}{8}\lambda_1(\phi^2-a^2)^2-\frac{\epsilon}{2a}(\phi-a) . \tag{2.5}$$

To leading order, the metastable minimum is given by $\phi_-=-a$ and the absolute minimum by $\phi_+=+a$. In addition, we will take an interaction term of the form

$$V_{\rm int}(\phi)=-\frac{1}{2a}Y(\psi)a^4(\phi-a) , \tag{2.6}$$

where Y is a dimensionless function which evolves in time and is independent of the ϕ field. In this case, the effective energy difference between the vacua [see ϵ in Eq. (2.5)] is given by

$$\epsilon_{\rm eff}=\epsilon+Y(\psi)a^4 . \tag{2.7}$$

Bubbles will nucleate at a rate given by Eq. (2.2). For the potential of Eq. (2.5) and in the limit that the nondegeneracy of the vacua is small (i.e., ϵ small), the Euclidean action can be obtained analytically[12] and is given by

$$S_E=\frac{\pi^2}{6}\frac{\lambda_1^2 a^{12}}{\epsilon_{\rm eff}^3} . \tag{2.8}$$

The limit of small ϵ is sometimes denoted as "the thin-wall limit" because the validity of the analytic expression (2.8) is limited to cases in which the wall thickness of the nucleated bubble is small compared to the bubble radius (see Ref. 12). If the function Y changes from a very small initial value (which leads to a small nucleation rate) to a large value at some later time t_f throughout space, a large nucleation rate will result and the phase transition can come to completion near the time t_f. If the end of this phase transition is sufficiently abrupt, bubbles of nearly equal size will nucleate simultaneously everywhere in space. Thus both percolation and thermalization can be more easily achieved. Any cluster of bubbles consists of equal-sized bubbles which can more easily thermalize one another than the wide variety of bubble sizes arising in old inflation.

III. DOUBLE-FIELD INFLATION

In this section, we implement the ideas discussed in the preceding section by presenting a particular model in which the interaction term is given by the interaction of the inflaton field ϕ with a second scalar field; the potential of this second field ψ is very flat and gives rise to slowly rolling behavior, just as in new inflation. The old inflation field ϕ, which is initially trapped in its metastable minimum and must tunnel through a potential barrier, dominates the dynamics and causes the Universe to inflate. The rolling field merely serves as a catalyst for an abrupt end to the inflationary epoch; i.e., the ψ field produces the desired time-dependent nucleation rate for bubbles of true ϕ vacuum. In this model, we find that bubbles of true vacuum can indeed percolate and we argue that thermalization of the interiors can more easily take place.

A. The model

The total Lagrangian (for both fields) has the form

$$\mathcal{L}=\tfrac{1}{2}(\partial_\mu\phi)^2+\tfrac{1}{2}(\partial_\mu\psi)^2-V_{\rm tot}(\phi,\psi)\ , \tag{3.1}$$

where the total potential can be written

$$V_{\rm tot}(\phi,\psi)=V_1(\phi)+V_2(\psi)+V_{\rm int}(\phi,\psi)\ . \tag{3.2}$$

For the sake of definiteness, we take $V_1(\phi)$ to be the potential of Eq. (2.5). The potential $V_2(\psi)$ can be any flat potential which leads to slow-rolling behavior of the ψ field [see Eq. (1.2)]. For convenience, we take the interaction term to be of the form

$$V_{\rm int}(\phi,\psi)=-\gamma(\phi-a)\psi^3\ , \tag{3.3}$$

where the dimensionless parameter γ determines the strength of the interaction and where a is the minimum of the potential $V_1(\phi)$. Notice that other forms for the interaction potential are possible (e.g., $V_{\rm int}\sim\phi^2\psi^2$); however, the resulting behavior should be qualitatively the same for a fairly wide variety of choices.

In the presence of the interaction term, the inflaton field ϕ will evolve according to the potential

$$V(\phi)=\tfrac{1}{8}\lambda_1(\phi^2-a^2)^2-\left[\frac{\epsilon}{2a}+\gamma\psi^3\right](\phi-a)\ . \tag{3.4}$$

We let the ϕ field be trapped in the false vacuum at the beginning of inflation. In the limit of nearly degenerate vacua (small ϵ) and sufficiently weak coupling (small γ), bubbles will nucleate at a rate given by Eq. (2.2), where the effective energy difference between the vacua is given by

$$\epsilon_{\rm eff}=\epsilon+2a\gamma\psi^3\ . \tag{3.5}$$

Notice that we have taken the small ϵ limit (or, equivalently, the "thin-wall limit," see Ref. 12) only for the sake of obtaining analytic results; larger values for ϵ (and γ) will lead to similar behavior.

During inflation, the equation of motion (1.2) for the rolling field ψ is approximately given by

$$3H\dot\psi=-\frac{\partial V}{\partial\psi}=-\frac{\partial V_2}{\partial\psi}+3\gamma(\phi-a)\psi^2\ , \tag{3.6a}$$

where we have neglected the $\ddot\psi$ term in accordance with the slow-rolling approximation.[15] In the limit that ϕ is in the false vacuum for essentially all of inflation (at least for purposes of determining the evolution of the ψ field), we can set $\phi\simeq-a$ and find the equation of motion

$$3H\dot\psi=-\frac{\partial V}{\partial\psi}=-\frac{\partial V_2}{\partial\psi}-6\gamma a\psi^2\equiv\mathcal{F}\ , \tag{3.6b}$$

where we have defined $\mathcal{F}$ as the sum of the two terms above. The first term in $\mathcal{F}$ is positive and causes the ψ field to roll down the hill; the second term, on the other hand, is a negative frictional term, and later we will demand that this term is small enough to allow the field to roll. The Hubble parameter is determined by

$$\left[\frac{\dot R}{R}\right]^2=H^2=\frac{8\pi G}{3}(\rho_\phi+\rho_\psi+\rho_{\rm rad})\ , \tag{3.7}$$

where ρ_ϕ and ρ_ψ are the false-vacuum energy densities of the ϕ and ψ fields, and where $\rho_{\rm rad}$ is the radiation energy density.

Initially, the value of ψ is small, $\epsilon_{\rm eff}$ is small, the nucleation rate of true vacuum bubbles $\Gamma_N(t)$ is small, and the Universe remains in the false ϕ vacuum and inflates. As the rolling field approaches its minimum, $\psi\to\psi_f$, the value of $\epsilon_{\rm eff}$ becomes larger, and many bubbles (with nearly equal sizes) of true vacuum nucleate throughout space. Our present Universe lies within one initially causally connected region which experiences an inflationary epoch; the end to this inflationary period occurs when the rolling field ψ approaches the minimum of its potential and thereby signals the old inflationary field ϕ to nucleate rapidly. Many bubbles of true vacuum nucleate simultaneously inside the region in which the ψ field is coherent [i.e., the entire region for which we can use a single evolution equation such as Eq. (3.6) to describe the behavior of the ψ field]. Our own Universe must lie within such a region of coherent ψ.

B. Constraints on the model

In order to obtain a successful epoch of double-field inflation, we must consider several constraints on the model parameters. First, we want the ϕ field to dominate the dynamics of the Universe and be responsible for the inflationary epoch; hence we require $V_1(\phi)>V_2(\psi)$. Since $\phi\simeq-a$ during inflation, this requirement becomes

$$\epsilon>V_2(\psi)\ . \tag{3.8a}$$

In particular, at the beginning of inflation when ψ is small [so that $V_2(\psi)$ is near its peak], the constraint takes the form

$$\epsilon>V_2(\psi_0)\ , \tag{3.8b}$$

where ψ_0 is the value of the ψ field at the beginning of the inflationary epoch. Given this constraint, the Hubble parameter is given by

$$H^2=\frac{8\pi G}{3}\rho_\phi\quad\text{where } \rho_\phi\simeq\epsilon+2\gamma a\psi^3\ , \tag{3.9}$$

where we have made the assumption that $\phi\simeq-a$ during inflation.

Second, in order for the coupling of the ψ field to influence the ϕ field and bring an end to inflation, we need the ratio $2\gamma a\psi^3/\epsilon$ to be sufficiently large at the end of inflation, i.e.,

$$\frac{2\gamma a\psi_f^3}{\epsilon}=\eta\ , \tag{3.10}$$

where η is a dimensionless constant; in practice we require $\eta\sim10^{-1}$ or larger.

Third, the slowly rolling field must be able to roll despite the frictional effect provided by the interaction term. We must have $\dot{\psi} > 0$, i.e.,

$$-\frac{\partial V_2}{\partial \psi} - 6\gamma a \dot{\psi}^2 = \mathcal{F} > 0 \ . \tag{3.11}$$

Fourth, we require that there be sufficient inflation; i.e, the total number N_T of e-foldings must satisfy $N_T \geq N_e$, where N_e is the number of e-foldings required to solve the original cosmological problems ($N_e \simeq 70$). For the slow-rolling ψ field, we can write the number N_T of e-foldings in terms of an integral and the constraint of obtaining sufficient inflation takes the form

$$N_T(\psi_0 \to \psi_f) = 3H^2 \int_{\psi_0}^{\psi_f} \frac{d\psi}{\mathcal{F}} \geq N_e \ . \tag{3.12}$$

Fifth, the quantum fluctuations in the field ψ render its value at any given time uncertain by the amount

$$\Delta\psi > \frac{H}{2\pi} \ , \tag{3.13}$$

which leads to a constraint on the initial value ψ_0, i.e.,

$$\Delta\psi_0 \geq H/2\pi \ , \tag{3.14}$$

which means that we cannot specify ψ_0 to an arbitrarily precise value.

The sixth constraint is that the density fluctuations in the slowly rolling field [see Eq. (1.3)] are not in conflict with the observed isotropy of the microwave background. This requirement can be written as

$$3H^3/\mathcal{F} \leq \delta \ , \tag{3.15}$$

where $\delta \equiv \delta\rho/\rho < 10^{-5}$ is the constraint on density perturbations.[6] We will also require all energy scales in the theory (e.g., the vacuum expectation values of the ϕ and ψ fields) to be below the Planck scale.

C. A simple example: The ramp potential

In this subsection, we will illustrate the model of double-field inflation by considering the simplest possible case for the potential of the rolling field ψ, i.e., we will take $\mathcal{F} = \text{const} > 0$. Many of the features of this simple case apply to any version of double-field inflation. We have chosen to present results for this simple case as it reveals many aspects of the double-field model with a minimal amount of algebra. In this model, the ψ field will move through a potential $V_{\text{eff}}(\psi)$ of the form

$$V_{\text{eff}}(\psi) = V_2(\psi) + V_{\text{int}}(\psi,\phi) = V_{\text{eff}}(\psi_0) - \mathcal{F}(\psi - \psi_0) \ . \tag{3.16}$$

Notice that for ψ near ψ_0 (i.e., near the beginning of inflation), the interaction term V_{int} is small and $V_2(\psi) \sim V_{\text{eff}}(\psi)$, which has a simple linear form. With this choice of potential, the constraint that the field ψ must be able to roll [see Eq. (3.11)] is automatically satisfied. Given this potential, the rolling field will begin at some initial value ψ_0 an roll to a final value ψ_f at the end of the inflationary epoch. The two most restrictive constraints are the density perturbation constraint

$$\frac{3H^3}{\mathcal{F}} \leq \delta \ , \tag{3.17}$$

and the constraint that sufficient inflation occurs

$$\frac{3H^2\psi_f}{\mathcal{F}} \geq N_e \ , \tag{3.18}$$

where we have taken $\psi_f \gg \psi_0$. For this theory, the fine-tuning parameter [as defined by Eq. (1.4)] can easily be evaluated and is given by

$$\lambda_2 = \frac{\mathcal{F}(\psi_f - \psi_0)}{(\psi_f - \psi_0)^4} \approx \frac{\mathcal{F}}{\psi_f^3} \ , \tag{3.19}$$

where the subscript denotes the second field ψ. Combining the constraints of Eqs. (3.17) and (3.18), we obtain an upper limit on the fine-tuning parameter:

$$\lambda_2 \leq 3\delta^2/N_e^3 \approx 10^{-15} \ , \tag{3.20}$$

where we have used $\delta = 10^{-5}$ and $N_e = 70$ to obtain the numerical value. Thus, we obtain a fine-tuning requirement similar to that of the standard new inflationary picture.

The fine-tuning of the potential arises in order to avoid overproduction of density fluctuations, which are produced by the rolling ψ field (this statement is generally true for models of inflation which involve slowly rolling fields.)[9] One might hope that the subsequent collisions of old inflation bubbles after the end of the inflationary period would dominate the resultant perturbation spectrum, especially since more energy density is associated with the ϕ field than with the ψ field. Unfortunately, these bubbles are tiny compared to scales of astrophysical interest (e.g., the scale of galaxies), which have gone outside the horizon well before the end of inflation and have ψ-field perturbations imprinted on them. In other words, the old inflation bubbles cannot affect structure on scales larger than the horizon size at the end of inflation, and this size scale is much smaller than galactic scales. Although dramatic bubble collisions can restructure the predicted anisotropy on small scales, these collisions cannot wipe out the unwanted large-scale perturbations produced by the rolling field.

We can now examine the remaining constraints by writing them in terms of the parameter λ_2 (which is constrained to be small); we will also define a nondimensional parameter for the vacuum energy density of the ϕ field, i.e.,

$$\tilde{\epsilon} \equiv \frac{\epsilon}{a^4} \ . \tag{3.21}$$

The constraint that the ϕ field dominates the energy density of the Universe [see Eq. (3.8)] can be written

$$\epsilon \geq V_2(\psi_0) \approx V_{\text{eff}}(\psi_0) \approx \lambda_2 \psi_f^4 \ , \tag{3.22a}$$

which now takes the form

$$\tilde{\epsilon} \geq \lambda_2 (\psi_f/a)^4 \ . \tag{3.22b}$$

The constraint that the coupling between fields is large enough to produce a time-dependent nucleation rate [see Eq. (3.10)] takes the form

$$\tilde{\epsilon} \sim 2\gamma(\psi_f/a)^3 \ . \tag{3.23}$$

Notice that if $\psi_f \sim a$ (i.e., the vacuum expectation values of the two fields are comparable), then $\tilde{\epsilon} \sim \gamma$.

The constraint of sufficient inflation [Eq. (3.18)] can now be written

$$\tilde{\epsilon} \approx \lambda_2 \frac{N_T}{8\pi} \frac{m_{\rm Pl}^2 \psi_f^2}{a^4} \ , \tag{3.24}$$

where $m_{\rm Pl}$ is the Planck mass and N_T is the total number of e-foldings ($N_T \geq N_e \approx 70$). If we combine this latter constraint with Eq. (3.23), we obtain the relation

$$\gamma \sim \frac{N_T}{16\pi} \frac{m_{\rm Pl}^2}{a\psi_f} \lambda_2 \ . \tag{3.25}$$

Since $N_T/16\pi$ is typically of order unity, the coupling constant γ is larger than the (small) parameter λ_2 by the factor $m_{\rm Pl}^2/a\psi_f$. In order to obtain $\tilde{\epsilon} \sim \gamma \sim 1$, we must have $\psi_f \sim a$ and $a/m_{\rm Pl} \sim 10^{-7}$ ($a \sim 10^{12}$ GeV). Thus, this model of double-field inflation can produce a reasonable scenario, provided that the small parameter λ_2 can be realized. Since the presence of such a small fine-tuning parameter is generic to theories of inflation which utilize slowly rolling fields (see Ref. 9 for a more complete discussion), this new model is comparable (in terms of fine-tuning) to existing models.

Notice that this model is described by seven parameters: the vacuum expectation value a of the potential of the inflaton field, the heights of the potentials λ_1 and λ_2, the energy difference ϵ, the coupling strength γ, and finally the initial and final values ψ_0 and ψ_f of the rolling field. Ideally one would like to explore fully the available range of parameter space; such a presentation with seven parameters subject to six constraints is beyond the scope of the present paper. However, some volume in this parameter space is allowed and will lead to successful inflation.

Many of the features of the simple $\mathcal{F}$=const model described above will hold in general for any version of double-field inflation. (1) Double-field inflation will involve the seven parameters described above (in general, the final value ψ_f of the rolling field corresponds to the vacuum expectation value of the ψ field). (2) Large-scale perturbations (i.e., on the scale of the present horizon down to the scale of galaxies) will be produced in a manner analogous to that of new inflation. These perturbations will not be erased through the nucleation and subsequent thermalization of bubbles of the ϕ field (these bubbles have size scales comparable to the horizon at the end of inflation, i.e., much smaller than the scale of galaxies). (3) To avoid overproduction of density perturbations on large scales, the potential of the slowly rolling field ψ must be very flat, with a fine-tuning parameter $\lambda \sim 10^{-15}$.

We have also considered more realistic choices for the potential of the ψ field in double-field inflation. For example, we have examined a potential $V_2(\psi)$ of the Coleman-Weinberg form,[16] i.e.,

$$V_2(\psi) = \tfrac{1}{2}B\sigma^4 + B\psi^4[\ln(\psi/\sigma)^2 - \tfrac{1}{2}] \ , \tag{3.26}$$

where σ is the vacuum expectation value of the ψ field and B characterizes the flatness of the potential and is analogous to the parameter λ_2 defined above. A discussion of double-field inflation with a Coleman-Weinberg potential is given in the Appendix. We find that successful double-field inflation can occur with this potential, although the constraints on the model are even more restrictive than in the simple case outlined above. In particular, the constraint that the ψ field can roll initially [see Eq. (3.11)] implies that the coupling parameter γ must be much smaller than unity; the constraint of Eq. (3.23) then implies that $\tilde{\epsilon}$ must also be small for this case. For example, if we take $\sigma \sim a \sim m_{\rm Pl}$, we find that $B \leq 3\delta^2/8N_T^3 \approx 10^{-15}$ and that $\tilde{\epsilon} \sim \gamma \sim B$. Thus, fine-tuning arises in this model. Alternatively, a model with two different inherent mass scales (similar to the case of schizons[17] or axions[18]) may provide the necessary flat potential.[19]

D. Evolution of the probability function

Once the necessary constraints are satisfied, we can solve for the evolution of the Universe. In particular. we can calculate the probability of finding the inflaton field ϕ in its false-vacuum state. This probability can be written [see Eq. (2.3) and Ref. 2]

$$p(t) = \exp\left[-\frac{4\pi}{3}\frac{A}{\chi_0^4}\int_{t_i}^{t} \chi_0 dt' e^{-S_E(t')}(1+\eta\tilde{\psi}^3)^{-3/2} \right] , \tag{3.27}$$

where S_E is the Euclidean action and is given by

$$S_E = S_0(1+\eta\tilde{\psi}^3)^{-3} \ , \tag{3.28}$$

where we have defined $\tilde{\psi} \equiv \psi/\psi_f$ and where the dimensionless constant η is given by Eq. (3.10) [notice that in this present notation, $\epsilon_{\rm eff} = \epsilon(1+\eta\tilde{\psi}^3)$].

Since the number of e-foldings (of the scale factor) is the relevant time variable for inflation, we change variables according to

$$d\tau = \chi\, dt = \chi_0(1+\eta\tilde{\psi}^3)^{1/2} dt \ ; \tag{3.29}$$

we can then write the probability as

$$p(\tau) = \exp\left[-\alpha \int_0^{\tau} d\tau\, e^{-S_0(1+\eta\tilde{\psi}^3)^{-3}}(1+\eta\tilde{\psi}^3)^{-2} \right] , \tag{3.30}$$

where we have defined

$$\alpha \equiv \frac{4\pi}{3}\frac{A}{\chi_0^4} \ , \tag{3.31}$$

which will generally be of order 1 (see Ref. 11). Equation (3.30) can be written as a differential equation,

$$\frac{dp}{d\tau} = -p(\tau)\alpha e^{-S_0(1+\eta\tilde{\psi}^3)^{-3}}(1+\eta\tilde{\psi}^3)^{-2} \ , \tag{3.32}$$

which can be integrated numerically once we have solved the evolution equation (3.6) for the $\tilde{\psi}$ field. As an example, we will consider the $\mathcal{F}$=const model presented in the preceding subsection; for this case, the equation of motion of the $\tilde{\psi}$ field takes the form

$$(1+\eta\tilde{\psi}^3)\frac{d\tilde{\psi}}{d\tau}=\frac{\mathcal{F}}{3\chi_0^2\psi_f}=\frac{\lambda_2\psi_f^2}{3\chi_0^2}\equiv\mathcal{B}\ , \tag{3.33}$$

where we have defined a new constant $\mathcal{B}$ [in the form of Eq. (3.33), the equation of motion can be easily integrated].

With this formulation of the problem, we must specify three parameters (S_0, η, and $\mathcal{B}$) to determine the evolution of the probability function (we have taken $\alpha=1$). The initial value S_0 of the action must be large enough to make the initial nucleation rate small, but small enough to allow for a sufficiently rapid nucleation rate at the end of inflation [see Eq. (3.28)]; we thus require $S_0\sim 10$. The interaction strength is given by η, which must be large enough to affect the evolution of the Universe but small enough not to dominate the dynamics; we thus require $\eta=0.1-1$. The constant $\mathcal{B}$ essentially determines the number of e-foldings [see Eq. (3.33)], so we must have $\mathcal{B}=O(1/N_e)$. If we choose $S_0=12$, $\eta=0.4$, and $\mathcal{B}=0.02$, the resulting probability evolution function $p(\tau)$ is shown in Fig. 2. By choosing parameters appropriately, we can arrange to have nucleation efficiency β [see Eq. (2.4)] subcritical initially and thereby obtain sufficient inflation. Since β is now time dependent, we can also have β supercritical for the latter part of the inflationary epoch and thereby allow for percolation of the true-vacuum bubbles. Notice also that the probability function $p(\tau)$ is much more like a step function (see Fig. 2) than for the case of old inflation (i.e., constant nucleation efficiency). This result implies that most of the bubbles of true vacuum which are nucleated will have sizes comparable to the horizon scale at the end of the inflationary epoch; since this size scale is small compared to size scales of astrophysical interest (e.g., the scale of galaxies), the only relevant density fluctuations produced by this inflation will result from the rolling field ψ and not from the inflaton field ϕ if the phase transition is infinitely sharp. Notice, however, that the end of the phase transition is not infinitely sharp; the width of the phase transition shown in Fig. 2 is approximately 20 e-foldings. We have not calculated the details of the end of this phase transition; we leave this study of the thermalization for future work. Notice, however, that the nucleation of the inflaton field may generate additional large-scale structure which may explain some of the features we observe today.

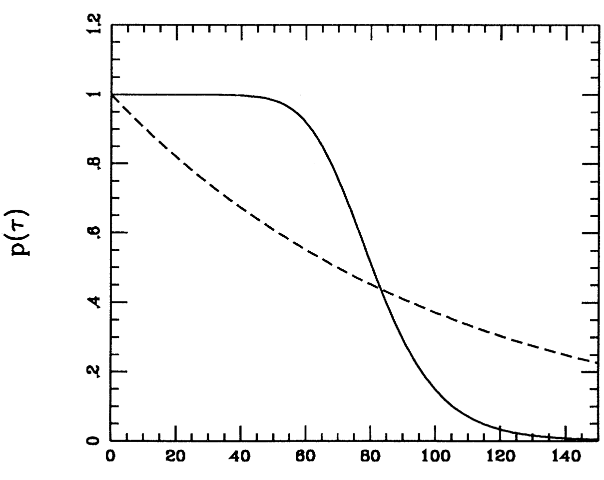

FIG. 2. Probability p of a point in space being in the false vacuum as a function of nondimensional time τ. Solid curve shows the double-field inflation model of Sec. III; for comparison, the dashed curve shows the case of constant nucleation efficiency (as in old inflation).

IV. DISCUSSION

We have studied inflationary scenarios which (ab)use two coupled real scalar fields; the coupling between fields can lead to a time-dependent nucleation rate. We thus obtain a successful inflationary scenario which ends through a first-order phase transition, i.e., through the nucleation of true-vacuum bubbles in the sea of false vacuum. The required degrees of flatness in the potential of the rolling field ψ is comparable to that required in "new inflation," i.e., $\lambda\sim 10^{-15}$. This present model is thus comparable in success to existing models but occurs in a different manner and may offer some advantages. For example, the inflationary epoch ends through the process of nucleation and topological defects and such as cosmic strings[20] can form at the end of the phase transition (provided that the potential of the ϕ field is complex).

"Extended" inflation[21] also revives some of the aspects of the "old" inflation models in that the inflation takes place at a supercooled first-order phase transition. The essential difference from old inflation is that gravity is described not by general relativity, but by Brans-Dicke[22] theory. Extended inflation also provides a time-dependent nucleation efficiency; however, the time dependence is achieved through a time-dependent Hubble parameter [see the denominator of Eq. (2.4)] rather than through a time-dependent nucleation rate [the numerator of Eq. (2.4)]. Studies of bubble nucleation, collisions, and percolation[23,24] restrict the allowed parameters of the model and the potential of the coupled field also must be fine-tuned (see Ref. 9). A generalized version of extended inflation ("hyperextended inflation"[25]) utilizes more complicated couplings of the rolling field to gravity to obtain a time-dependent Hubble parameter (and hence a time-dependent β).

The specific model of double-field inflation presented in this paper can produce a "successful" inflationary epoch. However, the theory must contain a very small parameter (namely $\lambda_2\sim 10^{-15}$) in order to satisfy constraints on density perturbations. Although this particular (highly simplified) model is unlikely to provide the ultimate inflationary scenario, the concept of a time-dependent nucleation rate provides a promising mechanism. Freese, Frieman, and Olinto[19] proposed a model using pseudo Nambu-Goldstone bosons tht naturally involves two disparate mass scales and thus gives very flat potentials without any fine-tuning of parameters; potentials such as these are ideal for the rolling field in double-field inflation.

Note added in proof. After the completion of this paper, we discovered that A. Linde has simultaneously suggested the possibility of a time-dependent nucleation rate through the coupling of scalar fields [Report No. CERN-TH.5806/90, 1990 (unpublished)].

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APPENDIX: THE COLEMAN-WEINBERG CASE

In this appendix, we will consider a more realistic model of double-field inflation using a Coleman-Weinberg form for the potential of the slowly rolling field; i.e., we will take

$$V_2(\psi)=\frac{B\sigma^4}{2}+B\psi^4\left[\ln\left[\frac{\psi^2}{\sigma^2}\right]-\frac{1}{2}\right] . \tag{A1}$$

The potential for the ϕ field is still described by Eq. (2.5) with the interaction term of Eq. (3.3). The ψ field starts rolling at some initial value $\psi_0 \geq H/2\pi$ and finally reaches its stable minimum at $\psi_f=\sigma$. With this choice of potential, the equation of motion (3.6) becomes

$$3H\dot{\psi}=-4B\psi^3\ln\left[\frac{\psi^2}{\sigma^2}\right]-6\gamma a\psi^2 . \tag{A2}$$

We will consider the constraints for this potential. We will consider the special case where $\sigma=a$ and will define $\tilde{\epsilon}=\epsilon/a^4$ as in Eq. (3.21). With these restrictions, the requirement that the ϕ field dominate the dynamics of the Universe and cause an inflationary epoch becomes

$$\tilde{\epsilon}\geq B/2 . \tag{A3}$$

In order for the coupling to the ψ field to influence the ϕ field we require that

$$\gamma\geq\tilde{\epsilon}/20 , \tag{A4}$$

where we have taken $\eta=0.1$ [see Eq. (3.10)]. The third constraint Eq. (3.11), the requirement that $\dot{\psi}>0$ in order for the ψ field to enter a slow-rolling epoch, becomes

$$\lambda_B(\psi_0/a)\geq 6\gamma , \tag{A5}$$

where ψ_0 is the initial value of the ψ field and where we have defined $\lambda_B\equiv 4B\ln(a^2/\psi_0^2)$. The condition of sufficient inflation then becomes

$$N(\psi_0\rightarrow\psi_f)\simeq\frac{3H^2}{2\lambda_B}\left[\frac{1}{\psi_0^2}-\frac{1}{\psi_f^2}\right]\approx\frac{3H^2}{2\lambda_B}\frac{1}{\psi_0^2}\geq N_e , \tag{A6}$$

where we have assumed that the final value $\psi_f>>\psi_0$ and where N_e is the required number of e-foldings. The quantum fluctuation constraint [Eq. (3.14)] remains the same. Finally, the constraint that the density fluctuations are sufficiently small [see Eq. (3.15)] takes the form

$$\frac{3H^3}{\lambda_B\psi_0^3}\leq\delta . \tag{A7}$$

The coupled constraints of Eqs. (A6) and (A7) can be combined to obtain a bound on the parameter λ_B:

$$\lambda_B\leq\frac{3\delta^2}{8N_e^3} . \tag{A8}$$

The numerical value of the right-hand side of Eq. (A8) is of order 10^{-14}; we thus obtain a "fine-tuning" requirement which is comparable in magnitude to that of new inflation. Let us now saturate the constraint of Eq. (A3); i.e., we will take $\tilde{\epsilon}=B/2$. If we then consider the specific case $a=m_{\rm Pl}$ and define $x=\psi_0/a=\psi_0/m_{\rm Pl}$ (where the dimensionless parameter x must be less than unity), we can write the remaining constraints in the form

$$\gamma\geq B/40 , \tag{A9a}$$

$$4Bx\ln(1/x)\geq 3\gamma , \tag{A9b}$$

$$\frac{\pi}{4N_e}\geq x^2\ln(1/x) . \tag{A9c}$$

If we saturate the third constraint [Eq. (A9c)] and solve for x we obtain $x=0.064$ (where we have taken $N_e=70$). With this value of x, the remaining two constraints [Eqs. (A9a) and (A9b)] confine the ratio B/γ to the range

$$4.3\leq B/\gamma\leq 40 . \tag{A10}$$

With these values of the parameters, the quantum fluctuation constraint [Eq. (3.14)] is satisfied. Thus, there exists a region of parameter space which allows successful inflation with two coupled scalar fields. Notice, however, that Eq. (A8) constrains λ_B (hence B) to be very small. In our specific example, $\tilde{\epsilon}=B/2$ (by assumption) and γ is within an order of magnitude of B [by Eq. (A10)], so that both $\tilde{\epsilon}$ and γ are also very small in this case.

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