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Perturbative renormalization of null-plane QED

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It has been recognized for some time that quantization on a null plane has several unique and remarkable advantages for the elucidation of quantum field theories. To date these unique features have not been exploited to solve strongly coupled, four-dimensional gauge theories. This is the first in a series of papers aimed at systematically formulating renormalizable gauge theories on the null plane. In order to lay down the groundwork for upcoming nonperturbative studies, it is indispensable to gain control over the perturbative treatment first. A discussion of one-loop renormalization of QED in the Hamiltonian formalism is presented. In this approach, one is faced with severe infrared divergences characteristic of the light-cone gauge. We show how to treat these divergences in a coherent fashion, and thus recover the usual results of the renormalization procedure such as Ward identities and coupling-constant renormalizations.

I. INTRODUCTION

The dynamics of a relativistic many-body system is specified completely by expressing the ten generators of the Poincaré group P^μ and $M^{\mu\nu}$ in terms of dynamical variables. The dependence of the generators upon the interaction between the particles is not unique, but there always exists a kinematic subgroup, the set of generators which are independent of the interactions. The kinematic subgroup is uniquely determined by the choice of an "initial surface" on which the state of a system is specified. The Poincaré generators that do not belong to the kinematic subgroup are called "Hamiltonians" and they contain information about the dynamics.

In the usual formulation ($H=P^0$) of dynamics, a physical state is given at $x^0=$const, which is left invariant by the six generators of the associated kinematic subgroup, translations, and rotations. In the null-plane formalism ($H=P^-$) a physical state is given on $x^+=(x^0+x^3)/\sqrt{2}=$const. There are seven generators that leave the null plane invariant, and this is the largest stability group of all possible schemes of relativistic evolution.[1]

Another important property of the null-plane kinematic subgroup is that it acts transitively on the mass shell $p^2=m^2$, $p^0>0$. Hence the null-plane wave function of a system is determined if it is known at rest. This property is not shared by theories based on a $x^0=$const initial surface because the boost operators depend on the dynamics. That is why the null-plane formulation is ideally suited for the relativistic bound-state problem. Finally the kinematic subgroup of the null plane contains the generator $P^+=(P^0+P^3)/\sqrt{2}$ of lightlike translations. The requirement that the spectrum of P^μ should be contained in the forward cone $p^2>0$, $p^0>0$ implies that for massive particles p^+ must be positive. This ensures that the exact ground state of the system is the bare Fock-space vacuum of the canonical quanta.

Thus if a viable nonperturbative approximation scheme is ever devised for the null-plane quantization approach, vacuum problems will be vastly simplified. In a future publication, we will investigate the approach invented independently by Tamm and Dancoff in the late 1940s.[2,3] The idea consists in diagonalizing the Hamiltonian in a Fock space truncated to some finite number of bare particles, yielding a finite number of coupled integral equations in the wave functions of the Fock components.[4] In doing this, one insists that the masses of the physical free particles are fixed once and for all, which requires the addition of counterterms to the Hamiltonian. The method will be successful if the number of required counterterms remains finite, and if the eigenvalues converge rapidly enough when the size of the allowed Fock space is increased.

The pursuit of the Tamm-Dancoff approach in the 1950s led to insoluble difficulties because, in a *space-time* frame, first the physical vacuum is not a Fock state, and second the dependence of the boost operators on the interaction makes a covariant treatment impractical. On the other hand, a Tamm-Dancoff method developed in a *null-plane* frame might very well, because of the properties described above, allow for a solution of quantum field theories.[5,6] A clear understanding of divergences in perturbation theory is a necessary first step towards formulating such a program. The objective of this work is precisely to clarify this issue, at least at order one loop. We expect it to provide guidance in the choice of counterterms in the null-plane Tamm-Dancoff treatment, as well as a check of our future results against perturbative ones in the small-coupling limit.

The null-plane (or light-front) formulation of quantum field theory is not to be confused with the infinite-momentum frame (IMF) description, in which a field theory is formulated in standard fashion, but in a frame moving at (almost) the speed of light [the renormalization of QED at order one loop in the IMF can be found in Refs. 7(a) and 7(d)]. In the general formulation of a field theory in a null-plane frame, canonical commutation relations and initial conditions are defined on the null plane $x^+=0$ and the evolution of the system with respect to x^+ is determined.[8,9] The canonical coordinates and independent degrees of freedom are very different in this approach from those in the standard formulation of field theory. In particular, QED develops four-point interactions, the so-called instantaneous terms, which introduce new singularities into the theory. It is not surprising therefore that the one-loop renormalization of QED quantized on the null plane looks very different from the standard treatment. For example, one finds[10] that, in the IMF limit, the disconnected vacuum contributions survive simply because they are Lorentz invariant; furthermore there is a delicate interplay between cutoffs and total-momentum limits, giving origin to tedious calculations. These difficulties do not arise in the null-plane treatment.

Quantization is done in the light-cone gauge $A^+=0$, and a "time"-ordered perturbation theory is developed in the null-plane Hamiltonian formalism. For gauge-invariant quantities, this is equivalent to the use of Feynman diagrams together with an integration over p^- by residues.[11] In addition to not being manifestly covariant, x^+-ordered perturbation theory is fraught with singularities, even at the tree level. In particular, there are two types of instantaneous four-point interactions, one of them proportional to $(p_{\rm tr}^+)^{-1}$ and the other to $(p_{\rm tr}^+)^{-2}$, where $p_{\rm tr}$ is the momentum transferred instantaneously across the vertex. The origin of these unusual, "spurious," infrared divergences is no mystery. Consider for example a free particle whose transverse momentum $\mathbf{p}_\perp=(p^1,p^2)$ is fixed, and whose third component p^3 is cut off at some momentum Λ. Using the mass-shell relation one sees easily that p^+ has a lower bound proportional to Λ^{-1}. Hence the light-cone spurious infrared divergences are simply a manifestation of space-time ultraviolet divergences. One of the goals of this and future work is to show how to treat these divergences in a self-consistent manner. Bona fide infrared divergences are of course also present, and can be taken care of as usual by giving the photon a small mass, consistent with null-plane quantization.[12] For simplicity in this paper we shall not provide an exhaustive treatment of the latter type of infrared divergences, but shall concentrate on the divergences requiring renormalization. The appearance of four-point interactions, that is of terms that are naively nonrenormalizable, indicates that a power-counting analysis of divergences works very differently in null-plane quantization from a conventional space-time treatment. To the order that is dealt with here (one loop), this did not cause any major problems. However a more careful study of higher-order corrections is necessary to understand the full structure of counterterms in null-plane QED.

In Sec. II we present some of the basics of null-plane theory including a brief review of the null-plane QED Hamiltonian based on the work of Kogut and Soper.[13] Some of our results in the next sections overlap with those of Bjorken, Kogut, and Soper,[14] who studied scattering from an external field in the same framework. In Sec. III we calculate the fermion mass shift and wave-function renormalization Z_2. Here we introduce our mixed regularization scheme which involves dimensional regularization in the transverse dimensions and cutoffs in the longitudinal direction $x^-=(x^0-x^3)/\sqrt{2}$. A three-dimensional cutoff regularization scheme is also presented. In the dimensional-regularization scheme, which will be adopted in most of this paper, Z_2 is found to be momentum dependent.

In Sec. IV we calculate the photon mass shift and wave-function renormalization Z_3. In Sec. V we calculate the various vertex corrections. We show that at zero photon momentum the Ward identity $Z_1=Z_2$ is satisfied. However in general, because of the momentum dependence of Z_2 found in Sec. III, the Ward identity takes the form $Z_1(p^+,\bar{p}^+)=\sqrt{Z_2(p^+)Z_2(\bar{p}^+)}$, where p^+ and $\bar{p}^+$ are the longitudinal momenta of the incoming and outgoing electrons.

The Heitler method[15] for defining single-particle reducible diagrams which have a vanishing "energy" denominator, is discussed. Finally we effect the renormalization of the electric charge and find the standard result $e_R=e\sqrt{Z_3}$.

In Sec. VI we present our summary and outlook.

II. BASICS

We define our light-cone coordinates by

$$x^+\equiv\frac{x^0+x^3}{\sqrt{2}},\quad x^-\equiv\frac{x^0-x^3}{\sqrt{2}},\quad \mathbf{x}_\perp=(x^1,x^2)\ , \tag{2.1}$$

and four-vectors are $x=(x^+,x^-,\mathbf{x}_\perp)$. The matrix tensor is

$$g^{\mu\nu}=\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix}. \tag{2.2}$$

Our Dirac matrices have the property

$$(\gamma^+)^2=(\gamma^-)^2=0\ . \tag{2.3}$$

We will use extensively the projection operators

$$\Lambda_\pm\equiv\frac{\gamma^\mp\gamma^\pm}{2}\ . \tag{2.4}$$

Fermion fields will often be decomposed as

$$\psi=\psi_++\psi_-,\quad\text{where }\psi_\pm\equiv\Lambda_\pm\psi\ . \tag{2.5}$$

The integration measure in light-cone coordinates is given by

$$\int d^2\mathbf{p}_\perp\int_0^\infty\frac{dp^+}{2p^+}=\int d^4p\,\delta(p^2-m^2)\theta(p^0)=\int\frac{d^3\mathbf{p}}{2\omega_p}\ . \tag{2.6}$$

A list of conventions can be found in Appendix A along with a number of useful identities.

The null-plane Hamiltonian is P^-, the operator conjugate to the "time" evolution variable x^+. P^μ is given in terms of the energy-momentum tensor

$$P^\mu=\int d^2\mathbf{x}_\perp dx^-T^{+\mu}\ . \tag{2.7}$$

The energy-momentum tensor is easily obtained from the standard QED Lagrangian

$$\mathcal{L}=\frac{i}{2}\bar{\psi}\overleftrightarrow{\partial}\psi-m\bar{\psi}\psi-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-J^{\mu}A_{\mu}\ , \tag{2.8}$$

where

$$J^{\mu}=e\bar{\psi}\gamma^{\mu}\psi\ . \tag{2.9}$$

One finds

$$T^{\mu\nu}=F^{\alpha\mu}(\partial^{\nu}A_{\alpha})+g^{\mu\nu}\tfrac{1}{4}F^{\alpha\beta}F_{\alpha\beta}+\bar{\psi}\left[\frac{i}{2}\gamma^{\mu}\overleftrightarrow{\partial}^{\nu}+g^{\mu\nu}\left[m-\frac{i}{2}\overleftrightarrow{\not\partial}\right]\right]\psi+g^{\mu\nu}J^{\alpha}A_{\alpha}\ . \tag{2.10}$$

It is convenient to separate out P^- into a purely bosonic part P_G^- plus a coupled fermionic part P_F^-, and working in the light cone gauge $A^+=0$ we find

$$P_G^-=\int d^2\mathbf{x}_{\perp}dx^-[(\partial_-A_k)(\partial_kA_+)-\tfrac{1}{2}(\partial_-A_+)^2+\tfrac{1}{2}(F_{12})^2]\quad(k=1,2)\ , \tag{2.11}$$

$$P_F^-=\int d^2\mathbf{x}_{\perp}dx^-\left[\bar{\psi}\left[-\frac{i}{2}\gamma^-\overleftrightarrow{\partial}_--\frac{i}{2}\gamma^k\overleftrightarrow{\partial}_k+m\right]\psi+J^{\mu}A_{\mu}\right]\ . \tag{2.12}$$

However from the equations of motion for the vector potential we see that

$$\partial_-^2A_+=\partial_-\partial_kA_k-J^+\ , \tag{2.13}$$

so that A_+ is not an independent variable. Solving for $A_+(x^+,x^-,\mathbf{x}_{\perp})$ we find

$$A_+(x)=\int dy^-\frac{|x^--y^-|}{2}[\partial_-\partial_kA_k(y^-)-J^+(y^-)]\ . \tag{2.14}$$

Similarly the fermion equation of motion gives

$$\partial_-\psi_-=-\frac{i}{2}[(i\partial_k-eA_k)\gamma^k+m]\gamma^+\psi_+ \tag{2.15}$$

so that ψ_- is not an independent variable. We find

$$\psi_-(x)=-\frac{i}{4}\int dy^-\epsilon(x^--y^-)\times\{[i\partial_k-eA_k(y^-)]\gamma^k+m\}\gamma^+\psi_+(y^-)\ . \tag{2.16}$$

Using the equations of motion and integrating by parts, P_F^- can be written more simply as

$$P_F^-=\int d^2\mathbf{x}_{\perp}dx^-\left[\frac{i}{2}\bar{\psi}\gamma^-\overleftrightarrow{\partial}_-\psi+J^+A_+\right]\ . \tag{2.17}$$

It is now clear that we have two independent boson degrees of freedom A_k, which we will now write $a_k=A_k$, and we define $A_+=a_++\varphi_+$ and $a_-=A_-=0$, where $\partial_-a_+=\partial_ka_k$ and $\partial_-^2\varphi_+=-J^+$. Only one of the projections of ψ is an independent degree of freedom. We define $\psi_-=\xi_-+\eta_-$, $\psi_+=\xi_+$, and $\xi=\xi_++\xi_-$, and use ξ_+ as our independent fermion degree of freedom. ξ_- and η_- are defined by

$$\partial_-\xi_-=-\frac{i}{2}(i\gamma^k\partial_k+m)\gamma^+\xi_+\ , \tag{2.18}$$

$$\partial_-\eta_-=\frac{i}{2}eA_k\gamma^k\gamma^+\xi_+\ . \tag{2.19}$$

Inserting these into P^- we find

$$P^-\equiv H=H_0+V_1+V_2+V_3\ , \tag{2.20}$$

where

$$H_0=\int d^2\mathbf{x}_{\perp}dx^-\left[\frac{i}{2}\bar{\xi}\gamma^-\overleftrightarrow{\partial}_-\xi+\frac{1}{2}(F_{12})^2-\frac{1}{2}a_+\partial_-\partial_ka_k\right] \tag{2.21}$$

is the free Hamiltonian,

$$V_1=e\int d^2\mathbf{x}_{\perp}dx^-\bar{\xi}\gamma^{\mu}\xi a_{\mu} \tag{2.22}$$

is a standard, order-e, three-point interaction,

$$V_2=\frac{i}{2}\int d^2\mathbf{x}_{\perp}dx^-\bar{\eta}\gamma^-\overleftrightarrow{\partial}_-\eta=-\frac{i}{4}e^2\int d^2\mathbf{x}_{\perp}dx^-dy^-\epsilon(x^--y^-)(\bar{\xi}a_k\gamma^k)(x)\times\gamma^+(a_j\gamma^j\xi)(y) \tag{2.23}$$

is an order-e^2 nonlocal effective four-point vertex corresponding to an instantaneous fermion exchange, and

$$V_3=\frac{e}{2}\int d^2\mathbf{x}_{\perp}dx^-\bar{\xi}\gamma^+\xi\varphi_+=-\frac{e^2}{4}\int d^2\mathbf{x}_{\perp}dx^-dy^-(\bar{\xi}\gamma^+\xi)(x)|x^--y^-|\times(\bar{\xi}\gamma^+\xi)(y) \tag{2.24}$$

is an order-e^2 nonlocal effective four-point vertex corresponding to an instantaneous photon exchange. Graphically V_2 and V_3 will be drawn as four-point interactions and a hash mark will be drawn on the instantaneous particle. All graphs will be representations of matrix elements of H in "old-fashioned" perturbation theory.[15] ξ and α_{μ} have standard expansions in terms of creation and annihilation operators:

$$\xi(x)=\int\frac{d^2\mathbf{p}_\perp}{(2\pi)^{3/2}}\int\frac{dp^+}{\sqrt{2p^+}}\sum_{s=\pm 1/2}[u(p,s)e^{-i(p^+x^- - \mathbf{p}_\perp \mathbf{x}_\perp)}b(p,s,x^+)+v(p,s)e^{i(p^+x^- - \mathbf{p}_\perp \mathbf{x}_\perp)}d^\dagger(p,s,x^+)]\ , \tag{2.25}$$

$$a_\mu(x)=\int\frac{d^2\mathbf{q}_\perp}{(2\pi)^{3/2}}\int\frac{dq^+}{\sqrt{2q^+}}\sum_{\lambda=1,2}\epsilon_\mu^\lambda(q)[e^{-i(q^+x^- - \mathbf{q}_\perp \mathbf{x}_\perp)}a(q,\lambda,x^+)+e^{i(q^+x^- - \mathbf{q}_\perp \mathbf{x}_\perp)}a^\dagger(q,\lambda,x^+)]\ , \tag{2.26}$$

where

$$\{b(p,s),b^\dagger(p',s')\}=\delta(p^+-p'^+)\delta^2(\mathbf{p}_\perp-\mathbf{p}'_\perp)\delta_{ss'}=\{d(p,s),d^\dagger(p',s')\}\ , \tag{2.27}$$

$$[a(q,\lambda),a^\dagger(q',\lambda')]=\delta(q^+-q'^+)\delta^2(\mathbf{q}_\perp-\mathbf{q}'_\perp)\delta_{\lambda\lambda'}\ . \tag{2.28}$$

(These relations hold at equal x^+, and this argument is suppressed for brevity.) In terms of these momentum-space operators, the free Hamiltonian has the form

$$H_0=\int d^2\mathbf{p}_\perp dp^+\left[\frac{\mathbf{p}_\perp^2+m^2}{2p^+}\sum_{s=\pm 1/2}(b^\dagger(p,s)b(p,s)+d^\dagger(p,s)d(p,s))+\frac{\mathbf{p}_\perp^2}{2p^+}\sum_{\lambda=1,2}a^\dagger(p,s)a(p,s)\right]\ . \tag{2.29}$$

Additional discussion of the interactions can be found in Appendix A (we do not attempt here to present a detailed set of diagrammatic rules).

III. ELECTRON MASS AND WAVE-FUNCTION RENORMALIZATION

Consider the amplitude T_{pp} of the transition matrix T between free electron states (p,s) and (p,σ). Note that the normalization of states

$$\langle p',s'|p,s\rangle=\delta(p^+-p'^+)\delta^2(\mathbf{p}_\perp-\mathbf{p}'_\perp)\delta_{ss'}\ , \tag{3.1}$$

which corresponds to Eq. (2.27), is not Lorentz invariant. For an invariant normalization one must use

$$|\tilde{p},s\rangle\equiv\sqrt{2p^+}|p,s\rangle\ . \tag{3.2}$$

Hence

$$2m\,\delta m\,\delta_{s\sigma}\equiv T_{\tilde{p}\tilde{p}}=2p^+T_{pp}\Longrightarrow\delta m\,\delta_{s\sigma}=\frac{p^+}{m}T_{pp}\ . \tag{3.3}$$

One can also identify a matrix $\Sigma(p)$ through

$$\delta m\,\delta_{s\sigma}\equiv\bar{u}(p,\sigma)\Sigma(p)u(p,s)\ . \tag{3.4}$$

Note that in this time-ordered formalism all particles are on shell; therefore, $\Sigma(p)$ has meaning only as defined by Eq. (3.4). In particular, the electron wave-function renormalization Z_2 must be obtained separately.

At order e^2, we find three contributions to this amplitude. First, the perturbation expansion

$$T=V+V\frac{1}{p^--H_0}V+\cdots \tag{3.5}$$

yields a contribution which is second-order in V_1, and is shown in Fig. 1(a) (in all our diagrams, time flows toward the top). Here

$$p=\left[p^+,\frac{\mathbf{p}_\perp^2+m^2}{2p^+},\mathbf{p}_\perp\right] \tag{3.6}$$

is the initial (or final) electron momentum. The longitudinal and transverse momenta are conserved at each vertex, and all particles are on shell. Also, we have first-order contributions from V_2 and V_3. These two-point vertices have been called "seagulls" or "self-induced inertias" in the literature. They are displayed in Figs. 1(b) and 1(c).

Detailed calculations are found in Appendix B. Here we briefly present the results along with a discussion of regularization procedures.

The other momenta appearing in Fig. 1(a) are

$$k=\left[k^+,\frac{\mathbf{k}_\perp^2}{2k^+},\mathbf{k}_\perp\right] \tag{3.7}$$

and

$$k'=\left[p^+-k^+,\frac{(\mathbf{p}_\perp-\mathbf{k}_\perp)^2+m^2}{2(p^+-k^+)},\mathbf{p}_\perp-\mathbf{k}_\perp\right]\ . \tag{3.8}$$

Using Eqs. (A14), (A17), and (A23), we obtain

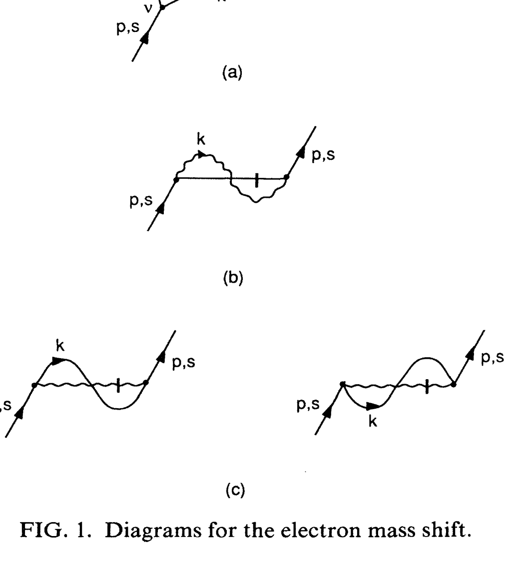

FIG. 1. Diagrams for the electron mass shift.

$$\delta m_a \delta_{s\sigma} = \frac{e^2}{m}\int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3}\int_0^{p^+} \frac{dk^+}{k^+(p^+-k^+)} \frac{\bar{u}(p,\sigma)\gamma^\mu(\not{k}'+m)\gamma^\nu u(p,s) d_{\mu\nu}(k)}{p^- - k^- - k'^-}$$
$$= \bar{u}(p,\sigma)\Sigma_a(p)u(p,s) \ . \tag{3.9}$$

For diagram 1(b), one gets, using Eq. (A24),

$$\delta m_b \delta_{s\sigma} = \frac{e^2 p^+ \delta_{s\sigma}}{2m}\int \frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\int_0^{+\infty}\frac{dk^+}{k^+(p^+-k^+)} = \bar{u}(p,\sigma)\Sigma_b(p)u(p,s) \ . \tag{3.10}$$

For diagram 1(c), one finds, using Eq. (A26),

$$\delta m_c \delta_{s\sigma} = \frac{e^2 p^+ \delta_{s\sigma}}{2m}\int \frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\left[\int_0^{+\infty}\frac{dk^+}{(p^+-k^+)^2} - \int_0^{+\infty}\frac{dk^+}{(p^++k^+)^2}\right]$$
$$= \bar{u}(p,\sigma)\Sigma_c(p)u(p,s) \ . \tag{3.11}$$

These integrals have potential singularities at $k^+=0$ and $k^+=p^+$, as well as an UV divergence in $\mathbf{k}_\perp$. To regularize them, in a first step we introduce small cutoffs α and β,

$$\alpha < k^+ < p^+ - \beta \ , \tag{3.12}$$

and get rid of the pole at $k^+=p^+$ in δm_b and δm_c by a principal-value prescription. Then one obtains (see Appendix B)

$$\delta m_a = \frac{e^2}{2m}\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\left[\int_0^{p^+}\frac{dk^+}{k^+}\frac{m^2}{p\cdot k} - 2\left[\frac{p^+}{\alpha}-1\right] - \ln\left[\frac{p^+}{\beta}\right]\right] ,$$
$$\delta m_b = \frac{e^2}{2m}\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\ln\left[\frac{p^+}{\alpha}\right] , \tag{3.13}$$
$$\delta m_c = \frac{e^2}{m}\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\left[\frac{p^+}{\alpha}-1\right] ,$$

where

$$p\cdot k = \frac{m^2(k^+)^2 + (p^+)^2\mathbf{k}_\perp^2}{2p^+k^+} \ . \tag{3.14}$$

Adding these three contributions yields

$$\delta m = \frac{e^2}{2m}\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\left[\int_0^{p^+}\frac{dk^+}{k^+}\frac{m^2}{p\cdot k} + \ln\left[\frac{\beta}{\alpha}\right]\right] . \tag{3.15}$$

Note the cancellation of the most singular infrared divergence.

To complete the calculation, we present two possible regularization procedures.

Transverse dimensional regularization. The dimension d of transverse space is continued from its physical value of two, and the resulting integrals are expressed in terms of

$$\epsilon \equiv 1 - \frac{d}{2} \ . \tag{3.16}$$

Some useful formulas can be found in Appendix A. In this method, α and β are treated as constants, so the rules give zero for the logarithmic term in Eq. (3.15), and we are left with

$$\delta m = \frac{e^2 m}{(2\pi)^3}\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{\mathbf{k}_\perp^2 + m^2x^2} , \tag{3.17}$$

where $x \equiv (k^+/p^+)$. Using the rules again, we get finally

$$\delta m = \frac{e^2 m}{8\pi^2\epsilon} \ . \tag{3.18}$$

Cutoffs. In this method, one restricts the momenta of any intermediate state by means of the covariant condition

$$P^2 < \Lambda^2 \ , \tag{3.19}$$

where P is the total four-momentum of the intermediate state, and Λ is a large momentum cutoff. Furthermore, we assume that all transverse momenta are smaller than a certain cutoff $\Lambda_\perp$,[16] with

$$\Lambda_\perp \ll \Lambda \ . \tag{3.20}$$

In the case of diagram 1(a), Eq. (3.19) reads

$$\frac{\mathbf{k}_\perp^2}{k^+} + \frac{(\mathbf{p}_\perp - \mathbf{k}_\perp)^2 + m^2}{p^+ - k^+} < \Lambda' , \tag{3.21}$$

where

$$\Lambda' \equiv \frac{\Lambda^2 + \mathbf{p}_\perp^2}{p^+} \ . \tag{3.22}$$

Hence

$$\alpha = \frac{\mathbf{k}_\perp^2}{\Lambda'}, \quad \beta = \frac{(\mathbf{p}_\perp-\mathbf{k}_\perp)^2+m^2}{\Lambda'} \Longrightarrow \frac{\beta}{\alpha} = \frac{(\mathbf{p}_\perp-\mathbf{k}_\perp)^2+m^2}{\mathbf{k}_\perp^2} \ . \tag{3.23}$$

In Appendix B, we show that

$$\int d^2\mathbf{k}_\perp \ln\left[\frac{\beta}{\alpha}\right] = \int d^2\mathbf{k}_\perp \int_0^{p^+}\frac{dk^+}{p^+}\frac{m^2}{p\cdot k} , \tag{3.24}$$

similar to Eq. (3.43) of Ref. 7(a). Now

$$\delta m=\frac{e^2}{2m}\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\int_0^{p^+}dk^+\frac{m^2}{p\cdot k}\left[\frac{1}{p^+}+\frac{1}{k^+}\right] . \qquad (3.25)$$

Upon integration, and dropping the finite part, one finds finally

$$\delta m=\frac{3e^2m}{16\pi^2}\ln\left[\frac{\Lambda_\perp^2}{m^2}\right] , \qquad (3.26)$$

which is of the same form as the standard result.[17] Since δm is not by itself a measurable quantity, there is no contradiction in finding different results, Eqs. (3.18) and (3.26). Note that the seagulls (especially δm_b) are required in order to obtain the conventional result Eq. (3.26).

Henceforth, we shall use only the dimensional regularization.

The wave-function renormalization Z_2, at order e^2, is given by[18]

$$1-Z_2=\sum_m{}' \frac{|\langle p|V_1|m\rangle|^2}{(p^- -P_0^-)^2} , \qquad (3.27)$$

where P_0^- is the total energy of the intermediate state $|m\rangle$. Note that this expression is the same as the one giving δm_a, except that here the denominator is squared. Thus similarly to Eq. (3.9), we have

$$\begin{aligned}(1-Z_2)\delta_{s\sigma}&=\frac{e^2}{p^+}\int\frac{d^2\mathbf{k}_\perp}{(4\pi)^3}\int_0^{p^+}\frac{dk^+}{k^+(p^+-k^+)}\frac{\bar u(p,\sigma)\gamma^\mu(\not k'+m)\gamma^\nu u(p,s)d_{\mu\nu}(k)}{(p^- -k^- -k'^-)^2}\\ &=\frac{e^2\delta_{s\sigma}}{(2\pi)^3}\int_0^1 dx\int\frac{d^2\mathbf{k}_\perp}{\mathbf{k}_\perp^2+m^2x^2}\left[\frac{2(1-x)\mathbf{k}_\perp^2}{x(\mathbf{k}_\perp^2+m^2x^2)}+x\right] ,\end{aligned} \qquad (3.28)$$

which is the same result as in Ref. 13. Naturally this integral is both infrared and ultraviolet divergent. Using our rules, we get

$$\begin{aligned}Z_2(p^+)=1&+\frac{e^2}{8\pi^2\epsilon}\left[\frac{3}{2}-\ln\left[\frac{p^+}{\alpha}\right]^2\right]\\ &+\frac{e^2}{(2\pi)^2}\ln\left[\frac{p^+}{\alpha}\right]\left[1-\ln\left[\frac{\mu^2}{m^2}\right]-\ln\left[\frac{p^+}{\alpha}\right]\right] ,\end{aligned} \qquad (3.29)$$

where μ^2 is the scale introduced by dimensional regularization. Note that Z_2 has an unusual dependence on the longitudinal momentum, not found in the space-time treatment (this may vary with the choice of regularization). A similar dependence was found by Thorn in the case of scalar QED.[11]

IV. PHOTON MASS AND WAVE-FUNCTION RENORMALIZATION

Consider now the amplitude T_{pp} of the transition matrix T between free photon states (p,λ) and (p,λ') at order e^2. Although the bare Lagrangian does not contain a photon mass term ($\mu^2=0$), the one-loop contributions from perturbation theory will require a counterterm

$$-\frac{\delta\mu^2}{2}A_\mu A^\mu , \qquad (4.1)$$

because we are not using a covariant, gauge-invariant regularization. As in Eq. (3.3),

$$\delta\mu^2\delta_{\lambda\lambda'}=2p^+T_{pp} , \qquad (4.2)$$

where

$$p=\left[p^+,\frac{\mathbf{p}_\perp^2}{2p^+},\mathbf{p}_\perp\right] \qquad (4.3)$$

is the initial (or final) photon momentum. One can also identify a "tensor" $\pi^{\mu\nu}(p)$ through

$$\delta\mu^2\delta_{\lambda\lambda'}\equiv\epsilon_\mu^\lambda(p)\pi^{\mu\nu}(p)\epsilon_\nu^{\lambda'}(p) . \qquad (4.4)$$

This "tensor" is not the usual covariant vacuum-polarization tensor, and will be used only in the context of Eq. (4.4). The corresponding diagrams are displayed in Fig. 2.

The seagulls generated by V_2, which are displayed in Fig. 2(b), yield

$$\delta\mu_b^2=e^2\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\int_0^\infty dk^+\left[\frac{1}{p^+-k^+}-\frac{1}{p^++k^+}\right] , \qquad (4.5)$$

where again a principal-value prescription is implied. Since this integral vanishes by dimensional regularization, we are left only with $\delta\mu_a^2$ corresponding to Fig. 2(a). Similarly to the fermion case, one finds easily

$$\delta\mu^2\delta_{\lambda\lambda'}=2e^2\int\frac{d^2\mathbf{k}_\perp}{(4\pi)^3}\int_\alpha^{p^+-\beta}\frac{dk^+}{k^+(p^+-k^+)}\frac{\mathrm{tr}[\not\epsilon^{(\lambda)}(p)(\not k+m)\not\epsilon^{(\lambda')}(p)(\not k'-m)]}{p^- -k^- -k'^-} , \qquad (4.6)$$

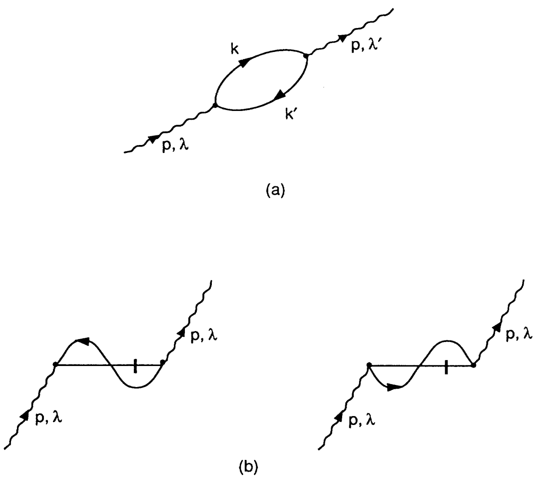

FIG. 2. Diagrams for the vacuum polarization.

where

$$k=\left[k^{+},\frac{\mathbf{k}_\perp^2+m^2}{2k^{+}},\mathbf{k}_\perp\right], \tag{4.7}$$

and

$$k'=\left[p^{+}-k^{+},\frac{(\mathbf{p}_\perp-\mathbf{k}_\perp)^2+m^2}{2(p^{+}-k^{+})},\mathbf{p}_\perp-\mathbf{k}_\perp\right]. \tag{4.8}$$

In Appendix C, we show that

$$\delta\mu^2=e^2\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\left[\ln\left[\frac{\alpha\beta}{(p^{+})^2}\right]+\frac{2\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2+m^2}\right]. \tag{4.9}$$

Using Eq. (A32), one obtains

$$\delta\mu^2=-\frac{e^2m^2}{4\pi^2\epsilon}. \tag{4.10}$$

As for the tensor defined in Eq. (4.4), it has the form

$$\pi^{\mu\nu}(p)=2e^2\int\frac{d^2\mathbf{k}_\perp}{(4\pi)^3}\int_0^{p^{+}}\frac{dk^{+}}{k^{+}(p^{+}-k^{+})}\frac{M^{\mu\nu}}{p^{-}-k^{-}-k'^{-}}. \tag{4.11}$$

Neglecting terms in $M^{\mu\nu}$ that will vanish by dimensional regularization, we show in Appendix C that

$$\frac{M^{\mu\nu}(p)}{4}=\frac{2k^{+}(p^{+}-k^{+})}{(p^{+})^2}p^\mu p^\nu+m^2d^{\mu\nu}. \tag{4.12}$$

Thus one sees immediately that

$$p_\mu M^{\mu\nu}(p)=0\Longrightarrow p_\mu\pi^{\mu\nu}(p)=0 \tag{4.13}$$

as required by current conservation. Now performing the integrations, one finds

$$\pi^{\mu\nu}(p)=-\frac{e^2}{4\pi^2\epsilon}\left[\tfrac{1}{3}p^\mu p^\nu+m^2d^{\mu\nu}(p)\right]. \tag{4.14}$$

Noncovariance introduces the transverse "tensor" $d^{\mu\nu}(p)$ in the expression of $\pi^{\mu\nu}$, which gives rise to a nonzero photon mass renormalization.

The wave-function renormalization Z_3 for the photon is given by the same formula as Z_2 in Eq. (3.27). Hence, similarly to Eq. (4.6), we have

$$\begin{aligned}(1-Z_3)\delta_{\lambda\lambda'}&=\frac{e^2}{p^{+}}\int\frac{d^2\mathbf{k}_\perp}{(4\pi)^3}\int_\alpha^{p^{+}-\beta}\frac{dk^{+}}{k^{+}(p^{+}-k^{+})}\frac{\mathrm{tr}[\not\epsilon^{(\lambda)}(p)(\not k+m)\not\epsilon^{(\lambda')}(p)(\not k'-m)]}{(p^{-}-k^{-}-k'^{-})^2}\\&=\frac{e^2\delta_{\lambda\lambda'}}{3}\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\left[\frac{2}{\mathbf{k}_\perp^2+m^2}+\frac{m^2}{(\mathbf{k}_\perp^2+m^2)^2}\right].\end{aligned} \tag{4.15}$$

Dropping the second term in the large parentheses which is finite, we find

$$Z_3=1-\frac{e^2}{12\pi^2\epsilon}, \tag{4.16}$$

which is of the same form as the space-time result.[19] Note that Z_3 can also be read off directly from the coefficient of $p^\mu p^\nu$ in Eq. (4.14).

V. VERTEX CORRECTIONS

The vertex corrections $\Lambda^\mu(p,\bar{p})$ are of order e^3, as they get contributions of the form V_1^3, V_2V_1 and V_3V_1. Diagrams are shown in Figs. 3–7.

Since we are interested here in calculating the vertex renormalization Z_1, checking the Ward identities and calculating the charge renormalization, it will be adequate for our purposes to calculate Λ^{+}. This leads to considerable simplification, because the tensor structure of the diagrams in Figs. 3, 6(a), and 6(c) all give zero. Also the photon mass renormalization does not enter.

The calculations follow in the manner discussed in earlier sections, and will be left almost entirely to Appendix D. The primitive vertex, properly normalized is

$$\Lambda_0^{+}=\frac{e\gamma^{+}}{(2\pi)^{3/2}\sqrt{2p^{+}}\sqrt{2\bar{p}^{+}}\sqrt{2q^{+}}}, \tag{5.1}$$

where p and $\bar{p}$ are the initial and final fermion momenta, respectively, and q is the photon momentum. We will first discuss the ultraviolet divergences, and return later to the treatment of the infrared divergences.

The diagram Fig. 4(a) gives a contribution

$$\Lambda_0^+ \frac{e^2}{8\pi^2\epsilon}\left[-\frac{3}{2}+\frac{1}{2}\left[\frac{p^+-\bar{p}^+}{p^+}\right]+\ln\left[\frac{\bar{p}^+}{\alpha}\right]^2\right], \tag{5.2}$$

while Figs. 4(b) and 4(c) together give

$$\Lambda_0^+ \frac{e^2}{8\pi^2\epsilon}\left[-\frac{1}{2}\left[\frac{p^+-\bar{p}^+}{p^+}\right]-\ln\left[\frac{\bar{p}^+}{p^+}\right]\right]. \tag{5.3}$$

Summing up the singular contributions to the diagrams in Fig. 4 yields the vertex correction

$$\Lambda_0^+ \frac{e^2}{8\pi^2\epsilon}\left[-\frac{3}{2}+\ln\frac{p^+\bar{p}^+}{\alpha^2}\right] \equiv \Lambda_0^+(Z_1^{-1}-1). \tag{5.4}$$

Thus

$$Z_1(p^+,\bar{p}^+)=1+\frac{e^2}{8\pi^2\epsilon}\left[\frac{3}{2}-\ln\frac{p^+\bar{p}^+}{\alpha^2}\right]. \tag{5.5}$$

Again we see that Z_1 like Z_2 is momentum dependent. Comparing the above result with the UV singular part of Eq. (3.27), one finds that the Ward identity is satisfied and that it takes the generalized form

$$Z_1(p^+,\bar{p}^+)=\sqrt{Z_2(p^+)}\sqrt{Z_2(\bar{p}^+)}. \tag{5.6}$$

The diagrams in Fig. 5 give the corrections to the final electron state. Figure 5(e) is the mass counterterm. Together they generate the fermion wave-function renormalization. Adding the contributions of the various diagrams we find

$$\Lambda_0^+ \frac{e^2}{8\pi^2\epsilon}\left[\frac{3}{2}-\ln\left[\frac{\bar{p}^+}{\alpha}\right]^2\right] \equiv -\Lambda_0^+(Z_2^{-1}-1). \tag{5.7}$$

Therefore

$$Z_2(\bar{p}^+)=1+\frac{e^2}{8\pi\epsilon}\left[\frac{3}{2}-\ln\left[\frac{\bar{p}^+}{\alpha}\right]^2\right] \tag{5.8}$$

in agreement with Eq. (3.27).

The diagrams in Fig. 6 give the correction to the photon line. Figure 6(a) gives zero and Fig. 6(b) gives

$$-\Lambda_0^+\frac{e^2}{12\pi^2\epsilon} \equiv \Lambda_0^+(Z_3-1). \tag{5.9}$$

Thus

$$Z_3=1-\frac{e^2}{12\pi^2\epsilon}, \tag{5.10}$$

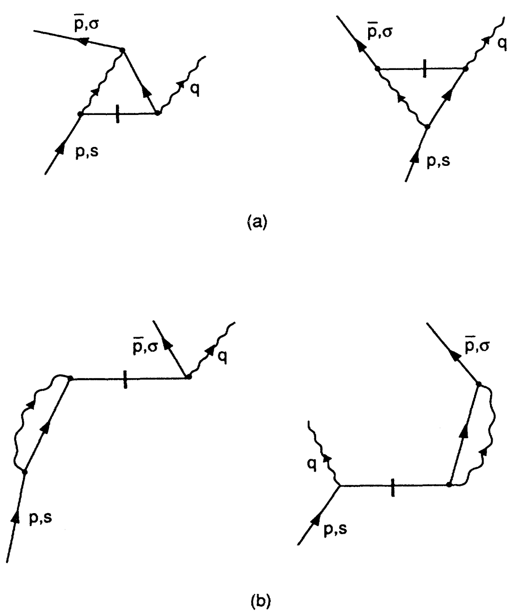

FIG. 3. Diagrams for the vertex corrections with instantaneous fermion exchange.

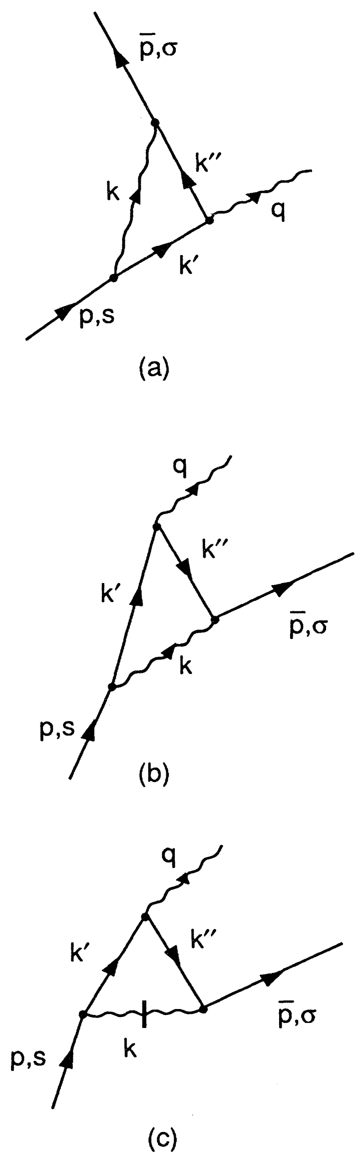

FIG. 4. Diagrams for the usual vertex correction and the associated instantaneous photon exchange.

in agreement with (4.9). The fact that (5.8) and (5.10) agree with previous determinations is a nontrivial result in Hamiltonian perturbation theory. The external lines do not factor from the vertex as they do in the covariant treatment because the energy denominator contains contributions from all particles in the intermediate states, thus mixing the final state with other parts of the diagram.

Consider the contributions from the diagrams in Fig. 7. They are all one-particle reducible, and therefore the energy denominator associated with the single-particle state vanishes. We shall use the Heitler method[15] for defining these diagrams. While being somewhat formal, it has the advantage of being straightforward to apply.

Let us study first Fig. 7(a). There are two intermediate states and therefore two energy denominators. To describe the method, we will write the integrals as discrete sums. The amplitude is of the form

$$T_{(7a)}=\sum_{n,m}\frac{V_{Am}V_{mn}V_{nB}}{(P_B^- - P_n^-)(P_B^- - P_m^-)}\ , \tag{5.11}$$

where B is the initial state, A the final state, n and m the intermediate states. The single-electron states m have $P_m^- = P_B^-$ because of momentum conservation, thus rendering Eq. (5.11) ill defined. We define it by means of the principal-value integral

$$T_{(7a)}=\int dE\,\delta(E-P_B^-)\times\sum_{n,m}\frac{\mathcal{P}}{(E-P_n^-)(E-P_m^-)}V_{Am}V_{mn}V_{nB}\ . \tag{5.12}$$

One then uses the relation between distributions

$$\frac{\mathcal{P}}{E-P_B^-}\delta(E-P_B^-)=-\frac{1}{2}\delta'(E-P_B^-)\ .$$

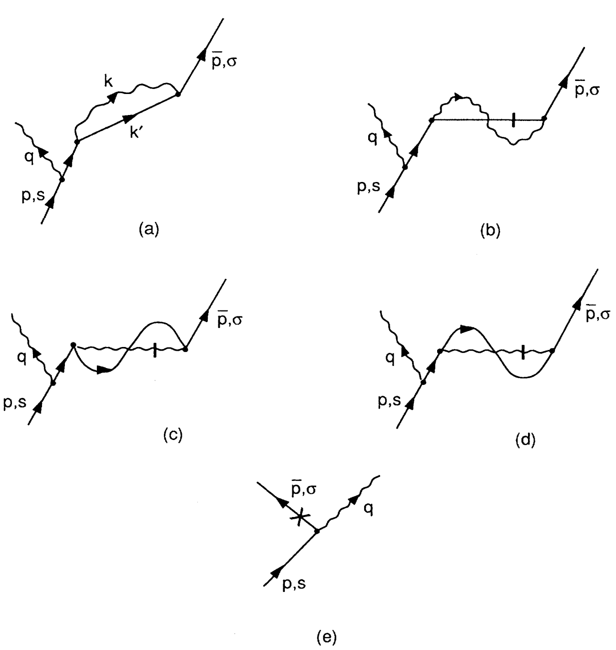

FIG. 5. Diagrams for the vertex corrections with outgoing-fermion renormalization.

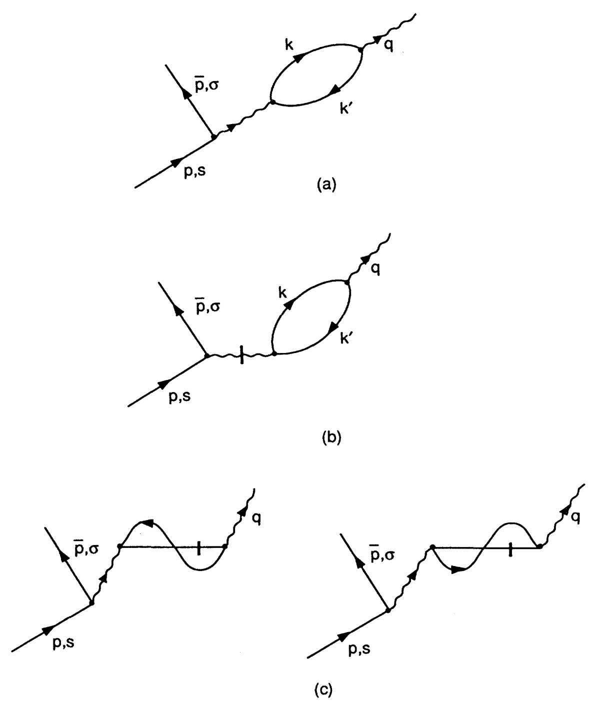

FIG. 6. Diagrams for the vertex corrections with vacuum polarization.

Performing formally the integration, one finds

$$T_{(7a)}=-\frac{1}{2}\sum_{nm}\frac{V_{Am}V_{mn}V_{nB}}{(P_B^- - P_n^-)^2}\ . \tag{5.13}$$

Here $B\equiv(p,s)$, $n\equiv(k,k')$, $m\equiv(p,\sigma)$. Returning now to the explicit integral form, one has

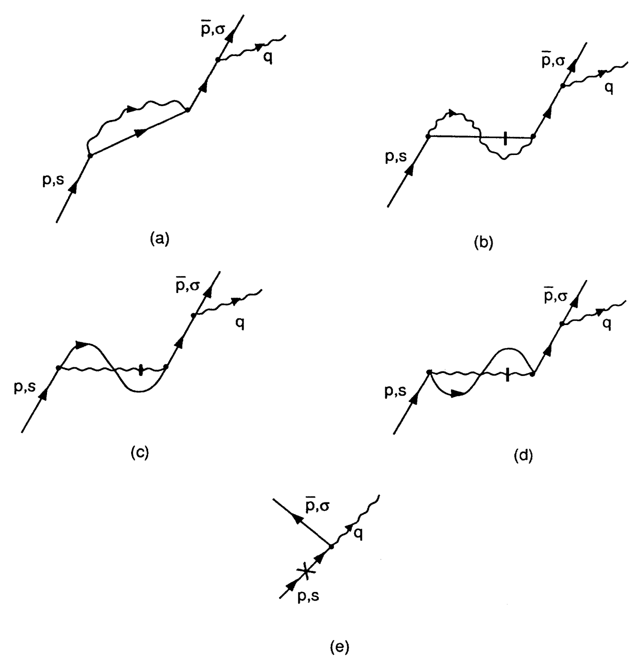

FIG. 7. Diagrams for the vertex corrections with incoming-fermion renormalization.

$$\sum_n \frac{V_{mn}V_{nB}}{(P_B^- - P_n^-)^2} = \frac{e^2}{p^+}\int \frac{d^2\mathbf{k}_\perp}{(4\pi)^3}\int_0^{p^+}\frac{dk^+}{k^+(p^+-k^+)}\frac{\bar{u}(p,\sigma)\gamma^\mu(\not{k}'+m)\gamma^\nu u(p,s)d_{\mu\nu}(k)}{(p^- - k^- - k'^-)^2} . \tag{5.14}$$

Similarly

$$\sum_m V_{Am} = \sum_\sigma \frac{e}{(2\pi)^{3/2}}\frac{\bar{u}(\bar{p},s)}{\sqrt{2p^+}}\frac{\gamma^+}{\sqrt{2q^+}}\frac{u(p,\sigma)}{\sqrt{2p^+}} . \tag{5.15}$$

So Eq. (5.13) reads

$$T_{(7a)} = -\frac{e^3}{2(2\pi)^{3/2}p^+}\sum_\sigma \frac{\bar{u}(\bar{p},s)}{\sqrt{2p^+}}\frac{\gamma^+}{\sqrt{2q^+}}\frac{u(p,\sigma)}{\sqrt{2p^+}}\frac{d^2\mathbf{k}_\perp}{(4\pi)^3}\int_0^{p^+}\frac{dk^+}{k^+(p^+-k^+)}\frac{\bar{u}(p,\sigma)\gamma^\mu(\not{k}'+m)\gamma^\nu u(p,s)d_{\mu\nu}(k)}{(p^- - k^- - k'^-)^2} . \tag{5.16}$$

Since the fermion self-energy is diagonal in spin, only $\sigma = s$ contributes. Note that the last part of Eq. (5.16) is identical to Eq. (3.28). Thus Eq. (5.16) yields

$$\Lambda^+(7a) = \left[-\frac{1}{2}\right]\Lambda_0^+[1-Z_2(p^+)] = \frac{e^2}{8\pi^2\epsilon}\Lambda_0^+\left[\frac{1}{2}\left[\frac{3}{2}\right]-\ln\left[\frac{p^+}{\alpha}\right]\right] . \tag{5.17}$$

Now applying the Heitler procedure to the other diagrams in Fig. 7 we see that they all have only one energy denominator. Thus the factor multiplying $\delta'(E-P_B^-)$ is independent of E, and upon integration by parts, all the integrals for diagrams (7b), (7c), (7d), and (7e) vanish. Adding the contributions of all the diagrams in Figs. 3–7 as well as the lowest-order vertex Λ_0^+, we obtain

$$\Gamma^+ = \Lambda_0^+\left[1+\frac{e^2}{8\pi^2\epsilon}\left\{\left[-\frac{3}{2}+\ln\frac{p_+\bar{p}_+}{\alpha}\right]+\left[\frac{3}{2}-\ln\left[\frac{p^+}{\alpha}\right]^2\right]+\left[-\frac{2}{3}\right]+\left[-\frac{1}{2}\right]\left[\ln\left[\frac{p^+}{\alpha}\right]^2-\frac{3}{2}\right]\right\}\right] . \tag{5.18}$$

To obtain the renormalized charge we must divide out the wave-function renormalization associated with the final particles. So dividing Γ^+ by $\sqrt{Z_2(\bar{p}^+)}\sqrt{Z_3}$, we find

$$\Gamma_R^+ = \Lambda_0^+\left[1-\frac{1}{2}\left[\frac{e^2}{12\pi^2\epsilon}\right]\right] = \Lambda_0^+\sqrt{Z_3} . \tag{5.19}$$

Therefore the renormalized charge is

$$e_R = e\sqrt{Z_3} \tag{5.20}$$

as in the space-time treatment.

Let us consider now the full infrared behavior of Z_1. For this discussion we will limit ourselves to $\bar{p}=p$, that is, zero photon momentum. Only the diagram in Fig. 4(a) will contribute. The UV-singular expression Eq. (5.2) already has an IR-singular contribution, viz., $\ln\alpha$. In addition however there are contributions that are IR singular but UV finite. They come from two sources. One of them is from terms of the form $[f(x)]^\epsilon = 1+\epsilon\ln f(x)+\cdots$, which we have previously neglected. The ϵ multiplying the logarithm cancels the UV singularity, but the integral of $\ln f(x)$ over p^+ may be IR singular. Second we consider contributions that were not UV singular but are IR singular. Details are given in Appendix D. We find

$$\Lambda^+(4a) = \Lambda_0^+\frac{e^2}{4\pi^2}\left\{\left[\ln\left[\frac{p^+}{\alpha}\right]\right]^2+\ln\left[\frac{p^+}{\alpha}\right]\ln\frac{\mu^2}{m^2} - \ln\left[\frac{p^+}{\alpha}\right]\right\} . \tag{5.21}$$

Therefore the complete expression for the vertex renormalization is

$$Z_1(p^+,p^+) = 1+\frac{e^2}{8\pi^2\epsilon}\left[\frac{3}{2}-\ln\left[\frac{p^+}{\alpha}\right]^2\right] + \frac{e^2}{4\pi^2}\ln\left[\frac{p^+}{\alpha}\right]\left[1-\ln\frac{\mu^2}{m^2}-\ln\frac{p^+}{\alpha}\right] . \tag{5.22}$$

Comparing Eqs. (5.22) and (3.27), we find that the Ward identity

$$Z_1(p^+,p^+) = Z_2(p^+) \tag{5.23}$$

is satisfied for both the UV-singular and UV-finite parts.

VI. SUMMARY AND OUTLOOK

We have carried out the renormalization of null-plane QED at order one loop, and have evaluated the electron and photon mass corrections, the wave-function renormalization constants Z_2 and Z_3, and the vertex correction Z_1.

During the course of our calculations several infrared singularities were encountered beyond those that can be eliminated by giving the photon a small mass. By adding the various contributions to each physical process, including some involving the four-point interactions characteristic of null-plane QED, these "spurious" divergences were shown to cancel, leading to conventional expressions. One feature, however, that distinguishes the null plane from space-time results is that in the former approach the ultraviolet-divergent parts of Z_1 and Z_2 exhibit momentum dependence. Again, for physical quantities such as the renormalized charge e_R, this momentum dependence cancels [in this case due to the Ward identity $Z_1(p^+,p'^+)=\sqrt{Z_2(p^+)Z_2(p'^+)}$]. On the other hand, momentum-dependent renormalization constants imply nonlocal counterterms. Given that the tree-level Hamiltonian is nonlocal in x^-, it is actually not surprising to find counterterms exhibiting the same property of nonlocality. In the future it will be important to carry out a more comprehensive study of the counterterm structure in null-plane QED. As mentioned in the Introduction, power counting works differently here from the space-time approach. This is already indicated by the presence of four-point interactions in the Hamiltonian. The momentum dependence in Z_1 and Z_2 is another manifestation of unusual power-counting laws. One of us (K.G.W.) has set up formal power-counting rules for null-plane gauge theories. It will be interesting to apply them systematically in the case of QED. Power counting alone does not provide information about cancellation of divergences between diagrams. So it is also important to gain more insight into when to expect cancellations such as those in the calculation of the electron mass shift [Eqs. (3.13)–(3.15)]. All of this is crucial, in order to successfully carry out nonperturbative studies and generalize the "Light-Front Tamm-Dancoff" program of Ref. 5 to (3+1)-dimensional gauge theories. This program is presently under investigation, and the current article represented the first step in this direction.

ACKNOWLEDGMENTS

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APPENDIX A: INTERACTION VERTICES

Dirac matrices:

$$\{\gamma^\mu,\gamma^\nu\}=2g^{\mu\nu}\ , \tag{A1}$$

$$(\gamma^0)^\dagger=\gamma^0,\quad (\gamma^k)^\dagger=-\gamma^k \quad (k=1,2,3)\ . \tag{A2}$$

For example, one can choose the representation

$$\gamma^0=\begin{bmatrix}0 & I\\ I & 0\end{bmatrix},\quad \gamma^k=\begin{bmatrix}0 & -\sigma_k\\ \sigma_k & 0\end{bmatrix}\ , \tag{A3}$$

$$\gamma^+=\frac{\gamma^0+\gamma^3}{\sqrt{2}},\quad \gamma^-=\frac{\gamma^0-\gamma^3}{\sqrt{2}}\ . \tag{A4}$$

In the representation Eq. (A3), these are

$$\gamma^+=\sqrt{2}\begin{bmatrix}0&0&0&0\\0&0&0&1\\1&0&0&0\\0&0&0&0\end{bmatrix},\quad \gamma^-=\sqrt{2}\begin{bmatrix}0&0&1&0\\0&0&0&0\\0&0&0&0\\0&1&0&0\end{bmatrix}\ , \tag{A5}$$

$$\gamma^+\gamma^-\gamma^+=2\gamma^+,\quad \gamma^-\gamma^+\gamma^-=2\gamma^-\ . \tag{A6}$$

Projection operators:

$$\Lambda_+=\tfrac{1}{2}\gamma^-\gamma^+,\quad \Lambda_-=\tfrac{1}{2}\gamma^+\gamma^-\ . \tag{A7}$$

In the representation Eq. (A3), these are

$$\Lambda_+=\begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix},\quad \Lambda_-=\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix}, \tag{A8}$$

$$\Lambda_+\Lambda_-=0,\quad \Lambda_++\Lambda_-=1\ , \tag{A9}$$

$$\Lambda_+^2=\Lambda_+,\quad \Lambda_-^2=\Lambda_-\ , \tag{A10}$$

$$(\Lambda_-)^\dagger=\Lambda_-,\quad (\Lambda_+)^\dagger=\Lambda_+\ . \tag{A11}$$

Dirac spinors:

$$(\not p-m)u(p,s)=0,\quad (\not p+m)v(p,s)=0\ , \tag{A12}$$

$$\begin{aligned}&\bar u(p,s)u(p,s')=-\bar v(p,s)v(p,s')=2m\delta_{ss'}\ ,\\ &\bar u(p,s)\gamma^\mu u(p,s')=\bar v(p,s)\gamma^\mu v(p,s')=2p^\mu\delta_{ss'}\ ,\end{aligned} \tag{A13}$$

$$\begin{aligned}&\sum_{s=\pm1/2} u(p,s)\bar u(p,s)=\not p+m\ ,\\ &\sum_{s=\pm1/2} v(p,s)\bar v(p,s)=\not p-m\ .\end{aligned} \tag{A14}$$

Photon polarizations:

$$\epsilon_\mu^\lambda p^\mu=0,\quad \epsilon_\mu^{\lambda'}\epsilon^{\lambda\mu}=-\delta_{\lambda'\lambda},\quad \epsilon_-^\lambda=0\ . \tag{A15}$$

For example, one can choose

$$\epsilon_\mu^1=\left[\frac{p^1}{p^+},0,-1,0\right],\quad \epsilon_\mu^2=\left[\frac{p^2}{p^+},0,0,-1\right]\ , \tag{A16}$$

$$\begin{aligned}d_{\mu\nu}(p)&=\sum_{\lambda=1,2}\epsilon_\mu^\lambda(p)\epsilon_\nu^\lambda(p)\\ &=-g_{\mu\nu}+\frac{\delta_{\mu+}p_\nu+\delta_{\nu+}p_\mu}{p^+}\ ,\end{aligned} \tag{A17}$$

$$\gamma^\alpha\gamma^\beta d_{\alpha\beta}(p)=-2\ , \tag{A18}$$

$$\gamma^\alpha\gamma^\nu\gamma^\beta d_{\alpha\beta}(p)=\frac{2}{p^+}(\gamma^+p^\nu+g^{+\nu}\not p)\ , \tag{A19}$$

$$\begin{aligned}\gamma^\alpha\gamma^\mu\gamma^\nu\gamma^\beta d_{\alpha\beta}(p)&\\ =-4g^{\mu\nu}+2\frac{p_\alpha}{p^+}(&g^{\mu\alpha}\gamma^\nu\gamma^+-g^{\alpha\nu}\gamma^\mu\gamma^++g^{\alpha+}\gamma^\mu\gamma^\nu\\ &-g^{+\nu}\gamma^\mu\gamma^\alpha+g^{+\mu}\gamma^\nu\gamma^\alpha)\ .\end{aligned} \tag{A20}$$

Interactions:

$$V_1=e\int d^2\mathbf{x}_\perp dx^-\int[dp][d\bar{p}][dk]$$
$$\times\sum_{s,s',\lambda}[e^{i\bar{p}\cdot x}\bar{u}(\bar{p},s')b^\dagger(\bar{p},s')+e^{-i\bar{p}\cdot x}\bar{v}(\bar{p},s')d(\bar{p},s')]\gamma^\mu[e^{-ip\cdot x}u(p,s)b(p,s)+e^{ip\cdot x}v(p,s)d^\dagger(p,s)]$$
$$\times\epsilon_\mu^\lambda(k)[e^{-ik\cdot x}a(k,\lambda)+e^{ik\cdot x}a^\dagger(k,\lambda)]\ , \tag{A21}$$

where

$$\int[dp]\equiv\int_{-\infty}^{+\infty}\frac{d^2\mathbf{p}_\perp}{(2\pi)^{3/2}}\int_0^\infty\frac{dp^+}{\sqrt{2p^+}}\ , \tag{A22}$$

which gives a vertex of the form

$$(\pm)\frac{e}{(2\pi)^{3/2}}\frac{\overline{\omega}(\bar{p},s')}{\sqrt{2\bar{p}^+}}\frac{\not\epsilon^\lambda(k)}{\sqrt{2k^+}}\frac{\omega(p,s)}{\sqrt{2p^+}}\delta^{(3)}(\text{momentum})\ . \tag{A23}$$

The initial $(\pm)$ sign is determined by the order of the creation and annihilation operators. $\omega(p,s)$ can be either a u or a v, and the δ^3 enforces conservation of p^+ and $\mathbf{p}_\perp$ at the vertex with the constraint that $p^+>0$:

$$V_2=-\frac{i}{4}e^2\int d^2\mathbf{x}_\perp dx^-dy^-dl\,[dp][d\bar{p}][dk][d\bar{k}]$$
$$\times\sum_{s,s',\lambda,\lambda'}\left[-\frac{i}{\pi l}\right]e^{il(x^--y^-)}[e^{ip\cdot x}\bar{u}(p,s)b^\dagger(p,s)+e^{-ip\cdot x}\bar{v}(p,s)d(p,s)]$$
$$\times\not\epsilon^\lambda(k)[e^{-ik\cdot x}a(k,\lambda)+e^{ik\cdot x}a^\dagger(k,\lambda)]\gamma^+\not\epsilon^{\lambda'}(\bar{k})$$
$$\times[e^{-i\bar{p}\cdot y}u(\bar{p},s')b(\bar{p},s')+e^{i\bar{p}\cdot y}v(\bar{p},s')d^\dagger(\bar{p},s')][e^{-i\bar{k}\cdot y}a(\bar{k},\lambda')+e^{i\bar{k}\cdot y}a^\dagger(\bar{k},\lambda')]\ , \tag{A24}$$

where $y=(x^+,y^-,\mathbf{x}_\perp)$, which will contribute a vertex of the form

$$(\pm)\frac{e^2}{(2\pi)^3}\frac{\overline{\omega}(p,s)}{\sqrt{2p^+}}\frac{\not\epsilon^\lambda(k)}{\sqrt{2k^+}}\frac{\gamma^+}{2(\pm p^+\pm\bar{k}^+)}\frac{\not\epsilon^{\lambda'}(\bar{k})}{\sqrt{2\bar{k}^+}}\frac{\omega(\bar{p},s')}{\sqrt{2\bar{p}^+}}\delta^3(\text{momentum})\ , \tag{A25}$$

where again signs vary according to the particles involved:

$$V_3=\frac{e^2}{4}\int d^2\mathbf{x}_\perp dx^-dy^-dl\,[dp][d\bar{p}][dk][d\bar{k}]$$
$$\times\sum_{s,s',\sigma,\sigma'}\frac{e^{il(x^--y^-)}}{\pi l^2}[e^{i\bar{p}\cdot x}\bar{u}(\bar{p},s')b^\dagger(\bar{p},s')+e^{-i\bar{p}\cdot x}\bar{v}(\bar{p},s')d(\bar{p},s')]$$
$$\times\gamma^+[e^{-ip\cdot x}u(p,s)b(p,s)+e^{ip\cdot x}v(p,s)d^\dagger(p,s)][e^{i\bar{k}\cdot y}\bar{u}(\bar{k},\sigma')b^\dagger(\bar{k},\sigma')+e^{-i\bar{k}\cdot y}\bar{v}(\bar{k},\sigma')d(\bar{k},\sigma')]$$
$$\times\gamma^+[e^{-ik\cdot y}u(k,\sigma)b(k,\sigma)+e^{ik\cdot y}v(k,\sigma)d^\dagger(k,\sigma)]\ , \tag{A26}$$

which will contribute a vertex of the form

$$(\pm)\frac{e^2}{2(2\pi)^3(\pm p^+\pm\bar{p}^+)^2}\frac{\overline{\omega}(\bar{p},s')\gamma^+\omega(p,s)}{\sqrt{2\bar{p}^+}\sqrt{2p^+}}\frac{\overline{\omega}(\bar{k},\sigma')\gamma^+\omega(k,\sigma)}{\sqrt{2\bar{k}^+}\sqrt{2k^+}}\delta^3(\text{momentum})\ . \tag{A27}$$

Again the signs depend on which particles are involved in the vertex.

Dimensional regularization:

$$\int d^2\mathbf{k}_\perp\to(\mu^2)^\epsilon\int d^d\mathbf{k}_\perp,\quad\text{where }\epsilon=1-d/2\ , \tag{A28}$$

$$(\mu^2)^\epsilon\int d^d\mathbf{k}_\perp\frac{1}{\mathbf{k}_\perp^2+M^2}=\left[\frac{\mu^2}{M^2}\right]^\epsilon\frac{\pi}{\epsilon}\ , \tag{A29}$$

$$(\mu^2)^\epsilon\int d^d\mathbf{k}_\perp\frac{1}{(\mathbf{k}_\perp^2+M^2)^2}=\left[\frac{\mu^2}{M^2}\right]^\epsilon\frac{\pi}{M^2}\ , \tag{A30}$$

$$(\mu^2)^\epsilon\int d^d\mathbf{k}_\perp(\mathbf{k}_\perp^2)^\alpha=0\quad\text{for }\alpha\geq 0\ , \tag{A31}$$

$$(\mu^2)^\epsilon\int d^d\mathbf{k}_\perp\frac{\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2+M^2}=-\left[\frac{\mu^2}{M^2}\right]^\epsilon\frac{\pi M^2}{\epsilon}\ . \tag{A32}$$

APPENDIX B: FERMION RENORMALIZATIONS

In order to derive the expression for δm_a given in Eq. (3.11) starting from Eq. (3.6), use Eqs. (A13), (A18), and (A19) to obtain

$$\bar{u}(p,\sigma)\gamma^{\mu}(\not{k}'+m)\gamma^{\nu}u(p,s)d_{\mu\nu}(k)=4\delta_{s\sigma}\left[\left[\frac{2p^+}{k^+}+\frac{k^+}{p^+-k^+}\right](p\cdot k)-m^2\right], \tag{B1}$$

then use

$$p^- - k^- - k'^- = \frac{p\cdot k}{k^+-p^+} . \tag{B2}$$

To prove the expression of δm_b in Eq. (3.11) starting from Eq. (3.8), write

$$\begin{aligned}\int_0^{\infty} dk^+ \frac{p^+}{k^+(p^+-k^+)} &= \int_0^{\infty} dk^+ \left[\frac{1}{k^+}+\frac{1}{p^+-k^+}\right]\\ &= \int_{\alpha}^{\infty}\frac{dk^+}{k^+}+\int_{p^++\eta}^{\infty}\frac{dk^+}{p^+-k^+}+\int_0^{p^+-\eta}\frac{dk^+}{p^+-k^+}\\ &= \left[\int_{\alpha}^{\infty}-\int_{\eta}^{\infty}+\int_{\eta}^{p^+}\right]\frac{dk^+}{k^+}=\int_{\alpha}^{p^+}\frac{dk^+}{k^+}=\ln\left[\frac{p^+}{\alpha}\right],\end{aligned} \tag{B3}$$

where we have identified η with α in order to bring the momenta of Fig. 1(b) into physical range.
To prove the expression of δm_c in Eq. (3.11) starts from Eq. (3.9), write

$$\begin{aligned}\int_0^{\infty}\frac{dk^+}{(p^+-k^+)^2}-\int_0^{\infty}\frac{dk^+}{(p^++k^+)^2} &= \int_0^{p^+-\eta}\frac{dk^+}{(p^+-k^+)^2}+\int_{p^++\eta}^{\infty}\frac{dk^+}{(p^+-k^+)^2}-\int_{p^+}^{\infty}\frac{dk^+}{(k^+)^2}\\ &= \left[\int_{\eta}^{\infty}+\int_{\eta}^{p^+}-\int_{p^+}^{\infty}\right]\frac{dk^+}{(k^+)^2}=2\int_{\eta}^{p^+}\frac{dk^+}{(k^+)^2}=\frac{2}{p^+}\left[\frac{p^+}{\alpha}-1\right],\end{aligned} \tag{B4}$$

where again we identified η and α.
In order to deduce Eq. (3.25) from Eq. (3.15), write

$$\frac{(\mathbf{p}_\perp-\mathbf{k}_\perp)^2+m^2}{\mathbf{k}_\perp^2}=\frac{(\mathbf{p}_\perp^2+m^2)^2-2(\mathbf{p}_\perp^2+m^2)(\mathbf{p}_\perp\cdot\mathbf{k}_\perp)+\mathbf{k}_\perp^2(\mathbf{p}_\perp^2+m^2)}{\mathbf{k}_\perp^2(\mathbf{p}_\perp^2+m^2)}=\frac{\gamma^2+\delta}{(\gamma-1)^2+\delta}, \tag{B5}$$

where

$$\gamma\equiv 1-\frac{\mathbf{p}_\perp\cdot\mathbf{k}_\perp}{\mathbf{p}_\perp^2+m^2},\quad \delta\equiv\frac{\mathbf{k}_\perp^2}{\mathbf{p}_\perp^2+m^2}-\left[\frac{\mathbf{p}_\perp\cdot\mathbf{k}_\perp}{\mathbf{p}_\perp^2+m^2}\right]^2 . \tag{B6}$$

Therefore

$$\begin{aligned}\ln\frac{\beta}{\alpha}=\int_{\gamma-1}^{\gamma}dy\frac{2y}{y^2+\delta} &= \int_0^1 dy\frac{2\left[y-\dfrac{\mathbf{p}_\perp\cdot\mathbf{k}_\perp}{\mathbf{p}_\perp^2+m^2}\right]}{\left[y-\dfrac{\mathbf{p}_\perp\cdot\mathbf{k}_\perp}{\mathbf{p}_\perp^2+m^2}\right]^2+\delta}\\ &= \int_0^1 dy\frac{2[y(\mathbf{p}_\perp^2+m^2)-\mathbf{p}_\perp\cdot\mathbf{k}_\perp]}{y^2(\mathbf{p}_\perp^2+m^2)-2y\mathbf{p}_\perp\cdot\mathbf{k}_\perp+\mathbf{k}_\perp^2}\\ &= \int_0^1 dy\frac{2ym^2}{y^2(2p^+p^-)-2y\mathbf{p}_\perp\cdot\mathbf{k}_\perp+2k^+k^-}+2\int_0^1 dy\frac{y\mathbf{p}_\perp^2-\mathbf{p}_\perp\cdot\mathbf{k}_\perp}{(\mathbf{k}_\perp-y\mathbf{p}_\perp)^2+y^2m^2}\\ &= \int_0^{p^+}\frac{dk^+}{p^+}\frac{m^2}{p\cdot k}-2\mathbf{p}_\perp\cdot\int_0^1 dy\frac{\mathbf{k}_\perp-y\mathbf{p}_\perp}{(\mathbf{k}_\perp-y\mathbf{p}_\perp)^2+y^2m^2} .\end{aligned} \tag{B7}$$

The second integral yields zero upon integration over $\mathbf{k}_\perp$.
Adding the contributions Σ_a, Σ_b, and Σ_c which lead to Eqs. (3.9), (3.10), and (3.11), one obtains

$$\Sigma(p)=\frac{e^2}{2}\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\int_0^{p^+}dk^+\left[\frac{-m}{k^+(p^+-k^+)(p^--k^--k'^-)}+\frac{m^2}{(p^+)^2}\frac{\gamma^+}{p\cdot k}\right]$$
$$+\frac{e^2}{2}\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\int_0^{p^+}\frac{dk^+}{k^+(p^+)^2}\frac{\not p p^+-m^2\gamma^+}{p^--k^--k'^-}\ . \tag{B8}$$

Using dimensional regularization, this is

$$\Sigma(p)=\frac{e^2}{8\pi^2\epsilon}\left[m+\frac{\gamma^+m^2}{2p^+}-\frac{1}{2}\left[\not p-\frac{m^2\gamma^+}{p^+}\right]\right]\ . \tag{B9}$$

APPENDIX C: PHOTON RENORMALIZATIONS

We want to deduce Eq. (4.9) from Eq. (4.6). As usual,

$$M^{\mu\nu}\equiv\mathrm{tr}[\gamma^\mu(\not k+m)\gamma^\nu(\not k'-m)]=4[k^\mu k'^\nu+k^\nu k'^\mu-g^{\mu\nu}(k\cdot k'+m^2)]\ , \tag{C1}$$

where

$$k\cdot k'+m^2=\frac{(p^+)^2}{2k^+(p^+-k^+)}\left[\left[\mathbf{k}_\perp-\frac{k^+}{p^+}\mathbf{p}_\perp\right]^2+m^2\right]\ . \tag{C2}$$

So using Eq. (A15), one finds

$$\frac{\mathrm{tr}[\not\epsilon^{(\lambda)}(p)(\not k+m)\not\epsilon^{(\lambda')}(p)(\not k'-m)]}{4}=-2\left[k^\lambda-\frac{k^+}{p^+}p^\lambda\right]\left[k^{\lambda'}-\frac{k^+}{p^+}p^{\lambda'}\right]+\delta_{\lambda\lambda'}(k\cdot k'+m^2)\ . \tag{C3}$$

With the help of

$$p^--k^--k'^-=-\frac{k\cdot k'+m^2}{p^+}\ , \tag{C4}$$

one gets

$$\delta\mu^2=e^2\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\int_\alpha^{p^+-\beta}\frac{dk^+}{k^+(p^+-k^+)}\frac{-\mathbf{k}_\perp^2+\dfrac{(p^+)^2}{2k^+(p^+-k^+)}(\mathbf{k}_\perp^2+m^2)}{-\dfrac{p^+}{2k^+(p^+-k^+)}(\mathbf{k}_\perp^2+m^2)}$$
$$=e^2\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\int_\alpha^{p^+-\beta}dk^+\left[\frac{p^+}{k^+(k^+-p^+)}+\frac{2\mathbf{k}_\perp^2}{p^+(\mathbf{k}_\perp^2+m^2)}\right]\ , \tag{C5}$$

yielding Eq. (4.9).

To find a more explicit expression of $M^{\mu\nu}$, it is convenient to parametrize k and k' in the following way:

$$k^\mu=xp^\mu+\sum_{\lambda=1}^{2}\epsilon^{\lambda\mu}k^\lambda+\delta^{\mu-}\frac{\mathbf{k}_\perp^2+m^2}{2p^+x}\ ,$$
$$k'^\mu=(1-x)p^\mu-\sum_{\lambda=1}^{2}\epsilon^{\lambda\mu}k^\lambda+\delta^{\mu-}\frac{\mathbf{k}_\perp^2+m^2}{2p^+(1-x)}\ . \tag{C6}$$

One finds easily from Eq. (C1) that $M^{\mu\nu}$ has the form

$$\frac{M^{\mu\nu}}{4}=2x(1-x)p^\mu p^\nu+\left\{m^2-(\mathbf{k}_\perp^2+m^2)\left[1-\frac{1}{2}\left[\frac{x}{1-x}+\frac{1-x}{x}\right]\right]\right\}d^{\mu\nu}(p)$$
$$-(\mathbf{k}_\perp^2+m^2)g^{\mu\nu}+\frac{\mathbf{k}_\perp^2+m^2}{2x(1-x)(p^+)^2}\delta^{\mu-}\delta^{\nu-}\ . \tag{C7}$$

Using this result, as well as

$$p^--k^--k'^-=-\frac{\mathbf{k}_\perp^2+m^2}{2p^+x(1-x)}\ , \tag{C8}$$

one obtains $\pi^{\mu\nu}$ given in Eq. (4.11). The terms in $M^{\mu\nu}$ proportional to $(\mathbf{k}_\perp^2+m^2)$ cancel the $(\mathbf{k}_\perp^2+m^2)$ dependence in the denominator, leaving a term independent of $\mathbf{k}_\perp^2$ which integrates to zero in dimensional regularization. Thus

$$\pi^{\mu\nu}(p)=-2e^2\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\int_0^1 dx\frac{2x(1-x)p^\mu p^\nu+m^2 d^{\mu\nu}(p)}{\mathbf{k}_\perp^2+m^2}\;, \tag{C9}$$

yielding Eq. (4.14) by dimensional regularization.

APPENDIX D: VERTEX CORRECTIONS

The diagram shown in Fig. 3(a) is typical of the diagrams in Fig. 3 and 6(c). The integrand has the form

$$\frac{\bar{u}(p,s)}{\sqrt{2p^+}}\frac{\not\epsilon^{\lambda}(k')}{\sqrt{2k'^+}}\frac{u(k'',\sigma)}{\sqrt{2k''^+}}\frac{\bar{u}(k'',\sigma)}{\sqrt{2k''^+}}\frac{\gamma^\mu}{\sqrt{2q^+}}\frac{\gamma^+}{2k^+}\frac{\not\epsilon^{\lambda}(k')}{\sqrt{2k^+}}\frac{u(p,s)}{\sqrt{2p^+}}\;. \tag{D1}$$

For $\mu=+$, the integrand contains $(\gamma^+)^2$ and vanishes.

The momenta in Fig. 4(a) are defined as follows:

$$p=\left[p^+,\frac{\mathbf{p}_\perp^2+m^2}{2p^+},\mathbf{p}_\perp\right]\;. \tag{D2}$$

The transverse momentum of the photon is parametrized as $(y\mathbf{p}_\perp+\mathbf{q}_\perp)$, and we choose a frame in which $\mathbf{q}_\perp=0$ (this choice does not affect the divergent contribution to Λ^+). Further,

$$\begin{aligned}
k&=\left[xp^+,\frac{(x\mathbf{p}_\perp+\mathbf{k}_\perp)^2}{2p^+x},x\mathbf{p}_\perp+\mathbf{k}_\perp\right]\;,\\
q&=y\left[p^+,\frac{\mathbf{p}_\perp^2}{2p^+},\mathbf{p}_\perp\right]\;,\\
k'&=\left[(1-x)p^+,\frac{[(1-x)\mathbf{p}_\perp-\mathbf{k}_\perp]^2+m^2}{2(1-x)p^+},(1-x)\mathbf{p}_\perp-\mathbf{k}_\perp\right]\;,\\
k''&=\left[(1-x-y)p^+,\frac{[(1-x-y)\mathbf{p}_\perp-\mathbf{k}_\perp]^2+m^2}{2(1-x-y)p^+},(1-x-y)\mathbf{p}_\perp-\mathbf{k}_\perp\right]\;,\\
\bar{p}&=\left[(1-y)p^+,\frac{(1-y)\mathbf{p}_\perp^2}{2p^+}+\frac{m^2}{2p^+(1-y)},(1-y)\mathbf{p}_\perp\right]\;.
\end{aligned} \tag{D3}$$

Applying V_1 three times, one gets

$$\Lambda^\mu(4\text{a})=\lambda\int\frac{d^2\mathbf{k}_\perp}{(4\pi)^3}\int\frac{dk^+}{k^+k'^+k''^+}\frac{N_a^\mu+yN_b^\mu}{(p^--k^--k'^-)(p^--k^--k''^-q^-)}\equiv\Lambda_a^\mu+y\Lambda_b^\mu\;, \tag{D4}$$

where

$$N_a^\mu+yN_b^\mu=\gamma^\alpha(\not k'+m)\gamma^\mu(\not k''+m)\gamma^\beta d_{\alpha\beta}\;, \tag{D5}$$

$$\lambda^{-1}=(2\pi)^{3/2}\sqrt{2p^+}\sqrt{2\bar{p}^+}\sqrt{2q^+}\;. \tag{D6}$$

Using Eqs. (A18), (A19), and (A20), we find

$$\begin{aligned}
\Lambda^{a^+}&=\lambda\int_\alpha^{1-y}dx\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\frac{\gamma^+\left[\dfrac{1-x}{x}\right]\left[\dfrac{1}{1-x}+1-x\right]}{(1-y)(\mathbf{k}_\perp^2+g_1m^2)}\\
&\quad+\lambda\int_\alpha^{1-y}dx\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\frac{2x(1-x)[(1-x)\not p p^+-(1-x)m^2\gamma^+-p^+m]}{(1-y)(\mathbf{k}_\perp^2+x^2m^2)(\mathbf{k}_\perp^2+g_1m^2)}\;,
\end{aligned} \tag{D7}$$

where $g_1\equiv x(x+y)/(1-y)$ and α is a dimensionless infrared cutoff, and

$$y\Lambda_b^+ = -\frac{\lambda y\gamma^+}{1-y}\int_\alpha^{1-y}dx\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\frac{\left[\frac{2}{x}-1\right]}{(\mathbf{k}_\perp^2+g_1m^2)}$$
$$+\lambda\frac{y}{1-y}\int_\alpha^{1-y}dx\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\frac{m^2x\gamma^+ + 2x(1-x)(p^+\not{p}-m^2\gamma^+)}{(\mathbf{k}_\perp^2+m^2x^2)(\mathbf{k}_\perp^2+g_1m^2)} \;. \tag{D8}$$

The ultraviolet-singular contribution to $\Lambda^+(4\text{a})$ is

$$\Lambda^+(4\text{a}) = \Lambda_0^+\int_\alpha^{1-y}dx\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\frac{\left[\frac{2}{x}+\frac{x+y-2}{1-y}\right]}{\mathbf{k}_\perp^2+g_1m^2} = \Lambda_0^+\frac{1}{8\pi^2\epsilon}\left[\frac{-3}{2}+\frac{y}{2}+2\int_\alpha^{1-y}\frac{dx}{x}\right], \tag{D9}$$

which can be rewritten as Eq. (5.2). At $y=0$ the infrared-singular contribution to $\Lambda^+(4\text{a})$ is

$$\Lambda^+(4\text{a})_{\text{IR}} = \Lambda_0^+\frac{e^2}{4\pi^2}\left[(\ln\alpha)^2 - \ln\alpha\ln\left[\frac{\mu^2}{m^2}\right]\right] - \frac{\lambda e^2}{4\pi^2}\frac{[(\not{p}-m)p^+ - m^2\gamma^+]}{m^2}\ln\alpha \;. \tag{D10}$$

Since the above expression is meant to be used between $\bar{u}(p,s)$ and $u(p,s)$, it is equivalent to

$$\Lambda^+(4\text{a})_{\text{IR}} = \Lambda_0^+\frac{e^2}{4\pi^2}\left[(\ln\alpha)^2 - \ln\alpha\ln\left[\frac{\mu^2}{m^2}\right] + \ln\alpha\right] \;. \tag{D11}$$

The contribution of the diagram in Fig. 4(b) is

$$\Lambda^+(4\text{b}) = -\lambda\int\frac{d^2\mathbf{k}_\perp}{(4\pi)^3}\int_{p^+-q^+}^{p^+}\frac{dk^+}{k^+k'^+k''^+}\frac{N^\mu}{(p^- - k^- - k'^-)(p^- - k^- - k'^- + k''^-)} \;, \tag{D12}$$

where N^μ is given in Eq. (D5), and λ in Eq. (D6). Note that here the photon is produced by the annihilation of a forward moving electron and positron. Simplifying further we find

$$\Lambda^+(4\text{b}) = -\frac{1}{2y}\int_{1-y}^{1}dx\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\frac{(1-x)N^\mu}{(\mathbf{k}_\perp^2+x^2m^2)(\mathbf{k}_\perp^2+g_2m^2)} \;, \tag{D13}$$

where $g_2 = x(2-x-y)/(1-y)$. Thus the UV-singular contribution is

$$\Lambda^+(4\text{b}) = \frac{\gamma^+}{y}\frac{1}{8\pi^2\epsilon}\int_{1-y}^{1}\frac{dx}{x}(1-x)[x^2-2x+2+y(x-2)] \;. \tag{D14}$$

The diagram in Fig. 4(c) receives two identical contributions from operator ordering, together with a minus sign. One gets

$$\Lambda^+(4\text{c}) = (-2)\lambda\int\frac{d^2\mathbf{k}_\perp}{(4\pi)^3}\int_{p^+-q^+}^{p^+}\frac{dk^+}{k'^+k''^+(k^+)^2}\frac{\gamma^+(\not{k}''+m)\gamma^\mu(\not{k}'+m)\gamma^+}{p^- - k^- - k''^- - q^-}$$
$$= \frac{\Lambda_0^+}{4\pi^2}\frac{1}{y}\int_{1-y}^{1}\frac{dx}{x^2}(1-x)(x+y-1) \;. \tag{D15}$$

Combining Eqs. (D14) and (D15), and performing the x integration yields Eq. (5.3). The calculation of Figs. 5(a), 5(b), 5(c), and 5(d) closely parallels the calculation of the fermion self-energy in Sec. III, and the result is very similar:

$$\Lambda^+5(\text{a})\text{–}5(\text{d}) = \left[\frac{1}{2}\int_0^{\bar{p}^+}dk^+\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\left[-\frac{m}{k^+(\bar{p}^+-k^+)D} + \frac{\gamma^+}{(\bar{p}^+)^2}\frac{m^2}{\bar{p}\cdot k}\right]\right.$$
$$\left. + \frac{1}{2}\int_0^{\bar{p}^+}dk^+\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\left[\frac{\bar{p}-m^2G\gamma^+/\bar{p}^+}{p^+k^+D}\right]\right]\left[-\frac{(\not{\bar{p}}+m)\gamma^+}{ym^2}\right] , \tag{D16}$$

where

$$G \equiv \frac{y}{2x(1-x)} + \frac{x+y}{2x} + \frac{1}{2}(1-y) \;, \tag{D17}$$

$$D \equiv -\frac{\mathbf{k}_\perp^2 + m^2x^2\bar{g}}{2\bar{p}^+x(1-x)} \;, \tag{D18}$$

$$x^2\bar{g} \equiv x(x+y-xy) \ . \tag{D19}$$

For $y=0$ (viz., $G=\bar{g}=1$), the object in the square brackets is just $\Sigma(\bar{p})$.

For the mass counterterm in Fig. 5(e) it is convenient to use an oversubtraction procedure since we are only looking at divergent contributions. We will take this mass counterterm to be $-\Sigma(\bar{p})$, hence,

$$\Lambda^+(5e) = -\Sigma(\bar{p})\left[-\frac{(\bar{\not p}+m)\gamma^+}{ym^2}\right] . \tag{D20}$$

Since we are using a dimensional-regularization procedure, Eq. (D20) is not sensitive to $\bar{g}\neq 1$. Only the difference between G and 1 contributes. Finally, adding Eqs. (D16) and (D20), we obtain

$$\Lambda^+(5) = \frac{\gamma^+}{\bar{p}^+}m^2\frac{1}{8\pi^2\epsilon}\int_0^1 dx\,(1-x)(G-1)\left[-\frac{(\bar{\not p}+m)\gamma^+}{ym^2}\right] . \tag{D21}$$

Doing the integration and using Eq. (A11) we find

$$\Lambda^+(5) = \gamma^+\frac{1}{8\pi^2\epsilon}\left[\frac{3}{2}-2\int_0^1\frac{dx}{x}\right] . \tag{D22}$$

This can be rewritten as Eq. (5.7).

The integrand of the diagram in Fig. 6(a) has the form $\bar{u}(p,s)\gamma^\alpha u(p,s')n_\alpha$, where

$$n_\alpha \equiv \tfrac{1}{2}d_{\alpha\beta}(q)\mathrm{tr}[(\not k+m)\gamma^\beta(\not k-m)\gamma^+] \ . \tag{D23}$$

Using Eq. (C7), one obtains

$$n_\alpha = \frac{1}{4}d_{\alpha\beta}(q)\left[2x(1-x)q^\beta q^+ + \left\{m^2-(\mathbf{k}_\perp^2+m^2)\left[1-\frac{1}{2}\left[\frac{x}{1-x}+\frac{1-x}{x}\right]\right]\right\}d^{\beta+}(q)-(\mathbf{k}_\perp^2+m^2)g^{\beta+}\right] . \tag{D24}$$

However $d_{\alpha\beta}(q)q^\beta=0$, $d^{\beta+}=0$, $d_{\alpha\beta}g^{\beta+}=d_{\alpha-}=0$; therefore, the contribution of Fig. 6(a) vanishes.

The diagram in Fig. 6(b) receives two identical contributions from operator ordering:

$$\begin{aligned}\Lambda^+(6b) &= 2\Lambda_0^+\int\frac{d^2\mathbf{k}_\perp}{(4\pi)^3}\int\frac{dk^+}{k^+k'^+(q^+)^2}\frac{\mathrm{tr}[\gamma^\mu(\not k+m)\gamma^+(\not k'-m)]}{p^--k^--k'^-}\\ &= -\frac{2\Lambda_0^+}{(q^+)^2}\int\frac{d^2\mathbf{k}_\perp}{(2\pi)^3}\int_0^{p^+}\frac{2p^+x(1-x)y}{\mathbf{k}_\perp^2+m^2\bar{g}'} ,\end{aligned} \tag{D25}$$

where $\bar{g}'\equiv x^2y+1$, $k^+=xyp^+$, $k'^+=(1-x)yp^+$, $q^+=yp^+$, and $\bar{p}=(1-y)p^+$. Integrating, one finds

$$\Lambda^+(6b) = -\frac{\Lambda_0^+}{12\pi^2\epsilon} . \tag{D26}$$

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