Trace anomaly via stochastic quantization

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We analyze the trace anomaly for a scalar, self-interacting, field theory in four-dimensional space-time in the framework of the stochastic quantization method. We adopt a regularization scheme at the Langevin equation level, obtaining the regularized expression for the anomaly, for a wide class of regularizing operators.

I. INTRODUCTION

A well-established result in field theory is that symmetries at the classical level cannot be conserved after quantization, giving rise to anomalies. Examples of this effect have been studied by using various techniques.[1] One of these anomalies, the conformal anomaly, is present in those massless field theories that exhibit conformal invariance at the classical level, which means that the energy-momentum tensor has a null trace at this level,[2] but this is not preserved by quantum fluctutations. In these cases, the quantum expectation value of $T_\mu{}^\mu$ is not zero, even at the first quantum correction. This is the so-called trace anomaly.[3]

In the curved-space-time case, we can apply the method developed by Fujikawa:[4] starting with the generating functional

$$Z[g_{\mu\nu}]=\int D\phi\, e^{iS[\phi]} \tag{1}$$

with

$$S[\phi]=\int d^4x\sqrt{-g}\,L(\phi)\ .$$

The quantum expectation value of $T_\mu{}^\mu$ can be expressed as[3]

$$\langle T_\mu{}^\mu\rangle=g^{\mu\nu}\langle T_{\mu\nu}\rangle=\frac{1}{Z[g_{\mu\nu}]}\frac{\delta Z[\tilde g_{\mu\nu}]}{\delta\alpha(x)}\bigg|_{\alpha=0}\ . \tag{2}$$

Here $\alpha(x)$ is the parameter of the conformal transformation:

$$\tilde g_{\mu\nu}=e^{2\alpha(x)}g_{\mu\nu}\ . \tag{3}$$

Let us consider the self-interacting scalar field theory, described by the action

$$S[\phi]=\frac{1}{2}\int d^4x\sqrt{-g}\left[\partial_\mu\phi\partial^\mu\phi-\frac{1}{6}R\phi^2+\frac{\lambda}{12}\phi^4\right]\ , \tag{4}$$

where R is the scalar curvature. Now, as a consequence of the conformal transformation (3), the scalar field must transform as

$$\tilde\phi(x)=e^{-\alpha(x)}\phi(x)\ . \tag{5}$$

It is easy to see that the action (4) is left invariant under transformations (3) and (5), but the generating functional is not, since the measure $D\phi$ changes with (6). This is the origin of the anomaly in Fujikawa's analysis. This effect can be understood in the following way: classically, the trace $T_\mu{}^\mu$ is proportional to the mass and energy density. To calculate its quantum expectation value by using (1) and (2), one has to integrate over all possible "trajectories" $\phi(x)$ in addition to the solution of the classical equation, which corresponds to $m=0$. The contributions of the nonclassical trajectories lead to a nonzero quantum expectation value for the trace of the energy-momentum tensor.[5] In this approach, the anomaly in flat space-time is obtained by taking the zero-curvature limit, after all the calculation has been done. However, we cannot use (2) to compute directly the anomaly in Euclidean space-time because, in this case, the generating functional (1) is a constant, commonly normalized to one.

The stochastic quantization method (SQM) proposed by Parisi and Wu,[6,7] affords a very interesting framework to analyze the anomalies problems.[8] In this method, Euclidean quantum field theory is considered as the equilibrium limit of a statistical system, coupled with a thermal reservoir. This system evolves in a fictitious time direction τ until it reaches the equilibrium limit as $\tau\to\infty$. Also, in the context of the SQM it is possible to implement various regularization schemes that arise from properties of the stochastic calculus.[9,10] In recent years, there have been some works dedicated to analyzing the conformal and Weyl anomalies, in different circumstances, within the context of the SQM.[11–13] In this paper, we present the complete calculation of the trace anomaly, for the scalar field theory described by the action (4), in both flat and curved space-times, by using purely stochastic techniques. This is performed by introducing the regularization scheme proposed by Bern *et al.*,[10] which has the advantage of being more widely applicable, including when gauge symmetries are present. We will see that in this framework the trace anomaly is computed directly as a quantum expectation value of the expression for the trace of the energy-momentum tensor, getting the regularized version in both cases. Also, we discuss the freedom of choosing the regularizing operator, showing how the SQM is able to drive us to different schemes, namely, the Fujikawa (heat kernel) and the negative power kernel.

Our interest in studying the self-interacting scalar field is, as we will see, because in the one-loop approximation, there will be a classical background, in addition to the gravitational one. The anomaly arises from the interaction between the quantum fields and these backgrounds. So, all theories that we have to linearize with the one-loop expansion will have this interaction and will yield the anomaly, even in the flat-space-time case.

In the second section, we present the general framework for the calculations. In the third and fourth sections, we analyze flat and curved space-times, respectively. Finally, we write some conclusions and remarks.

II. THE GENERAL FRAMEWORK

Let us consider a physical system on a d-dimensional space-time described by a field $\phi(x)$ and an action $S[\phi]$. The SQM[6,7] introduces an extra dimension supplied by a fictitious time τ, so that the field ϕ now takes values in a $(d+1)$-dimensional space-time. Then one imposes that the τ dependence of the field $\phi=\phi(x,\tau)$ be governed by the Langevin equation

$$\frac{\partial\phi(x,\tau)}{\partial\tau}=-a\frac{\delta S[\phi]}{\delta\phi(x,\tau)}+\eta(x,\tau) , \qquad (6)$$

where $\eta(x,\tau)$ is white noise, and a is a constant that is equal to 1 when one is dealing with Euclidean space-time, or $(-i)$ in general non-Euclidean space-time,[13] as we shall see in Sec. IV. The fundamental assertion of the SQM is that, for any function $F[\phi]$, one can obtain its quantum expectation value by evaluating the stochastic average of the random function $F[\phi_\eta]$, where ϕ_η are solutions of Eq. (6), in the limit $\tau\to\infty$:

$$\lim_{\tau\to\infty}\langle F[\phi_\eta]\rangle_\eta=\int D\phi\, F[\phi]e^{S[\phi]} . \qquad (7)$$

As has been mentioned, in the SQM it is possible to introduce various regularization schemes arising from properties of the stochastic processes.[9,10] In this work we adopt the method proposed by Bern *et al.*,[10] which is not an action regularization, preserving the Markovian character of the stochastic processes and avoiding some troubles, that are present in the stochastic regularization by fifth-time smearing,[9] in the quantization of gauge theories. This method consists in the introduction of a regularizing operator R_Λ in front of the white-noise source, in the Langevin Eq. (6), where Λ is a parameter so that, for a certain limit of it, R_Λ becomes the identity operator.

In this paper we are interested in calculating the trace anomaly for the system (4). In order to do this, we work in the one-loop approximation, which allows us to to deal with linearized equations of motion and Langevin equations. Therefore, we decompose ϕ in

$$\phi=\phi_c+\varphi , \qquad (8)$$

where ϕ_c is the solution of the classical equation of motion, and φ represent the quantum fluctuations. With this decomposition, one obtains the effective one-loop equation of motion:

$$\Delta\varphi=0 \qquad (9)$$

with

$$\Delta=g_{\mu\nu}\partial^\mu\partial^\nu+\frac{\lambda}{2}\phi_c^2+\frac{1}{6}R . \qquad (10)$$

At this order, we shall see that, in the general case, the trace of the energy-momentum tensor can be written as

$$T_\mu{}^\mu=\varphi\Delta\varphi , \qquad (11)$$

where one can see that $T_\mu{}^\mu$ is null on shell.

In order to get the quantum expectation value of (11), we introduce the SQM as a stochastic process, evolving in a fifth time τ, so that in the equilibrium limit will describe the quantum fluctuations φ. Then, our stochastic scheme is

$$\frac{\partial\phi(x,\tau)}{\partial\tau}=a\Delta\varphi(x,\tau)+R_\epsilon\eta(x,\tau) , \qquad (12)$$

$$\begin{aligned}&\langle\eta(x,\tau)\rangle_\eta=0 ,\\ &\langle\eta(x',\tau')\eta(x,\tau)\rangle_\eta=2\delta(x-x')\delta(\tau-\tau') ,\end{aligned} \qquad (13)$$

and all other higher correlation functions are zero. Here, we have already introduced an appropriate regularizing operator R_ϵ,[10] and we suppose that it is independent of the field φ. After the calculation has been done, one must take the limit value of ϵ for which R_ϵ becomes the identity operator.

With this prescription we proceed to calculate the quantum expectation value of $T_\mu{}^\mu$, Eq. (11), following the fundamental assertion (7). Then, the stochastic average of $T_\mu{}^\mu[\varphi_\eta]$ is

$$\langle T_\mu{}^\mu[\varphi_\eta(x,\tau)]\rangle_\eta=\langle\varphi_\eta(x,\tau)\Delta\varphi_\eta(x,\tau)\rangle_\eta . \qquad (14)$$

Inverting the Langevin Eq. (6), we get

$$\varphi_\eta(x,\tau)=\int_0^\tau dt\, e^{a\Delta(\tau-t)}R_\epsilon\eta(x,t) . \qquad (15)$$

Introducing this in $\langle T_\mu{}^\mu[\varphi_\eta]\rangle_\eta$, it is possible to explicitly calculate the stochastic average (14):

$$\langle\varphi_\eta(x,\tau)\Delta\varphi_\eta(x,\tau)\rangle_\eta=\left\langle\int d^4x'\delta(x-x')\int_0^\tau dt_1dt_2[e^{a\Delta'(\tau-t_1)}R'_\epsilon\eta(x',t_1)]\Delta[e^{a\Delta(\tau-t_2)}R_\epsilon\eta(x,t_2)]\right\rangle_\eta . \qquad (16)$$

Taking into account that the η average acts only on the η noise, one gets

$$\langle\varphi_\eta(x,\tau)\Delta\varphi_\eta(x,\tau)\rangle_\eta=\int d^4x'\delta(x-x')\int_0^\tau dt_1dt_2(e^{a\Delta'(\tau-t_1)}R'_\epsilon)\Delta(e^{a\Delta(\tau-t_2)}R_\epsilon)\langle\eta(x',t_1)\eta(x,t_2)\rangle_\eta . \qquad (17)$$

Now, we can use the second correlation function (13), obtaining

$$\langle \varphi_\eta(x,\tau)\Delta\varphi_\eta(x,\tau)\rangle_\eta = \int d^4x'\delta(x-x')\left[\int_{-\sqrt{2}\tau}^{\sqrt{2}\tau} dt\,\delta(t)\int_0^{\sqrt{2}\tau} dT\, e^{a\Delta(2\tau-\sqrt{2}T)}R_\epsilon^2\Delta\right]\delta(x-x')\ , \tag{18}$$

where we have performed the rotation $\sqrt{2}t=(t_1-t_2)$ and $\sqrt{2}T=(t_1+t_2)$. Now, the t and T integrals are immediate, obtaining

$$\langle \varphi_\eta(x,\tau)\Delta\varphi_\eta(x,\tau)\rangle_\eta = \int d^4x'\delta(x-x')(1-e^{2a\Delta\tau})R_\epsilon^2\delta(x-x')\ . \tag{18'}$$

Now, as we shall see, the second term on the right-hand side (RHS) behaves as a dumping factor that becomes zero in the limit $\tau\to\infty$, surviving only the first term. Then, performing this limit and expanding the δ distribution in some complete basis of eigenfunction $\psi_n(x)$, one gets

$$\langle \varphi(x)\Delta\varphi(x)\rangle = \lim_{\epsilon\to 0}\int d^4x\,\delta(x-x')\sum_n \psi_n(x')R_\epsilon\psi_n(x)\ ,$$

which can be written as

$$\langle T_\mu{}^\mu\rangle = \lim_{\tau\to\infty}\lim_{\epsilon\to 0}\langle \varphi_\eta(x,\tau)\Delta\varphi_\eta(x,\tau)\rangle_\eta = \lim_{\epsilon\to 0}\mathrm{Tr}(R_\epsilon^2)\ . \tag{19}$$

Here, Tr means functional trace and $\epsilon=0$ is the limit value for which $R_\epsilon=I$. Then, we have arrived at the final expression for the trace anomaly, by using only purely stochastic techniques shown as the regularized expression for the anomaly arising by regularizing the noise source.

Consistently with the one-loop approximation, one can see that the Fokker-Planck equation, for the stochastic system given by Eqs. (12) and (13),

$$\dot{P}[\varphi,\tau] = \int d^4x\frac{\delta}{\delta\varphi(x)}\left[a\frac{\delta S[\varphi]}{\delta\varphi(x)}+R_\epsilon\frac{\delta}{\delta\varphi(x)}\right]P[\varphi,\tau]\ , \tag{20}$$

can be solved at the equilibrium limit $\dot{P}[\varphi,\tau\to\infty]=\dot{P}_{\rm eq}[\varphi]=0$ by analyzing the integrable first-order equation:

$$\left[a\frac{\delta S[\varphi]}{\delta\varphi(x)}+R_\epsilon\frac{\delta}{\delta\varphi(x)}\right]P_{\rm eq}[\varphi]=0\ . \tag{21}$$

This equation leads to the equilibrium distribution

$$P_{\rm eq}[\varphi]=e^{-aS_{\rm eff}[\varphi]}\ , \tag{22}$$

where $S_{\rm eff}[\varphi]$ is the effective action:

$$S_{\rm eff}[\varphi]=\tfrac{1}{2}\int d^4x\,\varphi(x)R_\epsilon\Delta\varphi(x)\ . \tag{23}$$

In the limit $\epsilon\to 0$ one recovers the one-loop contribution to the quantum effective action. We remark that this formal manipulation must require some particular cares in nonEuclidean space-time, in order to dealt with a well-defined stochastic process. This will be explained in Sec. IV.

Observe that expression (19) is the usual result, obtained by the introduction of a regularization in the $\delta(0)$ arising in the naive calculus of the anomaly. This has been attained by regularizing the white noise source, responsible for the quantum fluctuations, with the operator R_ϵ. By choosing, the appropriate R_ϵ, we get different familiar results, such as, for instance,

$$R_\epsilon=e^{-\epsilon\Delta}\ , \tag{24}$$

which lead us to the Fujikawa[4] (heat kernel) scheme, and

$$R_\epsilon=\Delta^{-\epsilon}\ , \tag{25}$$

which corresponds to the negative-power kernel regularization. In both cases, in the limit $\epsilon\to 0$ one recovers the identity operator.

III. TRACE ANOMALY IN FLAT SPACE-TIME

In this case we can work in Euclidean coordinates for which the stochastic process, (12) and (13), with $a=1$, is well defined and reaches the right equilibrium limit. Evaluating the functional trace (19), with $R_\epsilon=e^{\epsilon\Delta}$ and using the plane-wave basis, the calculus of the Weyl anomaly is immediate.[11] Deriving the energy-momentum tensor from the Euclidean version of the action (4), we get

$$\Theta_{\mu\nu}=\delta_{\mu\nu}\left[\frac{1}{2}\partial_\rho\phi\partial_\rho\phi+\frac{\lambda}{24}\phi^4\right]+\partial_\nu\phi\partial_\nu\phi - \frac{1}{6}(\partial_\mu\partial_\nu\phi^2-\delta_{\mu\nu}\partial_\rho\partial_\rho\phi^2)\ , \tag{26}$$

where the last two terms have been added to ensure that the trace will be zero on shell. It is easy to see that in the one-loop approximation $\Theta_{\mu\nu}$ can be written as in Eq. (11):

$$\Theta_{\mu\nu}=\varphi\Delta\varphi$$

with $\Delta=\Box-(\lambda/2)\phi_c^2$.

Then, introducing the plane-wave basis in (19), and after some algebra one arrives at

$$\langle\varphi(x)[\Box-(\lambda/2)\phi_c^2]\varphi(x)\rangle = \lim_{\tau\to\infty}\lim_{\epsilon\to 0}\frac{2}{(2\pi)^4}\int d^4k\left(e^{2\epsilon[\Box-(\lambda/2)\phi_c^2-k^2]} - e^{2[\Box-(\lambda/2)\phi_c^2-k^2](\tau+\epsilon)}\right)\ .$$

Here we can see that, as we have mentioned, the second term in the RHS contains the dumping factor $e^{-k^2\epsilon}$. Then, after a bit of algebra,

$$\langle\varphi(x)[\Box-(\lambda/2)\phi_c^2]\varphi(x)\rangle = \lim_{\epsilon\to 0}\frac{2}{(2\pi)^4}\int d^4k\,\epsilon^{-2}e^{-k^2}\exp\left[\epsilon\left[\Box-\frac{\lambda}{2}\phi_c^2\right]\right]\ , \tag{27}$$

where we have only conserved those terms that will not cancel when the k integration is carried out, and the limits $\tau \to \infty$ and $\epsilon \to 0$ are taken. Developing the exponential of Δ, and taking the limit $\epsilon \to 0$ we get the regularized expression for the trace (Weyl) anomaly:

$$\langle \Theta_{\mu\mu} \rangle_{\text{reg}} = \frac{\lambda}{8\pi^2} \left[\Box - \frac{\lambda}{2} \phi_c^2 \right] \phi_c^2 \ . \tag{28}$$

In this expression, we are considering only the finite terms because in this case, in the absence of gravitation, only energy differences are observable. Thus, there is no problem in disregarding any quantity, even infinite, to write the energy-momentum tensor or its trace.

IV. THE CURVED-SPACE-TIME CASE

When one is dealing with non-Euclidean space-time, some cares must be taken in the application of the SQM.[14] Because in non-Euclidean space-time the Feynman path-integral shares the complex distribution $\epsilon^{iS[\phi]}$, one must choose $a = -i$ in the Langevin Eq. (6), in order to ensure the right distribution in the equilibrium limit. However, new problems arise from this modified Langevin equation. In its solution Eq. (15), and in Eq. (18′) one can see the crucial role played by the drift term. In the Euclidean case, the exponential of the drift term behaves as a dumping factor, dissipating the "energy" supplied by the noise source and driving the system to the equilibrium limit. In the present case we are dealing with imaginary exponentials that oscillates indefinitely, troubling the convergence. This fact also complicates the calculus of stochastic average, as those performed in the Sec. II, leading to nonconvergent expressions. A way to overcome this problem is by adding an imaginary mass term in the initial action, and, after all the stochastic averages have been done, it must be zero.

With this prescription and adopting the heat-kernel regularization (24), we start the calculus of Sec. II with the operator

$$\Delta_\epsilon = g_{\mu\nu} \partial^\mu \partial^\nu + \left[\frac{\lambda}{2} - i\kappa \right] \phi_c^2 + \frac{1}{6} R \ .$$

Then, we will get the result (19) containing the operator Δ_κ. Performing the limit $\kappa \to 0$, we obtain

$$\langle T_\mu{}^\mu \rangle = \lim_{\epsilon,\kappa \to 0} \mathrm{Tr}[e^{\epsilon \Delta_\kappa}] = \lim_{\epsilon \to 0} \mathrm{Tr}[e^{\epsilon \Delta}] \tag{29}$$

with Δ defined as in Eq. (10).

The expression (27) is the usual result and, in order to obtain the explicit form of the conformal anomaly, we proceed to analyze it from the "heat-kernel" approach. In Ref. 13, the calculus is made by using the plane-wave basis and expanding the metric $g_{\mu\nu}$ around the flat-space-time case, but this is not possible for arbitrary metrics.

By substituting $t = \Lambda^{-2}$ one can calculate (27), by means of the heat equation, the integral kernel $F(x,x',t)$:

$$\frac{dF(x,x',\epsilon)}{d\epsilon} = \Delta F(x,x',\epsilon) \ . \tag{30}$$

Taking into account the limit $\epsilon \to 0$, we are interested in the short time (ϵ) behavior of this kernel. Then, we can use the asymptotic expansion

$$F(x,x',\epsilon) = \frac{D^{1/2}(x,x')}{(4\pi\epsilon)^{a/2}} e^{-\sigma(x,x')/2\epsilon} \sum_n a_n(x,x')\epsilon^n \ , \tag{31}$$

$D(x,x')$ and $\sigma(x,x')$ are bidensities[15] and contain the information of space-time. The coefficients $a_n(x,x')$, in the limit $x' \to x$ are the Seeley coefficients.[16] Introducing the asymptotic expansion (29) in the heat equation (28), one obtains a recurrence equation for the $a_n's$. The finite part of the final result is obtained taking the limit $\epsilon \to 0$:

$$\langle T_\mu{}^\mu \rangle = \frac{1}{8\pi^2} \left[-\frac{1}{15} (\Box R + R_{\mu\nu} R^{\mu\nu} - R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}) + \lambda \left[\Box - \frac{\lambda}{2} \phi_c^2 \right] \phi_c^2 \right] \ . \tag{32}$$

Also, one gets an infinite quantity that must be absorbed in a renormalization scheme. The general motivation for the curved-space-time treatment for the theory (4) is to connect the expectation value of $T_{\mu\nu}$ with Einstein's equation. In this case, the renormalization of the cosmological and gravitational constants give us finite results for the full theory.[3]

In an analogous way we can regularize the noise with the negative-power kernel, which leads to the same result (32), but without the infinity quantities.[17] For this reason, the negative-power kernel is also called "regularization without renormalization."

V. CONCLUSIONS

We have analyzed the calculus of the trace anomaly in the framework of stochastic quantization, adopting a regularization scheme proposed by Bern *et al.*[10] This scheme has the benefit that it is widely applicable, including the dynamical gauge and gravitational field cases. We have showed that, starting from the regularized Langevin equation, one arrives straightforwardly at the regularized expression for the anomaly, which is given by the functional trace of the regularizing operator introduced in front of the white noise source. It is worthwhile emphasizing the general character of our calculus. Moreover, an analogous scheme can be applied in other types of anomalous theories, for instance, in the axial anomaly,[18] leading to the same familiar result: the anomaly is given by the functional trace of the regularizing operator introduced in the Langevin equation. The relevant fact of this scheme is that, due to the fact that the Markovian character of the stochastic process is preserved, one is dealing with a second-order (in functional derivatives) Fokker-Planck equation. Then, one is able to study the evolution of the probability distribution. Furthermore, in our case, at the one-loop approximation, we have showed the correctness of the equilibrium limit. The stochastic calculus proved how the quantum fluctuations, represented by the noise source, generate the anomaly and also how, by regularizing them, one can extract its finite part.

With respect to the specific model, in Secs. III and IV we have presented some particular facts of flat- and curved-space-time cases, showing that the nonzero contribution to the quantum expectation value of the trace of the energy-momentum tensor comes from the self-interaction term, considered as a classical background in the same spirit we use when the space-time is not flat. The anomaly arises, in both cases, from the backgrounds.

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[1]R. Jackiw, B. Zumino, and E. Witten, *Current Algebra and Anomalies* (World Scientific, Singapore, 1985).

[2]T. Fulton, F. Rohlich, and L. Witten, Rev. Mod. Phys. **34**, 442 (1962).

[3]N. D. Birrell and P. C. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1985).

[4]K. Fujikawa, Phys. Rev. Lett. **42**, 1195 (1979); Phys. Rev. D **21**, 2848 (1980); **22**, 1499(E) (1980); **29**, 285 (1984).

[5]M. S. Alves and J. Barcelos-Neto, Mod. Phys. Lett A **4**, 155 (1989).

[6]G. Parisi and Y. S. Wu, Sci. Sin. **24**, 483 (1981).

[7]For a good review, see, for example, P. Damgaard and H. Huffel, Phys. Rep. **152**, 227 (1987).

[8]M. B. Gavela and N. Parga, Phys. Lett. B **174**, 319 (1986); R. Kirshner, E. R. Nissimov, and S. J. Pacheva, *ibid.* **174**, 324 (1986); J. Alfaro and M. B. Gavela, Phys. Lett. **158B**, 473 (1985); J. A. Magpantay and M. Reuter, Phys. Lett. B **199**, 519 (1987); J. Webb, J. Phys. G **13**, 1307 (1987); M. Namiki, I. Ohba, S. Tanaka, and D. M. Yanga, Phys. Lett. B **194**, 530 (1987); H. Montani and F. A. Schaposnik, Ann. Phys. (N.Y.) **181**, 161 (1988); H. Montani, Phys. Rev. D **38**, 3 (1988).

[9]B. Sakita, in *Lattice Gauge Theory, Supersymmetry and Grand Unification,* proceedings of the 7th Johns Hopkins Workshop on Current Problems in Particle Theory, Bad Honnef, Germany, 1983, edited by G. Domokos and S. Kovesi-Domokos (World Scientific, Singapore, 1983); J. D. Breit, S. Gupta, and A. Zaks, Nucl. Phys. **B233**, 61 (1984).

[10]Z. Bern, M. B. Halpern, L. Sadun, and C. Taubes, Phys. Lett. **165B**, 151 (1985); Nucl. Phys. **B284**, 1 (1987); **B284**, 35 (1987); **B284**, 92 (1987).

[11]M. B. Halpern, Ann. Phys. (N.Y.) **178**, 272 (1987); Phys. Lett. B **185**, 111 (1987).

[12]J. W. Jun and J. K. Kim, Phys. Rev. D **37**, 2239 (1988).

[13]M. Namiki, H. Soshi, and S. Tanaka, Phys. Rev. D **38**, 1346 (1988).

[14]H. Hüffel and H. Rumpf, Phys. Lett. **148B**, 104 (1984); E. Gozzi, *ibid.* **150B**, 119 (1985); H. Hüffel and P. V. Landshoff, Nucl. Phys. **B260**, 545 (1985).

[15]B. S. DeWitt, *Dynamical Theory of Groups and Fields* (Gordon and Breach, New York, 1965); Phys. Rep. **19C**, 295 (1975).

[16]R. T. Seeley, Am. Nat. Soc. Proc. Pure Math. **10**, 288 (1967).

[17]M. S. Alves, C. Farina, and C. Wotzaseck, Phys. Rev. D (to be published); T. F. Treml, *ibid.* **35**, 2036 (1977).

[18]C. Farina, H. Montani, and L. C. Albuquerque, Phys. Rev. D (to be published).