Strings, texture, and inflation

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We examine mechanisms, several of which are proposed here, to generate structure formation, or to just add large-scale features, through either gauged or global cosmic strings or global texture, within the framework of inflation. We first explore the possibility that strings or texture form if there is no coupling between the topological theory and the inflaton or spacetime curvature, via (1) quantum creation, and (2) a sufficiently high reheat temperature. In addition, we examine the prospects for the inflaton field itself to generate strings or texture. Then, models with the string/texture field coupled to the curvature, and an equivalent model with coupling to the inflaton field, are considered in detail. The requirement that inflationary density fluctuations are not so large as to conflict with observations leads to a number of constraints on model parameters. We find that strings of relevance for structure formation can form in the absence of coupling to the inflaton or curvature through the process of quantum creation, but only if the strings are strongly type I, or if they are global strings. If formed after reheating, naturalness suggests that gauged cosmic strings correspond to a type-I superconductor. Similarly, gauged strings formed during inflation via conformal coupling $\xi=1/6$ to the spacetime curvature (in a model suggested by Yokoyama in order to evade the millisecond pulsar constraint on cosmic strings) are expected to be strongly type I. Type-II strings are possible if ξ is large or if the strength of direct coupling between the string field and inflaton is chosen appropriately. We improve upon the understanding of the formation process of strings and texture during inflation, and find that the alternative fractal string scenario put forth by Vishniac *et al.* is further restricted in parameter space, but the model still turns out to be much more plausible than the authors had realized. The fractal model, as originally proposed, may in fact lead to ordinary scaling scenarios of structure formation. However, new structure formation scenarios still appear to be quite possible, especially for global strings. Gauged strings leading to nonstandard scenarios must be type I, and the quartic coupling for global texture should satisfy $\lambda \lesssim 4\times10^{-4}$ if a new scenario of texture formation is to be possible. We find that the formation of strings and/or texture differs from the standard picture if $\xi \lesssim 1/12$, and that the string and/or texture-curvature interaction is not sufficient to place the characteristic formation scale of texture or strings within the horizon if $\xi \lesssim 0.002$. We estimate the likelihood P, in terms of model parameters, that structure in a universe of our size may be described by a nonstandard string or texture scenario. We find that P can be of order unity.

I. INTRODUCTION

At present, there are only a few well-received theories that can provide the large-scale fluctuations required for cosmological structure formation. Inflation is one such theory, which seems to be required to explain a host of cosmological problems, and as a bonus it can provide the density inhomogeneities needed to explain the observed structure in the Universe. For reviews of the inflationary scenario, see Ref. 1. Competitive topological models for structure formation include cosmic strings, reviewed in Ref. 2, and global texture.[3] These models are all generic, in the sense that they each arise in large classes of field theories containing appropriate scalar fields. (Of course, with inflation there are other possibilities for generating geometrical structures.[4])

In this paper we examine various means by which strings and texture can be made compatible with inflation and explore the constraints on model parameters that arise when the inflationary fluctuations are constrained to be small enough that inflation does not overproduce density inhomogeneities. Contrary to the conclusion from simple estimates, reviewed in the following paragraphs, it is possible to generate strings and texture in inflationary models via a number of different mechanisms.

Simple estimates indicate that strings and texture are not produced after inflation, as the reheating temperature after inflation is expected to be below the scale at which these topological structures form.[5–7] Then they form before or during inflation and are diluted into oblivion (scales outside our horizon), along with monopoles and other unwanted relics, by the inflationary expansion. In the simplest models of inflation, e.g., chaotic inflation,[8] there is a constraint on the energy density, or Hubble parameter H, during inflation from the requirement that inflation does not overproduce density inhomogeneities on large scales:[7,9]

$$H \lesssim 7\times 10^{14}\ \text{GeV} . \tag{1.1}$$

This constraint assumes a cold-dark-matter scenario of structure formation; it remains valid if there is biasing, and it applies to all scales below the normalization scale ($\simeq 8$ Mpc). The derivation of (1.1) follows from the relation between the power spectrum and the inflationary potential in the slow-roll approximation and a constraint on the potential that allows inflation to occur. Hence models that nearly violate slow-roll conditions, such as power-law inflation,[10] can saturate the bound given by (1.1). The constraint resulting from the observed isotropy of the microwave background[6,9,11] is $H \lesssim 8\times 10^{14}$ GeV, which is relatively close to the constraint (1.1).

The constraint (1.1) can easily be translated into a constraint on the reheating temperature T_r of ordinary radiation after inflation:

$$T_r \lesssim 2\times 10^{16}\epsilon_r\ \text{GeV} , \tag{1.2}$$

where the effective particle degrees of freedom were taken to be those of the standard model $g = 106.75$ [this may be increased somewhat by grand unified theories (G.U.T.'s) and supersymmetry]. The efficiency of reheating ϵ_r (≤ 1) is expected to be small since typical inflationary models have difficulty achieving efficient reheating as a consequence of the small potential couplings required for an observably acceptable fluctuation amplitude. The actual reheat temperature may be far below its maximum possible value. For completeness, we mention that additional assumptions about the inflationary model can significantly tighten the bound (1.2). For potentials corresponding to inflationary power spectra that do not exceed the constraint on the power spectrum at the normalization scale, as well as for scales below it, the following bound arises:[9]

$$T_r \lesssim 6\times 10^{15}\epsilon_r\ \text{GeV} . \tag{1.3}$$

Keep in mind that the above constraints correspond to the inflationary fluctuations having the largest values allowed if it is these fluctuations that form cosmological structures. If we want strings or texture to be solely responsible for structure formation, then constraints (1.1)–(1.3) would become even stronger. (The precise constraints would be dependent on the details of structure formation with strings and/or texture, which are not yet fully understood.)

The preferred value of the string energy per length μ for galaxy and large-scale structure formation is $G\mu\approx 10^{-6}$. The naive expectation is that the symmetry-breaking scale $\eta\approx\sqrt{G\mu}m_{\rm Pl}\approx 10^{16}$ GeV ($m_{\rm Pl}=G^{-1/2}$ $=1.22\times 10^{19}$ GeV is the Planck mass) and that the critical temperature T_c of the phase transition in which strings form is $\approx\eta$. It therefore appears marginal that strings could form after inflation, taking into consideration the constraint (1.2). Even if a sufficiently high reheat temperature was possible to generate strings (or texture), one is left with the disastrous possibility that monopoles also form after inflation. Therefore, one has to ensure that the mechanism leading to strings or texture with a high T_r does not automatically lead to monopoles; perhaps, certain higher-dimension models of gravity can do this.[5,12] An accurate determination of η for texture will not be available until it becomes clearer how galaxies and large-scale structure form with texture, but $\eta\simeq 5\times 10^{15}$ GeV is expected.[13] If this scale is much smaller than that for monopoles, and the reheat temperature is in between these scales, then topological structure can be relevant for structure formation in inflationary models.

The basic Lagrangian that we shall consider for strings is of the form

$$L=(D_\mu\Phi)^*(D^\mu\Phi)-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}-V(\Phi) , \tag{1.4}$$

where Φ is a complex scalar field, $D_\mu=\partial_\mu-iqA_\mu$, $F_{\mu\nu}=\partial_\nu A_\mu-\partial_\mu A_\nu$, and the U(1) potential is

$$V(\Phi)=-m_S^2|\Phi|^2+\lambda|\Phi|^4 . \tag{1.5}$$

The case of global strings can be obtained by setting the gauge coupling charge $q=0$, and dropping the terms involving the gauge field A_μ. Writing $\Phi=f\exp(i\chi)/\sqrt{2}$, where f and χ are real fields, the scale η (expectation value) that arises upon minimization of the potential with respect to f is $\eta^2=m_S^2/\lambda$. In the case of global texture, we generalize the complex global string field Φ to a complex doublet—which can form texture when the global SU(2) symmetry is broken. In fact, the majority of results for global strings and global texture are the same, owing to the similarity of the theories. We therefore concentrate on the string case, but note differences between producing global strings and global texture when they arise.

In Sec. II we explore several mechanisms for string and texture formation that do not involve coupling the symmetry-breaking field to other fields or to the gravitational curvature R. Then, in Sec. III, we consider the coupled case. In Sec. IV we discuss our results and mention other constraints that must be considered. The Appendix discusses the equilibrium distribution of scalar fields during inflation.

II. FORMATION OF STRINGS AND TEXTURE WITHOUT ADDITIONAL COUPLING

In this section we consider the formation of strings and texture in inflationary cosmologies for the case that the string and/or texture field is not coupled to the inflaton or curvature. We first consider what is perhaps the simplest possibility: lowering the critical temperature for local string formation by an appropriate choice of gauge and quartic couplings. Next, we discuss the formation of strings and/or texture via quantum fluctuations in the corresponding scalar fields during inflation and the possibility of nonstandard structure formation scenarios. Finally, we examine the possibility that a global string and/or texture theory may also be responsible for inflation, with decoherence of the relevant fields at the end of inflation leading to a scaling pattern of global strings and/or texture.

A. Adjusting the critical temperature

For global strings or global texture, the critical temperature T_c of the transition is determined by η alone. The value of η required for global texture structure formation is within a factor of a few of the bound (1.2) and is nearly ruled out by (1.3). A global string scenario of structure formation is easier to realize since η is somewhat smaller than that for texture (discussed later), and constraint (1.3) can be satisfied if $\epsilon_r \gtrsim 0.03$. Gauge strings have additional freedom owing to the gauge field, and perhaps one of the simplest proposals made to have a string scenario be compatible with inflation is to utilize the parameter space of the theory [Eqs. (1.4) and (1.5)] to adjust the critical temperature of the string-forming phase transition below the reheat temperature.[6,14,15]

The critical temperature of the gauge string-formation transition is given by (see, e.g., Ref. 16)

$$T_c^2 = \frac{12m_S^2}{4\lambda+3q^2} = \frac{12\eta^2}{4+3q^2/\lambda} , \tag{2.1}$$

and one has the freedom to make T_c well below the scale η for $q^2 \gg \lambda$, hence bypassing the naive problems discussed in the Introduction. The other critical feature of this scenario is that the energy per length of the string $\mu \simeq \eta^2$, practically independent of the ratio

$$\beta \equiv m_V^2/m_S^2 = q^2/2\lambda , \tag{2.2}$$

where $m_V = q\eta/\sqrt{2}$ is the vector-boson mass and m_S is the scalar mass. [If $\beta > 1$ ($\beta < 1$) the strings exhibit type-I (type-II) superconductivity in the Ginzburg-Landau theory.] More precisely,[14]

$$\mu \simeq 1.04\pi\eta^2/\beta^{0.195} , \tag{2.3}$$

for the range $0.01 \lesssim \beta \lesssim 100$; and in the limits $\beta \gg 1$, $\beta \ll 1$, the energy per length only varies logarithmically, $\mu \sim \eta^2/\ln(\beta)$, $\mu \sim \eta^2 \ln(1/\beta)$, respectively.

The model [Eqs. (1.4) and (1.5)] is subject to radiative corrections, unless canceled because of symmetry considerations (e.g., supersymmetry), which become important when $\lambda \lesssim q^4$. Also, (2.1) assumes $T_c \gg m_V, m_S$ or, equivalently, $\lambda \gg q^4$, $q^2 \ll 1$. If we require that the circle of degenerate vacua in the effective potential be at the global minimum of the potential, which is necessary for a plausible cosmic-string theory, then one finds that

$$\lambda > 3q^4/32\pi^2 \quad \text{or} \quad \beta < 16\pi^2/3q^2 . \tag{2.4}$$

Naturalness suggests that the gauge coupling $q = O(e)$, which implies $\beta \lesssim 10^2$, and that the critical temperature can at most be smaller than η by a factor $\sim 10^{-1}$. In this case the string network could form after inflation if there is fairly efficient reheating. Assuming the self-interaction of Φ is perturbative ($\lambda \lesssim 1$) leads to the additional naturalness constraint $\beta \gtrsim 10^{-2}$.

Acceptable values of β can be pinned down by requiring that the critical temperature (2.1) be below the reheating temperature. The constraint (1.2) allows the formation of strings for all values of β if reheating is very efficient. More stringent conditions can be placed on the wide class of inflationary models leading to constraint (1.3). Applying constraint (1.3), using Eqs. (2.1)–(2.3), and taking $G\mu \approx 10^{-6}$, we find that

$$\beta \gtrsim 3\epsilon_r^{-2.48} \quad \text{or} \quad \beta \lesssim 10^{-3}\epsilon_r^{10.24} . \tag{2.5}$$

Technically, the latter condition pushes the use of (2.3), and it suffices to say that this constraint cannot be satisfied for natural values of β. Since $\epsilon_r \leq 1$, the range $10^{-2} \lesssim \beta \lesssim 3$ is excluded. The entire "natural" range of β is excluded for type-II strings. Strongly type-II strings, though unnatural, are able to form after reheating because smaller values of β require lowering the scale η (and hence T_c) to maintain a fixed string energy per length μ. However, the weak (logarithmic) dependence of η on β, for $\beta \lll 1$ (unnaturally small), prevents η from being lowered below $\simeq 7\times 10^{14}$ GeV—in which case the horizon scale becomes a cutoff and the limit of a global string is achieved. It appears much more natural that the strings are type I. We note that if $\epsilon_r \lesssim 0.1$, even "natural" type-I strings are no longer a possibility. A large reheating efficiency is required for the formation of natural strings.

Even though the inflaton might deposit its energy into ordinary fields at a very-high-energy scale, it is still not clear that strings will form because our analysis assumes that the string field is in a state of thermal equilibrium. After inflation, and before the inflaton decays away, the string field could be coherently lying in one of its vacua and it is crucial that it become excited enough to hop over the potential barrier during the short time interval that it is energetically feasible. A suggestive, but by no means rigorous, analysis of whether or not the string field should be in equilibrium is to calculate the interaction rate of the scalar particles among themselves. The cross sections are $\sigma \simeq \lambda^4/T^2$ and $\sigma \simeq q^8/T^2 \simeq \lambda^4\beta^4/T^2$ for interactions via scalar and gauge particles, respectively. Comparison of the interaction rate with the expansion rate requires $T/m_{\rm Pl} \lesssim (q^8,\lambda^4)\sqrt{g}$ for equilibrium, where g is the particle degrees of freedom. This seems moderately difficult to satisfy in light of additional constraints and the fact that naturalness suggests that T_c be larger than $\approx 10^{15}$ GeV.

B. Quantum fluctuations in the string and/or texture field

A critical temperature above the reheating temperature does not necessarily preclude the formation of strings and/or texture at a cosmologically significant level if the effects of fluctuations in the corresponding scalar fields during the inflationary era are taken into account. It turns out, however, that the constraints on the amplitude of inflationary fluctuations necessitate that the Hawking temperature be sufficiently low during the last ≈ 60 e-folds of inflation, which corresponds to scales within the observable Universe, so that an infinite network of strings of relevance for structure formation cannot form via previously known mechanisms. However, we find, with a likelihood which we calculate, that the global structure of the Universe may have local regions (miniuniverses) where string scaling solutions are possible. In addition, there are regions containing strings that could give rise to a nonscaling scenario of structure formation. A nonscaling scenario was discussed by Vishniac, Olive, and Seckel[6] (cf. also Ref. 4) for models where there is coupling of the string and/or texture field to the inflaton (discussed further in Sec. III)—the novel feature here is that no extra couplings are required.

Let us recall a few of the basic properties of a real scalar field ϕ in de Sitter space. The effective (Hawking) temperature T of the Universe during inflation is simply related to the Hubble parameter H, i.e., $T=H/2\pi$. An effectively massless ($m^2 \lesssim H^2$) real scalar field ϕ fluctuates, on average, by an amount $\delta\phi=H/2\pi$ during an e-fold of expansion during inflation.[17] (Whenever we refer to fields, it is the long-wavelength $\lambda \gtrsim H^{-1}$ portion that we are really considering.) These fluctuations occur independently in domains of size $\sim H^{-1}$. In a time interval $\Delta t \sim H/m^2$, the field loses memory of its initial value and is Gaussian distributed about $\phi=0$ with

$$\langle \phi^2 \rangle \approx H^4/m^2 \ . \tag{2.6}$$

If the mass scale satisfies $m^2 \gtrsim H^2$, the field fluctuates by an amount $\delta\phi \approx H^2/m$ in each Hubble expansion time, but does not random walk.

Now we introduce topological features in the potential and restrict attention to the case that the field of interest has a strong bias to live near the minimum of the potential, and we presently assume that the field on the horizon scale was near the minimum. For a real scalar field with potential $V=\lambda(\phi^2-\eta^2)^2/4$, it is clear that such a restriction effectively excludes the possibility of making domain walls (unless very improbable fluctuations take the field to the top of the potential). (For a discussion of domain walls, see Vilenkin.[2]) Similarly, it is not topologically possible to make strings. However, texture-antitexture pairs could, in principle, form. To consider this possibility, it is simplest to consider the formation of texture in the U(1) model (1.5) in one spatial dimension and argue that analogous results should hold in our spacetime. Radial fluctuations in Φ are unimportant if

$$\langle (|\Phi|-\eta)^2 \rangle \approx H^4/\lambda\eta^2 \lesssim \eta^2 \ , \tag{2.7}$$

and the radial degrees of freedom are frozen out, yielding an effective Lagrangian given by $L=\eta^2\partial_\mu\chi\partial^\mu\chi/2$, where χ is the phase of Φ.

The typical phase fluctuation in an expansion time is given by $\delta\chi=H/2\pi\eta$, and texture-antitexture pairs will form on scales corresponding to $\delta\chi \approx 1$. That is, when H drops below

$$H_* \simeq 2\pi\eta \ , \tag{2.8}$$

in the course of the inflationary expansion, the fluctuations in the phase are no longer large enough to efficiently produce further pairs. Taking into consideration constraint (1.1) and values of η required for structure formation, this condition can only be satisfied on scales well outside our present horizon. We further note that the assumption of the scenario (2.7) coupled with condition (2.8) implies $\lambda \gtrsim 1$, which is distasteful if one believes in perturbative interactions. [The constraint $\lambda \gtrsim 1$ could be weakened if H remains approximately constant from the horizon scale to the end of inflation. The time and length scales associated with the creation of a texture-antitexture pair is related to the number of expansion times required to encircle the U(1) potential: $N \approx (\eta/H)^2$. If $N \lesssim 50$ or $H/\eta \gtrsim 0.14$, texture may still be relevant for large-scale structure—and the constraint on λ weakens to $\lambda \gtrsim 0.01$. The former constraint still cannot be satisfied, however, if texture is to be responsible for structure formation.] There is the additional possibility, however, that the string or texture field happened to be near the top of the potential on scales corresponding to our horizon. As we shall discuss later, these circumstances coupled with appropriate choices of the parameters of the theory, subject to the constraint (2.7), can in fact lead to texture and string scenarios—albeit they may be very different from the standard scaling scenarios.

We now explore the opposite inequality of (2.7), i.e.,

$$\lambda\eta^4 \lesssim H^4 \ , \tag{2.9}$$

in which case radial fluctuations are important, and we presently consider this case for the string model (1.5). In the vicinity of the degenerate vacua and the top of the potential $|(\Phi| \lesssim \eta)$, the magnitude of the effective mass squared $m^2_{\rm eff}$ of the string field is no larger than $\sim\lambda\eta^2$, which is smaller than the inflationary scale H^2 (if $\eta \gtrsim H$ or $\lambda\eta^2 \lesssim H^2$ for $\eta \lesssim H$), and the classical radial motion of Φ is friction dominated. However, the classical motion of the field is unimportant, and Φ effectively takes a random walk of typical size $H/2\pi$ in an expansion time. The field Φ will typically not walk too far up the outer sides of the potential, however, since the downward classical motion of the field will overtake a typical fluctuation in one expansion time if $|\Phi| \gtrsim H/\lambda^{1/3}$.

We now discuss, heuristically, the possibility of quantum fluctuations leading to a standard string scaling scenario or standard texture scenario. A string scaling model will arise if an infinite network of strings forms on scales below structure formation scales. The usual picture of string formation will arise if the string field is prevented from significantly random walking away from the origin, so that there will be small phase correlations between typical neighboring Hubble volumes. Infinite string formation should then be possible when the typical size of a field fluctuation $\sim H$ is comparable to the scale $\sim H/\lambda^{1/4}$, within which Φ is typically confined [the latter estimate is obtained by setting $m^2 \sim \lambda\langle|\Phi|^2\rangle$ in (2.6), which is appropriate if (2.9) is satisfied]. This leads to the condition that the field must be strongly coupled $\lambda \sim 1$, which further leads to the constraint $\eta \lesssim H$ [from (2.9)]. The latter constraint, along with (1.1), cannot be satisfied by ordinary or global strings, or texture, for values of η necessary for structure formation.

A more rigorous analysis of scaling scenarios leads to slightly less stringent results. In principle, scaling solutions might be expected whenever the symmetry is nearly restored. The crucial question, however, is the length scale associated with the restoration of symmetry, i.e., the scale over which different vacua are nearly uncorrelated. We estimate this scale by finding the minimum number of e-folds, N_l, before the end of inflation, which corresponds to a Hubble region in which the symmetry is typically restored, with the simplifying *assumption* that H is approximately constant (a number of inflationary models can lead to $H \simeq$const; see, e.g., Ref. 18). That is, the symmetry will be restored on the scale N if $N \gtrsim N_{\rm eq}$, where $N_{\rm eq}$ is the number of expansion times needed to realize an equilibrium distribution of fields, and $N=N_l=N_{\rm eq}$ is the smallest scale that satisfies this inequality. The equilibrium distribution of fields is given in the Appendix, and the time scale $N_{\rm eq}$ can be estimated by the number of Hubble expansion times it would take to go from $\phi^2=0$ to $\phi^2 \approx H^2/\sqrt{\lambda}$ (or, equivalently, when the dispersion of fields associated with random walking "covers" most of the equilibrium distribution phase space). For a $\lambda(\phi^2-\eta^2)^2/4$ theory (which we presently consider for the sake of simplicity), the effective mass squared is given by $m_{\rm eff}^2=3\lambda\phi^2-\lambda\eta^2$, and using $\langle\phi^2\rangle \approx H^4/\langle m_{\rm eff}^2\rangle$, we find that $m_{\rm eff}^2 \approx \sqrt{\lambda}H^2$. The symmetry is restored in less than a Hubble time if $m_{\rm eff}^2 \gtrsim H^2$ or $\lambda \gtrsim 1$. For $m_{\rm eff}^2 \lesssim H^2$, one easily finds

$$N_l=N_{\rm eq} \simeq (2\pi^2/\lambda)^{1/2} . \tag{2.10}$$

Therefore, if $N_l \lesssim 50$ or

$$\lambda \gtrsim 10^{-2} , \tag{2.11}$$

the symmetry is restored well within our horizon, and scaling scenarios relevant for structure formation may arise. This constraint is the most favorable that can be imagined and becomes more stringent in relation to when the Hubble parameter significantly changes from its value on structure formation scales. Constraints (2.9) and (2.11) may only be consistent with inflationary constraints if global strings give rise to structure formation.

Even if $N_{\rm eq} \gtrsim N_h$ (we define $N_h \approx 60$ to be the number of e-folds corresponding to the horizon scale and H_h to be the Hubble parameter at that time), within a universe of our size it may be possible to satisfy $N_l \lesssim N_h$. The initial value of the relevant field on the scale corresponding to our horizon will determine whether or not a string or texture scenario is possible and whether or not a scaling solution is even relevant for structure formation. We first examine the initial conditions required for a scaling scenario and then determine the likelihood of such conditions. Then we explore the possibility of a nonscaling scenario.

First, we point out that it is not at all obvious that a network of strings can even form on scales small enough to allow a scaling solution to be relevant for structure formation when $N_{\rm eq} \gtrsim N_h$. To consider this case, we return to the real scalar field model and examine the limit $N_* \lesssim N_h$; here

$$N_* \equiv (2\pi\phi_h/H)^2 , \tag{2.12}$$

which represents the time scale for the dispersion of the distribution to become larger than the initial horizon-scale value ϕ_h (equivalently, ϕ_h is the mean-field value in our Universe). (Note that if $N_* \gg N_h$, an inconsequential amount of walls, string, or texture forms.) We shall provide an estimate of the length scale l corresponding to the scale over which the vacua are essentially uncorrelated. We shall consider the domain-wall case for the purpose of simplicity—but our results for l are equally applicable to strings and texture.

Let us consider the correlations between two inflationary domains, at the end of inflation, of causal size Δ separated by a distance L. The domains were in causal contact $N_c=\ln(L/\Delta)$ e-folds before the end of inflation. We presently assume that the mean value of ϕ_c (ϕ_h) is less than dispersion in ϕ_c and later check for conditions of self-consistency. Then the value of the fields ϕ_c at the epoch N_c will typically lie in the range $-H\alpha\sqrt{N_h-N_c}/2\pi \lesssim \phi_c \lesssim H\alpha\sqrt{N_h-N_c}/2\pi$, where $\alpha \sim 1$. After N_c steps within a domain of initial field value ϕ_c, the field values at the end of inflation will be dispersed by an amount $\delta\phi=\sqrt{N_c}H/2\pi$. If this dispersion is larger than ϕ_c, by some factor γ (of order unity), we expect inflationary domains to be nearly uncorrelated on the scale L. The minimum scale L that satisfies $\delta\phi \gtrsim \gamma\phi_c$ we call l, and we find

$$l/\Delta \simeq \exp(N_l) , \tag{2.13}$$

where $N_l=N_h/[1+(\alpha\gamma)^{-2}]$, and we have taken $\phi_c=H\alpha\sqrt{N_h-N_c}/2\pi$. For a scaling solution to be relevant for large-scale structure, we require $N_l \lesssim 50$, which yields the constraint

$$\alpha\gamma \lesssim 2.2 . \tag{2.14}$$

A benchmark value of γ is the required value for percolation. If one of the two discrete vacuum states in a domain-wall theory is randomly assigned, with a probability p to inflationary domains, percolation occurs if $0.31 \lesssim p \lesssim 0.69$ (see, e.g., Ref. 19). This translates to the condition $\gamma \gtrsim 2$. Although our model is somewhat different, we do not expect significantly different results. The possibility of scaling solutions seems to be rather marginal, but because of the nature of the problem and our calculation, we are unable to make a definite conclusion one way or the other. Our difficulty primarily stems from a coincidence of numbers in which α and γ of order unity lead to a correlation length of the order of 10 e-folds below the horizon scale. For the purposes of classification, we shall simply refer to this case as a scaling solution. Our calculation is self-consistent (with respect to our choice of ϕ_c) if the dispersion on the scale l satisfies $\delta\phi(N_c=N_l) \gtrsim \gamma\phi_h$ or

$$N_* \lesssim N_h/(\gamma^2+\alpha^2\gamma^4) . \tag{2.15}$$

We briefly discuss these results and an alternative way of viewing our analysis. We have just examined the case that a field on the horizon scale is quite close to the top of the potential, with subsequent field values typically lying within a few deviations of a Gaussian distribution of width corresponding to how many e-foldings occurred. There is no characteristic length scale in our problem, and strings have a continuous range of length scales superposed upon them (fractal-like). It should be clear that a large-scale string should exist, but perhaps it is less obvious that (2.13) implies that, viewed from the comoving present horizon scale, the scale over which the long-string network is randomized gets continually smaller as inflation proceeds. This is easily understood if we recall that if we look inside a subdomain the symmetry will eventually be restored if one allows enough inflation to occur, and a large-scale string will traverse the subdomain. If one waits further until *most* of the subdomains go through restoration, then the long strings in each subdomain may connect (percolate) and an finite string network can appear with l corresponding to the subdomain scale. Hence the amount of inflation plays an important role in determining the scale of the network.

Now we consider initial conditions that lead to $\phi_c \approx \phi_h$. Following the previous calculations with this choice of ϕ_c, we find

$$N_l = \gamma^2 N_* \ . \tag{2.16}$$

This result applies when the dispersion of fields N_l e-folds before the end of inflation is less than ϕ_h or

$$N_h/(\gamma^2-1) \gtrsim N_* \gtrsim N_h/(\gamma^2+1) \ . \tag{2.17}$$

This corresponds to a l larger than that given by Eq. (2.13). Taking $\gamma \simeq 2$ implies $20 \gtrsim N_* \gtrsim 12$, which represents a much larger range of initial condition space

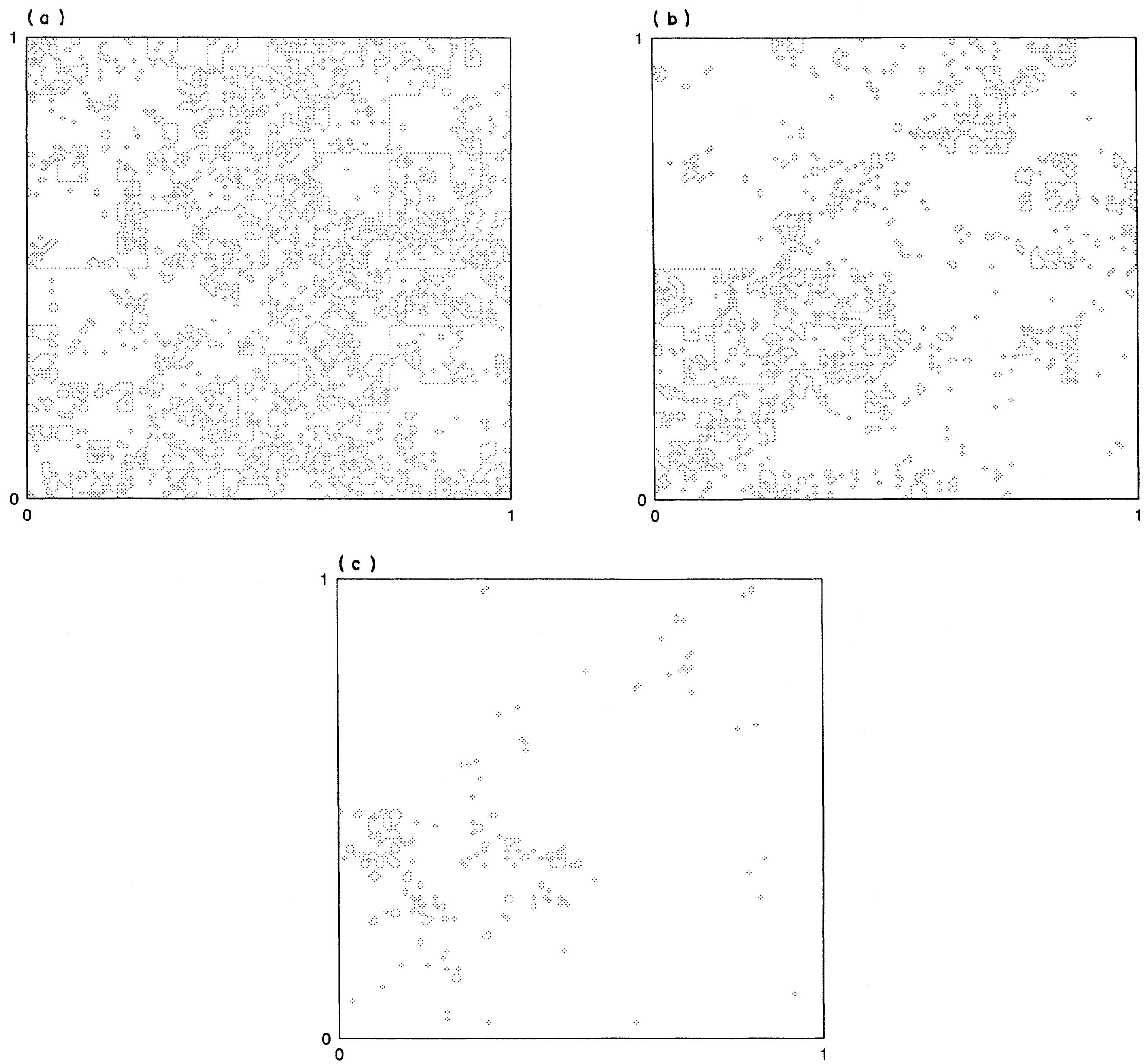

FIG. 1. Shown are the zero contours of a real scalar field in de Sitter space after $\simeq 5$ e-folds of expansion for various initial values of the field. In terms of the ratio of the initial field value to the expected field dispersion at the end of the simulation, (a)–(c) correspond to 0, 1, and 2, respectively. If this ratio is significantly smaller than unity, there is an abundant amount of large-scale defects. For ratios exceeding unity, defects become sparse.

than in our previous calculation—and typically leads to a long network of strings that will be useless for structure formation (l exceeds the horizon scale for $N_* \gtrsim N_h/\gamma^2 \approx 15$). However, there may still be plenty of string to initiate structure formation (see Sec. III for further details). We classify this set of initial conditions as leading to nonscaling scenarios.

To illustrate the effects of different values of ϕ_h on the distribution of defects, we have performed a simple numerical simulation of an effectively massless real scalar field in de Sitter space in two dimensions (which is equivalent to looking at a slice of a three-dimensional distribution). We fix our coordinates to the scale corresponding to ϕ_h and, after each twofold of expansion, divide a previous inflationary domain into four pieces and add fluctuations drawn from a Gaussian distribution of width $\sigma_G^2 \simeq H^2 \ln(2)/4\pi^2$. In Figs. 1(a)–1(c) we show the resulting wall distribution after $N=7$ twofoldings of inflation for $\phi_h/\sqrt{N}\,\sigma_G = 0,1,2$. To facilitate comparison, we have used the same set of quantum fluctuations (random numbers) in each figure. It should be recalled that we expect the wall network to be further randomized on small scales as N is increased, so that the networks of Fig. 1 are not representative of initial string or wall distributions in our Universe.

We point out that there exists strong connections between domain walls, strings, and monopoles in the limit that the fields are freely random walking and the potential can be ignored. A realization of a string distribution may be obtained from the intersections of two independent domain-wall distributions because zeros of a complex field Φ only occur where the two independent real field components are simultaneously zero. We then see that if an initial condition leads to large-scale walls (percolation), analogous conditions are likely to give rise to "infinite" strings. Similarly, monopoles could be described by the intersections of walls drawn from three independent domain-wall distributions or, equivalently, from the intersections of wall and string distributions.

Assuming, for simplicity, that H remains constant and that inflation has occurred long enough for an equilibrium distribution to be realized, we can obtain a rough estimate of the likelihood that we happen to live in a Universe that had the string field close enough to the point of symmetry on the horizon scale so that a scaling solution will develop. To estimate the likelihood P of string formation via this scenario, we merely take the ratio of the Φ-space area relevant for string formation about $\Phi=0$ [$\sim H^2 N_h/\gamma^4$ from Eq. (2.15)] to the typical area that is populated by Φ. The latter ingredient can be estimated by setting $m^2 \sim \lambda\langle\phi^2\rangle$ in (2.6), which implies an area $\sim H^2/\lambda^{1/2}$ (for a more rigorous analysis, see the Appendix). We then find

$$P \approx \lambda^{1/2} N_h/\gamma^4 \; . \tag{2.18}$$

This matches smoothly (at $P \simeq 1$) with Eq. (2.10). For our purposes, P is best thought of as representing the likelihood of universes of size $\sim 10^3$ Mpc that had Φ near the top of the potential on the horizon scale during inflation—and which correspond to universes that lead to structure formation via a string scenario. Of course, it may be questionable to apply (2.18) to our Universe directly as there may be an anthropic selection effect[20] that enhances the likelihood we live in a universe containing strings. However, we presently show that it is not terribly unreasonable, anthropic arguments aside, that we happen to live in a universe that had the string field close to $\Phi=0$ on the horizon scale during inflation.

Although (2.18) looks rather large for reasonable values of λ, we must recall the assumption that $\lambda\eta^4 \lesssim H^4$ (which could easily be relaxed in a more complete analysis—see the Appendix for further details) and constraint (1.1). For ordinary strings the scale $\eta \approx \sqrt{G\mu}\, m_{\rm Pl} \approx 10^{16}$ GeV is required to be useful for structure formation, and we find that Eq. (2.18) can only be valid if $\lambda \lesssim 10^{-5}$, and hence P can be as large as ~ 0.1. This value of λ corresponds to a strongly type-I superconductor, and it may be difficult to reconcile such a small value without supersymmetry. Global strings can form much more naturally than ordinary ones because of the fact that the energy per length of a global string is two orders of magnitude larger than that of ordinary strings (if η is the same) as a consequence of long-range interactions. The global string energy per length is $\mu \simeq \pi\eta^2 \ln(m\Lambda)$, where Λ is a large-scale cutoff comparable to the typical separation of strings, and presumably of order the horizon scale. Therefore, global strings require $\eta \simeq 7\times 10^{14}$ GeV; and $\lambda\eta^4 \lesssim H^4$ and constraint (1.1) implies $\lambda \lesssim 1$. Note that the case $P \gtrsim 1$ just leads to the parameter space (2.11), in which global strings may definitely form well within our present horizon.

Generalizing these results to texture, the likelihood of texture formation is found from taking the ratio of the Φ-space four-volume relevant for texture formation ($\sim N_h^2 H^4/\gamma^8$) to the total four-volume over which Φ is typically populated ($\sim H^4/\lambda$ if $\lambda\eta^4 \lesssim H^4$): $P \sim N_h^2 \lambda/\gamma^8$. The constraints on this scenario imply $\lambda \lesssim 4\times 10^{-4}$, and P can again be of order unity. It should be clear that a theory becomes less favorable as the number of real scalar field components increases (assuming η is the same) because the space occupied within $\sim H$ of the origin becomes an increasingly smaller portion of the available phase space.

We now examine the probability that strings or texture give rise to large-scale structure in a manner that is not described by the usual scaling solutions. The maximum allowed value of N_* for such a scenario to be plausible corresponds to $N_l \lesssim N_h$ (otherwise the string density will significantly drop), and following our previous calculation, the area relevant for a nonscaling scenario of string formation is $\approx N_h H^2/\gamma^2$ and the resulting probability is $P_{\rm nsc} \simeq N_h \lambda^{1/2}/\gamma^2$. The ratio of $P_{\rm nsc}$ to $P_{\rm sc}$ is just $\simeq \gamma^2$.

We briefly return to the parameter range specified by (2.7), i.e., $\lambda\eta^4 \gtrsim H^4$. Taking $\lambda\eta^2 \lesssim H^2$, one can use the results of the Appendix to see that the long-time distribution of fields favors the minima of the potential over the top by an exponential factor $\exp(2\pi^2\lambda\eta^4/3H^4)$ and that there is rather strong biasing for a field to be near the minimum of the potential. Although it is a remote possibility, the only way for string or texture formation to occur is if the field happened to be near the top of the potential on scales corresponding to our horizon. However, we briefly discuss it nonetheless because anthropic considerations could, in principle, give this case some significance. The equilibration time scale $t_{\rm eq} \sim H/m^2$ is of great relevance, as a field will quickly wind up in the minimum of the potential if this scale is shorter than an expansion time $\sim H^{-1}$, and strings or texture will not form. (Texture can form if $H \gtrsim \eta$, as previously discussed, but it will be irrelevant for structure formation.) This case corresponds to $\lambda\eta^2 \gtrsim H^2$. In the limit $\lambda\eta^2 \ll H^2$, the relevant field could random walk near the top of the potential for a relatively long time and, depending upon a local sample of the global universe, could lead to a scaling network of strings, no strings, or a structure formation scenario by strings that is not described by a scaling solution. To be more specific, we determine the necessary parameters for a field placed at the top of the scalar potential to random walk for $N_h \approx 60$ e-folds without suffering significant biasing. The change in potential energy from the top of the potential to a distance ϕ becomes comparable to H^4 when $\phi \approx H^2/\sqrt{\lambda}\eta$, and the number of e-folds N_b required to reach this characteristic value is $N_b \simeq (H/\eta)^2/\lambda$. Setting $N_b \gtrsim N_h$, we find that biasing is not a consideration if

$$\lambda\eta^2/H^2 \lesssim 1/N_h \ . \tag{2.19}$$

We summarize some of the results of this section below and in Fig. 2. A field can random walk from the minimum of the potential to the maximum without experiencing a bias due to an increased potential energy if $\lambda\eta^4 \lesssim H^4$ (and $\lambda\eta^2 \lesssim H^2$). And, if $\lambda \gtrsim 1-10^{-2}$, the time

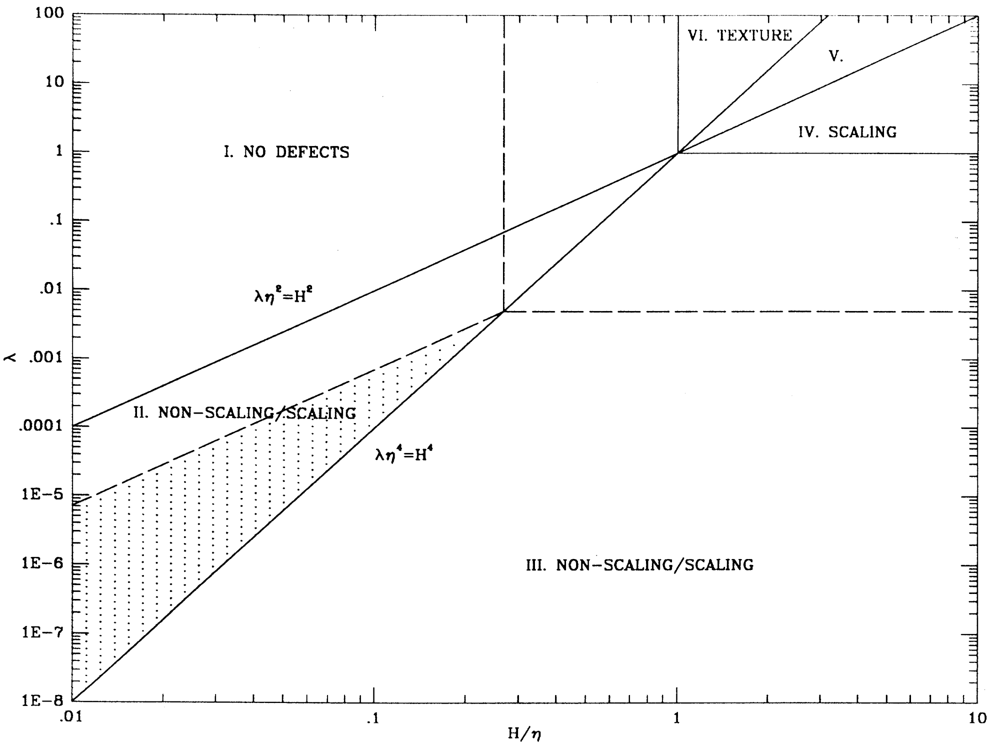

FIG. 2. Schematic diagram showing the types of scenarios that are possible as a function of the parameter space $H/\eta, \lambda$ for a string and/or monopole and/or wall and/or texture theory that is completely decoupled from the inflaton or curvature. In region I a field will quickly be driven (in an expansion time) toward the minimum of the potential, if it is not already there, and it will not be topologically possible to make strings once in the trough—texture-antitexture pairs can be created but the creation rate is too slow to be of interest; II corresponds to a region of parameter space that has an exponentially suppressed likelihood of the field being near the top of the potential, and there is a limited range over which the field can random walk before "falling down" the potential hill—a field at the origin on the horizon scale will not see the effects of the potential during 60 e-folds of inflation for parameter space in the shaded region; III corresponds to either a nonscaling or scaling scenario, occurring with probability $P \sim \lambda^{1/2} N_h$ for strings and $P \sim N_h^2 \lambda$ for texture; in region IV an infinite network of strings, or scaling pattern of texture, can form on scales relevant for structure formation—the dashed boundary applies if H remains nearly constant for $N_h \simeq 60$ e-folds of inflation; V corresponds to a fuzzy region for strings (texture-antitexture pairs can form), interpolating between the properties of regions IV and VI—it can have a large negative m^2 near the top of the potential, which would seem to hamper string formation; VI corresponds to texture-antitexture formation only, which can be extended (dashed line) if H remains constant for $\simeq 60$ e-folds of inflation. Inflationary constraints on H exclude strings or texture from playing a role in structure formation in regions IV (global strings may be allowed, however), V, and VI.

scale for an equilibrium distribution of fields, or restoration of symmetry, is sufficiently fast so that a scaling network may develop within our horizon. If $\lambda\eta^2 \gtrsim H^2$ and $\lambda\eta^4 \lesssim H^4$, string scaling solutions may not be possible for some potential parameters as the time scale to hop over the potential barrier should significantly increase with increasing $\lambda\eta^2/H^2$, but texture-antitexture pairs can be created. Observational constraints on H, for values of η required for a structure formation scenario, exclude the possibility of a scaling scenario for gauge strings or texture, but scaling scenarios for global strings might be possible. If $\lambda \lesssim 1-10^{-2}$ and $\lambda\eta^4 \lesssim H^4$, a string or texture structure formation scenario is possible, and the initial value of the relevant field on the horizon scale differentiates between a scaling scenario (which we are uncertain even exist for this range of parameters) and a nonscaling scenario. If $\lambda\eta^4 \gtrsim H^4$ and $\lambda\eta^2 \lesssim H^2$, strings and texture scenarios can also result, but the required initial conditions are exponentially disfavored.

Thus far, we have not explicitly considered the effects of the gauge field on the formation properties of strings; e.g., fluctuations in A_μ should give an effective-mass term for ϕ. Presumably, a simple estimate of such effects could be obtained by replacing T_c in (2.1) with $H_c/2\pi$, the Hawking temperature. Then strings roughly form when H drops below H_c during the course of the inflationary expansion. Application of constraint (1.1) implies that the characteristic formation scale is within our horizon if $\beta \gtrsim 10^4$, which is outside of the natural range of β, and also corresponds to the strong type-I limit.

Finally, we discuss some differences that can arise in models where H may significantly vary. For definiteness, we consider the inflationary model specified by the potential $V(\Psi)=m_\Psi^2\Psi^2/2$ (which will also be studied in Sec. III when we consider couplings between the inflaton and string and/or texture sectors). If $\lambda\eta^2 \lesssim m_\Psi^2$, we find that $N_l \simeq 1/\sqrt{\lambda}$ (recovering our previous estimate). This was derived from setting $(N_L - N_\Psi)H^2 \simeq H^2/\sqrt{\lambda}$ and finding the smallest value of $N_L(N_l)$ consistent with the condition $N_L - N_\Psi \lesssim N_\Psi$. The latter condition states that H should not significantly change during the approach to equilibrium. (If $\lambda\eta^2 \gtrsim m_\Psi^2$, we find $N_l \simeq \sqrt{\lambda}\eta^2/m_\Psi^2$, which is effectively determined by the smallest N_L consistent with $\lambda\eta^4 \lesssim H^4$.) Variation of H can affect our previous calculations on the likelihood that the string or texture field will be near the top of the potential. The region near the top of the string potential is specified by $\sim H^2$ on the horizon scale, but the dispersion of the field is given by $\delta\phi_l^2 \simeq H_l^2/\sqrt{\lambda}$, so that the probability of finding a string field near the top of the potential is $P \sim \lambda$. The dependence of P on λ is a factor $\sqrt{\lambda}$ larger than the case with H nearly constant. For general power-law inflationary potentials of the form $V(\Psi) \sim \Psi^n$, we find $P \sim \lambda^{(n+2)/4}$, and so we effectively recover the result (2.18) in the limit $n \to 0$.

C. Global strings and/or texture from the inflaton

We now discuss the possibility that the inflaton potential itself has a global U(1) or SU(2) symmetry, which is broken. The symmetries must be global so that radiative corrections associated with a gauge coupling do not destroy the flatness of the inflaton potential and lead to unacceptably large density fluctuations. Naively, one might think it is impossible for the inflating field to give rise to strings or texture as the inflaton is spatially very uniform, but we shall demonstrate that, under certain conditions, it is possible.

We consider the potential $V(\Phi)=\lambda(|\Phi|^2-\eta^2/2)^2$ during a coherent-oscillation stage at the end of inflation and, for the present, leave the form of the potential at higher energies, where inflation occurred, unspecified. We assume that the height of the potential barrier $\lambda\eta^4/4$ is less than the energy scale at the end of inflation, which is presumably very reasonable. If the inflaton were perfectly coherent, it would undergo a large number of oscillations, gradually losing energy through Hubble redshifting (we presently assume that reheating is unimportant for energy densities above the potential barrier), and eventually its energy density would fall below the barrier and the vacuum would be determined.

We now address the possibility that the inflationary density fluctuations produced near the *end* of inflation can sufficiently destroy the coherence of the field oscillations so that different vacua are chosen, in different regions of space, as the energy density drops below the potential barrier. Without loss of generality, we can think of the oscillations of the complex field occurring over its real portion ϕ, i.e., $\Phi=(\phi+i\psi)/\sqrt{2}$, with $\psi=0$, in which case the energy density $\rho=\dot\phi^2/2+V(\phi)$ and pressure $p=\dot\phi^2/2-V(\phi)$. The energy density evolves according to

$$\dot\rho = -3H(\rho+p) = -3H\dot\phi^2 , \qquad (2.20)$$

where we have assumed, for simplicity, that the decay rate Γ of the field is less than the expansion rate H. The time scale of an oscillation is given by the mass scale $t_{\rm osc} \simeq 1/m \simeq 1/\sqrt{\lambda}\eta$, which is much shorter than the expansion time scale $\sim H^{-1}$ in the proximity of the barrier energy density $\lambda\eta^4/4$—which we now focus upon. The change in energy density associated with a $\frac{1}{2}$ cycle of oscillation is

$$\delta\rho_{1/2} \approx -3H\rho t_{\rm osc} \approx \rho\eta/m_{\rm Pl} , \qquad (2.21)$$

where $m_{\rm Pl}$ is the Planck mass and we have used $\rho \approx \lambda\eta^4$ to determine the Hubble parameter. Therefore, if the inflationary density fluctuations satisfy $\delta\rho \gtrsim \delta\rho_{1/2}$, phase coherence will be lost, different horizons should end up in very different vacua, and strings or texture can form. This requirement can be rewritten, using (2.21), as

$$\delta\rho/\rho \gtrsim \eta/m_{\rm Pl} . \qquad (2.22)$$

Since this applies at the end of inflation, the initial scale of a string network will be of microscopic size and the usual scaling solution will ensue. Recall that for global strings $\eta/m_{\rm Pl} \simeq 6\times10^{-5}$ corresponds to $G\mu \simeq 10^{-6}$.

Whether or not the inflationary fluctuations satisfy this criterion near the end of inflation depends upon the precise form of the potential in the regime where inflation occurs and the type of defect being formed [e.g., global strings can satisfy (2.22) significantly easier than texture]. In some of the simplest inflationary models, e.g., $\lambda|\Phi|^4$ (where λ is also the string and/or texture quartic coupling), it is not possible to satisfy (2.22) and simultaneously have an acceptable fluctuation amplitude on large scales. Part of the difficulty is that $\delta\rho/\rho$ decreases in amplitude from large to small scales (the usual expectation[1]). However, the scale of the initial string network need not correspond to the scale at the end of inflation to be useful for structure formation, which may relax the constraint (2.22). It is also conceivable that the inflationary piece of the potential can lead to fluctuation spectra that either remain approximately Zel'dovich or even fall off with an increase in scale[9]—in which case (2.22) may be satisfied in addition to having inconsequential inflationary fluctuations on large scales.

III. FORMATION OF STRINGS AND TEXTURE VIA COUPLING TO THE INFLATON OR CURVATURE

Perhaps the most well-studied theories for achieving compatibility of strings and/or texture with inflation involve some sort of coupling between the two sectors. It has recently been pointed out[21,22] that such scenarios may actually save the cosmic-string scenario from running into problems with the millisecond pulsar constraints. (Significant evasion of the constraints may also occur in some of the models of Sec. II.) We will first briefly review the current status of these constraints[23] and also explain how the constraints can be evaded with inflation via coupling of the string field to the inflaton or to the curvature. In the following subsections, we clarify how either scaling or fractal networks of strings can form and the constraints on model parameters that can arise in such scenarios.

A. Millisecond pulsar constraints

Last year, a preliminary limit from millisecond pulsar timing $\Omega_g h^2 < 4\times10^{-8}$ on the density of gravitational radiation in units of critical density was used[24,25] to deduce a very restrictive limit on the string energy per unit length: $\mu < 10^{-7}/G$. This is probably an order of magnitude too small to permit sufficient structure formation and, if valid, would probably rule out the standard cosmic-string scenario for galaxy formation. More recently, the Taylor group has reported[26] that the present noise level in the millisecond pulsar timing residuals is consistent with $\Omega_g h^2 \lesssim 10^{-7}$, with a 95%-C.L. upper limit of $\Omega_g h^2 < 4\times10^{-7}$ per logarithmic frequency interval near $f=(7y)^{-1}$. This weakening of the preliminary limit by an order of magnitude would weaken the bound on $G\mu$ to about 3×10^{-7}, still bad for the cosmic-string scenario.

But theorists may have rescued it: Recent cosmic string simulations[27,28] find more small-scale structure than the earlier simulations, implying[29] that most of the gravitational radiation comes from very small loops of cosmic string. Bouchet and Bennett[29] assume that the initial loop size α (in units of the horizon size) does not exceed the gravitational radiation back-reaction scale $\Gamma G\mu$ (where $\Gamma\approx50$), from which it follows that $G\mu\approx10^2\Omega_g h^2$, or $G\mu<2\times10^{-5}$ (Ref. 30) using the new millisecond pulsar limit,[26] which is a very weak limit—not nearly as restrictive as the cosmic-microwave-background anisotropy limit $G\mu<5\times10^{-6}$.[31] Although in this case a cosmic-string scenario could still be viable, the scenario has changed: If loop sizes are indeed much smaller than the horizon, then the wakes of long strings, rather than loops, must be the dominant source of structure formation.

The assumption is that $\alpha\lesssim\Gamma G\mu$ is conservative, in the sense that the bound deduced on $G\mu$ is less restrictive than for larger α, but Turok[32] has recently argued that it may not be justified since the numerical simulations[27,28] on which it is based may suffer from a critical flaw: They allow small-scale string structure to be preserved on scales below those on which loops are allowed to be chopped off. It appears that higher-resolution simulations will be required to settle this issue.

If the string network forms during inflation, evasion of the millisecond timing constraint on $G\mu$ is possible. This arises because the string loops (in the old picture) or wakes (in the new picture) that form galaxies or clusters are much larger than those whose redshifted gravitational radiation is today in the frequency range to which the pulsar timing measurements are sensitive, around $f=(7y)^{-1}$. Indeed, unless $\alpha<\Gamma G\mu$, virtually all the gravitational radiation emitted by cosmic strings since matter-radiation equality has periods longer than 100 years today.[29] By coupling the string field Φ to the inflaton Ψ (Refs. 4 and 6) or to the curvature R,[21] it is possible to arrange for the string network to form when Ψ reaches a certain value, corresponding to a particular comoving length scale. Thus—at the cost of introducing this new scale and adjusting it appropriately—the large cosmic strings needed to form galaxies or large-scale structures can be formed without any need for the smaller loops whose gravitational radiation could be detected. Of course, this also evades any constraints from primordial nucleosynthesis on the density of gravitational radiation from cosmic strings. (It is also possible that a little inflation, just from a largest scale l much less than our present horizon scale l_h, could evade these constraints;[22] but such a small amount of inflation would not solve the horizon problem or other cosmological problems.) Global strings emit much less gravitational radiation than gauged strings, and the millisecond pulsar constraints do not significantly constrain the scenario. A model for the formation of global strings during inflation is given in Ref. 33.

B. Models with coupling to the inflaton or curvature

The inflationary potential we consider is

$$V(\Psi)=m_\Psi^2\Psi^2/2\ , \tag{3.1}$$

where Ψ is a real scalar field. We couple this field to the theory described by (1.4) and (1.5) via the interaction

$$V(\Phi,\Psi)=f\Psi^2|\Phi|^2, \tag{3.2}$$

in the spirit of Refs. 4 and 6. Yokoyama[21] considered coupling the string field to the spacetime curvature scalar $R \simeq 12H^2$ through a term

$$V(\Phi,\Psi)=\xi R\,|\Phi|^2\ , \tag{3.3}$$

and took $\xi=\frac{1}{6}$, corresponding to conformal coupling. However, with the potential (3.1) these interactions are of the same form and we shall primarily consider (3.2), although one can always translate to (3.3) through the relation

$$f=16\pi\xi(m_\Psi/m_{\rm Pl})^2\ . \tag{3.4}$$

In order that the inflationary model (3.1) lead to acceptably small fluctuations[1,34] on the normalization scale, one finds that

$$m_\Psi/m_{\rm Pl}\lesssim 9\times 10^{-7}\ . \tag{3.5}$$

Analogously, to avoid generating large inflationary fluctuations via an induced $\lambda_\Psi \sim f^2$ quartic inflaton coupling will require

$$f\lesssim m_\Psi/m_{\rm Pl}\lesssim 10^{-6}\ . \tag{3.6}$$

Assuming that the string or texture sector has little effect on the inflationary expansion implies

$$\lambda\eta^4\lesssim m_\Psi^2 m_{\rm Pl}^2\ . \tag{3.7}$$

The effective mass of the string or texture field will be of great importance. Expanding the string potential near the origin $\Phi=0$, dropping terms of order $|\Phi/\eta|^4$, and expressing Φ in terms of the components $\Phi=(\phi_1+i\phi_2)/\sqrt{2}$, the motion of the fields ϕ_1,ϕ_2 is effectively described by the Lagrangian

$$L=\tfrac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1+\tfrac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 +\tfrac{1}{2}(\lambda\eta^2-f\Psi^2)\phi_1^2+\tfrac{1}{2}(\lambda\eta^2-f\Psi^2)\phi_2^2\ .$$

So, to this order, we have two decoupled scalar fields with effective mass squared $m_{\rm eff}^2\simeq-\lambda\eta^2+f\Psi^2$; Φ-dependent corrections to the mass become important for $|\Phi|\gtrsim m_{\rm eff}/\lambda^{1/2}$. In what follows we shall always mean the effective mass near $\Phi=0$ when we refer to $m_{\rm eff}$.

We now consider separately further constraints arising in the standard scaling models and in alternative models that may lead to significantly different descriptions of structure formation.[6]

C. Scaling models

Central to the usual picture of string formation is the assumption that the field Φ is sufficiently near the minimum $\Phi=0$ (everywhere in space), before the effective mass squared becomes negative, so that fluctuations effectively randomly pick out vacua. A necessary, but insufficient, requirement for this to occur is that $m_{\rm eff}^2\gtrsim H^2$ is satisfied sometime before symmetry breaking. Using the asymptotic limit (large Ψ) of $m_{\rm eff}^2/H^2$, we find

$$f\gtrsim\frac{4\pi}{3}(m_\Psi/m_{\rm Pl})^2\ . \tag{3.8}$$

Translating the above constraint to the interaction (3.3), we see that one requires

$$\xi\gtrsim\tfrac{1}{12}\ . \tag{3.9}$$

It is readily verified that this result is independent of the form of the inflaton potential (3.1). Strings or texture form, in the standard picture, when $m_{\rm eff}^2=0$ or $\Psi_0^2=\lambda\eta^2/f$, which corresponds to

$$N_0\equiv 2\pi\Psi_0^2/m_{\rm Pl}^2 \tag{3.10}$$

e-folds before the end of inflation. If this scale is to be within our horizon, we require

$$f\gtrsim 2\pi\lambda\eta^2/N_h m_{\rm Pl}^2\ . \tag{3.11}$$

Therefore, the mass scales $\lambda\eta^2$ and m_Ψ^2 effectively determine which of the constraints, (3.8) or (3.11), is most relevant.

Scaling scenarios are still possible if condition (3.8), when more stringent than (3.11), is not strongly violated. The initial network may not be characterized, however, by a correlation length l corresponding to a Hubble size when $m_{\rm eff}^2\simeq 0$. To consider this case in greater detail, we follow the analysis of the previous section. We estimate l by finding the minimum number of e-folds, N_l, before the end of inflation that typically leads to a restoration of symmetry. Operationally, we take the minimum value of Ψ_L (Ψ_l) such that the dispersion $\delta\phi$ becomes comparable to the equilibration distribution width $\phi_{\rm eq}$, where

$$\phi_{\rm eq}^2\simeq 3H^4/4\pi^2 m_{\rm eff}^2\ , \tag{3.12}$$

from the Appendix, and we shall take $m_{\rm eff}^2=f\Psi^2-\lambda\eta^2$. Dropped from the latter expression is a term $\sim\lambda\phi^2$. Recall that this term was crucial in Sec. II and allowed scaling solutions for $\lambda\gtrsim 10^{-2}$ and $\lambda\eta^4\lesssim H^4$, even when $f=0$. We shall shortly determine conditions that allow our calculations to be self-consistent with the neglect of this term.

The notion of equilibrium, and our use of Eq. (3.12), hinges upon $\delta m_{\rm eff}^2$ and δH^2 between Ψ_L and the scale corresponding to $\delta\phi^2\approx\phi_{\rm eq}^2$ being smaller than $m_{\rm eff}^2$ and H^2, respectively (the former constraint is the most stringent). Then we equate $\delta\phi^2\simeq H^2(N_L-N_\Psi)/4\pi^2$ with Eq. (3.12) and find the smallest value of Ψ_L consistent with $\delta m_{\rm eff}^2/m_{\rm eff}^2\lesssim 1$. We found that

$$N_l/N_0\simeq 1+m_\Psi/m_S\quad\text{for}\quad m_S\gtrsim m_\Psi \tag{3.13}$$

and

$$N_l/N_0\simeq m_\Psi^2/m_S^2\quad\text{for}\quad m_S\lesssim m_\Psi \tag{3.14}$$

(recall that $m_S^2\equiv\lambda\eta^2$). These limits can be recovered from $N_l/N_0\simeq 1+m_\Psi/m_S+(m_\Psi/m_S)^2$, which we will use when convenient.

The scale of the network is similar to the scale set by N_0 (within a few e-folds) if N_l-N_0 is no larger than a few or

$$f\gtrsim(m_S/m_\Psi)(m_\Psi/m_{\rm Pl})^2\quad\text{for}\quad m_S\gtrsim m_\Psi \tag{3.15}$$

and

$$f \gtrsim (m_\Psi/m_{\rm Pl})^2 \quad \text{for } m_S \lesssim m_\Psi . \tag{3.16}$$

These constraints are consistent with our previous constraint (3.8). In terms of the curvature coupling, we find $\xi \gtrsim m_S/m_\Psi$ ($m_S \gtrsim m_\Psi$) and $\xi \gtrsim 1$ ($m_S \lesssim m_\Psi$). For scaling scenarios to be relevant for structure formation, we require $N_l \lesssim N_h$, which leads to the constraint

$$f \gtrsim 2\pi N_h^{-1}[1+m_S/m_\Psi+(m_S/m_\Psi)^2](m_\Psi/m_{\rm Pl})^2 . \tag{3.17}$$

In the limit $m_S \gg m_\Psi$, this reduces to (3.11). Even if this condition is not satisfied, it may still be possible to produce strings or texture via the probabilistic scenario discussed in Sec. II B, now extended to the case with coupling.

There is another way to view some of our results, which we mention for completeness. Naively, the infinite network of strings is set when $m_{\rm eff}^2 \simeq H^2$ (the smallest scale in which the Higgs phases get highly randomized over a Hubble distance in a Hubble time). However, subsequent fluctuations in the field can add small-scale structure to the long strings until the evolution of the field becomes dominated by the potential at $m_{\rm eff}^2 \simeq -H^2$, and the phases become effectively fixed. For reference, this scale corresponds to $N \simeq N_0(1+4\pi m_\Psi^2/3fm_{\rm Pl}^2)^{-1}$. Loops of string can also form in this regime. The duration and location of this interval are potentially important—if it is long and corresponds to scales within our horizon, the scale of the infinite network may differ substantially from the scale given by $m_{\rm eff}^2=0$. A spectrum of loops may also exist on scales significantly smaller than the scale of the infinite network, which may have important consequences for large-scale structure. The difference between the time $t_1=t(m_{\rm eff}^2 \approx H^2)$ and the time when the fluctuations become impotent (or when the potential becomes important), $t_2=t(m_{\rm eff}^2 \approx -H^2)$, can be characterized in terms of the number of e-folds, $\Delta N \equiv \int H\,dt$, between the two events. Using the slow-roll approximation for the evolution of the inflaton, i.e., $3H\dot\Psi+\partial V/\partial\Psi=0$, we find

$$\Delta N \simeq \frac{8\pi}{m_{\rm Pl}^2}\int_{\Psi_2}^{\Psi_1}\frac{V(\Psi)}{V'(\Psi)}d\Psi=\frac{16\pi^2\lambda\eta^2 m_\Psi^2}{3m_{\rm Pl}^4 f^2} . \tag{3.18}$$

This calculation assumes that $f \gtrsim (m_\Psi/m_{\rm Pl})^2$, in which case it is possible for $m_{\rm eff}^2$ to exceed H^2. Otherwise, ΔN corresponding to $|m_{\rm eff}^2| \lesssim H^2$ is formally infinite. So, if $\Delta N \lesssim 1$, the transition proceeds rapidly, and the initial network of strings can effectively be modeled by randomly assigned phases to inflationary domains of typical size $\sim H^{-1}$ at formation—identical to the procedure that was first used to obtain a rough look at strings formed in a thermal phase transition.[35] It is readily verified that the condition $\Delta N \lesssim 1$, along with $f \gtrsim (m_\Psi/m_{\rm Pl})^2$, reproduces conditions (3.15) and (3.16).

Implicit in our previous analysis is that strings form during inflation, which requires $N_0 \gtrsim 1$ or

$$f \lesssim 2\pi(m_\Psi/m_{\rm Pl})^2(m_S/m_\Psi)^2. \tag{3.19}$$

Comparison with (3.15) and (3.16) indicates that there exists a relatively narrow window in f that allows string formation during inflation. If $m_S \gtrsim m_\Psi$, the ratio of the limits of f, from Eqs. (3.15) and (3.16), is $N_h \simeq 60$, and in the limit $m_S \lesssim m_\Psi$ the range narrows until it disappears altogether when $m_S^2/m_\Psi^2 \simeq 1/N_h$.

We presently determine the conditions that allow our calculation of N_l to be self-consistent with the neglect of the $\lambda\phi^2$ term in the effective mass squared for the string field. The analysis should be self-consistent if the scale of the network in the absence of inflaton-string coupling is larger than the scale N_l. In either limit of m_S/m_Ψ, we find that our analysis is appropriate if

$$f \gtrsim \sqrt{\lambda}(m_\Psi/m_{\rm Pl})^2 \quad \text{or } \xi \gtrsim \sqrt{\lambda} . \tag{3.20}$$

In Fig. 3 we summarize constraints (3.15), (3.17), and (3.19) in terms of a plot of the parameter space characterized by $fm_{\rm Pl}^2/16\pi m_\Psi^2=\xi$ and m_S/m_Ψ.

A large portion of the parameter space in Fig. 3 corresponds to the formation of strings or texture after inflation. Before reheating occurs, the field Ψ can rapidly oscillate on the time scale $\sim m_\Psi^{-1}$, and the effective mass of the string field varies according to $m_{\rm eff}^2=f\Psi^2-\lambda\eta^2$. Strings form when $\langle\Psi^2\rangle$ drops below $\approx\lambda\eta^2/f$. The ratio of the energy densities in the inflaton ($\rho_\Psi \sim m_\Psi^2\langle\Psi^2\rangle$) and string field ($\rho_s \sim \lambda\eta^4$) at the string-forming epoch is just $\rho_\Psi/\rho_s \approx (m_\Psi/\eta)^2/f$. For $\eta \simeq 10^{16}$ GeV we see that this ratio approaches unity only if constraint (3.6) is near saturation.

The constraint (3.11) can be used to constrain λ and, particularly, provides insight to the case of Φ coupled to the spacetime curvature. [Technically, we should use (3.17)–this would have the effect of making our constraints even more stringent in the limit $m_S \lesssim m_\Psi$.] Applying (3.4) and (3.5) to (3.11), we find

$$\lambda \lesssim 3\times10^{-4}\xi(10^{16}\ \text{GeV}/\eta)^2 . \tag{3.21}$$

Using (2.2), (2.3), and (3.21), we see that

$$\beta \gtrsim 2\times10^3 q^{2.48}(G\mu/10^{-6})^{1.24}/\xi^{1.24} . \tag{3.22}$$

Therefore, strings formed during inflation via curvature coupling are expected to be strongly type I unless the coupling ξ is significantly larger than the conformally coupled case. Applying constraint (2.4) to (3.22), for nonsupersymmetric theories, we find that

$$q \lesssim 0.45\xi^{0.27} , \tag{3.23}$$

which is certainly within the realm of possibility if ξ is not made terribly small. Constraints on f are rather unrestrictive and are not particularly illuminating.

D. Nonscaling model

In this subsection we consider the case that $\Psi_0^2=\lambda\eta^2/f$ corresponds to scales well outside of our horizon or

$$f \lesssim 2\pi\lambda\eta^2/N_h . \tag{3.24}$$

As in Sec. II B, there can still be regions, which may encompass our Universe, with plenty of string and/or texture. However, the probabilities that we calculated in

Sec. II B for the string or texture field to be near the top of the potential may be significantly enhanced because the coupling term can help to localize the field near the origin. We presently calculate these probabilities and then discuss in greater detail the properties of strings that may arise in nonscaling models.

Assuming that the scale N_l [Eqs. (3.13) and (3.14)] is significantly larger than the scale of our horizon N_h, the field dispersion arising between N_l and N_h is just $\delta\phi_{lh}^2 \simeq m_\Psi^2 N_l^2$. Using (3.13) and (3.14), we find that the dispersion on the scale N_l, $\phi_{\rm eq}^2 \simeq H_l^4/m_{\rm Pl}^2 f N_l$, is similar to $\delta\phi_{lh}^2$ for $m_\Psi \gtrsim m_S$ and is less than $\delta\phi_{lh}^2$ for $m_S \lesssim m_\Psi$. So, in either limit, the probability that the field is within $\sim\sqrt{N_h}H_h$ of the top of the potential on the horizon scale is

$$P \approx N_h H_h^2/\delta\phi_{lh}^2 \approx N_h^2/N_l^2 \ , \qquad (3.25)$$

where $H_h^2 \approx m_\Psi^2 N_h$ is the value of the Hubble parameter on the horizon scale. Analogous to the analysis of Sec. II B, the likelihood P of texture formation is given by the square of (3.25). As discussed in Sec. II B, scaling scenarios may arise if the string and/or texture mean-field value in our Universe is very close to the origin, and (3.25) also gives a rough estimate of this possibility.

Our previous analysis only applies to a specified set of parameters in which the string or texture field does not feel the effects of the potential on scales from the horizon to N_l—otherwise P would be further suppressed. If $\lambda\eta^4 \lesssim H_h^4$, there is no suppression if the fields typically lie within a distance $\phi_*^2 \simeq H_h^2/\sqrt{\lambda}$ from the origin, and if $\lambda\eta^4 \gtrsim H_h^4$ there is no suppression if the fields typically lie within a distance $\phi_*^2 \simeq H_h^4/\lambda\eta^2$ from $\Phi=0$. Recall that in Sec. II B we found suppression if $\lambda\eta^4 \gtrsim H_h^4$; the difference in the case of coupling is that the fields can be sufficiently localized near the origin on the scale N_l so that the effects of the potential are still not felt by the time horizon scales are reached. Requiring $\delta\phi_{lh}^2 \lesssim \phi_*^2$, we find

$$\lambda \lesssim N_h^2/N_l^4 \ , \qquad (3.26)$$

for $\lambda\eta^4 \lesssim H_h^4$, and

$$\lambda\eta^2/m_\Psi^2 \lesssim (N_h/N_l)^2 \ , \qquad (3.27)$$

for $\lambda\eta^4 \gtrsim H_h^4$. [The latter two constraints imply that (3.27) is relevant when $m_\Psi/\eta \lesssim 1/N_l$.] Hence a necessary

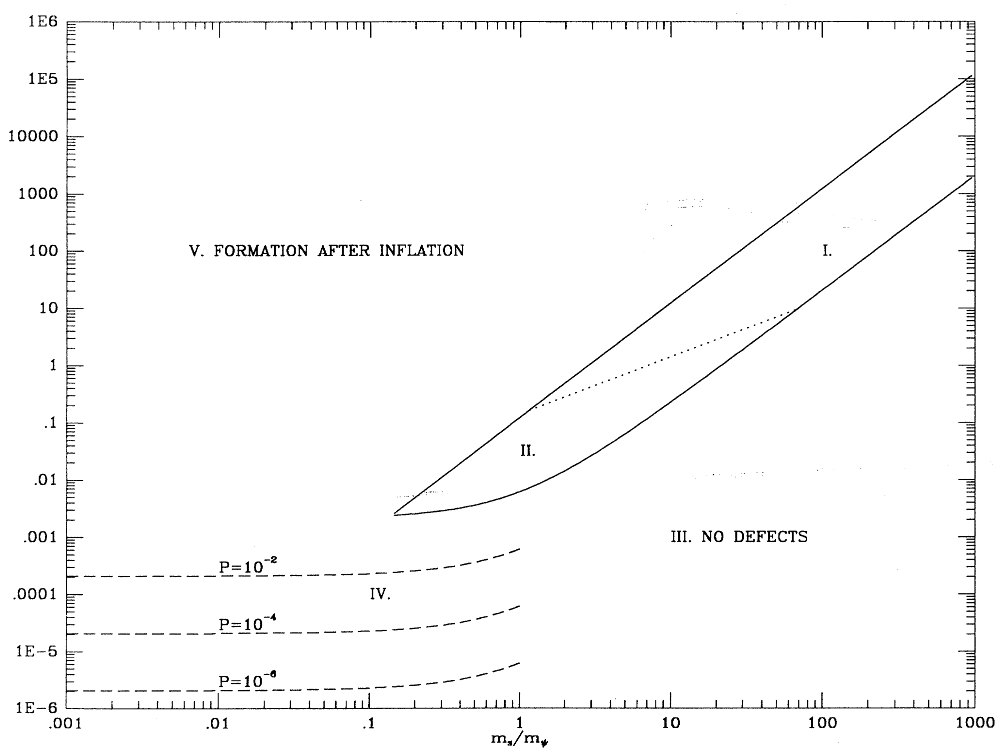

FIG. 3. Plot of the types of scenarios that are possible as a function of the parameter space m_S/m_Ψ, $fm_{\rm Pl}^2/16\pi m_\Psi^2=\xi$. In region I strings or texture form during inflation and on a well-defined scale within our horizon. Region II is similar to region I, except that the duration of the string- or texture-forming transition exceeds a Hubble expansion time. In region III structure formation scenarios are very unlikely. In region IV new structure formation scenarios might be possible, and the probability contours for a string to be near the top of the potential on horizon scales are plotted for $P=10^{-2}$, 10^{-4}, and 10^{-6} (note that additional constraints might be applicable in this parameter space—see the text). The probability contours are valid if $\xi \gtrsim \sqrt{\lambda}$; otherwise, $P\sim\lambda$ (independent of ξ). Region V corresponds to the formation of strings or texture after inflation is over.

condition for a nonstandard scenario ($N_l \gtrsim N_h$) with $\eta \approx 10^{16}$ GeV is that $m_S/m_\Psi \lesssim 1$ or

$$\lambda \lesssim (m_\Psi/\eta)^2 \lesssim 10^{-6} , \tag{3.28}$$

where we have used constraint (3.5). Here λ is sufficiently small so that strings should be strongly type I. We note that the above condition is consistent with Eq. (2.19). For global strings we find $\lambda \lesssim 3\times 10^{-4}$. Because this analysis also assumes that $N_l \lesssim 1/\sqrt{\lambda}$ (i.e., the $\lambda\phi^2$ term in the effective mass can be neglected), we see that $P \gtrsim \lambda N_h^2$ for $m_S \lesssim m_\Psi$. No matter how small the quartic coupling, the probability can always be made relatively large with an appropriate choice of the f coupling constant. In Fig. 3 we have plotted probability contours for string formation using Eqs. (3.13) and (3.14) for N_l.

We presently examine further features of the string or texture distribution that may arise in nonscaling scenarios. The likelihood of string formation occurring in an inflationary domain in one expansion time can easily be calculated numerically for a scenario in which Φ is random walking. In an expansion time, a domain splits up into $\simeq e^3$ domains, each of which acquire independent Gaussian fluctuations of the fields ϕ_1, ϕ_2. Focusing upon a cubic cell that has split into eight independent domains, we examine the probability that a string passes through a face of the cell as a function of the initial field configuration. Because of the U(1) symmetry, only the distance $r \equiv (\phi_1^2+\phi_2^2)^{1/2}(H/2\pi)$ from the point of symmetry, $\Phi=0$, is relevant for our calculation. For convenience, we measure the fields in terms of the width $\sigma = H/2\pi$ of the Gaussian distribution of fluctuations $\delta\phi_1, \delta\phi_2$. Upon specification of r, we then add field fluctuations to the four subcells, which comprise a face of the original cubic inflationary domain, calculate phases, and then determine if there is a net phase change (and hence a string) upon traversing a closed loop. The probability of finding a string through a face as a function of r is plotted in Fig. 4. As expected, the probability drops markedly for $r \gtrsim 1$.

Within an inflationary domain in which Φ is always random walking, the distribution of domains with a given field configuration r can also be easily calculated. If we start with Φ localized near the origin, then after ϕ_1 and ϕ_2 take a total of M random walks of typical size $\sigma = H/2\pi$ we find

$$\frac{dP}{dr} = \frac{r}{M}\exp(-r^2/2M) , \tag{3.29}$$

and hence most of the domains have $r \lesssim \sqrt{M}$. (We have assumed that the Hubble parameter is approximately constant, for simplicity.) This result is easily generalized to the case of T scalar fields ($T=3 \Longrightarrow$ monopoles, $T=4 \Longrightarrow$ texture, . . .):

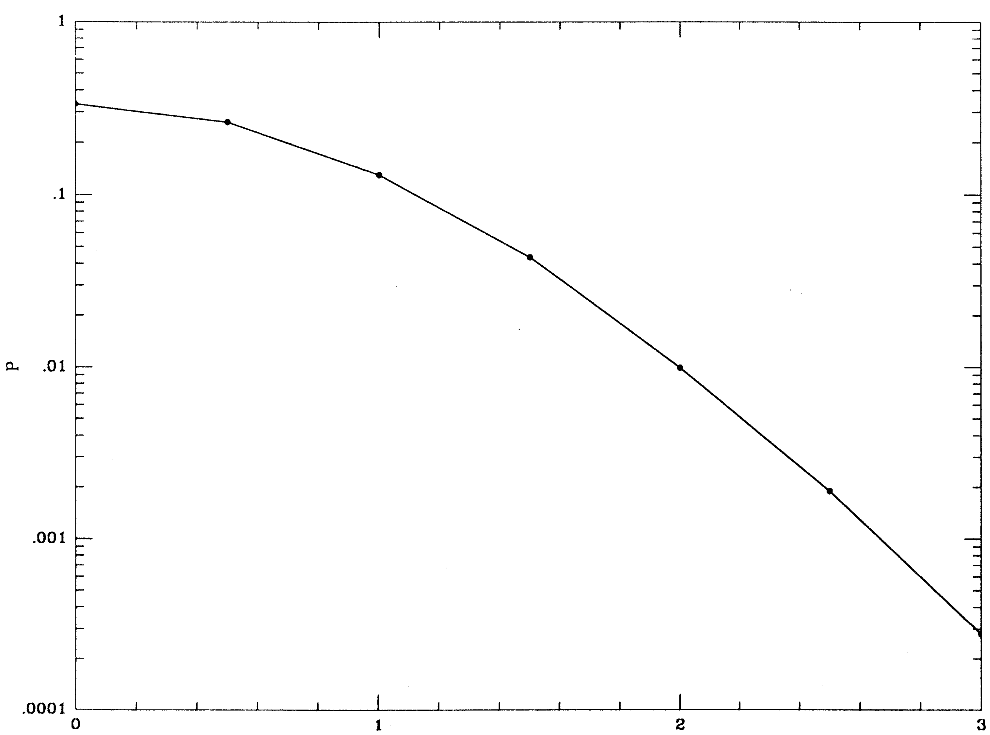

FIG. 4. Likelihood P of string formation occurring in an inflationary domain, in one Hubble expansion, as a function of the initial displacement r of the field Φ from the origin.

$$\frac{dP}{dr} = \frac{2r^{T-1}}{(2M)^{T/2}\Gamma(T/2)} \exp(-r^2/2M) , \quad (3.30)$$

where Γ is the gamma function and r is now generalized to $r^2 = \sum_{i=1}^{T} \phi_i^2/(H/2\pi)^2$. It follows from Fig. 4 that most of the strings that form will come from domains that had $r \lesssim 1$, and the likelihood of encountering such domains is

$$P \simeq \int_0^1 \frac{dP}{dr} dr = 1 - \exp\left[-\frac{1}{2M}\right] \simeq \frac{1}{2M} . \quad (3.31)$$

This generalizes to $P(M,T) \approx M^{-T/2}$. The expected number of string wiggles on a given scale t, within the original $\Phi=0$ domain characterized by a length scale L, is easily found from multiplying $P(M,T)$ by the number of inflationary domains $\simeq e^{3M}$ arising after M steps. Eliminating M via the relation $L/t = e^M$, we find the number density n of string wiggles, or texture or monopoles, formed on scale $\simeq t$, to be

$$n \approx t^{-3}/\ln^{T/2}(L/t) . \quad (3.32)$$

This is nearly a scale-invariant distribution, with only weak dependence on L.

We now extend the previous analysis to the more general case of Φ displaced by a distance r_h from $\Phi=0$ on the horizon scale t_h. Putting the angular dependence back into (3.29), the string forming region subtends an angle $\approx 1/r_h$, and the resulting density of string wiggles formed on scale t is

$$n \approx t^{-3} r_h^{-1} \{ \exp[-(r_h+1)^2/2\ln(t_h/t)] - \exp[-(r_h-1)^2/2\ln(t_h/t)] \} . \quad (3.33)$$

This density reduces to (3.32) if $r_h^2 \lesssim \ln(t_h/t)$ (which corresponds to a time interval that allows the field to random walk from r_h to the origin); otherwise, there is exponential suppression. It is crucial to note that although there may be few or even no pieces of string with curvature $\sim t$ on large scales according to Eq. (3.33), strings formed on smaller scales can connect to form infinite strings. In Sec. II B we found that infinite strings appeared on a scale corresponding to when the field dispersion became comparable to the displacement of the field from the origin on the horizon scale. This suggests that the string distribution may be described by two scales when the field is significantly displaced from the origin on horizon scales.

IV. DISCUSSION

We have seen that the formation of cosmic strings or texture is consistent with inflationary cosmology in a number of different ways. Perhaps the simplest is the possibility of lowering the critical temperature for local string formation by choosing the quartic string field coupling λ somewhat smaller than the squared gauge coupling q^2; as we have pointed out, $\lambda \ll q^2$ corresponds to strongly type-I superconductivity. However, this mechanism only works if there is efficient reheating. Global strings may also form after reheating, without too much difficulty. We have also explored in some detail the formation of topological defects as a result of quantum fluctuations of the associated scalar fields during the inflationary epoch, reviewed further below. With coupling to the inflaton (or coupling to the curvature, which is essentially equivalent for this purpose), an infinite network of strings and/or texture leading to a scaling solution can also arise. Finally, we considered the possibility that the string and/or texture field was also responsible for inflation, with a network of global strings and/or texture arising from decoherence of the inflaton at the end of inflation.

In Sec. II B we examined the quantum creation of strings and texture during inflation, without extra couplings to the inflaton or curvature. For the case of a nearly constant Hubble parameter H, we fully delineated the properties associated with the parameter space λ, η (string and/or texture quartic coupling and vacuum expectation value, respectively), and H. We found that gauge strings or global texture, of relevance for structure formation, cannot be guaranteed to form via this mechanism. However, we found that there was a likelihood $P \sim \sqrt{\lambda}$ (strings) and $\sim \lambda$ (texture) that such a scenario could occur, which might be further enhanced by anthropic considerations. The assumptions behind this scenario require $\lambda \lesssim 3\times10^{-5}$ (gauge strings), $\lambda \lesssim 1$ (global strings), and $\lambda \lesssim 4\times10^{-4}$ (texture). Hence the gauge strings must be strongly type I. Our upper limit on λ is much larger than that suggested in Ref. 6 (although they focused on the case of strings coupled to the inflaton) even though they used a weaker constraint ($\lambda\eta^2 \lesssim H^2$) to demarcate this scenario, because they took an overly stringent value of H. If $\lambda\eta^4 \lesssim H^4$, the characteristic scale of string and/or texture at formation corresponds to $N_l \simeq 1/\sqrt{\lambda}$ e-folds before the end of inflation, and for global strings there exists the possibility that this scale is within our horizon.

In Sec. III we explored the formation of strings and texture in a specific inflationary model $[V(\Psi) = m_\Psi^2\Psi^2/2]$ with specific interactions (either $f\Psi^2|\Phi|^2$ or $\xi R|\Phi|^2$, Φ being the string and/or texture field and R the gravitational curvature), which were shown to be effectively equivalent $[f \Longleftrightarrow 16\pi\xi(m_\Psi/m_{\rm Pl})^2]$. We mapped out the formation properties of strings and texture in the interaction coupling ξ and mass ratio $\sqrt{\lambda}\eta/m_\Psi$ parameter space. If ξ is of order unity, only small quartic couplings are allowed and the strings must be strongly type I. For large ξ or $m_\Psi/m_{\rm Pl} \gtrsim f \gg (m_\Psi/m_{\rm Pl})^2$, type-II strings are possible. If the interaction coupling becomes too small ($\xi \lesssim \sqrt{\lambda}$ if $\lambda\eta^4 \lesssim H^4$), then the mechanism for string and/or texture formation discussed in Sec. II B may be relevant. If $\xi \lesssim 0.002$, the characteristic formation scale of gauge strings or texture lies outside of our horizon. However, the probabilistic scenario is again possible, and the probabilities calculated in Sec. III can be significantly enhanced over the probabilities calculated in Sec. II B because of the effects of the interaction term.

In scenarios where cosmological structure forms solely from strings or texture that arose during inflation, it is necessary to fully suppress the amplitude of the usual inflationary density fluctuations. We have not considered the constraints from this, because they are dependent on the details of how structure forms in such scenarios, which are not yet fully understood. However, it may in fact be desirable to combine effects from inflationary and string and/or texture fluctuations. In one of the most attractive theories of structure formation, the cold-dark-matter scenario, an inflationary Harrison-Zel'dovich spectrum can be normalized to fit a great number of observations on small scales $\lesssim 10$ Mpc, but the predicted large-scale features are not nearly as prominent as the observations would suggest. Adding large-scale power via the formation of strings and/or texture on large-scale structure scales might do the job. One could go even further and speculate that the characteristic scale suggested by the "cosmic picket fence"[36] is directly related to the scale of string and/or texture at formation. For example, a quartic coupling of $\lambda \approx 10^{-2}$ for global strings can translate to a characteristic length scale on large-scale structure scales.

The possibility of new string or texture structure formation scenarios may be worthy of further investigation. Rather small quartic couplings are required, except for global strings. Depending upon how far the strings and/or texture field is displaced from the origin on scales corresponding to the horizon, strings and/or texture may form in a very sparse pattern. At some point there will be inconsistencies with constraints from limits on fluctuations of x-ray sources (see, e.g., Ref. 37) and number counts of galaxies (see, e.g., Ref. 38). We expect that the formalism in Sec. III can be extended so that a relatively straightforward comparison of these data with the inhomogeneity predicted by these models can be made, on the assumption that x-ray sources or galaxies are distributed in roughly the same way as the strings and/or texture.

In many of the cases we have considered, we have found that gauged strings need to be strongly type I in order to be consistent with inflation. In this regime it is not energetically favorable for a winding number $W=2$ string to split into two $W=1$ strings. Indeed, two overlapping $W=1$ strings are bound by an energy per unit length comparable to that of an isolated $W=1$ string.[39] One might well ask if this interaction is sufficient to have an effect on the usual string scenario. What is the probability of string intercommutation for strongly type-I strings? To our knowledge this limit has not been adequately explored in simulations of intercommutation. A numerical difficulty inherent to this limit is that a large grid is required to handle the disparity of length scales associated with the scalar and vector masses. It has been argued,[40] however, that intercommutation is rather insensitive to energetics and is more a question of topology. Another question is whether or not higher winding number $|W| \geq 2$ strings can form, and if so, how do they interact with other strings? It has been found that two $W=1$ strings join into a $W=2$ string in about 1 out of every ~ 5 correlation domains at formation.[41] However, simple energetics seem to indicate that further evolution of the string network would lead to the destruction of such joinings on a time scale corresponding to the correlation length at formation. Simulations of the interaction of strings with different winding numbers, for weakly type-I strings, indicates that higher-winding-number strings tend to peel into lower-winding-number strings.[42] In view of the several cases that we found to be strongly type I, we believe that it would be an interesting exercise to continue this analysis, especially considering the interactions of ordinary $W=1$ strings, well into the type-I regime.

Note added. After this paper was submitted, several recent related works have come to our attention. The possibility of the quantum creation of axionic domain walls is investigated in Ref. 43. In Ref. 44 constraints on string and texture models were explored via inflationary constraints. Lyth found that the string quartic coupling in Yokoyama's model is significantly more constrained and agrees with our Eq. (3.21). Also, we mention that the constraint (1.3) on the reheating temperature used in Sec. II A (taken from Hodges and Blumenthal[9]) is very close to the one presented by Lyth.

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APPENDIX

Here we present further details concerning "typical" values of fields in an inflationary universe. We make the simplifying assumption that the Hubble parameter is fixed. To begin, let us consider the simplest case of a single real scalar field ϕ with a double-well potential $V=\lambda(\phi^2-\eta^2)^2/4$ and decoupled from any other fields. If $|V''| \lesssim 9H^2$ (which we presently assume to be the case), where primes denote derivatives with respect to ϕ, the classical motion of the field is friction dominated. The effects of fluctuations can be included by adding a noise term to the equation of motion, which leads to the slow-roll Langevin equation[45] for ϕ:

$$3H\dot{\phi}+V'=3H^{5/2}\eta(t)/2\pi \ , \qquad (A1)$$

where η is Gaussian distributed noise with $\langle \eta(t) \rangle = 0$ and $\langle \eta(t)\eta(t') \rangle = \delta(t-t')$. The field ϕ is smoothed over a Hubble distance $\sim H^{-1}$. Explicit gradient terms are of higher order, and the spatial distribution of the field is effectively determined through the noise term.

The equilibrium distribution of fields can easily be explored via the associated time-independent Fokker-

Planck equation[46]

$$P''+\frac{8\pi^2}{3H^4}(V'P)'=0\ , \tag{A2}$$

where P is the probability distribution of ϕ. This equation can be directly integrated twice and yields

$$P=C\exp[-8\pi^2V(\phi)/3H^4]\ , \tag{A3}$$

when the appropriate boundary conditions are applied. The constant C is obtained by normalizing the probability to unity. The form of this expression should not be too surprising if one remembers that the effective temperature of de Sitter space is $T=H/2\pi$ and that one expects the energy in a Hubble volume, $\approx V/H^3$, to be an exponentially (Boltzmann) unsuppressed configuration only if $V\lesssim H^4$. Therefore, only in the limit $\lambda\eta^4\lesssim H^4$ can Hubble volumes be populated with ϕ at the top of the potential as often as it is found near one of the minima $|\phi|=\eta$. Hubble volumes with $|\phi|\gtrsim H/\lambda^{1/4}$ will be suppressed in likelihood.

For strings and texture, one obtains analogous results. The Fokker-Planck equation for strings is similar to that used in a description of the statistical properties of laser light,[46] and one finds that the long-time distribution of $|\Phi|^2$, for the potential $V=\lambda(|\Phi|^2-\eta^2/2)^2$, is given by

$$P=C\exp[-8\pi^2\lambda(|\Phi|^2-\eta^2/2)^2/3H^4]\ . \tag{A4}$$

Therefore, the Φ-space area which is typically populated in the limit $\lambda\eta^4\lesssim H^4$ is given by $\approx H^2/\lambda^{1/2}$. One can go beyond the parameter space constrained by $\lambda\eta^4\lesssim H^4$ by using (A4) to calculate directly the likelihood of string formation, and one will find exponential suppression. Such results could be trusted as long as (A1) and (A2) are valid (i.e., while the slow-roll approximation is valid) between the top of the potential and the degenerate vacua, which requires $\lambda\eta^2\lesssim H^2$.

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