Charged black holes in string theory

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A family of solutions to low-energy string theory representing static, spherically symmetric charged black holes is described. They are labeled by their mass, charge, and asymptotic value of the scalar dilaton. The presence of the dilaton is found to have important consequences. In particular, the extremal charged "black holes" are found to be geodesically complete spacetimes with no event horizons and no singularities. Implications of these new solutions for black-hole thermodynamics and open questions in general relativity are also discussed.

Static uncharged black holes in general relativity are described by the well-known Schwarzschild solution. If the mass parameter is large compared to the Planck mass, then this spacetime also describes (to a good approximation) uncharged black holes in string theory, except in the region near the singularity. This is because the classical equation of motion for string theory takes the form of Einstein's equation plus Planck-scale correction terms. As long as the curvature is small compared to the Planck scale, all vacuum solutions of general relativity are approximate solutions of string theory. Near a black-hole singularity, the curvature becomes large and the full string equation is not well approximated by Einstein's equation. However, for a large black hole, the curvature is small in a neighborhood of the horizon and everywhere outside. So this region is indeed an approximate solution.

This is not the case for Einstein-Maxwell solutions. The dilaton in heterotic string theory has a linear coupling to F^2 so every solution with nonzero $F_{\mu\nu}$ must have a nonconstant dilaton. Thus the Reissner-Nordström solution which describes charged black holes in general relativity is not even an approximate solution of string theory. The purpose of this paper is to present the solution for static charged black holes in string theory, valid for curvature below the Planck scale. We will see that the addition of the dilaton dramatically changes certain properties of the black hole. We will also discuss the implications for black-hole thermodynamics and the stability of inner horizons. Previous work on black holes in string theory can be found in Refs. 1–5 and black holes in general relativity with scalar fields are discussed in Refs. 6 and 7.

The four-dimensional low-energy Lagrangian obtained from string theory is

$$S=\int d^4x\sqrt{-g}\,[-R+2(\nabla\phi)^2+e^{-2\phi}F^2]\ , \qquad (1)$$

where $F_{\mu\nu}$ is the Maxwell field associated with a U(1) subgroup of $E_8\times E_8$ or Spin(32)/Z_2 and we have set the remaining gauge fields and antisymmetric tensor field $H_{\mu\nu\rho}$ to zero. Extremizing with respect to the U(1) potential A_μ, ϕ and $g_{\mu\nu}$ yield the field equations

$$\nabla_\mu(e^{-2\phi}F^{\mu\nu})=0\ , \qquad (2)$$

$$\nabla^2\phi+\tfrac{1}{2}e^{-2\phi}F^2=0\ , \qquad (3)$$

$$R_{\mu\nu}=2\nabla_\mu\phi\nabla_\nu\phi+2e^{-2\phi}F_{\mu\rho}F_\nu{}^\rho-\tfrac{1}{2}g_{\mu\nu}e^{-2\phi}F^2\ . \qquad (4)$$

We wish to find static, spherically symmetric solutions to these equations that are asymptotically flat and have a regular horizon. The most general such metric can be written in the form

$$ds^2=-\lambda^2dt^2+\frac{dr^2}{\lambda^2}+R^2d\Omega\ , \qquad (5)$$

where λ and R are functions of r only. This is different from the usual ansatz for a spherically symmetric metric, but turns out to simplify the equations. We first consider a purely magnetic Maxwell field $F=Q\sin\theta\, d\theta\wedge d\varphi$, where Q is the magnetic charge. [Of course a U(1) magnetic charge Q must be an integer multiple of $\frac{1}{2}$ due to the Dirac quantization condition. As described in Refs. 8 and 9, if the gauge field F is embedded in a larger unbroken non-Abelian group G, the topological charge takes values in $\pi_1(G)$ [rather than the $\pi_1(\mathrm{U}(1))=Z$], so not all allowed values of Q correspond to a topologically stable charge. For example, the center of Spin(32)/Z_2 is Z_2, so the topological charge is defined mod 1. Thus black holes with Spin(32)/Z_2 magnetic charge greater than $\frac{1}{2}$ are not prevented topologically from decaying into black holes with smaller magnetic charge, and indeed can decrease their mass by emission of non-Abelian radiation.[8] Since $\pi_1(E_8)$ is trivial, there are no topologically stable black holes for $G=E_8$. Of course, since the area of the event horizon is always nondecreasing,[10] the black hole cannot disappear by this classical radiation, but it can lose its non-Abelian charge.] Then $F^2=2Q^2/R^4$. Since ϕ is also a function of r only, Eq. (2) is automatically satisfied. By symmetry there are only three independent nonzero components of the Ricci tensor. In an orthonormal basis, these are the timelike R_{00}, radial R_{11}, and spherical R_{22} components. From (4) we see that $R_{00}=R_{22}$. This yields an equation for λ and R which is just $(\lambda^2R^2)''=2$ where a prime denotes derivative with respect to r. From (3) and (4) we obtain $R_{00}=-\nabla^2\phi$. This yields an equation for

$Z \equiv \lambda^2 e^{2\phi}$ which is $(\lambda^2 R^2 Z'/Z)'=0$. Solving these two equations together with (4) and imposing our boundary conditions, we obtain a remarkably simple expression for the charged-black-hole solution:

$$ds^2 = -\left[1-\frac{2M}{r}\right]dt^2 + \left[1-\frac{2M}{r}\right]^{-1}dr^2 + r\left[r - \frac{Q^2 e^{2\phi_0}}{M}\right]d\Omega \ , \tag{6}$$

$$e^{-2\phi} = e^{-2\phi_0} - \frac{Q^2}{Mr} \ , \tag{7}$$

$$F = Q \sin\theta \, d\theta \wedge d\varphi \ , \tag{8}$$

where ϕ_0 is the asymptotic constant value of the dilaton. Note that the metric is almost identical to the Schwarzschild metric. The only difference is that the areas of the spheres of constant r and t now depend on Q and are decreased from their Schwarzschild values. In particular, the area goes to zero when $r = Q^2 e^{2\phi_0}/M$ causing this surface to be singular. Since the metric, for fixed θ and φ, is the same as Schwarzschild, $r=2M$ is a regular event horizon.

In addition to their mass M and magnetic charge Q, static asymptotically flat solutions of (2)–(4) are also characterized by a dilaton charge

$$D = \frac{1}{4\pi}\int d^2\Sigma^\mu \nabla_\mu \phi \ , \tag{9}$$

where the integral is over a two-sphere at spatial infinity. For the charged black holes one finds

$$D = -\frac{Q^2 e^{2\phi_0}}{2M} \ . \tag{10}$$

This contrasts with the case where the coupling between ϕ and F in (1) is absent, for which it is known that the scalar charge must vanish.[6] However, D is not a new free parameter in these solutions: once the asymptotic value of ϕ is fixed, it is determined by M and Q, and is always negative. The dilaton charge is also responsible for a long-range, attractive force between black holes.

So far only magnetically charged solutions have been discussed. Electrically charged solutions may be obtained by a duality rotation. Define

$$\tilde{F}_{\mu\nu} = \tfrac{1}{2} e^{-2\phi} \epsilon_{\mu\nu}{}^{\lambda\rho} F_{\lambda\rho} \ . \tag{11}$$

It is easy to verify that the equations of motion (3) and (4) are invariant under $F \to \tilde{F}$ and $\phi \to -\phi$. Furthermore, Eq. (2) ensures that $\tilde{F}$ is curl-free. Electrically charged solutions may therefore be obtained by simply changing the sign of ϕ while keeping the metric fixed. This implies that the dilaton charge is *positive* for electrically charged black holes.

It is an interesting open problem to find solutions with both electric and magnetic charge. In that case, $F \wedge F$ will be nonzero. Since $F \wedge F$ is a source for the axion field strength H in string theory, such solutions will have a nontrivial axion field and carry axion charge. The $F \wedge F$ term in the Lagrangian will also affect the quantization conditions on the electromagnetic charge.[11]

We now compare these solutions to the Reissner-Nordström solutions of the Einstein-Maxwell theory:

$$ds^2 = -\left[1-\frac{2M}{r}+\frac{Q^2}{r^2}\right]dt^2 + \left[1-\frac{2M}{r}+\frac{Q^2}{r^2}\right]^{-1}dr^2 + r^2 d\Omega \ . \tag{12}$$

Both metrics (6) and (12) describe black holes of mass M and charge Q only when Q/M is sufficiently small. For large Q, they both describe naked singularities.

Despite this similarity, there are several significant differences. When the dilaton is present, there is no analog of the inner horizon present in the Reissner-Nordström metric. This is reminiscent of arguments that the inner horizon is unstable, and we will return to this point shortly. Also, the transition between black holes and naked singularities occurs at $Q^2 = 2e^{-2\phi_0}M^2$ in (6) rather than $Q^2 = M^2$ as in the Reissner-Nordström case. (We shall see that this is related to the existence of the dilaton charge.) More importantly, the horizon in (6) becomes singular for the extremal value of Q/M, unlike the case of (12).

For string theory, the statement that the horizon is singular when $Q^2 = 2e^{-2\phi_0}M^2$ is actually irrelevant. This is because strings do not couple to the metric $g_{\mu\nu}$ but rather to $e^{2\phi} g_{\mu\nu}$. This is the metric which appears in the string σ model. In terms of the string metric, the Lagrangian (1) becomes

$$S = \int d^4x \sqrt{-g}\, e^{-2\phi}[-R - 4(\nabla\phi)^2 + F^2] \ , \tag{13}$$

and the charged black-hole metric is

$$ds^2_{\text{string}} = -\frac{(1-2Me^{\phi_0}/\rho)}{(1-Q^2 e^{3\phi_0}/M\rho)}d\tau^2 + \frac{d\rho^2}{(1-2Me^{\phi_0}/\rho)(1-Q^2 e^{3\phi_0}/M\rho)} + \rho^2 d\Omega \ . \tag{14}$$

For $Q^2 < 2e^{-2\phi_0}M^2$, this again describes a black hole with an event horizon at $\rho = 2Me^{\phi_0}$. We have simply rescaled the metric by a conformal factor which is finite everywhere outside (and on) the horizon. However at the extremal value, the metric becomes

$$ds^2_{\text{string}} = -d\tau^2 + (1-2Me^{\phi_0}/\rho)^{-2}d\rho^2 + \rho^2 d\Omega \ . \tag{15}$$

The geometry of a τ=const surface in this spacetime is identical to that of a static slice in the extreme Reissner-Nordström metric. But the horizon, along with the singularity inside it, have completely disappeared. In its place is a bottomless hole. This metric, with $\rho > 2Me^{\phi_0}$, is globally static and geodesically complete. Furthermore, since there is no longer any singularity, the curvature is weak everywhere in this spacetime (for large M), and it is therefore expected to be a good approximation to an exact solution of string theory.

Note that even when $\phi_0 = 0$ there is a factor of $\sqrt{2}$

difference in the extremal value of Q/M for string theory and for the Einstein-Maxwell theory. This can be understood as follows. For the Reissner-Nordström metric, the extremal value corresponds to the case where the gravitational attraction exactly balances the electromagnetic repulsion. In fact one can find exact multi-black-hole solutions in this case.[12] In string theory, the dilaton contributes an extra attractive force, so for a given M, one needs a larger Q to balance the forces between two black holes. Since the electromagnetic force depends on ϕ_0, if $\phi_0 \neq 0$, the forces are balanced at the extremal value $Q^2 = 2e^{-2\phi_0}M^2$, and one can find multi-black-hole solutions in string theory as well. These are most easily described in isotropic coordinates. The dilaton is given by

$$e^{2\phi} = e^{2\phi_0} + \sum_i \frac{2M_i e^{\phi_0}}{|x - x_i|} , \tag{16}$$

where x are Cartesian coordinates on R^3, $M_i(x_i)$ is the mass (location) of the ith extremal black hole, and the charges must all be of the same sign. The charge distribution then uniquely determines the magnetic field. The metric is then simply

$$ds^2_{\text{string}} = -dt^2 + e^{4\phi} dx^2 . \tag{17}$$

This metric is also free of horizons and singularities.

The metric (17) bears a marked similarity to the multi-five-brane configurations of ten-dimensional string theory described in Ref. 13. This was in fact a motivation for the present work, and lends plausibility to the idea that the five-brane solitons of Ref. 13 are simply higher-dimensional extended black holes which can be produced classically via gravitational collapse.

In addition to their importance in string theory, the black-hole solutions (6) are of some interest in general relativity. We remarked earlier that the Reissner-Nordström solution has an inner horizon. Several calculations have shown that nonspherically symmetric perturbations tend to blow up on this horizon indicating that it is unstable.[14] (The outer horizon, i.e., event horizon, is known to be stable.) We consider here a slightly different question: Is the inner horizon stable against small changes in the matter fields in the theory? To investigate this, we consider the action

$$S = \int d^4x \sqrt{-g}\, [-R + 2(\nabla\phi)^2 + e^{-2a\phi}F^2] , \tag{18}$$

where a is an arbitrary parameter governing the strength of the coupling between the dilaton and the Maxwell field. Such Lagrangians arise in string theory with a taking values other than unity if F is a Maxwell field arising in the compactification process, or in the type IIa string. We know that for $a=0$, the static black-hole solutions have an inner horizon, and for $a=1$ (the case considered above) they do not. What do static black holes look like for small a? Following the method outlined above, one can find exact solutions for all a. Fixing $\phi_0=0$, there is again a two-parameter family of solutions. As before, the Maxwell field is given by (8). The scalar field is

$$e^{-2\phi} = \left[1 - \frac{r_-}{r}\right]^{2a/(1+a^2)} , \tag{19}$$

and the metric takes the form of (5) with

$$\lambda^2 = \left[1 - \frac{r_+}{r}\right]\left[1 - \frac{r_-}{r}\right]^{(1-a^2)/(1+a^2)} \tag{20}$$

and

$$R = r\left[1 - \frac{r_-}{r}\right]^{a^2/(1+a^2)} , \tag{21}$$

where r_+, r_- label the two free parameters. They are related to the physical mass and charge by

$$M = \frac{r_+}{2} + \left[\frac{1-a^2}{1+a^2}\right]\frac{r_-}{2} , \tag{22}$$

$$Q = \left[\frac{r_+ r_-}{1+a^2}\right]^{1/2} . \tag{23}$$

When $a=0$ or $a=1$ one can verify that these expressions reduce to Eqs. (12) and (6) (with $\phi_0=0$), respectively. These solutions all have a regular event horizon at $r=r_+$. But the most important feature for our purposes is that for any nonzero value of a, the inner horizon at $r=r_-$ is singular. Moreover, one can show that this singularity is spacelike; i.e., most timelike curves which approach it cannot stay in causal contact. Thus it appears that the inner horizon is not generic, even among static black-hole solutions in general relativity.

We conclude with a puzzle. The Hawking temperature of a Reissner-Nordström black hole is given by[15]

$$T_H = \frac{\sqrt{M^2 - Q^2}}{2\pi(M + \sqrt{M^2 - Q^2})^2} . \tag{24}$$

This expression smoothly goes to zero as the black hole evaporates and $M \to Q$. This is fortunate because, in the Einstein-Maxwell theory, were the black hole to lose any more mass after reaching $Q=M$ a naked singularity would appear. Thus the extreme Reissner-Nordström solutions are stable end points of Hawking evaporation. These observations are in harmony with the fact that the extreme Reissner-Nordström solutions are, in the context of $N=2$ supergravity, supersymmetric.[16] (Supersymmetry further implies[16] the existence of a Bogolmony bound bounding the mass by the charge, and explains the existence of static multi-black-hole solutions.) Generically one expects a supersymmetric configuration to be (perturbatively) quantum-mechanically stable.

In contrast, the Hawking temperature of our solutions (14), as inferred from the periodicity of the Euclidean sections, is simply

$$T_H = \frac{1}{8\pi M e^{\phi_0}} \tag{25}$$

independent of Q, for $\sqrt{2}e^{-\phi_0}M > Q$. When $\sqrt{2}e^{-\phi_0}M = Q$, the Euclidean section is smooth without identifications, so T_H vanishes. Although the Hawking temperature seems to change discontinuously, preliminary calculations indicate that the flux of radiation at infinity smoothly goes to zero as M approaches its critical value. (This is a result of a large effective barrier near the horizon.) This is fortunate since further evaporation would again seem to lead to a naked singularity. However, it is easily seen (from the photino transformation law) that these solutions are *not* supersymmetric configurations of heterotic string theory, and we do not know in general if our extremal solutions minimize the mass for fixed Q and ϕ_0. We are left with something of a mystery: If the extremal solutions are quantum-mechanically stable, what is the underlying reason? If not, what do they decay into?

Note added. After this paper was submitted for publication we were informed that many of our results, including the general solution (14), were previously obtained by G. W. Gibbons [Nucl. Phys. **B207**, 337 (1982)] and G. W. Gibbons and K. Maeda [Nucl. Phys. **B298**, 741 (1988)].

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[1]C. G. Callan, R. C. Myers, and M. J. Perry, Nucl. Phys. **B311**, 673 (1988).
[2]R. C. Myers, Nucl. Phys. **B289**, 701 (1987).
[3]R. C. Myers and M. J. Perry, Ann. Phys. (N.Y.) **172**, 304 (1986).
[4]H. J. de Vega and N. Sanchez, Nucl. Phys. **B309**, 552 (1988).
[5]P. O. Mazur, Gen. Relativ. Gravit. **19**, 1173 (1987).
[6]J. Bekenstein, Phys. Rev. D **5**, 1239 (1972); Ann. Phys. (N.Y.) **91**, 75 (1975); N. Bocharova, K. Bronnikov, and V. Melnikov, Vestn. Mosk. Univ. Fiz. Astron. **6**, 706 (1970).
[7]B. Xanthopoulos (unpublished).
[8]S. Coleman, in *The Unity of the Fundamental Interactions*, edited by A. Zichichi (Plenum, London, 1983).
[9]R. Brandt and F. Neri, Nucl. Phys. **B161**, 253 (1979).
[10]S. Hawking, Phys. Rev. Lett. **26**, 1334 (1971).
[11]E. Witten, Phys. Lett. **86B**, 283 (1979).
[12]A. Papapetrou, Proc. R. Irish Acad. **A51**, 191 (1947); S. D. Majumdar, Phys. Rev. **72**, 930 (1947).
[13]C. G. Callan, J. A. Harvey, and A. Strominger (unpublished).
[14]S. Chandrasekhar and J. Hartle, Proc. R. Soc. London **A384**, 301 (1982), and references therein.
[15]G. W. Gibbons, Commun. Math. Phys. **44**, 245 (1975).
[16]G. W. Gibbons and C. M. Hull, Phys. Lett. **109B**, 190 (1982).