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Wave-front distortion in laser-interferometric gravitational-wave detectors

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(Received 10 December 1990)

We discuss the limitations placed upon the sensitivity of laser-interferometric gravitational-wave detectors by imperfections in the shape or alignment of the optics, for various different detector configurations. Wave-front distortion is seen to be a serious problem for interferometers which recycle only the light power. We suggest that the use of dual recycling can confer greater tolerance of distortion, and therefore better gravitational-wave sensitivity, upon an interferometer. This suggestion is backed up with results from an experimental implementation of dual recycling. However, in its simple form, dual recycling only helps significantly when the signal bandwidth is narrowed. We propose a new optical arrangement, dual recycling with a compound mirror, which gives greater tolerance of distortion without restricting the bandwidth. This may improve the sensitivity of future gravitational-wave detectors by a factor as high as 5, while at the same time improving their operational flexibility.

I. INTRODUCTION

There are several current proposals for the construction of large laser interferometers for the observation of gravitational waves[1–4]. Possible sources include such exotic objects as supernovas, coalescing neutron stars or black holes, pulsars, or even the early stages of the big bang; see Thorne[5] for a recent review. While these sources are extremely energetic, the gravitational radiation has only a weak effect upon terrestrial objects, so very sensitive detectors are required.

If a gravitational wave distorts a region of spacetime, any light passing through this region will experience a change in travel time, or phase. This may be converted to a change in intensity by interference with light of a different history, as indicated in Fig. 1. If this power change at the output of the interferometer is to be detectable, it must be larger than the statistical fluctuations which, at least for unsqueezed vacuum, may be regarded as being due to the finite number of photons observed in the period of measurement. The significance of this photon counting error (or shot noise) is, therefore, reduced if the power level of the light is high. This led Drever[6] to suggest that, with the interferometer operating on a dark fringe, the light effectively reflected back from the interferometer might be recycled so as to increase the circulating power. This is achieved by adding a suitably chosen and controlled mirror, as shown in Fig. 1. In addition, the output power change, hence the possible gravitational-wave sensitivity, may be enhanced even further for continuous signals by ensuring that at least one of the sidebands induced on the light by the gravitational wave is perfectly resonant within the optical system. The original version of this idea of resonant recycling[6] used a rather different optical arrangement. Recently, a more flexible variant using the same basic optical layout of Fig. 1 has been suggested: *dual recycling*[7] adds a mirror at the output port of the interferometer, positioned so that a gravitational-wave-induced sideband

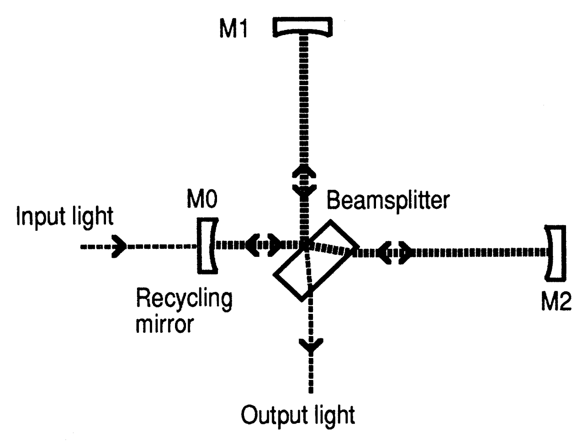

FIG. 1. The optical arrangement of an interferometer using power recycling. Differential phase shifts change the power of the output light. With the interferometer on a dark fringe, most of the light is directed back towards the laser, is caught by mirror M_0 and added coherently to the incoming laser light.

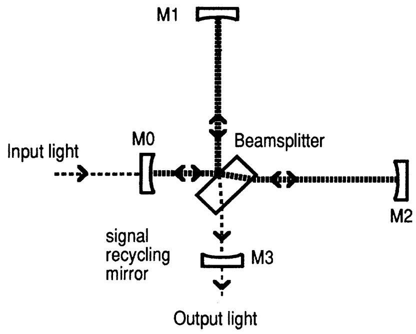

FIG. 2. The optical arrangement for dual recycling. The partially transmitting signal recycling mirror M_3 is placed at the output to resonate signal sidebands induced by a gravitational wave.

is resonant. The optical arrangement is indicated in Fig. 2. The transmission of the signal recycling mirror M_3 determines the signal storage time and therefore the sensitivity-bandwidth combination of the detector.[7,8] If recycling systems are to best improve the sensitivity of interferometric gravitational-wave detectors, the optical losses within the interferometer must be very low. Not only must the absorption and scattering at the mirrors be small, but little light can be allowed to leak out due to imperfect interference at the beam splitter. In this paper we will discuss the significance of poor interference resulting from distortion of the interfering beams and evaluate the requirements that this places upon the quality of the optical components. We will assume that the sensitivity of the detector is limited by photon counting statistics; this should be true for gravitational-wave frequencies above ~ 100 Hz.[1-5] Much of the discussion will be concerned with the less obvious case of dual recycling, in both broadband and narrow-band modes. We will describe an experiment that we have performed that tests our conclusions. We will also suggest a variant of dual recycling that greatly eases the requirements on mirror figure and alignment. This may allow significantly better gravitational-wave sensitivity.

II. DISTORTION IN POWER RECYCLING

If there is imperfect interference between the beams from the two arms of the interferometer, light that would have been directed back towards the laser and recycled will instead leak out towards the photodetector. This has two undesirable consequences: the loss reduces the power buildup; and the extra light hitting the photodiode will increase the level of photon noise. The signal is reduced and the noise is increased.

In order to calculate the reduction in signal-to-noise ratio produced by poor fringe contrast, consider an interferometer such as that one shown in Fig. 1. The action of the gravitational wave may be regarded as being a phase modulation of the circulating light, imposing sidebands that exit the interferometer at the beam splitter. These sidebands are then detected by beating them with some reference beam. This reference beam, or local oscillator, is usually derived from within the interferometer, either by differential phase modulation of the interfering beams (internal modulation) or by splitting off a small fraction of the circulating light, which can then be modulated (external modulation[9,2,10]). The latter system avoids problems with losses and distortions associated with modulators, so is the system that we will explictly consider here (see also Sec.III D). Let us assume that a fraction ϵ of the power is taken out by reflection off a surface (such as the back face of the beam splitter) in one arm of the interferometer. We will be conservative and assume that only one of the reflections is used. Now, if the incident laser power is I_0, then the power circulating within the interferometer is $\mathcal{F}I_0$, where $\mathcal{F}$ is the recycling factor. The observed signal is proportional to the internal field and the reference field (see, e.g., Ref. 8), so

$$\text{signal} \propto \sqrt{\mathcal{F}I_0}\sqrt{\tfrac{1}{2}\epsilon\mathcal{F}I_0}\ . \tag{1}$$

The noise power is proportional to the light power:

$$\text{noise}^2 \propto \mathcal{F}I_0(C+\tfrac{1}{2}\epsilon)\ , \tag{2}$$

where C is the contrast ratio between the power leaking out and that circulating. So the signal-to-noise ratio S/N is given by

$$(S/N)^2 \propto \frac{\frac{1}{2}\epsilon\mathcal{F}I_0}{C+\frac{1}{2}\epsilon}\ . \tag{3}$$

If all the incident power is coupled into the interferometer, the power gain $\mathcal{F}$ is just the reciprocal of the total losses for one round trip of the interferometer:[7]

$$\mathcal{F} = \frac{1}{\epsilon + C + NA^2}\ , \tag{4}$$

NA^2 being the loss associated with N reflections off mirrors of loss coefficient A^2 in the arms of the interferometer. This gives a signal-to-noise ratio of

$$(S/N)^2 \propto \frac{\frac{1}{2}\epsilon I_0}{(C+\frac{1}{2}\epsilon)(\epsilon + C + NA^2)}\ . \tag{5}$$

It is clear that there will be an optimum value for the fraction ϵ of the power taken out for the reference beam: too low a value will allow the noise from the light leaking out to dominate, too high a value will reduce the power buildup. The optimum value is

$$\epsilon_{\rm opt} = \left[2C(C+NA^2)\right]^{1/2}\ . \tag{6}$$

The resultant improvement in signal-to-noise ratio S compared to a nonrecycled system is

$$S_{\rm opt} = \frac{1}{(C+NA^2)^{1/2} + (2C)^{1/2}}\ . \tag{7}$$

(The factor of 2 is absent for an arrangement, such as internal modulation, that uses all of the light reflected out of the interferometer.[11]) This equation tells us that the contrast ratio should be considerably less than the loss NA^2 in the arms of the interferometer if the best sensitivity is to be achieved. It is, perhaps, easier to see this if we express the result in terms of the normalized contrast $C_n = C/NA^2$. The signal-to-noise ratio can then be compared to the maximum value it could have if the contrast was perfect:

$$\frac{(S/N)}{(S/N)_{\max}} = \frac{1}{(C_n+1)^{1/2} + (2C_n)^{1/2}} . \tag{8}$$

This relation is plotted in Fig. 3. Note the slow variation of S/N with contrast. Poor contrast may significantly degrade the sensitivity even when the power leaking out is somewhat smaller than the losses in the arms of the interferometer: it is necessary to have a normalized contrast of $\leq 3\times10^{-2}$ if no more than 20% of the signal-to-noise ratio is to be lost.

We need to evaluate the implications for the mirror figure and alignment. In order to do this, it is helpful to expand the beam in terms of its normal spatial modes. The reasons for doing this are threefold: any distortion of the wave front may be represented as the introduction of higher-order modes, the magnitudes of which remain constant as the beam propagates through free space; modes in one beam only interfere with modes in another if they are of the same order (i.e., modes are orthogonal); and it is relatively easy to calculate how the different modes resonate if they are enclosed in a cavity (such as the signal recycling cavity). We can choose to use a Cartesian coordinate system, in which case the field distribution $E(x,y)$ may be written as

$$E(x,y) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_m h_m(x) A_n h_n(y) . \tag{9}$$

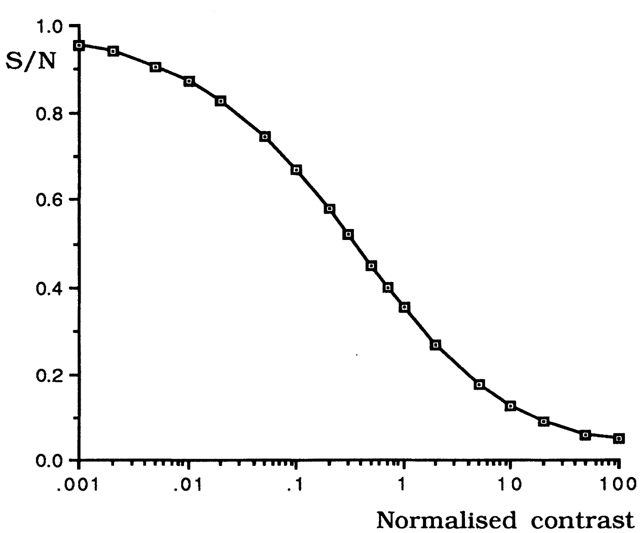

FIG. 3. The signal-to-noise ratio S/N, compared to its maximum value, for different fringe contrasts.

A_m is the amplitude coefficient for the mth mode. The normal modes $h_m(x)$ are Hermite-Gaussian functions for spherical mirrors.[12] If we normalize the coordinate to the beam radius w, so that $X = \sqrt{2}\,x/w$, then

$$h_m(X) = \frac{\Gamma(m/2+1)}{\Gamma(m+1)} H_m(X) e^{-X^2/2} , \tag{10}$$

where $H_m(X)$ is a Hermite polynomial. The gamma functions Γ are just normalization factors.

Now, if we imagine two beams interfering, one of which consists purely of the fundamental mode while the other has higher modes introduced by some distortion, the field emerging from the output port of the beam splitter is

$$E_{\text{out}} = \frac{A_0}{\sqrt{2}} \left(\frac{\Delta A_0}{A_{00}} h_0 + \sum_{m=1}^{\infty} \frac{A_m h_m}{A_{00}} \right) , \tag{11}$$

where A_0 is the incident amplitude, A_{00} is the amplitude in each arm and ΔA_0 is the change in fundamental mode amplitude resulting from the distortion. Note that E_{out} is normalized to the field amplitude incident on the beam splitter. Using the fact that the original distortion conserved energy, we can say that the field leaking out gives a fractional power loss of

$$\frac{\Delta I}{I} = \frac{\Delta A_0}{A_{00}} . \tag{12}$$

So we can calculate the effective loss if we know the mode amplitudes. But, for an arbitrary field distribution $E(X)$, we can use the mode orthogonality to write

$$A_m = \frac{\int_{-\infty}^{\infty} E(X) h_m(X) dX}{\int_{-\infty}^{\infty} h_m h_m^* dX} . \tag{13}$$

The $*$ indicates complex conjugation. This useful formula may also be written as

$$A_m = \frac{1}{\Gamma(m/2+1) 2^m \sqrt{\pi}} \int_{-\infty}^{\infty} E(X) H_m(X) e^{-X^2/2} dX . \tag{14}$$

Note that $H_0(0) = \Gamma(1) = 1$ and that $\Gamma(N+\frac{1}{2}) = (\sqrt{\pi}/2^N)(2N-1)!!$. The corresponding power in the mth-order mode, relative to that in the fundamental, is

$$I_m \equiv g_m A_m^2 = A_m^2 \frac{\int_{-\infty}^{\infty} h_m h_m^* dX}{\int_{-\infty}^{\infty} h_0 h_0^* dX} \tag{15}$$

with the expression for the geometry factor g_m reducing to

$$g_m = \left(\frac{\Gamma(m/2+1)}{\Gamma(m+1)} \right)^2 2^m m! . \tag{16}$$

For example, $g_0 = 1$, $g_1 = \pi/2$, and $g_2 = 2$. These relations allow us to calculate the amplitudes and powers of the various modes corresponding to an arbitrary wavefront distortion.

Let us take an example, that of one arm of the interferometer being misaligned by an angle θ, corresponding to a phase gradient $\phi(x) = 2\pi\theta x/\lambda$ across the beam, λ being the wavelength of the light. Application of Eq. (14) then gives an expression for the resultant mode amplitudes:

$$\frac{A_m}{A_{00}} = \frac{(i\theta/\theta_c)^m e^{-(\theta/\theta_c)^2}}{\Gamma(m/2+1)} , \tag{17}$$

where the characteristic angle $\theta_c = \lambda\sqrt{2}/\pi w$. So most of the additional excitation produced by a small angular misalignment ($\theta \ll \theta_c$) is of the first-order mode:

$$|A_1| \approx \frac{2}{\sqrt{\pi}}(\theta/\theta_c)A_{00} , \tag{18}$$

with the fundamental mode amplitude being reduced to

$$A_0/A_{00} = e^{-(\theta/\theta_c)^2} \approx 1 - (\theta/\theta_c)^2 . \tag{19}$$

The fractional power loss out of the beam splitter is then

$$\Delta I/I = 1 - e^{-(\theta/\theta_c)^2} \approx (\theta/\theta_c)^2 , \tag{20}$$

with the approximation valid for small misalignments. Note that most of the emerging energy is contained in higher-order modes. The power in the fundamental is $\sim (\theta/\theta_c)^2$ times that in the first-order mode. This is a point that we will return to later.

If this power loss is not to degrade the potential sensitivity of the interferometer, it should be considerably smaller than the losses associated with arms of the interferometer. Thus, if an interferometer of length l has N reflections in each arm, with a mirror loss coefficient A^2, the requirements on angular alignment are

$$\theta \ll (2\lambda N A^2/\pi l)^{1/2} \approx 4\times 10^{-7}\left(\frac{N}{30}\right)^{1/2}\left(\frac{l}{3\,\text{km}}\right)^{-1/2}\left(\frac{A^2}{5\times 10^{-5}}\right)^{1/2}\left(\frac{\lambda}{0.5\,\mu\text{m}}\right)^{1/2} \text{rad.} \tag{21}$$

The value for the mirror loss coefficient $A^2 = 5\times 10^{-5}$ is one that is good but not exceptional with current technology. It is the fact that so little power is dissipated in the mirrors that allows the various recycling schemes to enhance the interferometer sensitivity so much; but it also means that very little power can be allowed to be lost by other means if the full potential sensitivity is to be achieved. If we remember that our discussion of the effect of poor contrast suggested that the safety factor by which the inequality (21) is satisfied should be ~ 5 (if we are to lose only 20% of the potential signal to noise ratio), we can see that the interfering beams should be aligned to within an angle of 8×10^{-8} rad. This is quite a tough requirement with suspended mirrors: an automatically aligning servo system with both good sensing and high loop gain will probably be needed.

Two lessons can be drawn immediately from this analysis. First, that the required alignment accuracy is very high and will not be easy to achieve. Second, that the problem is so severe because, although interference gives the beam splitter a very high effective reflectivity for the fundamental mode of the system, the higher-order modes representing any wave-front distortion see a low reflectivity and are therefore able to transport power out of the optical system very easily.

This loss of power via other spatial modes can also be seen when we try to estimate the required mirror figure. If we consider a single spatial Fourier component of a mirror deformation which imposes a phase ripple $\Phi_0 \cos\Omega X$ on the light, Ω being the normalized angular spatial frequency, then Eq. (14) tells us that the amplitude in the mth mode is

$$A_m = \frac{1}{\pi}\int_{-\infty}^{\infty} H_m(X)e^{-X^2}e^{-i\Phi_0\cos\Omega X}\,dX . \tag{22}$$

If we assume that the imposed phase ripple is small ($\Phi_0 \ll 1$), this integral reduces to a standard form. A little algebra then gives the amplitude remaining in the fundamental mode:

$$|A_0| \approx [1 - \tfrac{1}{2}\Phi_0^2(1-e^{-\Omega^2/2})^2]^{1/2} , \tag{23}$$

while the amplitude in higher modes is, for even m,

$$|A_m| \approx \frac{\Phi_0}{2^{m/2+1}m!}(\Omega/\sqrt{2})^m e^{-\Omega^2/2} . \tag{24}$$

So deformations of spatial frequency Ω typically excite modes of order $m \approx \Omega$. The power lost from the system due to this cosinusoidal phase ripple is

$$\Delta I/I \approx \tfrac{1}{4}\Phi_0^2(1-e^{-\Omega^2/2})^2 . \tag{25}$$

Note the dependence on the spatial scale of the deformation ($\lambda_{\text{spatial}} = \sqrt{2}\,\pi w/\Omega$): there is a sharp reduction in the power loss when the spatial wavelength becomes about 2.5 times the beam radius. It is figure errors on length scales of the beam size and less that matter. So big beams require accurate figure on large length scales.

If we take the case of small scale deformation $\Omega \gg 1$, and express Φ_0 in terms of a mirror figure error z_0 ($\Phi_0 = 4\pi z_0/\lambda$), then the fractional power loss is just

$$\Delta I/I \approx (2\pi z_0/\lambda)^2 . \tag{26}$$

This loss of power is particularly serious for an interferometer with delay lines in its arms, for the figure error seen on each bounce will probably be essentially uncorrelated, taking about the same amount of power out of the fundamental mode on each reflection. In this case, the mirror figure required for the power loss to be less than that due to absorption and scattering is just

$$\frac{z_0}{\lambda} \ll \left(\frac{A^2}{4\pi^2}\right)^{1/2} \approx \frac{1}{1000}\left(\frac{A^2}{5\times 10^{-5}}\right)^{1/2} . \tag{27}$$

While current technology can produce mirrors of this accuracy on small scales (<1 mm), it is not yet clear whether it is possible to make mirrors with good enough figure on scales of several centimetres (as would be needed for 1-km-long interferometers). That this may well be a more serious problem than the requirement for beam alignment reflects the fact that it is very difficult to actively control mirror figure. Any relaxation of the requirements on the figure on scales comparable with the beam size would be welcome.

This last result (27) is only correct if the mirror figure errors at each reflection are uncorrelated. If they are not, then the amplitudes generated at each bounce must be added with appropriate phase. In this way it is possible for mirror figure to be somewhat less critical. An example is when a cavity forms the multibounce system: here the beam sees the same shape at each bounce on a particular mirror, the mode couplings are coherent, the result being that a new shape of fundamental mode is formed, with phase fronts following the mirror surfaces. (This point will be considered further when dual recycling is discussed.) The requirement on mirror figure for a cavity is

$$\frac{z_0}{\lambda} \ll \left(\frac{NA^2}{4\pi^2}\right)^{1/2} \approx \frac{1}{150}\left(\frac{N}{30}\right)^{1/2}\left(\frac{A^2}{5\times 10^{-5}}\right)^{1/2} \tag{28}$$

It is straightforward to use the mode formalism to calculate the power lost due to other types of wave-front distortion. For example, a problem of some practical importance is the required curvature accuracy of the mirrors. If the two interfering beams have curvatures $\mathcal{R}_1, \mathcal{R}_2$, the fractional power emerging from the beam splitter will be

$$\frac{\Delta I}{I} = \left[\frac{\pi w^2}{2\lambda}\left(\frac{1}{\mathcal{R}_1} - \frac{1}{\mathcal{R}_2}\right)\right]^2 , \tag{29}$$

or, if the beams have sizes w_1, w_2, then

$$\frac{\Delta I}{I} = \frac{(1 - w_1/w_2)^2}{2[1 + (w_1/w_2)^2]} \approx \left(\frac{\Delta w}{2w}\right)^2 . \tag{30}$$

All of these examples indicate that very high accuracy of mirror curvature, figure and alignment will be needed if the full potential benefits of power recycling are to be obtained. This accuracy is necessary because it is so easy to lose power out of the beam splitter: any distorted light sees a high transmission to the outside world. We would expect this latter property to be modified if a mirror is placed in the output beam, as it is in dual recycling. We shall discuss this in the next section. Before we do so, we should, perhaps, remind the reader that detuned recycling,[13] requiring as it does a very high power buildup in the recycling cavity, is even less tolerant of wave-front distortion than simple power recycling.

III. DISTORTION IN DUAL RECYCLING

If a simple two-mirror cavity is misaligned or its mirrors are slightly distorted, then the normal modes of the cavity will also change in direction, position or shape. Perfectly spherical mirrors are not required for a stable mode to exist,[14] though sufficiently large deviations will induce instability. In such a situation, the requirements on mirror figure and alignment for near-maximal power buildup are much less severe than for power recycling. For example, we have seen that $(\theta/\theta_c)^2 \ll NA^2 \sim 10^{-3}$ is needed in power recycling, whereas $(\theta/\theta_c)^2 \ll 1$ is sufficient for a simple cavity. Roughly speaking, this is because in a simple cavity, any light scattered out of the fundamental mode sees a high reflectivity when it next encounters a mirror (as long it is not deviated by enough to miss the mirror). The deviated light is therefore reflected around the system many times, forming a new mode in the process. This does not happen in power recycling because distorted light is immediately lost, with no opportunity to retraverse the optical system and contribute to a new normal mode. This argument suggests that dual recycling, in which a mirror (M_3 in Fig. 2) is placed at the output of the detector to partially reflect light back in, should be much more like a simple cavity in its behavior. We shall show that this is indeed the case, though with some significant complications.

In order to determine the power buildup inside the recycling system and the power leakage out of it we need to calculate how the light that is directed towards the signal recycling mirror M_3 resonates in the signal recycling cavity (the cavity formed by M_3 and the mirrors in the arms of the interferometer). We also need to know how this distorts the internal mode, and how efficiently this mode couples to both the local oscillator at the output and the input laser beam. Let us first consider only the problem of how much power is lost via the output port. As we saw in the last section, the light emerging from the output port in power recycling consists mainly of higher-order modes resulting from the distortion, with a smaller amount of energy contained in the fundamental mode. The amplitude of the mth mode emerging from the signal recycling mirror M_3 will be modified by how it resonates in the signal recycling cavity, being multiplied by the resonance factor γ_m:

$$|\gamma_m| = \frac{T_{3m}}{(1 - R_{3m}R_{Am})[1 + F'_{sm}\sin^2(\delta_m/2)]^{1/2}} , \tag{31}$$

where T_{3m} and R_{3m} are the amplitude transmission and reflection coefficients of the signal recycling mirror M_3 for the mth mode, R_{Am} is the corresponding reflectivity of the arms of the interferometer, δ_m is the phase offset in the signal recycling cavity for the mth mode, and the signal finesse F_{sm} is given by

$$\frac{4F_{sm}^2}{\pi^2} = F'_{sm} = \frac{4R_{3m}R_{Am}}{(1-R_{3m}R_{Am})^2} \tag{32}$$

The total fractional power loss is then

$$\frac{\Delta I}{I} = \frac{1}{2}\left[\left(\frac{\Delta A_0}{A_{00}}\right)^2\gamma_0^2 + \sum_{m=1}^{\infty}\left(\frac{\Delta A_m}{A_{00}}\right)^2\gamma_m^2 g_m\right], \tag{33}$$

or, if γ_m is approximately the same for all higher modes containing significant power,

$$\frac{\Delta I}{I} \approx \frac{\Delta A_0}{2A_{00}}\left[\frac{\Delta A_0}{A_{00}}\gamma_0^2 + \gamma_m^2\left(2-\frac{\Delta A_0}{A_{00}}\right)\right]. \tag{34}$$

The signal recycling cavity will have a nonconfocal geometry, so all modes other than the fundamental will be off resonance. The power loss in higher-order modes is therefore reduced by a factor $\sim 1/T_{3m}^2$. Since higher modes contain most of the leaking power at low distortion levels, the wasted power will be smaller. As long as the deformations are sufficiently small ($\frac{\Delta A_0}{A_{00}}\gamma_0^2 < 1$), the power loss in dual recycling will be less than in power recycling. The value of γ_0, and hence, the amount of power leaking out in the fundamental will depend on how the detector is operated.

A. Broadband operation

In the broadband mode, the signal recycling cavity is arranged to be resonant at the original laser frequency, with a bandwidth comparable to the observing frequency. The amplitude of any of the fundamental mode leaking out because of distortion is then *enhanced* by the about same factor as the gravitational-wave signal. This means that the advantages of dual recycling are restricted to situations in which the distortion levels are quite low. This is illustrated in Fig. 4, which shows how much the power loss due to a misalignment is reduced in broadband dual recycling, for various different signal recycling mirror transmissions. The power loss reduction factor is the ratio of the fractional power loss with only power recycling to that with dual recycling, at the same distortion level. Remember that the normalized misalignment is θ/θ_c. This calculation, together with all of the others in this section, assumes a 30-bounce, 3-km interferometer. While we will take angular misalignment as our example, similar results should hold for more general distortions. The precise value of the improvement factor with dual recycling will depend upon the geometry of the signal recycling cavity, which determines how the various modes resonate. For this calculation, we have assumed $\sin\delta_1 = 1$. As expected, the resonant suppression of the first-order mode leads to a considerable improvement in the power loss for small misalignments. Sensible values for the transmission T_3 are determined by the requirement for the signal bandwidth to be approximately equal to the gravitational-wave frequency ν_{g0} for which the detector is optimized. This implies[8] that $(F_s/\pi)\sin(\pi\nu_{g0}Nl/c) \approx 1$, with l the length of the interferometer and c the speed of light, giving a transmission

$$T_3^2 \approx 1 - [1 + 2\sin(\pi\nu_{g0}Nl/c)]^{-2}. \tag{35}$$

For example, if $N = 30$, $l = 3\,\mathrm{km}$, and $\nu_{g0} = 150\,\mathrm{Hz}$, then $T_3^2 \approx 0.4$. In this case, the misalignment angle which gives a fractional power loss of 2×10^{-4}, for example, is $0.035\theta_c$, a factor of 2.5 larger than with the same power recycling system. So, while broadband dual recycling does give greater tolerance of distortion, the benefits are limited by the requirement that the bandwidth should not be narrowed too much.

B. Narrow-band operation

It can be seen in Fig. 4 that for large misalignments, high signal finesses give worse power losses. This is because the component of the light at the laser frequency in the fundamental mode is resonantly enhanced. In this situation, it is to be expected that lower power loss will be obtained if the signal recycling cavity is tuned so that it is resonant for one of the gravitational-wave induced sidebands. The original laser frequency will then not be perfectly resonant, but will have a mode amplitude enhancement factor γ_0 given by Eq. (31) with $\delta_0 = 2\pi Nl\nu_{g0}/c$. A higher signal finesse can then reduce the power in all modes at the original frequency. This is illustrated in Fig. 5. The distortion tolerance of narrow-band systems is clearly significantly better than that of broadband systems, especially at high distortion levels. High signal

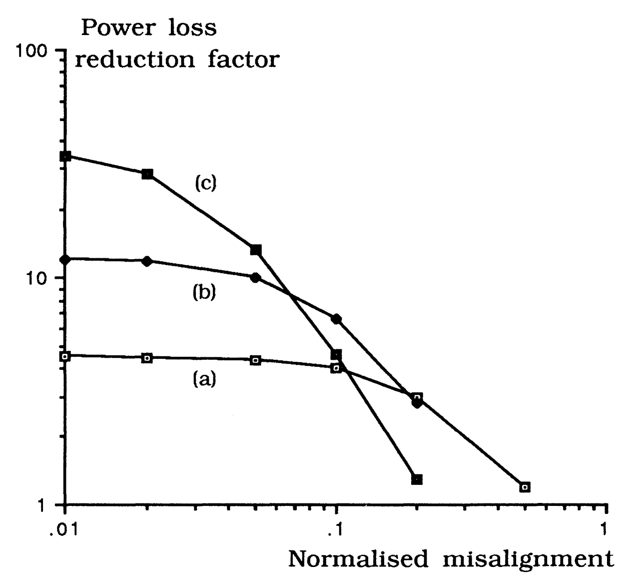

FIG. 4. The reduction of power leakage in broadband dual recycling compared with power recycling: (a) $T_3^2 = 0.5$; (b) $T_3^2 = 0.25$; (c) $T_3^2 = 0.1$. Angles are in units of θ_c.

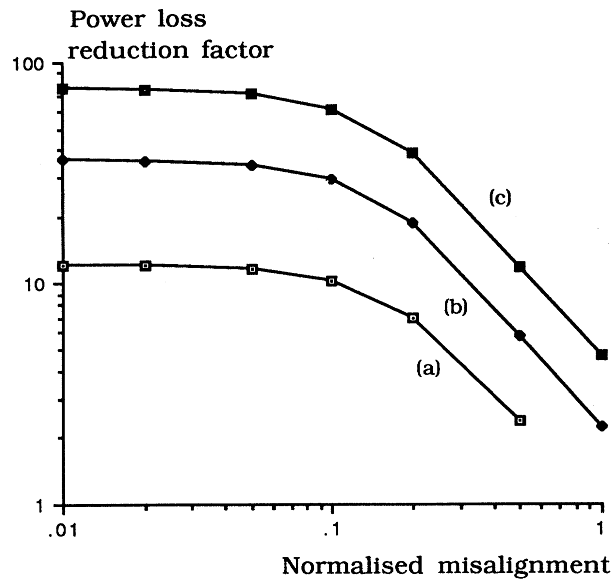

FIG. 5. The reduction factor for power loss in narrow-band dual recycling, tuned to 150 Hz: (a) $T_3^2 = 0.25$; (b) $T_3^2 = 0.1$; (c) $T_3^2 = 0.05$.

finesses now always give lower power loss, the power-loss reduction factor at low distortion levels being $\sim 4/T_3^2$. This improvement factor could easily be $\sim 10^3$ for genuinely narrow-band detectors, corresponding to a relaxation of the requirements for mirror figure by a factor of ~ 30.

In narrow-band mode, a slightly lower value of the transmission T_3^2 of the signal recycling mirror is required to give a particular bandwidth than in broadband mode.[8] This is because the signal sidebands at the frequency at which the detector has been optimized are at the center of the detector tuning curve rather than being down the side of it. This means that a choice of $T_3^2 = 0.1$ will give a bandwidth of $\sim$75 Hz, centered on 150 Hz, for our 3-km, 30-bounce interferometer. Such a system would reduce the power loss due to small deformations by a factor $\sim$30 compared with power recycling. The alignment stability required for a fractional power loss of 2×10^{-4} is relaxed by a factor of $\sim$5. The advantages of such a tuned optical system may well be significant.

Consider the case of a detector with an optimally narrow band, with $T_0^2 = T_3^2 = NA^2$. The fractional power loss produced by a small misalignment θ (each arm being oppositely misligned by $\theta/2$) is then

$$\Delta I/I \approx \tfrac{1}{2}(\theta/\theta_c)^2 NA^2 \,. \tag{36}$$

This will limit the overall power gain $\mathcal{F}$ to

$$\mathcal{F} \approx \frac{1}{NA^2[1+\frac{1}{2}(\theta/\theta_c)^2]} \approx \mathcal{F}_{\max}[1-\tfrac{1}{2}(\theta/\theta_c)^2] \,. \tag{37}$$

But this reduction in power buildup is just the same as that of a simple two-mirror cavity which is misaligned with respect to the input beam by an angle $\theta/2$ [cf. Eq. (19)]. So a dual recycling system with equal high reflectivity mirrors does indeed look like a simple two-mirror cavity for small misalignments.

It is interesting to see why the similarity of behavior of a dual recycling system and a two-mirror cavity breaks down at high distortion levels, or when the fundamental mode is resonant in the signal recycling cavity. In such situations, the deformation significantly changes the level of the fundamental mode. In a simple cavity, the interference of the fundamental mode with the incoming laser beam stops power leaking out and allows power to couple in efficiently. However, in dual recycling the component of the fundamental mode which emerges through the output mirror M_3 has no beam to interfere with, so power is lost relatively easily. In principle, the broken symmetry might be recovered if some of the laser light was injected into the interferometer through M_3, with just the right amplitude and phase to cancel the emerging fundamental mode. The dual recycling system would then be exactly equivalent to a two-mirror cavity.

C. Mode distortion

We have seen that dual recycling does reduce the power loss due to distortion from an interferometer. We still need to consider the formation of new internal modes and how these modes couple to the incoming laser and reference beams.

At this point the reader should be reminded that the argument about the formation of new normal modes is only applicable for distortions which are of large enough scale for the light removed from the fundamental mode to still hit the signal recycling mirror—light which is scattered at high angles is gone for good. However, the scales for which it works are probably those that are of the most practical significance. Nevertheless, it may well be sensible to use recycling mirrors of somewhat larger diameter than might naively be thought necessary.

Just as in a two-mirror cavity, any distortion of the wave front inside the dual recycling system will lead to the normal modes having a different shape. We will not attempt here to rigorously derive the form of the new normal modes. However, several important features can be seen without this. First, if the distortion level within the interferometer is $\sim \lambda/100$, say, the distortion of the mode shape will be of the same order. Also, the result of two beams adding at the beam splitter, with one misaligned by θ, will look like a beam misaligned by $\theta/2$. Similarly, if the curvatures are mismatched by $\Delta\mathcal{R}/\mathcal{R}$, the curvature of the new mode will differ by $\Delta\mathcal{R}/2\mathcal{R}$. More generally, if the beam in one arm of the interferometer is distorted so that the field distribution in that arm is

$$E(X) = \frac{A_0}{A_{00}} h_0(X) + \sum_m \frac{\Delta A_m}{A_{00}} h_m(X) \,, \tag{38}$$

then the shape of the new mode will look roughly like

$$h_0'(X) \approx \left(1 - \frac{\Delta A_0}{A_0}\right) h_0(X) + \sum_m \frac{\Delta A_m}{2A_{00}} h_m(X) , \tag{39}$$

with

$$\left(1 - \frac{\Delta A_0}{A_0}\right)^2 + \sum_m \frac{\Delta A_m^2}{4A_{00}^2} g_m = 1 . \tag{40}$$

This last equation just represents conservation of energy. To see that this gives the correct answer, consider the case of a small misalignment θ in one arm. Only the fundamental and first-order modes have significant amplitude, so

$$\left(1 - \frac{\Delta A_0}{A_0}\right)^2 + \left(\frac{\Delta A_1}{2A_{00}}\right)^2 g_1 = 1 , \tag{41}$$

or

$$\frac{\Delta A_0}{A_0} \approx \frac{1}{2}\left(\frac{\Delta A_1}{2A_{00}}\right)^2 g_1 = \left(\frac{\theta}{2\theta_c}\right)^2 . \tag{42}$$

We have used Eq. (18) to obtain the last result. Now a change in $\Delta A_0/A_0$ at the input must correspond, by Eq. (19), to an angular change θ_i of

$$(\theta_i/\theta_c)^2 = \Delta A_0/A_0 , \tag{43}$$

which means that $\theta_i = \theta/2$. So the angle of the new mode changes by half the angular change of only one arm, as expected.

We can now calculate the fraction of the energy in the incident laser beam that will be coupled into the distorted internal mode. This may be considered obvious, or Eqs. (13) and (15) may be used to confirm that the coupling of a pure fundamental mode is just

$$\frac{I_0'}{I_0} = \left(1 - \frac{\Delta A_0}{A_0}\right)^2 = 1 - \sum_m \frac{\Delta A_m^2}{4A_{00}^2} g_m , \tag{44}$$

where g_m is the geometrical factor defined in Eq. (16). For an angular misalignment θ,

$$\frac{I_0'}{I_0} = 1 - \tfrac{1}{2}(\theta/\theta_c)^2 . \tag{45}$$

It is evident that the coupling efficiency is only reduced significantly when the distortion is quite large. Small deformation levels ($\theta/\theta_c \ll 1$ or phase deviation $\Phi_0 \ll 1$) leave enough overlap of the mode shapes for efficient coupling. In most situations, the major problem will be the loss of power through the output of the interferometer.

The distortion of the internal mode may reduce the efficiency with which the signal is detected. If the output signal is produced by beating the emerging signal sidebands with a reference beam which is derived from the incident laser light, or if the output beam is passed through a mode selector, then the output coupling efficiency is the same as that at the input. On the other hand, if the reference beam is derived from the internal beam and there is no mode cleaner on the output, then there is no additional loss of signal. In addition, there is a minor difference between the characteristics of interferometers with delay lines or cavities in the arms. In a delay line interferometer, the circulating mode is always the one that generates the signal. A cavity interferometer, on the other hand, can have different mode structure in its different cavities. This will depend on the place at which the distortion occurs. It might be thought that this would severely limit the increase in power build up in the arms of a cavity interferometer if the distortion is at the beam splitter, for example. However, this will only be a serious problem if the distortion level is large enough to result in poor overlap between the mode incident upon and the mode inside the cavity. For reasonably small distortion, the enhancement of signal-to-noise ratio due to dual recycling will only be slightly smaller for cavity interferometers than for those using delay lines.

We have seen that the component of the circulating light in the original fundamental mode may be considerably increased by dual recycling, even though the light reflected back by the signal recycling mirror is predominantly in higher-order modes. This paradox may be resolved by realizing that the original modes of the system are *not* normal modes of the new, distorted system. The new modes are not orthogonal to the old modes. The new fundamental mode contains both the original fundamental mode and the modes representing the distortion, thus producing coupling between the original modes which allows power to be transferred back to the original mode. So distorted light reflected back from the signal recycling mirror couples with high efficiency into the new fundamental mode. We will present some experimental evidence for this in Sec. III D, but an explicit calculation of the coupling is also interesting.

The change $\Delta A_0'$ in the amplitude of the new fundamental mode produced by reflecting back distorted light that would otherwise be lost is

$$\Delta A_0' = \frac{\sum_m \int_{-\infty}^{\infty} \Delta A_m h_m(X) \frac{\Delta A_m}{2A_{00}} h_m^* dX}{\int_{-\infty}^{\infty} h_0'(X) h_0'^*(X) dX} . \tag{46}$$

So the distorted light couples back to the fundamental to produce an amplitude enhancement of

$$\frac{\Delta A_0}{A_{00}} = \sum_m \frac{\Delta A_m^2}{2A_{00}^2} g_m , \tag{47}$$

which is an increase in energy of the new mode of

$$\Delta I_0' = 2A_0' \Delta A_0' = \sum_m \Delta A_m^2 g_m . \tag{48}$$

But this is just the energy in the original distortion. So the distorted light does, indeed, couple into the new normal mode efficiently. Not only does dual recycling increase the circulating power, it also increases the useful power.

We have seen that dual recycling does, indeed, give the interferometer a greater tolerance to distortions of the wave front. For tuned, narrow-band systems this improvement factor can be very high. For broadband systems, the improvement factor is still significant, but the requirement that the transmission of the signal recycling mirror must be kept fairly high in order to retain the bandwidth is a limitation. Before discussing a method of improving this situation, we shall descibe an experimental test of some of the ideas that we have been considering.

D. A dual recycling experiment

We have recently completed an experimental demonstration that dual recycling does, indeed, work. The main purpose of this experiment was to test the predictions concerning the enhancement of signal-to-noise ratio, and the functioning of the control systems. These results will be described elsewhere.[15] The experiment also provided an opportunity to test predictions of the tolerance of a dual recycling interferometer to wave-front distortion.

The basic layout of the experiment is indicated in Fig. 6. It consists essentially of a simple, one-bounce Michelson interferometer with two recycling mirrors, each of 10% transmission. The end mirrors had a radius of curvature of 5 m, the recycling mirrors were of curvature 70 cm and the length of the recycling cavities was 59 cm. The rest of the optics was there to make sure that the interferometer functioned correctly, with an external modulation scheme to extract the signal. The light used to sense the length of the signal recycling cavity was frequency shifted in a double-passed acousto-optic modulator by two free spectral ranges of the signal recycling cavity ($\sim 500\,\mathrm{MHz}$): this was done primarily for noise reasons, but alteration of the drive frequency to the acousto-optic modulator was a convenient way of adjusting the tuning of the signal recycling cavity. An argon-ion laser (Spectra-Physics model 165, at 514.5 nm) was used as the light source for the experiment.

One of the mirrors in the arms of the interferometer was mounted on a piezoelectric transducer (Burleigh PZAT 80) that enabled its angle to be changed in a controllable way. This allowed us to study the effects of misalignments. We measured the fractional power loss out of the beam splitter as a function of misalignment angle for both power recycling and dual recycling. The results are shown in Fig. 7. The top set of points represents the observations with only power recycling. It can be seen that, at very small misalignment angles, the fractional power loss was constant at $\sim 4 \times 10^{-4}$. This was probably the result of a distortion of a mirror. The curve that is plotted is a prediction of the power loss assuming both this constant loss and the alignment dependence of Eq. (20). This model seems to be in good agreement with observation. The lower set of points in Fig. 7 represent the observations with broadband dual recycling. The first thing to notice is that they are lower—dual recycling does improve the fringe contrast. The fractional power loss at small misalignments was only $\sim 2 \times 10^{-5}$, a factor of ~ 20 better than with only power recycling. The curve shows the theoretical prediction of this power loss, with the assumption that essentially all of the power leaking out at very small misalignments was in the second-order mode. For our cavity geometry, $\gamma_1 = 0.175$ and $\gamma_2 = 0.236$ (with $\gamma_4 = 0.163$). Once more, the agreement between prediction and observation seems excellent.

We also wanted to test whether the power build up was better within the dual recycling system when the interferometer was misaligned. The circulating power was monitored by measuring the power reflected off the back face of the beam splitter. The maximum power enhancement from recycling was a factor of 32, the difference between this and the value of 37 predicted from the transmission

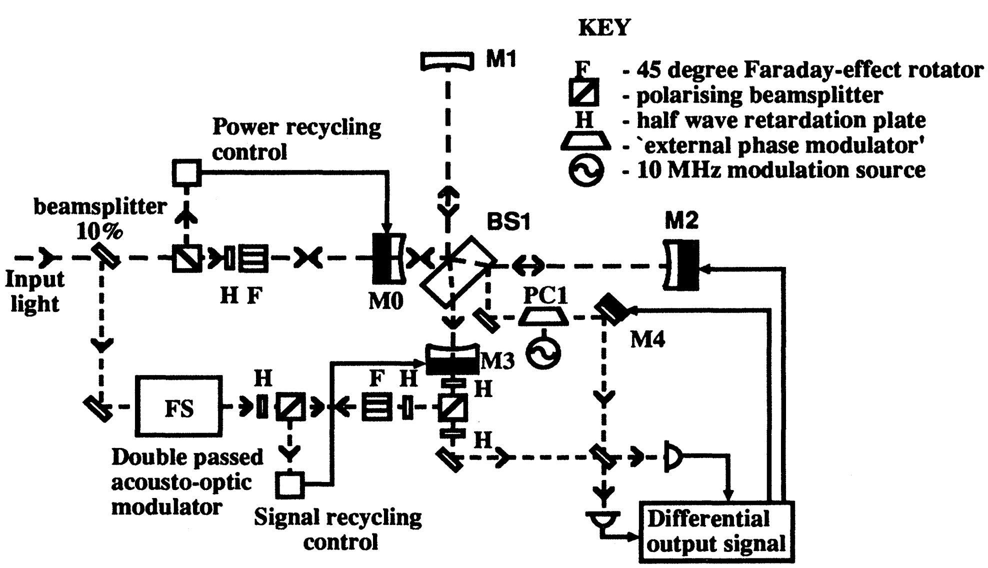

FIG. 6. The optical layout of the dual recycling experiment.

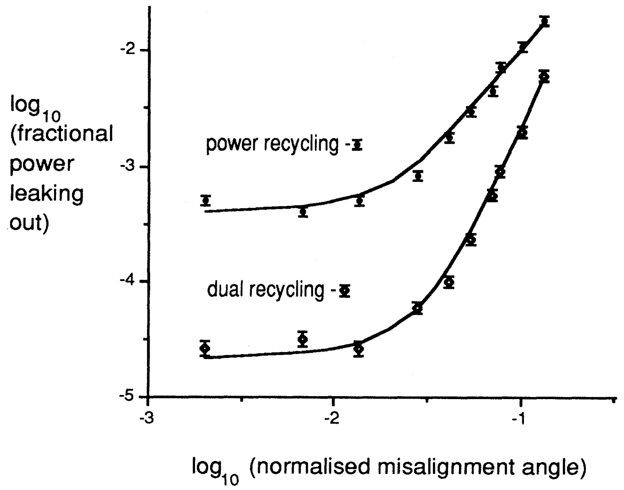

FIG. 7. Fractional power loss for different misalignments, with angles again in units of θ_c. The top curve indicates the results for power recycling. The lower points compare observation and expectation for dual recycling.

of the recycling mirror being due to slightly imperfect mode-matching and the presence of rf phase modulation. This rather modest power gain necessitated quite large misalignments if the effects on the power buildup were to be evident. The results for power recycling alone are shown in Table I. For the dual recycling interferometer, we measured both the power build up and the size of the signal produced by a modulation of the length of one of the arms of the interferometer at 6 kHz. The results of these measurements are summarized in Table II. So dual recycling does increase the power build up within the interferometer, and in a way that is consistent with improvement in the contrast. Furthermore, the *signal* level is also increased. Reflecting light back from the signal recycling mirror really does increase the useful power.

In Fig. 7 it is noticeable how the factor by which dual recycling improves the contrast decreases at high misalignment angles. This is the same falloff that we saw in Fig. 4. We argued earlier that this was due to a larger fraction of the power leaking out at high distortion levels being in the fundamental mode, which is resonant in the signal recycling cavity. To test this idea, we adjusted

TABLE I. The power within the recycled interferometer as a function of the normalised misalignment angle. The power has been scaled so that the maximum observed power equals unity.

Misalignment angle θ_n	Predicted power	Measured power
0.05	0.96	0.95 ± 0.05
0.1	0.84	0.80 ± 0.05
0.2	0.53	0.60 ± 0.05

TABLE II. The effect of relatively large misalignments on the power build-up and signal size with broadband dual recycling. The last three columns have been scaled to a maximum value of unity, as before.

Misalignment θ_n	Expected power or signal	Measured power	Measured signal
0.10	0.93	0.95 ± 0.05	0.90 ± 0.03
0.14	0.89	0.90 ± 0.05	0.85 ± 0.03

the resonant frequency of the signal recycling cavity by altering the drive frequency to the acousto-optic modulator. This reduced the amplitude of light leaking out in the fundamental and so improved the fringe contrast. The results are summarized in Table III. The external amplitude contrast κ is the ratio of the magnitude of the amplitude approaching the beam splitter from M_0 to that emerging from the interferometer. The agreement between the observed effects of detuning the signal recycling cavity and those predicted indicate that our model of resonating modes is a good representation of reality.

Another characteristic of the interferometer that we wanted to check was the coupling of motion of the signal recycling mirror to the output signal. This is important both as a test of our understanding and for the determination of tolerable noise levels. We can imagine a movement of M_3 phase modulating any light hitting it, generating sidebands which resonate in the same way as a genuine signal. Motion of M_3 should, therefore, be less important than motion of the beam splitter by a safety factor

$$s = \eta\alpha \, , \tag{49}$$

where α is the internal contrast (the ratio of the magnitude of the amplitude approaching the beam splitter from M_0 to that leaving the beam splitter towards M_3) and η is a factor which describes the efficiency of coupling of the distorted light incident on the signal recycling mirror into the detected mode. We have argued throughout this paper that this coupling is high, $\eta \approx 1$, and the experimental observations seem entirely consistent with this. The effect of a 3-kHz displacement of M_3 on the output of the interferometer was measured for two values (10 and 28) of the internal contrast (the interferometer being slightly misaligned) and compared with the 6-kHz calibration signal. These results are summarized in Table IV. It can be seen that Eq. (49) describes the results well with $\eta = 1$, within the experimental accuracy of $\sim$20%. So the distorted light reflected back from the signal recycling mirror does indeed merge efficiently with the new fundamental mode of the system.

In this experiment we have tested all of the essential features of our model of how distortion affects an interferometer. The excellent agreement between theory and experiment give us confidence in both our model and in ideas which are stimulated by it. We shall discuss one of these in the next section.

TABLE III. The external amplitude contrast κ and power build up in the misaligned dual recycling interferometer. The power is scaled to a maximum value of unity.

Detuning frequency (MHz)	Misalignment angle	Predicted contrast κ	Measured contrast κ	Power (normalized)
4	0.1	37.7	40 ± 3	1.00 ± 0.05
4	0.2	11.3	10 ± 1	0.80 ± 0.05
6	0.1	44.7	48 ± 3	1.00 ± 0.05
6	0.2	15.1	15 ± 2	0.90 ± 0.05
8	0.2	17.7	21 ± 2	0.95 ± 0.05

E. Dual recycling with a compound mirror

We have seen that most of the energy loss from the interferometer due to reasonably small distortions is contained in higher-order modes of the beam; that the power in these modes is suppressed in a dual recycling system by a factor of $\sim 1/T_{3m}^2$, but that this factor is limited in the usual, broadband dual recycling arrangement by the requirement that T_{30}^2 should not be so small as to reduce the signal bandwidth too much. This suggests that what we want is an optical system which gives a very small transmission for higher-order modes while allowing quite a high transmission for the fundamental mode: *we need a geometry-selective mirror.* A device which satisfies this requirement is a nonconfocal optical cavity. For example, if such a cavity is chosen to have mirrors of equal transmission coefficients which are large compared to the losses, then when the cavity is on resonance for the fundamental mode it has a high transmission $T_{30}^2 \approx 1$ for the fundamental, but a low transmission for higher modes which are off resonance in the cavity. If this cavity were to take the place of the signal recycling mirror, as indicated in Fig. 8, the signal could still be extracted quickly, with high bandwidth, while higher-order modes would be reflected back to contribute to the new internal mode. Note that there is no particular requirement on the length of the extra cavity. Choice of a short ($\sim$1 m) cavity will be convenient (although mode-matching will be required) and the large cavity bandwidth will leave the signal frequency response unaffected.

The amplitude transmission coefficient for the mth mode through the output cavity or compound mirror M_3 is

$$|T_{3m}| = \frac{T_c^2}{(1-R_c^2)[1+F_c'\sin^2(\delta_{cm}/2)]^{1/2}}\ , \tag{50}$$

where T_c and R_c are the amplitude transmission and reflection coefficients, respectively, of the mirrors in M_3 and δ_{cm} is the phase offset for the mth mode in the compound mirror. The fundamental transmission $|T_{30}|$ may be varied from ~ 1 down to quite small values by tuning the operating point δ_{c0} of the cavity. This will alter the sensitivity-bandwidth combination of the detector. Modes of order greater than zero will be well off resonance for all cavity tunings of interest, so their transmission coefficients will be

$$|T_{3m}| \approx T_c^2/2\ , \tag{51}$$

which means that the power emerging in higher-order modes will be suppressed by a factor of

$$\gamma_m^2 \approx (T_c^2/2)^2\ . \tag{52}$$

So even a conservative choice of $T_c^2 = 2 \times 10^{-2}$, which would give a finesse of 150, would reduce the power emerging in higher-order modes by a factor of $\sim 10^4$. This is clearly very good. There is still the power leaking out in the fundamental, but this will usually be a very small fraction of the original power loss. Rewriting Eq. (34), the expression for the power leaking out is

$$\frac{\Delta I}{I} \approx \frac{\Delta A_0}{2A_{00}}\left[\frac{\Delta A_0}{A_{00}}\gamma_0^2 + \left(\frac{T_c^2}{2}\right)^2\left(2-\frac{\Delta A_0}{A_{00}}\right)\right]\ . \tag{53}$$

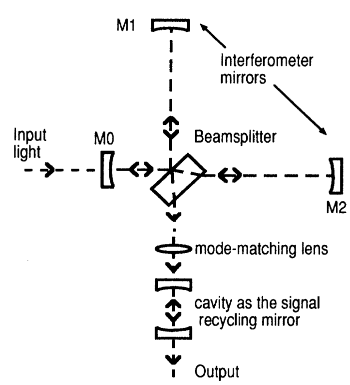

FIG. 8. The optical arrangement for dual recycling with a compound signal recycling mirror.

TABLE IV. The size of the signal produced by motions of one of the interferometer mirrors compared to that produced by movement of the signal recycling mirror. The expected ratio assumes mode coupling efficiency $\eta = 1$, while the measured ratio allows for the different sizes of the motions which were imposed.

External contrast κ	Expected ratio	Observed ratio
30 ± 2	9.5 ± 1	12 ± 2
85 ± 4	28 ± 2	24 ± 4

If we take our example of a small misalignment, the power leaking out, mostly in the fundamental, is only

$$\Delta I/I \approx \tfrac{1}{2}(\theta/\theta_c)^4\gamma_0^2\ , \tag{54}$$

which is considerably better than in the conventional mode of operation of dual recycling. This is illustrated in Figs. 9 and 10, which show the improvement in the power loss over a power recycling system by using dual recycling with a cavity of finesse 150 as the signal recycling mirror. Figure 9 shows the performance for broadband systems with no frequency offset, Fig. 10 assumes tuning of the 3-km, 30-bounce interferometer to a center frequency of 150 Hz. Remember that the normalized misalignment angle is $\theta/\theta_c = \theta\pi w/\sqrt{2}\,\lambda$. These two figures should be compared to Figs. 4 and 5—but notice the compression of the scale. It is clear that significantly better performance is obtained with the cavity as the signal recycling mirror, especially in broadband arrangements.

Some values of the fractional power loss as a function of misalignment angle, for various different optical arrangements, are plotted in Fig. 11. This both quantifies the required alignment accuracy in power recycling and shows how the different types of dual recycling ease this requirement. Equivalent plots for other types of distortion will not differ greatly. It can be seen, for example, that if the fractional power loss resulting from distortion is 1% with no signal recycling, the corresponding loss with a cavity as the output mirror, set to perfectly transmit the fundamental mode, would be less than 10^{-4} Looking back to Fig. 3, we can see that this would increase the signal-to-noise ratio for a detector with losses $NA^2 \sim 10^{-3}$ by a factor of more than 5. Somewhat smaller, but still significant, improvements will be achieved with better initial contrasts. The tolerable distortion is increased by an order of magnitude. Yet this improvement in performance is achieved without any restriction of the frequency response of the detector. If dual recycling with a compound mirror can be implemented in practice, it is evident that it may greatly improve the performance of

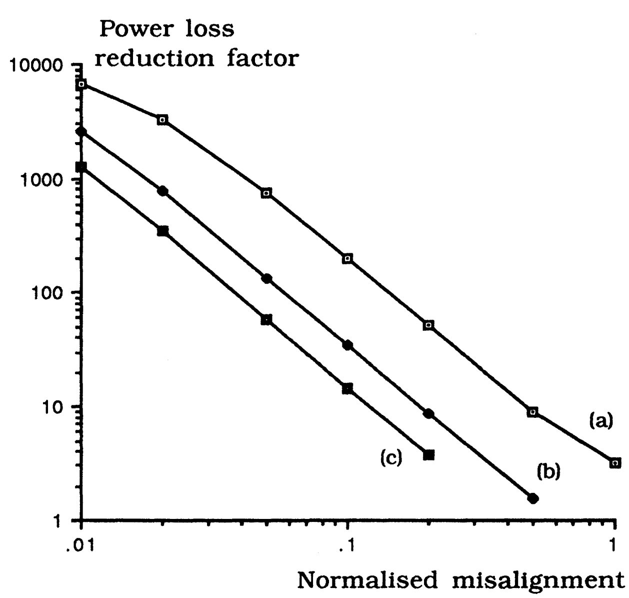

FIG. 9. The improvement of power loss in broadband dual recycling with a compound mirror: (a) $T_{30}^2 = 1$; (b) $T_{30}^2 = 0.5$; (c) $T_{30}^2 = 0.25$.

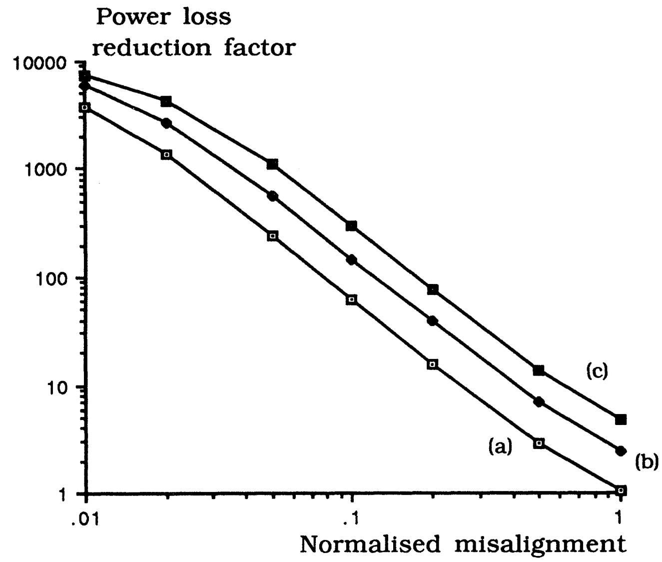

FIG. 10. The improvement of power loss in narrow-band dual recycling with a compound mirror: (a) $T_{30}^2 = 0.25$; (b) $T_{30}^2 = 0.1$; (c) $T_{30}^2 = 0.05$.

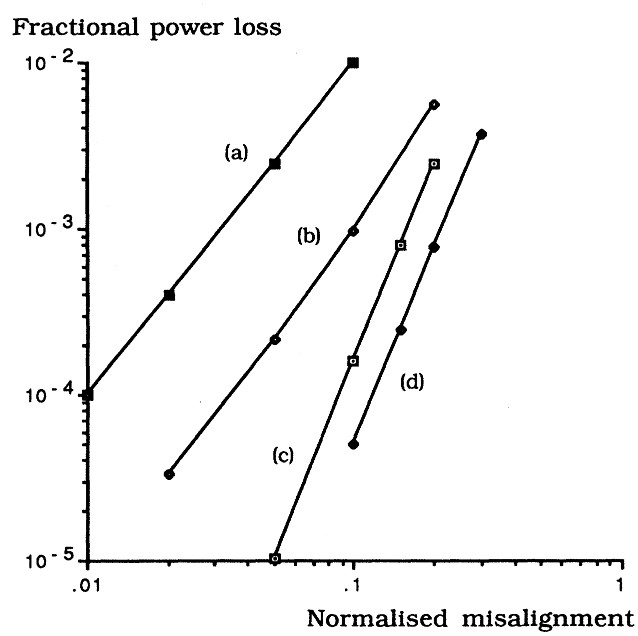

FIG. 11. Power loss due to misalignment for several optical systems: (a) standard recycling; (b) broadband dual recycling with $T_3^2 = 0.25$; (c) dual recycling using a compound mirror with $T_{30}^2 = 0.25$, tuned to 150 Hz; (d) dual recycling using a compound mirror with $T_{30}^2 = 1$. Angles are in units of θ_c.

laser-interferometric gravitational-wave detectors.

The fundamental mode of the output cavity would normally be matched to the original, undistorted fundamental mode of the interferometer. The coupling of the distorted mode to the cavity would then be just the same as the coupling of the incident laser beam. So as long as the initial distortion is small (spatial deviation$\ll \lambda/2$ in a beam radius), this coupling will be high, the signal will be enhanced and the frequency response unaffected.

The requirements for the properties of the output cavity do not seem severe. We have seen that a finesse of about 150 is perfectly adequate which, with good quality mirrors should not introduce significant extra loss. The cavity must be nonconfocal, but there is no particular requirement on its length apart from ease of mode matching. With a length of $\sim$ 1 m, the linewidth would be $\sim$ 1 MHz, much larger than the frequency of the gravitational waves: this means that the two gravitational-wave-induced sidebands will experience the same phase shift on reflection off this cavity, so the frequency response of the system will be the same as if the cavity was a simple mirror of the same reflectivity. The most difficult practical problem associated with using a cavity as the signal recycling mirror currently seems to be the extra complication of controlling the cavity. Its length must be adjusted to give the correct reflectivity (hence bandwidth), while its overall position determines the gravitational-wave frequency that is resonant. These two adjustments are coupled, for a change in cavity length will alter the phase shift on reflection off the cavity, changing the resonant frequency of the signal recycling cavity. We will not discuss details of the control systems here. However, it will probably be necessary to sense the state of resonance of the signal recycling cavity with light containing two frequencies: one which resonates in a similar way to a signal sideband, giving information on the tuning of the system; and one which resonates mainly in the output cavity, giving information on the bandwidth.

The use of a geometry-selective signal recycling mirror may have some other advantages. The amount of light that is scattered off a mirror onto the walls of the vacuum pipe and back to hit the final photodiode should be considerably reduced. Any worries about noise generated by this scattered light[16] should be correspondingly less serious. Furthermore, the improvement in fringe contrast in a nondissipative fashion may increase the factor by which the use of squeezed light can improve the sensitivity.[17]

It is clear that dual recycling with a compound signal-recycling mirror has the potential to give very good performance. In view of the possible importance of this system, it is to be hoped that a direct experimental test of the predictions will be made in the not too distant future. Tests using a numerical simulation are underway at the Australian National University, Canberra, and should produce results quite soon.

F. Conclusion

We have evaluated the mirror alignment and figure required if the full sensitivity of interferometric gravitational-wave detectors is to be attained. It is clear that in power recycling it is very easy for distortions of the beam to allow light to leak out of the interferometer, reducing the power build up, increasing the light hitting the photodetector and so limiting the sensitivity. The requirements on mirror figure and alignment are severe in this case. Lowering the gravitational wave frequency for which the detector is optimized helps somewhat. Mirror figure requirements are relaxed substantially if the dual recycling system is tuned or of narrow band. Truly narrow-band detectors are tolerant of distortion. In addition, we have seen that there is a solution which allows greater distortion without reducing the detector bandwidth—the use of a nonconfocal optical cavity as the signal recycling mirror. This should relax the requirements on mirror alignment and figure by an order of magnitude. The result may well be significantly better gravitational-wave sensitivity, perhaps by a factor as high as 5. Indeed, since the use of a cavity as the signal recycling mirror allows the sensitivity-bandwidth combination of the detector, as well as its tuning, to be varied without physically changing any components, this arrangement should lead to a gravitational-wave detector which is both sensitive and very flexible in its operation.

ACKNOWLEDGMENTS

We would like to gratefully acknowledge the support of the University of Glasgow, the Science and Engineering Research Council and (for B.J.M.) the Royal Society.

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