Limits on mixing angle and mass of Z' using $\Delta\rho$ and atomic parity violation

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We discuss the effect of an extra neutral boson Z' on atomic parity violation. Using the limit on $\Delta\rho$ from the Collider Detector at Fermilab and CERN LEP experiments and limits on ΔQ_W from the experiment of Noecker, Masterson, and Wieman on atomic parity violation, we obtain limits on the mixing angle of Z' with the canonical Z and the mass of Z'. Eleven models with an extra Z' have been analyzed. The limits obtained are comparable to the corresponding limits extracted from the Z-decay experiments at LEP.

Any extension of the standard model with a gauge group of rank bigger than four entails one or more extra neutral vector bosons. The lightest of these is generally known as Z' in the literature. The Z' through its mixing with the canonical Z gives rise to various predictable deviations from the standard-model (SM) results. Most of the effects of the extra neutral boson have been studied in great detail;[1-3] many of these studies are exclusively concerned with the effects in high-energy experiments and a few deal with atomic parity violation (APV). The experiment[4] of Noecker, Masterson, and Wieman (NMW) on atomic parity violation in the cesium atom provides the least expensive but very sensitive test of the standard model and any deviation from it. But its recent result has not been exploited phenomenologically to study deviations from the standard model. In this Brief Report we analyze the effect of Z' on APV in detail and its phenomenological consequences.[5] So far all the experimental observations agree very well with the standard-model predictions within the limits of experimental and theoretical uncertainties. Hence, at the moment, one can only put limits on the two parameters, the mixing angle of Z' with the canonical Z and the mass of Z'. Until now all such limits are based on the decay widths of the Z boson.[6,7] We make the observation that by taking the experimental limits on $\Delta\rho$ from the Collider Detector at Fermilab (CDF) and CERN LEP experiments and ΔQ_W from the NMW APV experiment one can put limits on these parameters. We have analyzed eleven models with Z'. In most of the models the limits gotten are comparable with the limits derived from Z decays. These limits depend very little on the uncertainties due to the masses of the top quark and Higgs boson.[8,9] We defer a global analysis of these parameters based on all neutral-current experiments to a later date.

The mixing between the canonical Z and Z' is model dependent, but certain features are generic. Let us call the two neutral vector bosons before mixing as $Z^{(0)}$ and B, the latter corresponding to the extra U(1) symmetry. A generic mass-square matrix takes the form

$$\begin{bmatrix} m^2_{Z^{(0)}} & m^2 \\ m^2 & M^2 \end{bmatrix} . \tag{1}$$

$Z^{(0)}_\mu$ mixes with B_μ giving Z_μ and a new neutral boson Z'_μ:

$$\begin{bmatrix} Z \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\chi & \sin\chi \\ -\sin\chi & \cos\chi \end{bmatrix} \begin{bmatrix} Z^{(0)} \\ B \end{bmatrix} , \tag{2}$$

where $\tan(2\chi) = -2m^2/(M^2 - M^2_{Z^{(0)}})$. Then the neutral-current interaction for any fermion ψ is given by

$$\mathcal{L}_{\rm NC} = -i\bar\psi\gamma^\mu \left[eQA_\mu - \frac{e}{\sin\theta_W\cos\theta_W} Z_\mu \left[(I_3 - Q\sin^2\theta_W)\cos\chi + \frac{\cos\theta_W g_2}{g} X\sin\chi \right] - \frac{e}{\sin\theta_W\cos\theta_W} Z'_\mu \left[-(I_3 - Q\sin^2\theta_W)\sin\chi + \frac{\cos\theta_W g_2}{g} X\cos\chi \right] \right] \psi , \tag{3}$$

where g and g_2 are the coupling constants for $SU(2)_L$ and the extra U(1), respectively; X is the extra U(1) charge of ψ.

Since $\tan\chi$ is expected to be small, we can write the neutral current in the form

$$\mathcal{L}_{\rm NC} = -i\bar\psi\gamma^\mu \left[eQA_\mu - \frac{e}{\sin\theta_W\cos\theta_W} Z_\mu [(I_3 - Q\sin^2\theta_W) + \lambda X] - \frac{e}{\sin\theta_W\cos\theta_W} Z'_\mu \left[\frac{\cos\theta_W g_2}{g} X \right] \right] \psi , \tag{4}$$

where $\lambda = (\cos\theta_W g_2/g)\sin\chi$. Here the X charges of the various fermions and the coupling constant of the extra U(1), g_2, are model dependent. For any given model the X charges are exactly determined, up to a constant multiplicative factor which can be absorbed into the coupling constant. The coupling constant g_2 is related to g and the relation depends on the details of the model such as the original gauge structure and the representations of

TABLE I. X charges of the fermions for the various extra U(1) models. N is the normalization factor.

Model	q_L	u_R	d_R	l_L	e_R	ν_R	$1/N$
$S^{(0)}$	1	4	−2	−3	−6	0	6
$S^{(1)}$	1	−1	3	−3	−1	−5	$\sqrt{40}$
$S^{(2)}$	2	−2	−4	4	−2	0	$\sqrt{160}$
$S^{(3)}$	4	−4	2	−2	−4	−10	$\sqrt{240}$
$S^{(4)}$	0	−2	0	2	2	0	4
$S^{(5)}$	2	4	−2	−4	−8	−2	$\sqrt{96}$
$S^{(6)}$	0	−1	1	0	1	−1	2
$S^{(7)}$	1	1	1	−3	−3	−3	$\sqrt{24}$
$S^{(8)}$	0.29	0.11	−0.05	0.19	−0.16	0	1
$S^{(9)}$	0.25	0	0	0.25	0	0	1
$S^{(10)}$	0.24	−0.11	0.053	0.35	0.16	0	1

Higgs bosons responsible for symmetry breaking. We have tabulated the X charges of the fifteen standard fermions in each family for the eleven models we have chosen for our analysis in Table I. We assume the extra fermions to be heavy.

$S^{(0)}$ is the model recently proposed by us[3] in which the X charges are exactly proportional to the weak hypercharges (Y) for the fifteen standard fermions. $S^{(i)}$ ($i=1,2,3$) come from E_6. They are respectively called χ, I, and η in the literature (see, for example, the review by Hewett and Rizzo[1]). $S^{(i)}$ ($i=4,5$) come from the flipped SU(5)⊗U(1) broken to the standard model with the Higgs fields residing in $(\mathbf{27}+\overline{\mathbf{27}})$- and $\mathbf{78}$-dimensional representations of E_6, respectively. $S^{(6)}$ is the doubly flipped SU(5)⊗U(1)⊗U(1) with the Higgs fields residing in $(\mathbf{27}+\overline{\mathbf{27}})$-dimensional representations of E_6. $S^{(7)}$ has its origin in the Pati-Salam group. $S^{(i)}$ ($i=8,9,10$) refer to the SU(3)⊗U(1) models in which the extra third quark has electric charge $\frac{2}{3}$, $\frac{1}{6}$, and $-\frac{1}{3}$, respectively.[2] The models S^{i} ($i=1,\ldots,7$) have been studied by Brahm and Hall[10] in connection with dark matter. For the models $S^{(i)}$ ($i=0,\ldots,7$) $g_2=0.8cg$ with the factor c being close to unity and for the models $S^{(i)}$ ($i=8,9,10$) $g_2=g/\cos\theta_W$.

The mixing shifts the mass of the canonical Z which leads to a change in the ρ parameter, $\rho=(1+\Delta\rho_M)$. $\Delta\rho_M$ is given by

$$\Delta\rho_M=\left[\frac{1}{\eta}-1\right]\tan^2\chi\ , \tag{5}$$

where $\eta=(m_Z/m_{Z'})^2$. A model-independent limit on $\Delta\rho_M$ can be directly extracted from the recent Collider Detector at Fermilab (CDF),[11] UA2,[12] and CERN LEP (Ref. 13) data. One obtains $\Delta\rho=0.005\pm0.007$ and so $\Delta\rho\le0.019$. In this paper all the limits are gotten with the experimental numbers taken with an error of two standard deviations. Taking into account the radiative corrections due to the top quark one gets[14]

$$\Delta\rho_M=\left[\frac{1}{\eta}-1\right]\tan^2\chi\le0.019-0.002(m_t/80\ \text{GeV})^2\ . \tag{6}$$

In the above bound we have omitted the contribution of the Higgs boson to radiative corrections as it is less by more than 1 order of magnitude compared to that of the top quark. Also, (6) is valid if the extensions of SM have only $SU(2)_L$ doublets and singlets in its scalar content and if one ignores radiative corrections beyond SM. The quantity of importance to APV is Q_W given as

$$Q_W=2[(2Z+N)C_{1u}+(Z+2N)C_{1d}]\ , \tag{7}$$

where $C_{1q}=2g_{1Ae}g_{1Vq}$; Z is the number of protons and N is the number of neutrons in the nucleus of the atom under consideration. It occurs in the low-energy effective parity-violating Hamiltonian as

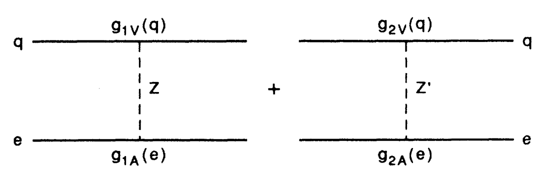
$$H^e_{\rm PV}=(G_\mu/\sqrt{2})Q_W\gamma_5\rho_{\rm nucl} \tag{8}$$

which is obtained from the Feynman diagram of Fig. 1(a) after summing it over all the u and d quarks in the nucleus.

The value of Q_W for the cesium atom in the standard model including all radiative and hadronic corrections (with $\sin^2\theta_W=0.231$) is[8,9]

$$Q_W^{(0)}=-73.1\pm0.15\ . \tag{9}$$

This value of Q_W is essentially independent of masses of the top quark and the Higgs boson. The experimental number is[4,15]



(a) (b)

FIG. 1. Feynman diagrams for the parity-violating interaction for (a) the standard model and (b) the extra contribution due to Z'. In the latter case g_{1A} and g_{1V} get replaced by g_A and g_V, appropriately modified due to mixing given by Eq. (3).

TABLE II. $\Delta Q'_W$ for the various extra U(1) models.

Model	General	$^{133}_{55}$Cs
$S^{(0)}$	$(-0.40N-1.19Z)\eta+(1.26N+1.84Z)\sin\chi$	$-96\eta+200\sin\chi$
$S^{(1)}$	$(0.64N+0.32Z)\eta+(1.20N+0.83Z)\sin\chi$	$67\eta+139\sin\chi$
$S^{(2)}$	$(0.24N+0.12Z)\eta+(0.20N-0.25Z)\sin\chi$	$25\eta+2\sin\chi$
$S^{(3)}$	$(-0.16N-0.08Z)\eta+(1.14B+0.48Z)\sin\chi$	$-17\eta+115\sin\chi$
$S^{(4)}$	$-(0.63N+1.26Z)\sin\chi$	$-118\sin\chi$
$S^{(5)}$	$(-0.40N-0.79Z)\eta+(1.29N+1.50Z)\sin\chi$	$-75\eta+183\sin\chi$
$S^{(6)}$	$(0.40N-0.40Z)\eta-(0.58Z)\sin\chi$	$9\eta-32\sin\chi$
$S^{(7)}$	$(1.54N+1.54Z)\sin\chi$	$205\sin\chi$
$S^{(8)}$	$(-1.22N-1.44Z)\eta+(2.45N+2.02Z)\sin\chi$	$-175\eta+303\sin\chi$
$S^{(9)}$	$(-0.77N-0.77Z)\eta+(2.02N+1.48Z)\sin\chi$	$-102\eta+239\sin\chi$
$S^{(10)}$	$(-0.54N-0.42Z)\eta+(1.82N+1.09Z)\sin\chi$	$-65\eta+202\sin\chi$

$$Q_W^{(\mathrm{expt})}=-71.04\pm1.58\pm0.88 \ . \qquad (10)$$

The last error is due to uncertainty in atomic theory calculations. Thus we get

$$\Delta Q_W=Q_W^{\mathrm{expt}}-Q_W^{(0)}=2.06\pm1.81 \ . \qquad (11)$$

There are four different ways in which the Z' will change the standard-model value for Q_W. (i) The mixing of Z' with Z changes the mass of the canonical Z and hence the effective low-energy interaction through $\rho=(1+\Delta\rho_M)$. (ii) The mixing also changes the standard-model couplings by an amount proportional to $\sin\chi$ as given in Eq. (3). (iii) There will also be contributions to the parity-violating interaction due to the exchange of Z' as in Fig. 1(b), and this is proportional to η. (iv) $\sin^2\theta_W$ extracted from experiment using models with and without Z' differ by an amount proportional to $\Delta\rho_M$. More explicitly

$$(\sin^2\theta_W)_{Z'}=(\sin^2\theta_W)_{\mathrm{SM}}-\Delta\rho_M(\cos^2\theta_W)_{\mathrm{SM}}$$

where subscript Z' (SM) indicates the value with (without) Z'. $(\sin^2\theta_W)_{Z'}$ is the one we have to use to find the deviation from SM. Thus

$$\Delta Q_W=\Delta\rho_M Q_W^{(0)}-4Z(\cos^2\theta_W)_{\mathrm{SM}}\Delta\rho_M+\Delta Q'_W \ . \qquad (12)$$

Here the first and the second terms are due to (i) and (iv), respectively. $\Delta Q'_W$ contains the effects of (ii) and (iii) and is of the form $\Delta Q'_W=n_1\eta+n_2\sin\chi$. We have compiled the values of $\Delta Q'_W$ for the various models in Table II. We have given the results for any general atom and $^{133}_{55}$Cs atom in particular. Taking the limit on $\Delta\rho_M$ with $m_t\geq 89$ GeV and the limit on ΔQ_W we can get the following limits for $\Delta Q'_W$:

$$-1.57\leq\Delta Q'_W\leq 9.68 \ . \qquad (13)$$

We now combine (6) and (13) to obtain limits on the mixing angle and η. We have given a typical plot of the parameter space for the two parameters $\sin\chi$ and η in Fig. 2. This particular one is for the model $S^{(0)}$. The two straight lines are given by $n_1\eta+n_2\sin\chi=\Delta Q'_W$ with $\Delta Q'_W$ equal to the two limits -1.57 and 9.68. The region of parameter space outside the two lines is ruled out. The parabolic (for small χ) curve denotes $\Delta\rho_M=[(1/\eta)-1]\tan^2\chi=0.0165$, which is the bound on $\Delta\rho_M$ for $m_t\geq 89$ GeV. The limit on $\Delta\rho_M$ rules out the parameter space below this curve. From this figure we can immediately read out the limits on $\sin\chi$ and η. We have compiled all these limits for the various models in Table III.

For the models $S^{(4)}$ and $S^{(7)}$, $\Delta Q'_W$ is independent of η and hence we are not able to get any limit on η. But we can get limits on $\sin\chi$ for these two models. For the models $S^{(3)}$ and $S^{(6)}$ the straight line with positive intercept intersects $\Delta\rho_M$ curve only at the lower end and there is no intersection at the upper end. So we get a lower limit for $\sin\chi$ and no limit on η. [For these two models one can use the limits on $\sin\chi$ from LEP data and combining it with (15) get an upper limit on η.[6]] We have also presented in Table III limits on $\sin\chi$ obtained from the decay widths of Z^0 from LEP for comparison.[6]

It is expected that the NMW APV experiment would be four times more precise within the next year.[16] Such high-precision data could yield better limits on the parameters than the corresponding ones from LEP data. We can use such an analysis in combination with the LEP data to further restrict the parameter space and possibly rule out some extra U(1) models. If the APV experi-

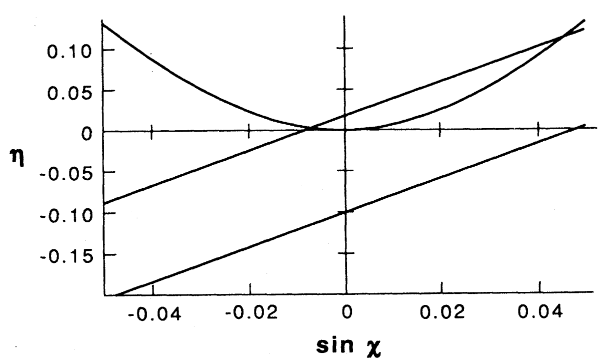


FIG. 2. A typical plot of $\Delta\rho_M=[(1/\eta)-1]\tan^2\chi=0.0165$ and $\Delta Q'_W=n_1\eta+n_2\sin\chi$ with $\Delta Q'_W=-1.57$ and 9.68. The allowed region of the parameter space is the intersection of the region above the "parabolic" curve and the region between the straight lines.

TABLE III. The limits for APV on $\sin\chi$ and η from the limits on $\Delta\rho$ and ΔQ_W for the various extra U(1) models and limits on $\sin\chi$ from Z-decay widths.

Model	APV				Z decays	
	$(\sin\chi)_{min}$	$(\sin\chi)_{max}$	η_{max}	$(M_{Z'})_{min}$	$(\sin\chi)_{min}$	$(\sin\chi)_{max}$
$S^{(0)}$	−0.007	0.045	0.11	274	−0.028	0.073
$S^{(1)}$	−0.090	0.035	0.33	158	−0.024	0.008
$S^{(2)}$	−0.100	0.100	0.40	144	−0.008	0.022
$S^{(3)}$	−0.012				−0.059	0.021
$S^{(4)}$	−0.082	0.013			−0.013	0.037
$S^{(5)}$	−0.007	0.055	0.16	235	−0.032	0.084
$S^{(6)}$	−0.140				−0.036	0.013
$S^{(7)}$	−0.008	0.047			−0.031	0.011
$S^{(8)}$	−0.004	0.035	0.07	344	−0.008	0.022
$S^{(9)}$	−0.006	0.050	0.13	252	−0.008	0.024
$S^{(10)}$	−0.007	0.077	0.26	178	−0.008	0.023

ment is done with another atom with the same level of accuracy, then the data from two atoms will yield limits on $\sin\chi$ and η and hence on $\Delta\rho_M$ without appealing to any high-energy experiment. (We note that, if the atom chosen is a different isotope of cesium, higher experimental precision is demanded.) Thus there exists an exciting possibility of high-precision APV experiments verifying the standard model to a very high accuracy.

Note added. After the completion of this paper, we received a paper by W. J. Marciano and J. L. Rosner [Phys. Rev. Lett. **65**, L963 (1990)] which uses APV to study deviations from the standard model and put constraints on technicolor models. P. Langacker has used the results of Ref. 4 in his analyses reported in various workshops.

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