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Poisson-type multiplicity distribution in e^+e^- annihilation

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A modified Poisson distribution in analogy with the photon statistics due to Scully and Lamb is used to fit the charged multiplicity distribution of e^+e^- annihilation. A remark is made on approximate Koba-Nielsen-Olesen scaling.

The remarkable property of particle production by e^+e^- annihilation predicted by the geometrical model of Chou and Yang[1] is that the total angular momentum of the process is either zero or $1\hbar$, the same as that of the virtual photon in the intermediate state. Thus, the production is expected to be coherent, and the multiplicity follows a Poisson distribution.

However, recent experiments of the TASSO Collaboration[2] and the AMY Collaboration[3] indicate that the charged multiplicity distributions of e^+e^- deviate from the Poisson distribution and approximately follow Koba-Nielsen-Olesen (KNO) scaling.[4] Indeed, their results indicate that the width parameter

$$f_2=\langle n(n-1)\rangle-\langle n\rangle^2$$

increases monotonically from 0.12 to 9.54 for increasing $\sqrt{s}$ from 14 to 57 GeV. It follows that the production is *partially* coherent and that, in analogy with photon counting, we may use the following Poisson-type distribution, due to Scully and Lamb,[5] to describe the multiplicity of hadrons from e^+e^- annihilation:

$$P_n=C\frac{\alpha^n}{(n+\beta)!}, \tag{1}$$

where α and β are two parameters and C the normalization coefficient:

$$C=\beta!/{}_1F_1(1;\beta+1;\alpha), \tag{2}$$

${}_1F_1$ being the confluent hypergeometric function.

We note that this distribution (1) has been used in a previous investigation of the negative multiplicity from the inclusive $pp\to\pi^-+\cdots$ collisions[6] and that the application of quantum optical model to hadron production has been considered by other authors[7] using, instead, the generalized Glauber-Lachs formula involving Laguerre polynominals.[8]

As regards the characteristics of (1), we note that the maximum of the distribution occurs at $\hat{n}$ given by

$$\Psi(\hat{n}+\beta+1)=\ln\alpha, \tag{3}$$

Ψ being the digamma function. The first and the second moments of (1) are

$$\langle n\rangle=(\alpha-\beta)+\varepsilon, \tag{4}$$

$$\langle n^2\rangle=\alpha+(\alpha-\beta)^2+(\alpha-\beta)\varepsilon \tag{5}$$

with

$$\varepsilon=C\beta/\beta!=\beta\frac{\sigma_0}{\Sigma\sigma_n}, \tag{6}$$

where σ_n denotes the cross-section of multiplicity n. As the percentage of zero-prong events is very small, the ε terms in (4) and (5) are practically negligible, so that

$$\alpha-\beta\cong\langle n\rangle, \tag{7}$$

$$\beta\cong\langle n(n-1)\rangle-\langle n\rangle^2\equiv f_2, \tag{8}$$

indicating that α and β are correlated and that β is determined by the width of the distribution under consideration. We note that, in the context of the maser theory of Scully and Lamb,[5] β represents the ratio of the emission coefficient to that of saturation.[9]

We now use the modified Poisson distribution (1) to analyze the e^+e^- data at $\sqrt{s}=57$ GeV of the AMY Collaboration.[3] As the initial state is *CP* invariant, we shall limit ourselves to the *charged* multiplicity; their data are reproduced in Fig. 1. As, here,

$$f_2=8.11\pm0.59\gg\Delta n_{\rm ch}=2,$$

whereas β should be less than 2 according to (1), we therefore fit the AMY data leaving both α and β as free parameters and find[10]

$$\alpha=19.21\pm0.29, \quad \beta=1.64\pm0.30.$$

The fit is shown by the solid curve in Fig. 1.

A comparison with the data indicates that the fit is satisfactory up to $n_{\rm ch}\le 24$, beyond that, i.e., for $\sim 7.2\%$ of the remaining data, there appears a systematic deviation suggesting that the width of the fit is actually slightly narrower than the experimental distribution, we get $D=4.48$ instead of 5.03 ± 0.26 from the data, but comparable to $D=4.77$ of the negative-binominal distribution fit reported by the AMY Collaboration.[3] The maximum of our fit is found at $\hat{n}_{\rm ch}=17.06$ according to (3) compared to ~ 17.5 estimated from their data, whereas $\alpha-\beta=17.57\pm0.42$ agrees well with $\langle n_{\rm ch}\rangle=17.19\pm0.49$ as it should be according to (7), in spite of the fact that $\beta\neq f_2$ according to (8).

In order to further check the fit, we have computed various scaled moments

$$C_l=\frac{\langle n^l\rangle}{\langle n\rangle^l} \tag{9}$$

to compare with the experimental values. The results are

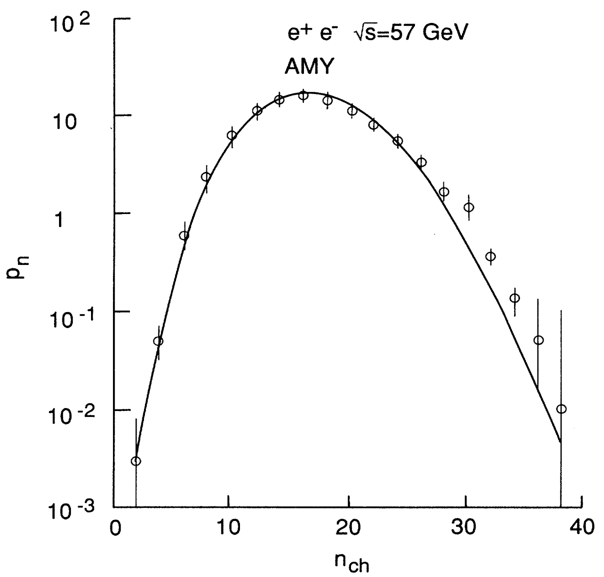

FIG. 1. Poisson-type distribution (1) fit to the charged multiplicity of e^+e^- annihilation at $\sqrt{s}=57$ GeV, AMY data, Ref. 3.

summed up in Table I. We find good agreement within about one-half standard errors, indicating that the distribution (1) is indeed adequate to describe the charged multiplicity data of e^+e^- annihilation of the AMY Collaboration.[3]

Finally, we note that the modified Poisson distribution (1) accounts for the *approximate* KNO scaling reported by the TASSO (Ref. 2) and the AMY (Ref. 3) Collaborations. In this regard, we recall that the necessary condition for KNO scaling is that the scaled moments C_l are energy independent, whereas the scaling may be approximate in the case $C_2=\text{const}$ up to $O(1/\langle n\rangle)$.[11] From (1) we find

$$C_2=1+\frac{1}{\langle n\rangle}+\frac{\beta(\beta+1)}{\langle n\rangle^2} . \tag{10}$$

As $\beta\simeq f_2$, it follows that the KNO scaling requires $f_2/\langle n\rangle$ to be $\ll 1$.

For the AMY data, $\beta/\langle n\rangle=0.097$ is rather small, so that an approximate KNO scaling may hold. This is also the case of the HRS data at $\sqrt{s}=29$ GeV (Ref. 12) and the TASSO data at $\sqrt{s}=34.8$ GeV of the KNO plot presented by the AMY Collaboration (Fig. 8, Ref. 3), the corresponding parameters being[13]

TABLE I. Comparison of scaled moments $C_l=\langle n_{\text{ch}}^l\rangle/\langle n_{\text{ch}}\rangle^l$. Experimental values of the AMY data, Ref. 3, and computations from the fit with the Poisson-type distribution Eq. (1).

	Experiment	Fit
$\langle n_{\text{ch}}\rangle$	17.19±0.07	17.57
C_2	1.084±0.086	1.062
C_3	1.266±0.093	1.190
C_4	1.577±0.156	1.403
C_5	2.080±0.261	1.902

$$\beta/\langle n_{\text{ch}}\rangle=0.60/12.87=0.046 \quad \text{for } \sqrt{s}=29 \text{ GeV}$$

and

$$\beta/\langle n_{\text{ch}}\rangle=0.80/13.59=0.059 \quad \text{for } \sqrt{s}=34.8 \text{ GeV} ,$$

indicating that the approximation for KNO scaling holds better at these energies.

We may understand this behavior by rewriting the distribution (1) as a product in terms of the KNO scaling variable $z(n)=n/\langle n\rangle$, namely,

$$P_n=C\prod\left[\frac{1+\beta/\langle n\rangle}{z(n)+\beta/\langle n\rangle}\right] . \tag{11}$$

We see that, in the vicinity of the maximum of the distribution, which contains most data, $n\simeq\langle n\rangle$, i.e., $z\simeq 1$, the factors are approximately equal, irrespective of $\beta/\langle n\rangle$, so that the KNO plot of scaled distributions is superposed in this region and that their differences, because of different $\beta/\langle n\rangle$, manifest mostly near the origin, i.e., in the region of small multiplicity. We note that such derivations already appear in the KNO plots presented in the papers by the TASSO (Ref. 2) and the AMY (Ref. 3) Collaborations, and that similar derivations are also found, if we include in their plots the latest OPAL data at $\sqrt{s}=91$ GeV.[14] Therefore, a more crucial test of this important property of KNO scaling needs data of higher energies, i.e., larger $\beta/\langle n\rangle$ and higher statistics to investigate the behavior in the end regions of the scaled distribution.

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[9] For an excellent expose of the derivation of this modified Poisson distribution (1), see R. Loudon, *The Quantum Theory of Light* (Clarendon, Oxford, 1983), p. 238 and the following.

[10] We have tried a parameter-free fit assuming $\beta=f_2=8.11$ and $\alpha=\langle n_{\rm ch}\rangle+\beta=25.30$ and found the result very bad, especially for the low-multiplicity part $n_{\rm ch}<12$.

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