Effects of πNN form factor on pionic contributions to $\bar{u}(x)-\bar{d}(x)$ distribution in the nucleon

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Effects of the cutoff parameter in πNN and $\pi N\Delta$ vertices on pionic contributions to $\bar{u}(x)-\bar{d}(x)$ in the nucleon and the Gottfried sum rule are investigated. A typical pionic contribution to the deviation from the Gottfried sum rule is found around -0.04 (about 50% of the discrepancy found by the New Muon Collaboration) for the dipole cutoff parameter $\Lambda_2 \sim 1$ GeV. This negative contribution is due to an excess of $\bar{d}$ over $\bar{u}$ in π^+ and it is (partly) canceled by a positive contribution due to an excess of $\bar{u}$ over $\bar{d}$ in an extra π^- in the $\pi N\Delta$ process. We find that the cancellation is significant especially for the large cutoff ($\Lambda_2=1.2$–2.0 GeV); thus, the distribution $\bar{u}-\bar{d}$ is less sensitive to the cutoff parameter than an $\mathrm{SU}(3)_f$-breaking distribution $(\bar{u}+\bar{d})/2-\bar{s}$. For the same reason, it is very important to include the $\pi N\Delta$ process in addition to the πNN process. Pionic contributions to $\bar{u}(x)-u(x)$, $d(x)-\bar{d}(x)$, and $d(x)-u(x)$ distributions in the nucleon are also discussed. Because pionic contributions produce an asymmetric sea, $(u_s \neq \bar{u}_s)_{\text{pionic}}$ and $(d_s \neq \bar{d}_s)_{\text{pionic}}$, we must be careful in discussing valence- and sea-quark distributions in the nucleon.

The New Muon Collaboration[1] (NMC) recently measured the proton and neutron structure functions at very small x. They found a significant discrepancy between the NMC experimental results and the Gottfried sum rule.[2] This discrepancy could suggest interesting new mechanisms. The Gottfried sum rule is described in the quark-parton model as follows. Integrating the proton and neutron structure-function difference over x and using assumptions $u \equiv u_p = d_n$, $d \equiv d_p = u_n$, and $s_p = s_n$ (and analogously for antiquarks) for quark distributions in the proton and neutron, we obtain

$$\int \frac{dx}{x}[F_2^{ep}(x)-F_2^{en}(x)] = \tfrac{1}{3}\int dx\,[u(x)+\bar{u}(x)-d(x)-\bar{d}(x)] \,. \quad (1)$$

We use "valence" distributions defined by[2,3]

$$u_v(x) \equiv u(x)-\bar{u}(x), \quad d_v(x) \equiv d(x)-\bar{d}(x) \,, \quad (2)$$

which satisfy sum rules $\int dx\, u_v(x)=2$ and $\int dx\, d_v(x)=1$ by considering proton and neutron charges. Using these sum rules, we write Eq. (1) as

$$\int \frac{dx}{x}[F_2^{ep}(x)-F_2^{en}(x)] = \tfrac{1}{3}+\tfrac{2}{3}\int dx\,[\bar{u}(x)-\bar{d}(x)] \,. \quad (3)$$

If we have an $\mathrm{SU}(2)_f$-symmetric sea ($\bar{u}=\bar{d}$), Eq. (3) becomes the Gottfried sum rule:

$$\int dx/x\,(F_2^{ep}-F_2^{en})=\tfrac{1}{3} \,.$$

Using measured structure functions, the NMC reported a significant deviation[1]

$$\int_{0.004}^{0.8} \frac{dx}{x}[F_2^{ep}(x)-F_2^{en}(x)] = 0.230 \pm 0.013(\text{stat}) \pm 0.027(\text{syst}) \,, \quad (4)$$

from the Gottfried sum rule. Possible candidates for explaining this discrepancy are valence-quark distributions at very small x (<0.004) as used in typical parametrizations[4] and an excess of $\bar{d}$ over $\bar{u}$ due to the Pauli exclusion principle.[5]

Another possible explanation in terms of pions has been proposed[6] by investigating the processes in Figs. 1(a) and 1(b). Using these processes, we can investigate pionic contributions to an $\mathrm{SU}(3)_f$-breaking distribution $[(\bar{u}+\bar{d})/2-\bar{s}]$ in the nucleon.[6–9] It has been found that the πNN form factor for explaining the $\mathrm{SU}(3)_f$-breaking distribution is very soft.[6,9] Namely, a limit for the monopole cutoff parameter is $\Lambda_1 < 0.6$–0.7 GeV,[6] which is much smaller than those frequently used in nuclear physics.[10] The investigation for $\mathrm{SU}(3)_f$ was applied to an $\mathrm{SU}(2)_f$-breaking distribution $(\bar{u}-\bar{d})$ in Ref. 6. It is especially noteworthy that the $\pi N\Delta$ process contribution to $\bar{u}-\bar{d}$ partly cancels the πNN contribution, although πNN and $\pi N\Delta$ contributions are both positive in the $(\bar{u}+\bar{d})/2-\bar{s}$ distribution. Therefore, an investigation for the πNN form factor by studying only the πNN process is not quite appropriate. If we are interested in investigating the discrepancies between the Gottfried sum rule and the experiments, it is very important to include the $\pi N\Delta$ process. In this paper, we investigate the πNN (and $\pi N\Delta$) cutoff-parameter effects on the $\bar{u}-\bar{d}$ distribution and the Gottfried sum rule.

We can directly apply the same procedure in the $\mathrm{SU}(3)_f$-breaking case for an $\mathrm{SU}(2)_f$-breaking distribution, $\bar{u}-\bar{d}$, in the nucleon.[6] The pionic contribution to an antiquark distribution in the nucleon is given by a pion momentum distribution $f_\pi(y)$ in an infinite momentum frame and an antiquark distribution in the pion $\bar{q}_\pi(x,Q^2)$:

$$x\bar{q}_N^{(\pi NN)}(x,Q^2) = \int_x^1 dy\, f_\pi^{(\pi NN)}(y)\frac{x}{y}\bar{q}_\pi(x/y,Q^2) \,, \quad (5a)$$

$$x\bar{q}_N^{(\pi N\Delta)}(x,Q^2) = \int_x^1 dy\, f_\pi^{(\pi N\Delta)}(y)\frac{x}{y}\bar{q}_\pi(x/y,Q^2) \,, \quad (5b)$$

where $f_\pi^{(\pi NN)}(y)$ and $f_\pi^{(\pi N\Delta)}(y)$ are given by

$$f_\pi^{(\pi NN)}(y)=I_{\pi NN}\frac{g_{\pi NN}^2}{16\pi^2}y\int_{-\infty}^{t_{\max}}dt\frac{-t}{(-t+m_\pi^2)^2}[F_{\pi NN}(t)]^2\ , \tag{6a}$$

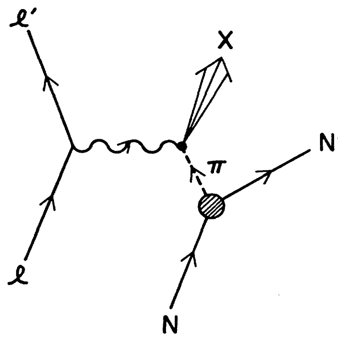
$$f_\pi^{(\pi N\Delta)}(y)=I_{\pi N\Delta}\frac{1}{24\pi^2}\left[\frac{g_{\pi N\Delta}}{2m_N}\right]^2 y \times\int_{-\infty}^{t'_{\max}}dt\frac{[F_{\pi N\Delta}(t)]^2}{(-t+m_\pi^2)^2}[(m_N+m_\Delta)^2-t] \times\left[\frac{(m_N^2-m_\Delta^2-t)^2}{4m_\Delta^2}-t\right]\ , \tag{6b}$$

where $I_{\pi NN}$ and $I_{\pi N\Delta}$ are the isospin factors given by $|\tilde{\phi}_\pi^*\cdot\tilde{\tau}|^2$ and $|\tilde{\phi}_\pi^*\cdot\tilde{T}|^2$.[6,11] In the above equations, t is the pion four-momentum square; $t_{\max}$ and $t'_{\max}$ are the maximum t given by

$$t_{\max}=-m_N^2y^2/(1-y)$$

and

$$t'_{\max}=m_N^2y-m_\Delta^2y/(1-y)\ ;$$



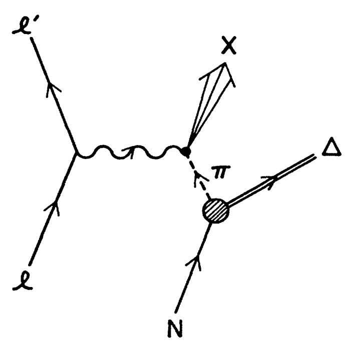


FIG. 1. Pionic contributions in deep inelastic scattering from (a) πNN process and (b) $\pi N\Delta$ process.

and the πNN coupling constant is given by $g_{\pi NN}=13.5$. The $\pi N\Delta$ coupling constant is given by the Δ decay width (Γ_Δ) by[7,12]

$$g_{\pi N\Delta}=2m_N\left[\frac{12\pi m_\Delta\Gamma_\Delta}{|\mathbf{p}_\pi|^3(m_N+E_N)}\right]^{1/2}\ , \tag{7}$$

where $|\mathbf{p}_\pi|=225.4$ MeV and $E_N=966.6$ MeV. Equations (6b) and (7) are derived by taking the $\pi N\Delta$ coupling as

$$g_{\pi N\Delta}/(2m_N)\tilde{\phi}_\pi^*\cdot\tilde{T}\,F_{\pi N\Delta}(p_\pi^2)\bar{\psi}_\Delta^\mu p_\mu^\pi\psi_N\ ,$$

where ψ_Δ^μ is the Rarita-Schwinger spinor[13] and $\tilde{T}$ is the transition isospin.[11] We assume $F_{\pi N\Delta}(t)=F_{\pi NN}(t)$ in Eq. (6b) for simplicity. Equations (5a) and (5b) indicate that the antiquark distribution $[\bar{q}_N(x,Q^2)]$ in the nucleon is a convolution of a probability $[f_\pi(y)]$ of finding a pion with a fraction y of the nucleon momentum with a probability $[\bar{q}_\pi(x/y,Q^2)]$ of finding an antiquark with a fraction x of the pion momentum.

We now discuss the $SU(2)_f$-breaking distribution ($\bar{q}_N$) and the pionic effects. There are significant contributions to the sea-quark distributions from the gluon splitting into a $q\bar{q}$ pair at large Q^2. Since deep-inelastic experiments are done at large Q^2, the gluon splitting process is significant in the nucleon's sea-quark distributions. Assuming that the sea quarks ($\bar{u},\bar{d}$) from the gluon splitting are flavor independent at large Q^2, we investigate $\bar{u}-\bar{d}$ distribution in Eqs. (5a) and (5b), where we expect that the gluonic splitting contribution is subtracted out. Then, the $x\bar{q}_N=x(\bar{u}-\bar{d})$ could be partly identified as the pionic contribution to the antiquark distribution given by Eqs. (5a) and (5b). In this way, the pionic contributions give a reasonable explanation for the $SU(2)_f$-breaking distribution and the deviations from the Gottfried sum rule.

We use the pion structure function measured by the E615 Collaboration[14] in this investigation. Other measured structure functions[15,16] are about 20% smaller in the region, $0.2<x<0.6$. We did not try to use Q^2 evolved pion structure function[17] because our program[18] indicates that the changes due to the evolution ($Q^2=25$ $\text{GeV}^2\to 4$ GeV^2) are about 20% which is about the same as the differences (20%) in the pion-structure-function measurements among different groups. In the pion structure function, the $SU(2)_f$ is assumed for sea quarks in the pion. Although it may seem contradictory to the $SU(2)_f$-breaking physics, which we try to investigate in the nucleon, we find this is not a problem as long as x is not small ($x>0.1$).[6] In investigating the pionic contribution to the deviation from the Gottfried sum rule, this may be problematic. This is because significant contribution to the integral $\int(dx/x)x(\bar{u}-\bar{d})$ (note the $1/x$ factor) comes from the small-x region. Nonetheless, we study the pionic contributions to the Gottfried sum rule for rough estimates.

Noting relations[19]

$$\bar{d}_{\pi^+}=\bar{u}_{\pi^-}=V_\pi+S_\pi,\quad \bar{u}_{\pi^+}=\bar{d}_{\pi^-}=S_\pi\ ,$$

and

$$\bar{u}_{\pi^0}=\bar{d}_{\pi^0}=V_\pi/2+S_\pi\ ,$$

we obtain $(\bar{u}-\bar{d})_{\pi^+}=-V_\pi$, $(\bar{u}-\bar{d})_{\pi^0}=0$, and $(\bar{u}-\bar{d})_{\pi^-}=+V_\pi$. Thus, we have summations (π^+ and π^0 in the πNN case, π^+, π^0, and π^- in the $\pi N\Delta$ case by assuming the proton as the initial nucleon) of the isospin times pion-structure-function factors ($I_{\pi NN}\bar{q}_\pi$ and $I_{\pi N\Delta}\bar{q}_\pi$) as $-2V_\pi$ in the πNN case and $+2/3V_\pi$ in the $\pi N\Delta$ case. In this way, we find that we have negative contribution from the πNN process and have positive contribution from the $\pi N\Delta$ process.[6] This is due to an excess of $\bar{d}$ over $\bar{u}$ in π^+ in the πNN process and to an excess of $\bar{u}$ over $\bar{d}$ in an extra π^- in the $\pi N\Delta$ process. Then, using the E615 pion structure function ($xV_\pi=F_\pi^v$ in Ref. 14) and the dipole πNN ($\pi N\Delta$) form factor,

$$F_{\pi NN}^{(2)}(t)=\frac{[1-m_\pi^2/(\Lambda_2)^2]^2}{[1-t/(\Lambda_2)^2]^2} \tag{8}$$

with $\Lambda_2=0.8$, 1.2, and 1.6 GeV, we obtain "theoretical" $\bar{u}-\bar{d}$ distributions in Fig. 2. To illustrate the cancellation between the πNN and $\pi N\Delta$ contributions, we also show each contribution in Fig. 2 for $\Lambda_2=0.80$ GeV. At $\Lambda_2=0.80$ GeV, the $\pi N\Delta$ process contribution is approximately 40% compared with the πNN; however, it is positive and partly cancels the πNN contribution. As we find in Fig. 2, the $\bar{u}-\bar{d}$ distribution is not very sensitive to the cutoff parameter if it is in the range $0.8<x<1.6$ GeV. We should note that theoretical results in the small-x region ($x<0.1$) are not very reliable, because the convolution model is problematic in such a small-x region due to shadowing phenomena,[20,21] and the assumption of $SU(2)_f$ for sea-quark distributions in the pion is also not without problems, as discussed earlier.

Calculating the integral $\frac{2}{3}\int dx(\bar{u}-\bar{d})\equiv\Delta S_G$, we find the pionic contribution to the deviation from the Gottfried sum rule [see Eq. (3)] as shown in Fig. 3. We find that the contribution from the πNN process is negative and it is canceled by the positive contribution from the $\pi N\Delta$ process. Therefore, total contributions are rather small compared with each contribution. Taking $\Lambda_2\sim 1$ GeV, which gives reasonable explanation[6] for $(\bar{u}+\bar{d})/2-\bar{s}$, we find that the contribution to the deviation from the Gottfried sum rule is about -0.04. This value could explain part (about 50%) of the discrepancy indicated by the NMC experiment. At large cutoff ($\Lambda_2=1.2$–2.0 GeV) in Fig. 3, both contributions cancel each other almost completely. Because the $\bar{u}-\bar{d}$ is not very sensitive to the πNN cutoff parameter compared with the $(\bar{u}+\bar{d})/2-\bar{s}$ case,[6] it does not give tighter restriction for the cutoff at this stage.

Finally, we briefly discuss pionic contributions to other quark distributions $\bar{u}-u$, $d-\bar{d}$, and $d-u$ in the nucleon. Using $SU(2)_f$ symmetry for sea quarks in the pion, we obtain quark distributions in the pion as

$$[\bar{u}-\bar{d}]_{\pi^+}=[\bar{u}-u]_{\pi^+}=[d-\bar{d}]_{\pi^+}=[d-u]_{\pi^+}=-V_\pi\ , \tag{9a}$$

$$[\bar{u}-\bar{d}]_{\pi^0}=[\bar{u}-u]_{\pi^0}=[d-\bar{d}]_{\pi^0}=[d-u]_{\pi^0}=0\ , \tag{9b}$$

$$[\bar{u}-\bar{d}]_{\pi^-}=[\bar{u}-u]_{\pi^-}=[d-\bar{d}]_{\pi^-}=[d-u]_{\pi^-}=+V_\pi\ . \tag{9c}$$

From the above equations, we find that pionic contributions to $\bar{u}-u$, $d-\bar{d}$, and $d-u$ distributions in the nucleon are the same as those to $\bar{u}-\bar{d}$:

$$[\bar{u}(x)-\bar{d}(x)]_N^{\text{pionic}}=[\bar{u}(x)-u(x)]_N^{\text{pionic}}=[d(x)-\bar{d}(x)]_N^{\text{pionic}}=[d(x)-u(x)]_N^{\text{pionic}} \tag{10}$$

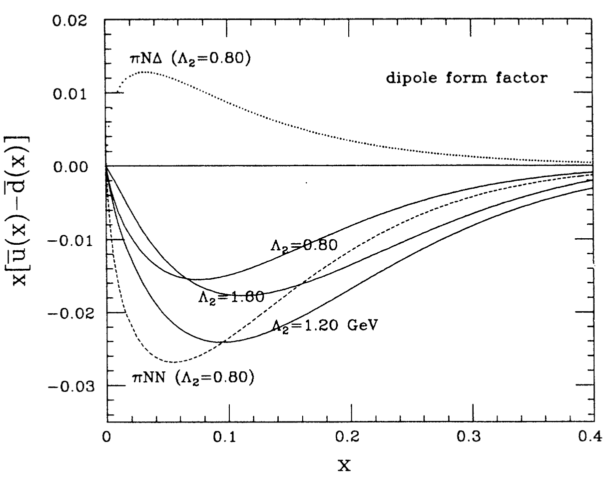


FIG. 2. Antiquark distributions $[\bar{u}(x)-\bar{d}(x)]$ in the nucleon obtained by using the dipole πNN ($\pi N\Delta$) form factor with cutoff parameters, $\Lambda_2=0.80$, 1.20, and 1.60 GeV, and the E615 pion structure function. Note ambiguities in the small-x region (see text). The dashed (dotted) curve is the contribution from the πNN ($\pi N\Delta$) process. Solid curves are summations of the πNN and $\pi N\Delta$ contributions. Pionic contributions to other distributions are the same, $\bar{u}-\bar{d}=\bar{u}-u=d-\bar{d}=d-u$, as shown in Eq. (6).

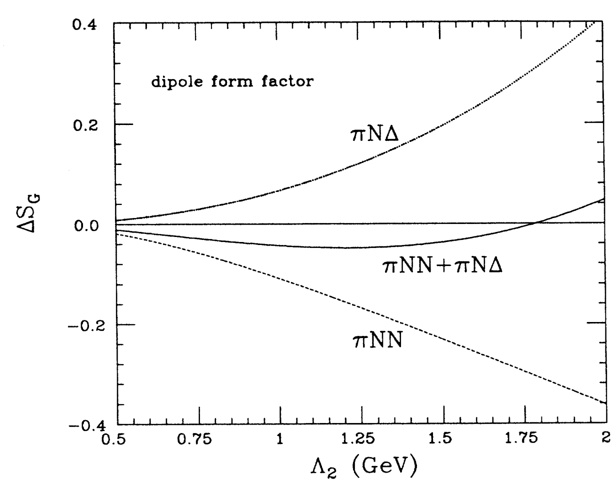


FIG. 3. Pionic contributions to the deviation from the Gottfried sum rule. The $\bar{u}(x)-\bar{d}(x)$ distribution is integrated over x and $\Delta S_G=\frac{2}{3}\int dx(\bar{u}-\bar{d})$ is shown as a function of the dipole cutoff parameter (Λ_2).

if the sea in the pion is $SU(2)_f$ symmetric. From Eqs. (1) and (10), we find that there is no pionic contribution[22] to the integral in Eq. (1). For example, the πNN process in Fig. 1(a) produces the excess of $\bar{d}$ over $\bar{u}$ due to π^+, but it also produces the same amount of u excess over d. These contributions cancel each other completely according to Eqs. (1) and (10), so that there is no contribution to the integral in Eq. (1). However, there are pionic contributions to the deviation from the Gottfried sum rule as discussed earlier. This is because the pionic contributions are included in the "valence" (actually, valence +a part of the sea) distributions defined in Eq. (2). Because pionic contributions produce an asymmetric sea,[23] $(u_s \neq \bar{u}_s)_{\text{pionic}}$ and $(d_s \neq \bar{d}_s)_{\text{pionic}}$, we must be careful how the valence- and sea-quark distributions in the nucleon are defined[24] and how they are compared with distributions in quark models.

Although the results presented in this paper are rough estimates due to problems associated with the convolution formula (shadowing) and the pion structure function [$SU(2)_f$ assumption in the sea] in the small-x region ($x<0.1$), they are encouraging for further investigations.

When the author was writing this paper, he received a paper by Henley and Miller[25] on the pionic contributions to $\bar{u}-\bar{d}$.

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