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Electroweak symmetry breaking by fourth-generation condensates and the neutrino spectrum

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We consider a new mechanism for dynamical symmetry breaking of the electroweak symmetries involving condensates of fourth-generation quarks and leptons. A dynamical generalization of the seesaw mechanism is proposed based upon the BCS theory in which a neutrino condensate gives rise to right-handed-neutrino Majorana masses and all associated spin-zero bosons are composite. The fourth-generation neutrino is naturally heavier than $M_Z/2$ and the scale of new physics is bounded above. The renormalization-group equations for the effective Lagrangian of this model are derived and used to solve the model. Implications for neutrino masses are discussed.

I. INTRODUCTION

A. Electroweak symmetry breaking by quark and lepton condensates

Recently there has been considerable interest in the possibility that a vacuum condensate involving the top quark, $\langle \bar{t}t \rangle$, is generated dynamically by new physics at a scale Λ, leading to the symmetry breaking of the standard model.[1-3] This can be treated in a fashion similar to the BCS theory of superconductivity, or of the Nambu–Jona-Lasinio (NJL) model of chiral-symmetry breaking. However, at scales $\mu \ll \Lambda$ the effective Lagrangian becomes exactly that of the standard model, and the renormalization group (RG) is an effective, if not essential, tool in obtaining reliable predictions in the scheme.[3] The minimal model with a single $\bar{t}t$ condensate leads to a prediction for the top-quark mass of $m_t \sim 230$ GeV for $\Lambda \sim 10^{15}$ GeV corresponding to the infrared quasifixed point,[4] and a Higgs boson appears as a bound state of $\bar{t}t$ with a mass of order 260 GeV.[3,4]

This minimal model suffers from several potential defects. First, the predicted m_t is large compared to indirect experimental limits when the radiative corrections of the standard model (ρ parameter constraints) are considered. Indeed, in global fits to all experimental data available at present, one finds $m_t \lesssim 200$ GeV.[5] If $m_t < 200$ GeV, then the top quark should be found within the next few years at the Fermilab Tevatron, and the minimal model would be ruled out. The minimal predictions seem to be fairly resilient to new interactions in the desert, at least in some particular models.[6,7] While it is conceivable that $m_t < 200$ GeV and a $\bar{t}t$ condensate still drives electroweak symmetry breaking, this would involve unknown dynamics for which more experimental input of physics beyond the electroweak scale would be needed. It has been emphasized, however, that in realistic technicolor schemes a substantial $\bar{t}t$ condensate seems to occur owing to the large mass of the top quark.[8] Hence, while $m_t < 200$ GeV would rule out the minimal scenario, it would not rule out the relevance of top-quark condensates in general.

A second, more theoretical, objection to the minimal scheme is the inherently large degree of fine-tuning. The scale Λ enters quadratically into the gap equation, in analogy to the radiative corrections to the Higgs-boson mass in the standard model. $m_t \ll \Lambda$ requires a delicate fine-tuning of the coupling constants of the effective theory at the scale Λ. In order to have a large hierarchy, one must demand that the theory lie very close to the critical point.[3] When Λ is taken sufficiently small to alleviate the fine-tuning, the predicted value of m_t becomes unacceptably large, so that fine-tuning is *inherent* to the minimal model.

Of course, the issue of fine-tuning may be a red herring. Perhaps some unknown dynamical mechanism will allow one to explain why the theory can naturally lie near the critical point, and the fine-tuning mechanism may "commute" with the successful predictions internal to the theory. In a sense this is what must happen for our most successful theory, QED. In the absence of fine-tuning, QED predicts a cosmological constant that is in gross conflict with observation, and whatever mechanism fine-tunes the cosmological constant to zero does not upset the other successful predictions of the theory. ("Wormhole calculus" gives us a sketch as to how this might go for both the cosmological constant and scalar-boson masses.[9]) Nonetheless, the great virtue of theories such as technicolor is that they embody a natural solution to the electroweak hierarchy problem, in which M_W/M_{Planck} is small and in principle calculable. This is lacking in the minimal model with a $\bar{t}t$ condensate.

Thus, in the present paper we wish to turn to a scheme in which electroweak symmetry breaking is driven by a condensate of conventional quarks and leptons, but the scale Λ of new dynamics is not far beyond the electroweak scale. For such a scheme we must invoke a fourth generation. This is apparent already in the analysis of Ref. 3 in which one sees that as $\Lambda \to 10$ TeV then $m_t \to 500$ GeV, clearly incompatible with the indirect limits. For a degenerate fourth-generation quark doublet, the ρ-parameter limits are not very stringent, and the mass of the fourth-generation doublet can be ~ 1 TeV. Here we are abandoning the large mass of the top quark as a *raison d'être* for quark and lepton condensates breaking the electroweak symmetry. Nonetheless, the heaviness of the top quark may arise because of its mixing to the fourth generation. In this sense the top quark is still a harbinger of this kind of symmetry-breaking scheme.

In a fourth-generation scheme the issue of the nonobservation of a fourth neutrino species at CERN LEP and the SLAC Linear Collider (SLC) must be faced. This is an issue of the origin of neutrino masses, which we turn to next.

B. Neutrino masses

Despite the fact that all quarks and charged leptons have both left- and right-handed components, there is currently no evidence for the existence of right-handed neutrinos. Even if a nonzero neutrino mass were found it would not necessarily imply the existence of a right-handed neutrino, since left-handed neutrinos may have Majorana masses. Moreover, right-handed neutrinos are all but impossible to detect, since they are decoupled from all known interactions except gravity. Such "sterile" neutrinos would thermally decouple in the very early Universe and would not contribute sufficient entropy to influence cosmological processes such as big-bang nucleosynthesis.

Nevertheless, there are good reasons for invoking the existence of right-handed neutrinos. For example, in some extensions of the standard model such as left-right-symmetric models or grand unified theories such as SO(10), right-handed neutrinos must exist to complete the matter multiplets. If right-handed neutrinos exist, then the most natural explanation for the smallness of the observed left-handed neutrino masses is the seesaw mechanism:[10] Small left-handed neutrino masses are naturally explained by assuming (1) conventional Dirac mass terms for the neutrinos linking left- and right-handed neutrinos and (2) a large Majorana mass term for the right-handed neutrinos. No known gauge interaction is broken by the presence of the large Majorana mass for the right-handed neutrinos. The sterility of the right-handed neutrinos then ensures that the large mass hierarchy between the left- and right-handed masses can be maintained without fine-tuning. After transforming to mass eigenstates, the induced Majorana mass for the left-handed neutrino is of order m_D^2/M_M, where m_D is the Dirac mass and M_M is the Majorana mass.

Of course, one can invoke the existence of a fourth generation without the seesaw mechanism by simply tuning the Dirac mass of ν_4 to be sufficiently large, i.e., $m_{\nu 4} > M_Z/2$. This is logically acceptable, but not aesthetically pleasing. Three somewhat arbitrary alternatives come to mind: (1) Nature may choose an exact $SU(3)_R$ chiral symmetry for the triplet of (e,μ,τ) right-handed neutrinos, while ν_4 is a singlet, thus enforcing masslessness for all but ν_4; (2) all right-handed neutrinos may have conventional Dirac masses, but only the (e,μ,τ) right-handed neutrinos have a very large [perhaps SO(3)-invariant] Majorana mass, so the seesaw mechanism applies only to them, ν_4 being left with a large physical Majorana mass; (3) ν_{4L} alone gets a large Majorana mass, though this possibility will be severely constrained by the standard-model ρ-parameter limits. However, all of these possibilities clearly beg the question of why the fourth-generation neutrino should be different from the others. Obviously these schemes can be implemented by fiat, but we prefer at present to consider the possibility that the fourth generation is fundamentally no different than the others. Hence apart from the details of the ordinary family hierarchy and its dynamical consequences, we propose a principle of "neutrino democracy," and insist that the ν_4 is not special. Then how do we evade the LEP and SLC limits on neutrino counting?

Here we find an intriguing, perhaps unique, possibility which we will incorporate at present.[11] We will assume the existence of a fourth generation, and assume that (1) all neutrinos have Dirac masses of order their charged-lepton counterpart and (2) all neutrinos have a large right-handed Majorana mass M *of order the electroweak scale.* In this scenario, the seesaw mechanism assures that the (e,μ,τ) neutrinos are light while ν_4 is naturally heavy.[11] The fact that M can be taken close to the electroweak scale has been emphasized by Glashow in the context of three generations.[12] Thus, the LEP-SLC limits do *not* imply that there are only three generations of quarks and leptons, even if "neutrino democracy" is invoked. These assumptions also imply that the light neutrinos have masses not far from their current experimental upper limits, opening up the possibility that neutrino masses could be discovered experimentally in the near future. In the simplest version which we present here there will be a massive Majorana-Higgs boson and a massless "Majoron" associated with the spontaneously broken global right-handed neutrino number.[13] The scenario appears to be nicely compatible with all laboratory constraints, and astrophysical considerations may make the existence of Majorons rather attractive.[11,14] Much of the present paper will focus upon a dynamical mechanism for generating the neutrino Majorana and Dirac masses at the electroweak scale, while neatly accommodating the LEP-SLC results.

C. Renormalization-group approach

In Appendix A a toy model exhibiting a dynamical seesaw mechanism is solved in the large-N limit using conventional Schwinger-Dyson techniques. However, this is simply for illustrative purposes, and we will show in the next section that equivalent results follow by using the RG equations when the compositeness conditions are properly implemented. The compositeness conditions are boundary conditions on the full RG equation that may be derived from the effective Lagrangian at the scale Λ. The renormalization group can be used as a dynamical tool to include *all* of the effects of the full theory and generate reliable and precise predictions of its consequences. This goes beyond the limited approaches of large-N fermion bubble sums, or planar QCD calculations. Moreover, the results of these "brute force" analyses can be easily reproduced by including only those terms in the renormalization-group equations that correspond to effects included in the "brute force" calculations. The important element which makes the renormalization group applicable is the fact that *the compositeness of certain dynamically generated multiplets, e.g., the Higgs multiplet and the majoran, implies UV boundary conditions on the renormalization-group equations of the effective field theory.*

Of course, the power of the RG lies ultimately in the existence of a long running in scales, i.e., a "desert," which occurs when we fine-tune the model. The compositeness conditions depend upon the details of the physics at Λ, and only if there is a desert will the low-energy predictions be insensitive to the presence of irrelevant operators in the effective Lagrangian at scale Λ. Since we are ultimately interested here in $\Lambda \sim 1$ TeV, we really can only use the RG as an approximate tool in obtaining results which we cannot trust in detail. In any case, we know of no better way to obtain these results.

We will thus analyze the full dynamical model of electroweak symmetry, and right-handed neutrino number breaking in detail by the RG methods of Ref. 3. Here the renormalization-group equations are solved implementing the boundary conditions that follow from compositeness.

II. BCS THEORY OF THE SEESAW MECHANISM AND THE MAJORON

We begin by considering a simple model which illustrates the dynamical generation of a Majorana mass for right-handed neutrinos. The model contains N generations of right-handed neutrinos ν_{Rj}, where j is the generation index. The Lagrangian is

$$\mathcal{L} = \bar{\nu}_{Rj} i\partial\!\!\!/ \nu_{Rj} + G_0 (\bar{\nu}^c_{Rj}\nu_{Rj})(\bar{\nu}_{Rk}\nu^c_{Rk}) \; , \tag{2.1}$$

where repeated indices are summed from 1 to N. Here ψ^c denotes charge conjugation, and our spinor conventions are described in Appendix B. This nonrenormalizable Lagrangian should be viewed as an effective field theory in the presence of a momentum cutoff Λ. Λ and G_0 are, strictly speaking, independent, since we would have in general a dimensionless coupling constant g and $G_0 \sim g^2/\Lambda^2$. On scales above Λ the four-fermion interaction softens and is viewed to be generated by some new interactions, such as a new gauge interaction.

The theory has a global $SO(N)_R \times U(1)$-flavor symmetry. This theory can be solved exactly in the large-N limit where only fermion loops are important, and we will argue that the qualitative features of the large-N limit are retained for small N. The full Schwinger-Dyson equation solution is presented in Appendix A. When G_0 exceeds a certain critical value, there is a vacuum condensate

$$\langle \bar{\nu}^c_{Rj}\nu_{Rj} + \text{H.c.} \rangle \neq 0 \; , \tag{2.2}$$

which breaks U(1), while preserving $SO(N)$ symmetry, and gives all of the neutrinos a Majorana mass. We see that, in a sense, the model (2.1) is more like the BCS theory than the NJL model: the condensate (2.2) breaks a (ungauged) U(1) symmetry which acts just like the U(1) of electromagnetism broken in the BCS theory [the NJL model, on the other hand, contains a condensate of the form $\langle \bar{\psi}\psi \rangle$, which breaks a chiral U(1)].

In addition to giving rise to a Majorana mass, the fact that the U(1)-flavor symmetry is spontaneously broken implies that there is a massless Nambu-Goldstone mode (the "Majoron") in the spectrum.[13] Also, there is a massive collective mode analogous to the "σ mode" in the NJL model which we will refer to as the Majorana-Higgs boson. In the large-N limit it has a mass exactly twice the neutrino Majorana mass, but there are significant corrections to this result at small N or in the presence of additional interactions.

We now discuss the solution to the theory defined in Eq. (2.1) in an effective Lagrangian framework using the block-spin renormalization group. The effective Lagrangian of Eq. (2.1) is equivalent to

$$\mathcal{L} = \bar{\nu}_{0Rj} i\partial\!\!\!/ \nu_{0Rj} + (\Phi_0 \bar{\nu}^c_{0Rj}\nu_{0Rj} + \text{H.c.}) - M_0^2 \Phi_0^\dagger \Phi_0 \; , \tag{2.3}$$

provided we identify

$$G_0 = 1/M_0^2 \; . \tag{2.4}$$

since integrating out Φ_0 yields the four-fermion interaction. Note that this technical trick contains some physics: it only works for an attractive interaction, and only such an interaction can form low-energy bound states.

As we consider scales $\mu \ll \Lambda$ we may obtain the effective Lagrangian from Eq. (2.3) by block-spin renormalization-group methods; i.e., we compute the coefficients of the lowest dimension terms in the effective Lagrangian for the theory defined by Eq. (2.3) by integrating out field modes with momenta p^2 with $\mu^2 < p^2 < \Lambda^2$. The effective Lagrangian at the scale μ becomes[3]

$$\mathcal{L}_\mu = Z_\Phi \partial^\mu \Phi_0^\dagger \partial_\mu \Phi_0 - \widetilde{M}^2 \Phi_0^\dagger \Phi_0 - \frac{\widetilde{\lambda}}{2}(\Phi_0^\dagger \Phi_0)^2 + Z_\nu \bar{\nu}_{0Rj} i\partial\!\!\!/ \nu_{0Rj} + \widetilde{\kappa}(\Phi_0 \bar{\nu}^c_{0Rj}\nu_{0Rj} + \text{H.c.}) + \cdots \; . \tag{2.5}$$

Note the induced kinetic and quartic interaction terms which follow from fermion loops as in Fig. 1.

In the large-N limit we obtain

$$Z_\Phi = \frac{N}{8\pi^2} \ln \frac{\Lambda^2}{\mu^2} \; , \tag{2.6}$$

$$\widetilde{M}^2 = M_0^2 - \frac{N}{4\pi^2}(\Lambda^2 - \mu^2) \; , \tag{2.7}$$

$$\widetilde{\lambda} = \frac{N}{\pi^2} \ln \frac{\Lambda^2}{\mu^2} \; , \tag{2.8}$$

$$Z_\nu = 1 \; , \tag{2.9}$$

$$\widetilde{\kappa} = 1 \; . \tag{2.10}$$

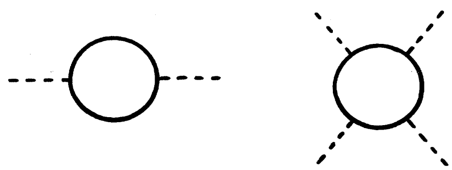

FIG. 1. Diagrams leading to the induced kinetic and quartic interaction terms for the scalar fields.

We may now exercise our freedom of renormalizing the fields to write

$$\mathcal{L}_\mu = \partial^\mu \Phi^\dagger \partial_\mu \Phi - M^2 \Phi^\dagger \Phi - \frac{\lambda}{2}(\Phi^\dagger \Phi)^2 + \bar{\nu}_{Rj} i \partial\!\!\!/ \nu_{Rj} + \kappa(\Phi \bar{\nu}^c_{Rj} \nu_{Rj} + \text{H.c.}) + \cdots , \quad (2.11)$$

where we have defined rescaled fields

$$\Phi = Z_\Phi^{1/2} \Phi_0, \quad \nu_R = Z_\nu^{1/2} \nu_{0R} , \quad (2.12)$$

and

$$\lambda = \tilde{\lambda}/Z_\Phi^2, \quad \kappa = \tilde{\kappa}/Z_\nu Z_\Phi^{1/2}, \quad M^2 = \tilde{M}^2/Z_\Phi . \quad (2.13)$$

The resulting renormalized coupling constants κ and λ take the form

$$\kappa = \left[\frac{N}{8\pi^2} \ln \frac{\Lambda^2}{\mu^2} \right]^{-1/2} , \quad (2.14)$$

$$\lambda = \left[\frac{N}{64\pi^2} \ln \frac{\Lambda^2}{\mu^2} \right]^{-1} . \quad (2.15)$$

The fine-tuning of the gap equation is equivalent to demanding an approximate cancellation between the quadratic divergence in Eq. (2.7) against M_0^2. Thus, when $\mu^2 \to 0$ we demand that $M^2 \to M_\Phi^2$, the desired low-energy value of the Φ mass. The interesting physics predictions are then contained in the quantities $\tilde{\lambda}$ and Z_Φ, or equivalently, in λ and κ.

The compositeness conditions are just those implied by the bare Lagrangian of Eq. (2.3),

$$Z_\Phi(\mu) \to 0|_{\mu \to \Lambda} , \quad (2.16)$$

$$\tilde{\lambda}(\mu) \to 0|_{\mu \to \Lambda} , \quad (2.17)$$

or equivalently, for the physically normalized coupling constants,

$$\kappa(\mu) \to \infty|_{\mu \to \Lambda} , \quad (2.18)$$

$$\lambda(\mu) \to \infty|_{\mu \to \Lambda} . \quad (2.19)$$

These may be taken as the boundary conditions on the solution to the RG equations.

The predictions of the model are obtained as follows. The low-energy effective potential for the field Φ with the physical normalization takes the form, as $\mu \to 0$,

$$V(\Phi) = M_\Phi^2 \Phi^\dagger \Phi + \frac{\lambda}{2}(\Phi^\dagger \Phi)^2 . \quad (2.20)$$

We *assume* (as a consequence of our choice of fine-tuning of M_0^2) that the symmetry is spontaneously broken and rewrite, for Φ,

$$\Phi = \left[v_\Phi + \frac{\phi}{\sqrt{2}} \right] e^{i\chi/v_\Phi} , \quad (2.21)$$

where $\langle \Phi \rangle = v_\Phi$. Here, χ is a massless Nambu-Goldstone model, the Majoron,[13] and ϕ is the Majorana-Higgs boson with mass

$$m_\phi^2 = 2\lambda v_\Phi^2 . \quad (2.22)$$

Also, we see from the Majorana-Yukawa coupling to the neutrinos,

$$\bar{\nu}_{Rj} i \partial\!\!\!/ \nu_{Rj} + \kappa(\Phi \bar{\nu}^c_{Rj} \nu_{Rj} + \text{H.c.}) , \quad (2.23)$$

that we have a Majorana mass for the right-handed neutrinos:

$$m_M = 2\kappa v_\Phi . \quad (2.24)$$

(Note that m_M is larger by a factor of 2 than what one naively expects. This comes from deriving the equation of motion for the neutrino field from the Lagrangian, since variations with respect to ν_R and $\bar{\nu}_R^c$ are not independent.) By using the results for λ and κ from Eqs. (2.14) and (2.15) we find

$$\frac{m_\phi}{m_M} = \left[\frac{\lambda}{2\kappa^2} \right]^{1/2} = 2 . \quad (2.25)$$

This is the conventional Nambu–Jona-Lasinio result, but we have derived it here in the BCS model.[15]

We can also derive this result from the usual one-loop differential RG equations satisfied by the physical couplings in Eqs. (2.14) and (2.15). These can be obtained directly in the usual way (though we alert the reader that the Majorana-Yukawa vertices lead to tricky factors of 2 when Wick contractions are performed). The results are

$$16\pi^2 \mu \frac{\partial \kappa}{\partial \mu} = (2N+4)\kappa^3 , \quad (2.26)$$

$$16\pi^2 \mu \frac{\partial \lambda}{\partial \mu} = 8N\kappa^2\lambda - 32N\kappa^4 + 8\lambda^2 . \quad (2.27)$$

Consider the solution to Eqs. (2.26) and (2.27) keeping only the leading large-N terms. We find

$$\frac{1}{\kappa^2(\mu)} = \frac{2N}{(4\pi)^2} \ln \frac{\Lambda^2}{\mu^2}, \quad \frac{1}{\lambda(\mu)} = \frac{N}{4(4\pi)^2} \ln \frac{\Lambda^2}{\mu^2} , \quad (2.28)$$

where we have used the compositeness boundary conditions (2.18) and (2.19). The second result follows upon assuming that $\lambda(\mu) \propto \kappa^2(\mu)$ and demanding that Eqs. (2.26) and (2.27) be consistent. These results are equivalent to Eqs. (2.14) and (2.15) and thus we find again

$$m_\phi = 2 m_M . \quad (2.29)$$

This tells us that in the large-N limit, the low-energy effective theory defined using the one-loop RG equations is *exactly* equivalent to the four-Fermi theory of Eq. (2.1), provided we impose the boundary conditions (2.18) and (2.19). The point of this exercise is to show that the effective Lagrangian defined by the one-loop RG equations (2.26) and (2.27), together with the compositeness boundary conditions contains *all* of the essential physics of the dynamical symmetry breaking. In the large-N limit, the one-loop effective Lagrangian is equivalent to the exact effective Lagrangian, and, for finite N, it contains corrections to the large-N results.

III. A REALISTIC MODEL

Our present goal is to specify a realistic effective Lagrangian similar to Eq. (2.1) which drives the formation of fourth-generation right-handed neutrino condensates and the quark and lepton condensates which break the electroweak symmetries. This theory must contain the observed spectrum of quark and lepton masses and mixing angles.

A. The model

Our model contains four standard generations of quarks and leptons, together with four right-handed neutrinos. At the scale Λ we have a four-fermion effective Lagrangian which may be represented by introducing auxiliary fields H and Φ. The fermions are assumed to have couplings to the auxilliary field H given by

$$\mathcal{L}_{\rm Dirac}=g_{jk}^{(-1)}\bar{L}_{Lj}He_{Rk}+g_{jk}^{(0)}\bar{L}_{Lj}\tilde{H}\nu_{Rk}+g_{jk}^{(2/3)}\bar{Q}_{Lj}Hu_{Rk}+g_{jk}^{(0)}\bar{Q}_{Lj}\tilde{H}d_{Rk}+\text{H.c.}-M_{H0}^2H^\dagger H+\cdots \; . \tag{3.1}$$

In addition, we assume that the right-handed neutrinos couple to the auxiliary field Φ:

$$\mathcal{L}_{\rm Majorana}=\kappa_{jk}(\Phi\bar{\nu}^c_{Rj}\nu_{Rk}+\text{H.c.})-M_{\Phi 0}^2\Phi^\dagger\Phi+\cdots \; . \tag{3.2}$$

Here we define $Q_{Li}=(u_L d_L)^T$ $[L_{Li}=(\nu_L e_L)^T]$ to be the ith quark (lepton) electroweak doublet, and $\tilde{H}=i\sigma_2 H^*$. Note that $\bar{\nu}^c_j\nu_k=\bar{\nu}_k\nu^c_j$ implies $\kappa_{jk}=\kappa_{kj}$. The above ellipses refer to the possible "irrelevant" operators of $d>4$, such as four-fermion terms that are suppressed by $1/\Lambda^2$ with numerical coefficients of order unity.

Ultimately H and Φ become dynamical fields at low energies and develop vacuum expectation values (VEV's). Through these VEV's the quarks and leptons acquire Dirac mass terms and the right-handed neutrinos acquire Majorana mass terms. The matrices g^α_{ij} will determine the mass spectrum and the pattern of mixing angles in the hadronic and leptonic weak currents.

B. The effective Lagrangian at low energies

We now consider the descent in the full theory to low energies in analogy to our treatment of the BCS-Majorana theory in Sec. II. The most general induced Lagrangian for both of the scalar fields is

$$\begin{aligned}\mathcal{L}_S=&Z_H(D_\mu H_0^\dagger D^\mu H_0)+Z_\Phi\partial_\mu\Phi_0^\dagger\partial^\mu\Phi_0\\&-M_{H0}^2H_0^\dagger H_0-M_{\Phi 0}^2\Phi_0^\dagger\Phi_0\\&-\frac{\tilde{\lambda}_1}{2}(H_0^\dagger H_0)^2-\frac{\tilde{\lambda}_2}{2}(\Phi_0^\dagger\Phi_0)^2-\tilde{\lambda}_3H_0^\dagger H_0\Phi_0^\dagger\Phi_0 \; .\end{aligned} \tag{3.3}$$

The RG boundary conditions can be derived using the same reasoning used for the toy model of the previous section. As $\mu\to\Lambda$, we demand

$$Z_\Phi\to 0 \; , \tag{3.4}$$

$$Z_H\to 0 \; , \tag{3.5}$$

$$\tilde{\lambda}_i\to 0 \; . \tag{3.6}$$

with all other couplings finite (and nonzero) in this normalization. The masses also evolve as before, but now we assume that the low-energy values are such as to trigger the appropriate symmetry breaking as described below. In the physical normalization, $H=Z_H^{1/2}H_0$ and $\Phi=Z_\Phi^{1/2}\Phi_0$; the Lagrangian becomes

$$\begin{aligned}\mathcal{L}_S=&D_\mu H^\dagger D^\mu H+\partial_\mu\Phi^\dagger\partial^\mu\Phi-M_H^2H^\dagger H-M_\Phi^2\Phi^\dagger\Phi\\&-\frac{\lambda_1}{2}(H^\dagger H)^2-\frac{\lambda_2}{2}(\Phi^\dagger\Phi)^2-\lambda_3H^\dagger H\Phi^\dagger\Phi \; ,\end{aligned} \tag{3.7}$$

with the physical coupling constants defined by

$$\lambda_1=\tilde{\lambda}_1/Z_H^2 \; , \tag{3.8}$$

$$\lambda_2=\tilde{\lambda}_1/Z_\Phi^2 \; , \tag{3.9}$$

$$\lambda_3=\tilde{\lambda}_1/Z_HZ_\Phi \; , \tag{3.10}$$

$$M_H^2=M_{H0}^2/Z_H \; , \tag{3.11}$$

$$M_\Phi^2=M_{\Phi 0}^2/Z_\Phi \; . \tag{3.12}$$

The boundary conditions can therefore be rewritten as

$$\kappa\to\infty \; , \tag{3.13}$$

$$\lambda_i\to\infty \; , \tag{3.14}$$

$$d_{44}^\alpha\to\infty \; . \tag{3.15}$$

The masses M_H^2 and M_Φ^2 are tuned to have low-energy values that are negative. This is equivalent to demanding the symmetry-breaking solution to the gap equations and thus trigger the formation of the vacuum expectation values of H and Φ. Therefore, we simply parametrize these VEV's at low energies:

$$\langle H^0\rangle=v_H=175\text{ GeV}, \quad \langle\Phi\rangle=v_\Phi\equiv\beta v_H \; . \tag{3.16}$$

where the parameter β is *a priori* arbitrary.

The Higgs-Yukawa coupling constants will have low-energy values:

$$d^{(-1)}=\frac{1}{v_H}\mathrm{diag}(m_e,m_\mu,m_\tau,m_{E4}) , \tag{3.17}$$

$$d^{(+2/3)}=\frac{1}{v_H}\mathrm{diag}(m_u,m_c,m_t,m_{U4}) , \tag{3.18}$$

$$d^{(-1/3)}=\frac{1}{v_H}\mathrm{diag}(m_d,m_s,m_b,m_{D4}) . \tag{3.19}$$

For the neutrinos we make the assumption $d_{ii}^{(0)}\approx d_{ii}^{(-1)}$ for $i=(1,2,3)$, while $d_{44}^{(0)}$ is determined by the RG equations. Here, m_{E4} is the mass of the fourth-generation lepton, etc. All large coupling constants will be determined in this model in terms of the scale Λ by using the RG equations with the assumption of the compositeness boundary conditions. Taking $d^{(0)}\approx d^{(-1)}$ for the light neutrinos is our special assumption of "neutrino democracy;" we certainly do not predict the three light-mass generation Higgs-Yukawa couplings, but it is reasonable to expect the usual generational hierarchy to apply in the real world for neutrinos. Of course, we allow for the overall scale difference, i.e., $d^{(0)}=\epsilon d^{(-1)}$ with $0.1\lesssim\epsilon\lesssim 1.0$ as in Ref. 11.

The low-energy Majorana-Yukawa coupling constants are assumed all to be large and will therefore all be predicted. We will find

$$\kappa=\mathrm{diag}(\kappa_1,\kappa_1,\kappa_l,\kappa_h) \tag{3.20}$$

where κ_l refers to the light neutrinos. Hence the light three generations will have approximately degenerate Majorana-Yukawa couplings. $\kappa_h\neq\kappa_l$ arises because of the renormalization effects due to the large Higgs-Yukawa couplings of the fourth generation.

C. The strong broken horizontal gauge theory

One might ask what kind of underlying theory can give rise to strong four-Fermi interactions at a scale Λ. We can imagine that this theory arises from a strong broken horizontal gauge theory (SBHGT), a broken gauge theory which is sufficiently strongly coupled to drive the formation of chiral condensates. We will not say much here about the form of the SBHGT; however, we do not have to commit ourselves to any particular underlying theory, since we will work solely with the effective Lagrangian. Integrating out the scalar fields of Eqs. (3.1) and (3.2) will generate the equivalent effective Lagrangian at Λ, which is then viewed as the starting point. Hence, by working backwards, we can specify a simple solution for the desired effective Lagrangian for the SBHGT by integrating out H and Φ

$$\mathcal{L}_{\mathrm{SBHGT}}=G_{ijkl}^{(-1,-1)}\overline{L}_{Li}e_{Rj}\overline{e}_{Rk}L_{Ll}+G_{ijkl}^{(0,0)}\overline{L}_{Li}\nu_{Rj}\overline{\nu}_{Rk}L_{Ll}+G_{ijkl}^{(-1,0)}\overline{L}_{Li}e_{Rj}\overline{\nu}_{Rk}L_{Ll}+G_{ijkl}^{(0,-1)}\overline{L}_{Li}\nu_{Rj}\overline{\nu}_{Rk}L_{Ll} \tag{3.21}$$

$$+K_{ijkl}\overline{\nu}^{\,c}_{Ri}\nu_{Rj}\overline{\nu}_{Rk}\nu^{c}_{Rl}+\cdots . \tag{3.22}$$

We have not explicitly written the analogous terms for the quark-quark four-fermion, and the quark-lepton four-fermion interactions. The tensor coefficients must then have the *approximate* factorization properties

$$G_{ijkl}^{(\alpha,\beta)}=g_{ij}^{\alpha}(g_{lk}^{\beta})^*/M_{H0}^2 , \tag{3.23}$$

$$K_{ijkl}=\kappa_{ij}\kappa_{kl}/M_{\Phi 0}^2 . \tag{3.24}$$

The factorization properties select a particular low-energy spectrum of composite Higgs and Majorana-Higgs bosons. One can throw the theory into a different mode by relaxing these conditions. For example, setting $G_{ijkl}^{(\alpha,\beta)}\approx 0$ for $\alpha\neq\beta$ would lead to a four-Higgs-doublet version of the scheme, allowing one doublet per charge species of right-handed quark or lepton. This is a far more complicated low-energy model than the single-Higgs-doublet version which we will presently study, but it is potentially interesting since it contains the largest set of low-energy composite states, yet naturally avoids the presence of off-diaognal neutral vertices. The two-doublet version of the minimal dynamical-symmetry-breaking scheme has been studied by Luty and by Suzuki.[16] We will presently make the simplifying assumption that the factorization properties are such that only one dynamical Higgs doublet is generated by the SBHGT.

If the factorization holds the $g_{ij}^{(\alpha)}$ can be brought to a positive diagonal form $d_{ij}^{(\alpha)}$ by performing SU(N)-flavor transformations on the fermion fields. The statement that we want the fourth generation to dominate the symmetry breaking is really the requirement that the $g^{(\alpha)}$ matrices have single large eigenvalues, which can be taken in an appropriate basis as the fourth diagonal elements of $d^{(\alpha)}$. This can be understood as a consequence of a symmetry principle, as emphasized by Fritzsch, Meshkov, and Kaus,[17] but one which pertains to the details of the SBHGT.

We emphasize that the factorization properties are expected to be only approximate to leading order in the largest terms. For example, we have $d_{ii}^{(\alpha)}\approx\epsilon_i d_{44}^{(\alpha)}$ with $\epsilon_i\ll 1$ for $i\neq 4$. We demand only that the factorization conditions of Eq. (3.24) hold to order ϵ. The $O(\epsilon^2)$ terms then become $O(1/\Lambda^2)$ contact interactions in the low-energy effective theory. The relevant structure of the effective Lagrangian for Dirac masses then takes the following schematic form, e.g., written here only for the $+\frac{2}{3}$ quarks:

$$\begin{aligned}\mathcal{L}=&G_0\overline{U}_L U_R\overline{U}_R U_L+\epsilon_2 G_0\overline{U}_L U_R\overline{t}_R t_L\\&+\epsilon_3 G_0\overline{U}_L U_R\overline{c}_R c_L+\epsilon_4 G_0\overline{U}_L U_R\overline{u}_R u_L\\&+O(\epsilon_i\epsilon_j)G_0(\overline{q}_{iL}q_{iR})(\overline{q}_{jR}q_{jL}) ,\end{aligned} \tag{3.25}$$

where $q_2=t$, $q_3=c$, and $q_4=U$. Unfortunately, here the fermion mass hierarchy is unexplained, arising because of the values of the ϵ_i which are relegated to the details of the SBHGT symmetry-breaking pattern. The Lagrangian is safe with respect to the generation of $\Delta S=2$ transitions. The contact terms are stronger for the heavier quarks.

Alternatively, it is possible that the quark and lepton hierarchy can be viewed as the consequence of a dynamical symmetry breaking where the effective Lagrangian contains only a single small parameter. The idea is that the lighter fermions get their masses from "nearest neighbor" couplings to heavier fermions. We have analyzed a simple model which contains only a single small parameter ϵ, but which gives only crudely realistic results:

$$\begin{aligned}\mathcal{L}=&G_0\overline{U}_L U_R \overline{U}_R U_L+\epsilon G_0\overline{U}_L U_R \overline{t}_R t_L\\&+\epsilon G_0\overline{t}_R t_L \overline{c}_R c_L+\epsilon G_0 \overline{c}_R c_L \overline{u}_R u_L\\&+O(\epsilon_i\epsilon_j)G_0(\overline{q}_{iL}q_{iR})(\overline{q}_{jR}q_{jL}) \ .\end{aligned} \tag{3.26}$$

Here, the fourth-generation quarks are the leading large condensate, and the third generation couples with strength ϵ; the second generation then couples to the third with strength ϵ, and so forth. The gap equations are now coupled and may be solved to find "tumbling" solutions, e.g., $m_{i+1}\approx\epsilon m_i$ with predictions such as $m_b\approx m_s^2/m_d$ and $m_t\approx m_c^2/m_u$ (unrenormalized). These are qualitatively reasonable estimates, yet it should be emphasized that we are taking this only as an *approximate* form of the interaction.

D. The full RG equations for fermion masses

We begin by studying the RG equations that pertain to the fermion Dirac and Majorana masses. In what follows we will shift the notation for ease of writing the RG equations. Let us define the matrices

$$\begin{aligned}&g_{ij}^{(-1)}\equiv E_{ij}, \quad g_{ij}^{(0)}\equiv N_{ij} \ ,\\&g_{ij}^{(+2/3)}\equiv U_{ij}, \quad g_{ij}^{(-1/3)}\equiv D_{ij} \ .\end{aligned} \tag{3.27}$$

The full one-loop renormalization-group equations for the coupling constant matrices as defined above are

$$\mathcal{D}\kappa=[2\,\mathrm{tr}(\kappa^\dagger\kappa)+4\kappa\kappa^\dagger]\kappa+(N^\dagger N)^T\kappa+\kappa N^\dagger N \ , \tag{3.28}$$

$$\begin{aligned}\mathcal{D}E=&[\tfrac{3}{2}EE^\dagger-\tfrac{3}{2}NN^\dagger+\mathrm{tr}(E^\dagger E)+\mathrm{tr}(N^\dagger N)\\&+3\,\mathrm{tr}(UU^\dagger+DD^\dagger)-\tfrac{15}{4}g_1^2-\tfrac{9}{4}g_2^2]E \ ,\end{aligned} \tag{3.29}$$

$$\begin{aligned}\mathcal{D}N=&[\tfrac{3}{2}NN^\dagger-\tfrac{3}{2}EE^\dagger+\mathrm{tr}(E^\dagger E)+\mathrm{tr}(N^\dagger N)\\&+3\,\mathrm{tr}(UU^\dagger+DD^\dagger)-\tfrac{3}{4}g_1^2-\tfrac{9}{4}g_2^2+2\kappa^\dagger\kappa]N \ ,\end{aligned} \tag{3.30}$$

$$\begin{aligned}\mathcal{D}U=&[\tfrac{3}{2}UU^\dagger-\tfrac{3}{2}DD^\dagger+\mathrm{tr}(EE^\dagger)+\mathrm{tr}(NN^\dagger)\\&+3\,\mathrm{tr}(UU^\dagger+DD^\dagger)-\tfrac{17}{12}g_1^2-\tfrac{9}{4}g_2^2-8g_3^2]U \ ,\end{aligned} \tag{3.31}$$

$$\begin{aligned}\mathcal{D}D=&[\tfrac{3}{2}DD^\dagger-\tfrac{3}{2}UU^\dagger+\mathrm{tr}(EE^\dagger)+\mathrm{tr}(NN^\dagger)\\&+3\,\mathrm{tr}(UU^\dagger+DD^\dagger)-\tfrac{5}{12}g_1^2-\tfrac{9}{4}g_2^2-8g_3^2]D \ .\end{aligned} \tag{3.32}$$

The parts that do not involve the Majorana couplings are contained in Refs. 4 and 18. Here, g_1, g_2, and g_3 are the $\mathrm{U}(1)_Y$, $\mathrm{SU}(2)_W$, and SU(3) gauge couplings, respectively, and we have used the abbreviation

$$\mathcal{D}\equiv 16\pi^2\mu\frac{\partial}{\partial\mu} \ . \tag{3.33}$$

Note that the RG coefficients can be computed in the massless limit. The Feynman rules for ν_R then reduce to the familiar ones for two-component spinors. We have given the equations for arbitrary complex coupling matrices, even though we will assume that the matrices are real and diagonal in what follows.

To simplify the RG equations, we assume that the Yukawa coupling matrices are real and diagonal, and satisfy

$$\begin{aligned}&E_{44}\gg E_{jj}, \quad N_{44}\gg N_{jj}, \quad D_{44}\gg D_{jj} \quad \text{for } j=1,2,3 \ ,\\&U_{44},U_{33}\gg U_{jj} \quad \text{for } j=1,2 \ .\end{aligned} \tag{3.34}$$

This is clearly a good approximation at low energies. The diagonal entries of κ are then split, or equivalently the SO(4) symmetry is broken. It is sufficient to consider only the fourth generation $\kappa_4\equiv\kappa_{44}$ and the three light generation $\kappa_l\equiv\kappa_{ii}$ independently.

The physical fermion Dirac masses are now determined as

$$m_{\nu 4}=N_{44}(\mu)v_H, \quad m_E=E_{44}(\mu)v_H \ , \tag{3.35}$$

$$m_U=U_{44}(\mu)v_H, \quad m_D=D_{44}(\mu)v_H, \quad \mu\sim 100\ \mathrm{GeV} \ , \tag{3.36}$$

while the Majorana masses are given by

$$M_{M4}=2\kappa_h(\mu)v_\Phi=2\kappa_h(\mu)\beta v_H, \quad M_{Ml}=2\kappa_l(\mu)\beta v_H \ , \tag{3.37}$$

where again we choose $\mu\sim 100$ GeV as an approximation to the threshold condition that determines the masses, i.e., $m=g(m)v$, but it is sufficient for our purposes. Here, m_E is the mass of the fourth-generation charged lepton, and $m_{\nu 4}$ is the Dirac mass of the fourth-generation neutrino. M_{M4} is the fourth-generation Majorana mass, and M_{Ml} is the Majorana mass of all other neutrinos.

The RG evolution of the light quark and lepton masses is irrelevant insofar as the coupling constants are small. We therefore will use the known values of the Dirac masses for these. For the light neutrinos we will follow Ref. 11 and assume that the neutrino Dirac masses are given by $m_\nu=\epsilon m_D$ (e.g., for the muon we assume $m_{\nu\mu}=\epsilon_\mu m_\mu$), where ϵ is an arbitrary parameter.

The physically observable neutrino masses are then

$$\begin{aligned}&m_{\nu R}=\tfrac{1}{2}(M_M+\sqrt{M_M^2+4m_D^2}) \ ,\\&m_{\nu L}=\tfrac{1}{2}(M_M-\sqrt{M_M^2+4m_D^2}) \ ,\end{aligned} \tag{3.38}$$

with analogous formulas holding for the first three generations. For the case of the light generations we may use the approximate forms

$$m_{\nu R}\approx M_l, \quad m_{\nu L}\approx\epsilon^2 m_E^2/M_l \ . \tag{3.39}$$

E. Scalar-boson interactions

The quartic interaction terms are found to satisfy the RG equations:

$$\mathcal{D}\lambda_1=12\lambda_1^2+2\lambda_3^2+4\lambda_1[\mathrm{tr}(E^\dagger E)+\mathrm{tr}(N^\dagger N)+3\,\mathrm{tr}(U^\dagger U)+3\,\mathrm{tr}(D^\dagger D)]-3\lambda_1(g_1^2+3g_2^2)+\tfrac{3}{2}g_2^4+\tfrac{3}{4}(g_1^2+g_2^2)^2$$
$$-4[\mathrm{tr}(E^\dagger EE^\dagger E)+\mathrm{tr}(N^\dagger NN^\dagger N)+3\,\mathrm{tr}(U^\dagger UU^\dagger U)+3\,\mathrm{tr}(D^\dagger DD^\dagger D)]\ , \quad (3.40)$$

$$\mathcal{D}\lambda_2=10\lambda_2^2+4\lambda_3^2+8\lambda_2\mathrm{tr}(\kappa^\dagger\kappa)-32\,\mathrm{tr}(\kappa^\dagger\kappa\kappa^\dagger\kappa)\ , \quad (3.41)$$

$$\mathcal{D}\lambda_3=6\lambda_1\lambda_3+4\lambda_2\lambda_3+2\lambda_3[\mathrm{tr}(E^\dagger E)+\mathrm{tr}(N^\dagger N)+3\,\mathrm{tr}(U^\dagger U)+3\,\mathrm{tr}(D^\dagger D)+4\,\mathrm{tr}(\kappa^\dagger\kappa)]-\tfrac{3}{2}\lambda_3(g_1^2+3g_2^2)-8\,\mathrm{tr}(N^\dagger N\kappa^\dagger\kappa)\ . \quad (3.42)$$

We integrate these equations with the compositeness conditions

$$\lambda_i\to\infty\,|_{\mu\to\Lambda} \quad (3.43)$$

(where in practice we take $\lambda_i=6$ for $\mu\to\Lambda$), and we integrate down from Λ to $\mu=100$ GeV. The effective potential at low energies takes the form

$$V_S=M_H^2(H^\dagger H)+M_\Phi^2(\Phi^\dagger\Phi)+\frac{\lambda_1}{2}(H^\dagger H)^2+\frac{\lambda_2}{2}(\Phi^\dagger\Phi)^2+\lambda_3 H^\dagger H\Phi^\dagger\Phi \quad (3.44)$$

and we demand a symmetry-breaking solution at low energies such that

$$H=\begin{bmatrix}H^0\\H^-\end{bmatrix}\ , \quad (3.45)$$

$$H^0=v_H+\frac{h}{\sqrt{2}}+\frac{ih'}{\sqrt{2}}\ , \quad (3.46)$$

$$\Phi=\beta v_H+\frac{\phi}{\sqrt{2}}+\frac{i\chi}{\sqrt{2}}\ , \quad (3.47)$$

where $v_H=175$ GeV. The fields H^+ and h' are the Nambu-Goldstone bosons which give mass to the W and Z bosons. The phase χ is an exactly massless Majoron in this model, and it exists as a physical state since we do not gauge the right-handed neutrino number. The potential is minimized for β and v_H:

$$M_H^2+v_H^2(\lambda_1+\tfrac{1}{2}\beta^2\lambda_3)=0\ , \quad (3.48)$$

$$M_\Phi^2+v_H^2(\beta^2\lambda_2+\tfrac{1}{2}\lambda_3)=0\ , \quad (3.49)$$

and we readily obtain the mass matrix for the Higgs boson h and the Higgs-Majoron ϕ. The states mix into physical mass matrix eigenstates given by

$$\Sigma_1=h^0\cos\alpha+\phi\sin\alpha\ , \quad (3.50)$$

$$\Sigma_2=\phi\cos\alpha-h^0\sin\alpha\ , \quad (3.51)$$

where the mixing angle α is determined by

$$\sin 2\alpha=\frac{2\beta^2\lambda_3}{S},\quad \cos 2\alpha=\frac{\lambda_1-\beta^2\lambda_2}{S}\ , \quad (3.52)$$

and where

$$S=\sqrt{(\lambda_1-\beta^2\lambda_2)^2+4\beta^2\lambda_3^2}\ . \quad (3.53)$$

The masses of the physical states are

$$M_{\Sigma_1}^2=\frac{v_H^2}{2}(\lambda_1+\beta^2\lambda_2+S)\ , \quad (3.54)$$

$$M_{\Sigma_2}^2=\frac{v_H^2}{2}(\lambda_1+\beta^2\lambda_2-S)\ . \quad (3.55)$$

The physical masses are real and hence the solution is stable provided that

$$\lambda_1,\lambda_2>0,\quad \lambda_1\lambda_2>\lambda_3\ . \quad (3.56)$$

F. NUMERICAL RESULTS

We now discuss the predictions of the model obtained by numerically integrating the RG equations supplemented with the composite boundary conditions. In Fig. 2 we show the evolution of the Higgs-Yukawa and Majorana-Yukawa coupling constants as a function of scale μ evolving downwards from a compositeness scale of $\Lambda=10^6$ GeV. We have multiplied all Dirac couplings by v_H, and Majorana couplings by $2v_\Phi=2v_H$ corresponding to $\beta=1$. The dashed lines represent the M_h, M_l, and $m_{\nu 4}$ as indicated, while $m_{\nu R}$ and $m_{\nu L}$ are the physically observable values as given in Eqs. (3.40) and (3.41). The purpose of this figure is to show the attraction from the large initial values down to the low-energy fixed points. In practice we used $\kappa_i=d_i=6$ at $\mu=\Lambda$, but the resulting low-energy values are very stable for a wide range of initial conditions. In practice the fourth-generation U and D quarks are degenerate to within a few GeV.

In Figs. 3–5 we show the fourth generation masses as a function of the scale of new physics, Λ, for various values of $\beta=v_\Phi/v_H$. We have indicated the lower limit $m_{\nu L}\geq M_Z/2$ and we thus see from the figures *that all schemes are ruled out for sufficiently large* Λ, for example, when $\beta=1.0$ we require $\Lambda\leq 10^3$ TeV. The schemes with $\beta>2.0$ are essentially the limiting case; for larger β one cannot escape the LEP-SLC neutrino-counting limit. In this case we see from Fig. 5 that $\Lambda<10$ TeV is required. Of course, in the small Λ limit our RG approximation is much less reliable. As can be seen from Fig. 3, for small values of β the upper bound on Λ becomes weaker.

In order to make definite predictions, we assume throughout that $m_t=130$ GeV. With the latter value of m_t it is unnecessary to consider the evolution of g_t, which we then treat as a constant independent of scale. All results are computed at the low-energy scale of $\mu=100$ GeV for simplicity. The largest uncertainties in these results arise from the uncertainty in the nonperturbative running of the Yukawa couplings at high energies. As

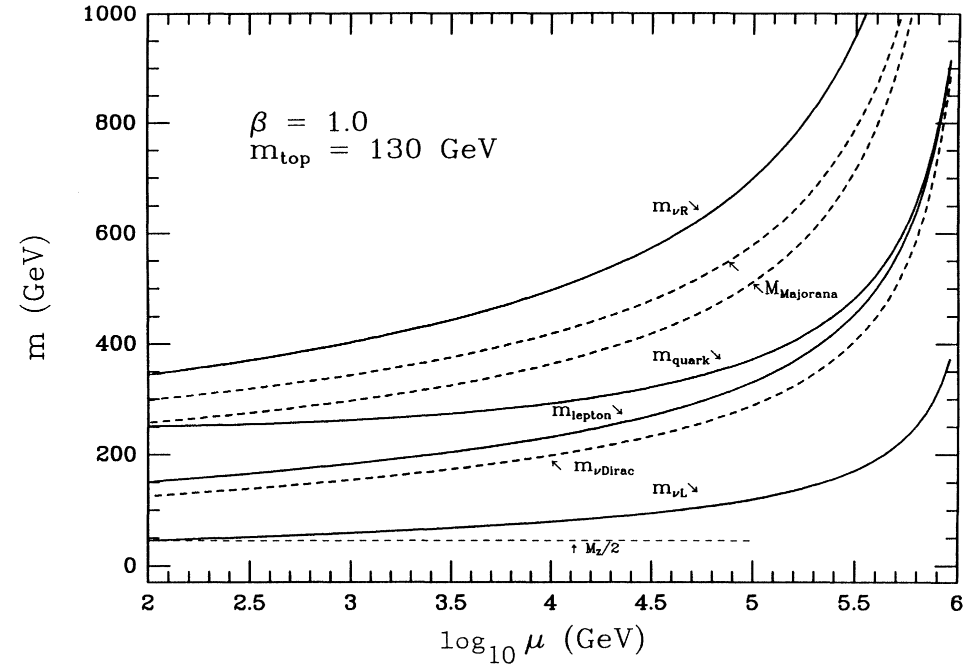

FIG. 2. Evolution of Higgs-Yukawa and Majorana-Yukawa coupling constants with scale μ from initial values $g_i=6$ at $\mu=\Lambda=10^6$ GeV to $\mu=100$ GeV. The couplings are translated into masses by multiplying by v_H as described in the text. The approach to the infrared fixed points is demonstrated. The larger Majorana masses apply to the light generations.

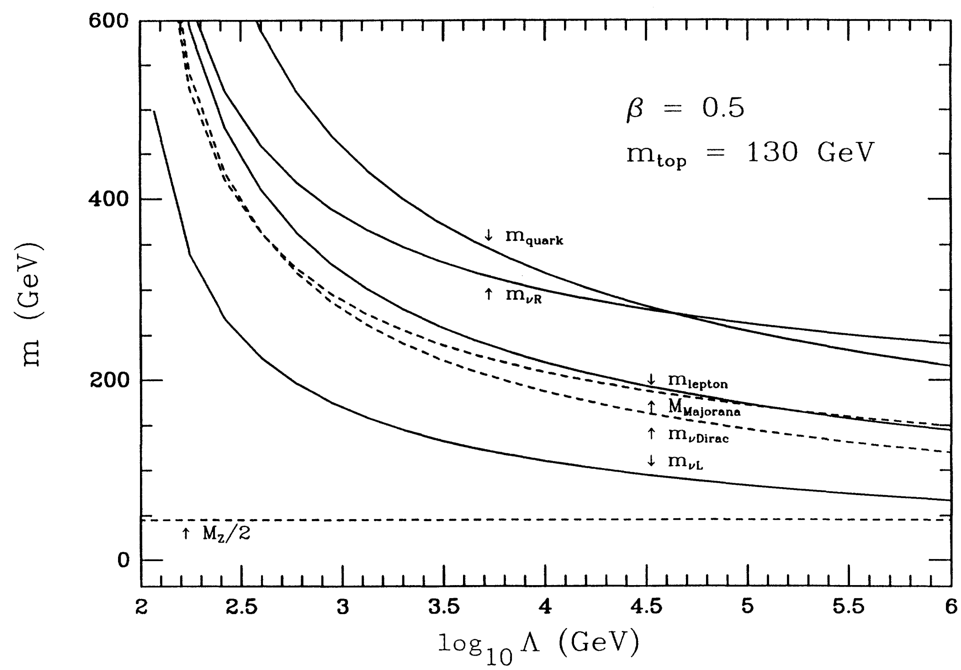

FIG. 3. Physical masses (solid lines) as predicted in the model as a function of composite scale Λ, for $\beta=v_\Phi/v_H=0.5$. The dashed lines indicate the heavy Majorana M_4 and neutrino-Dirac masses $m_{\nu 4}$ separately, before combining to form the physical combinations $m_{\nu R}$ and $m_{\nu L}$.

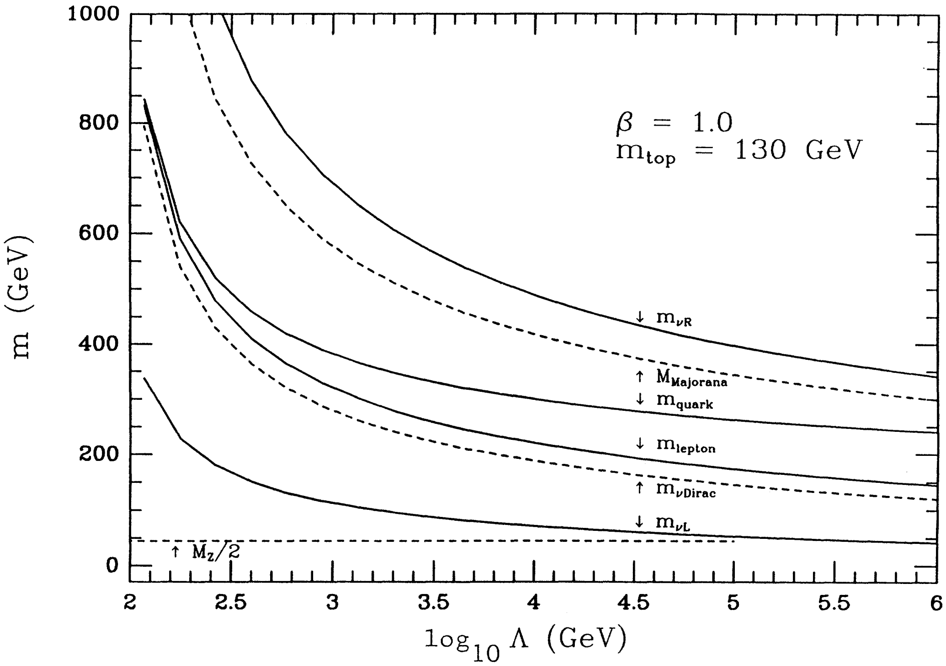

FIG. 4. As in Fig. 3, but for $\beta = v_\Phi / v_H = 1.0$.

discussed earlier, this is essentially an uncertainty in the precise high-energy boundary conditions.

In Fig. 6 we give the complete neutrino spectrum as a function of Λ for the case $\epsilon = 1$. Thus, the light neutrino masses as plotted are actually $m_\nu(\beta/\epsilon^2)$. Thus, for $\epsilon_\mu = 0.1$ one must multiply the plotted $m_{\nu\mu}$ by 0.01.

The evolution with scale μ of the quartic coupling constants is shown in Fig. 7 where we use the compositeness boundary conditions imposed at $\Lambda = 10^6$ GeV. Here we consider two independent sets of boundary conditions to probe the sensitivity. The solid lines show all Higgs- and Majorana-Yukawa couplings g_i are set $g_i = 6$, and $\lambda_i = 12$ for $\mu = \Lambda$; the dashed lines show the boundary conditions $g_i = 2$ and $\lambda_i = 6$ as $\mu = \Lambda$. The low-energy results con-

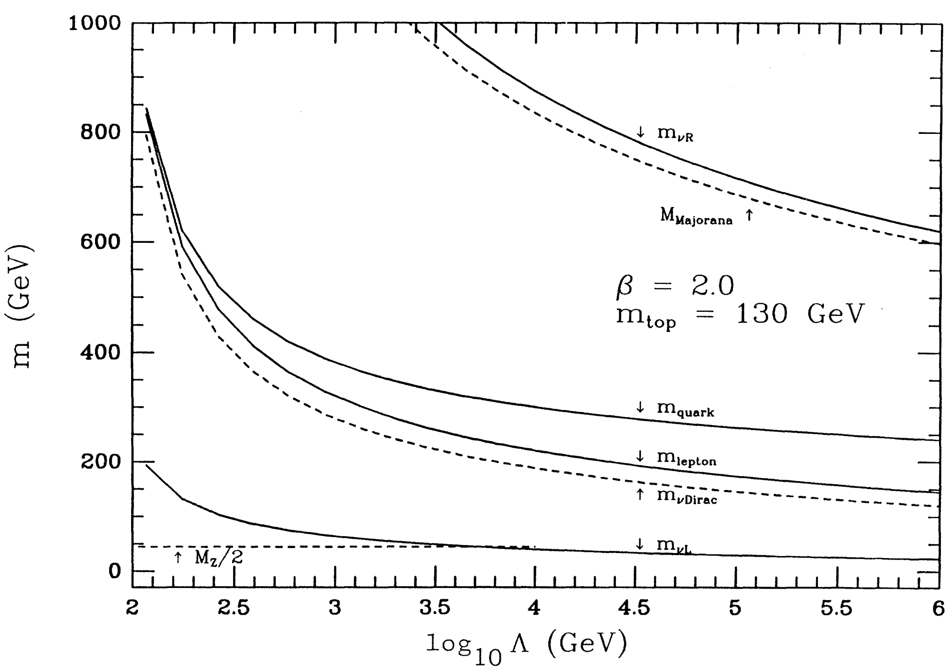

FIG. 5. As in Fig. 3, but for $\beta = v_\Phi / v_H = 2.0$.

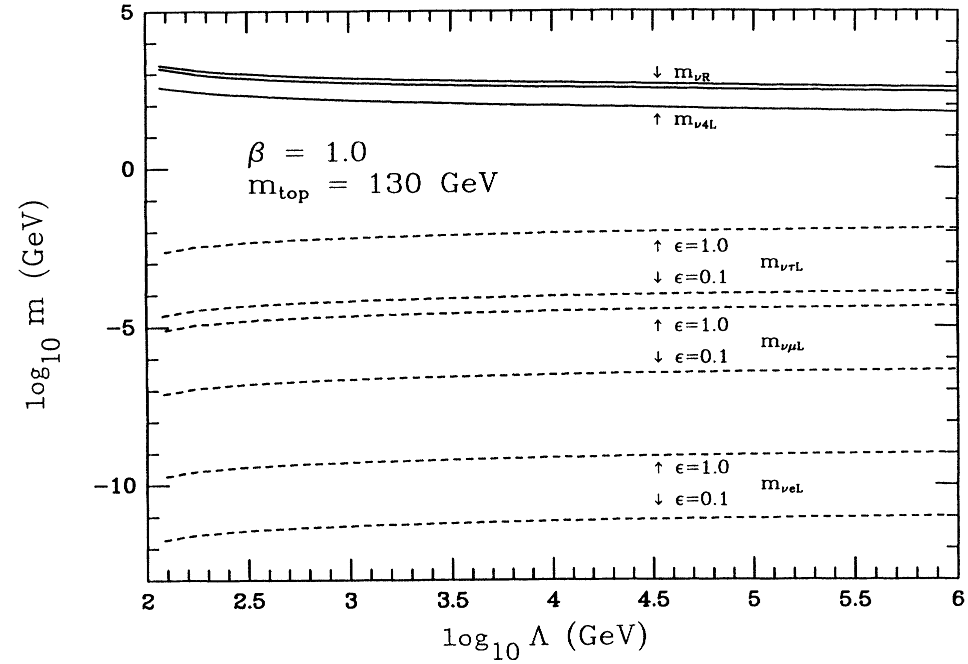

FIG. 6. Physical light neutrino masses (solid) as predicted in the model as a function of composite scale Λ for $\beta = v_\Phi / v_H = 1.0$ and we plot for the light masses the range $0.1 \leq \epsilon \leq 1.0$.

verge on fairly universal fixed points over a large range of initial conditions. Moreover, we see that in general the coupling constant λ_3 is very small compared to λ_1 and λ_2. This leads to the simplification for the masses,

$$M_{\Sigma_1} \approx v\sqrt{\lambda_1}\ , \tag{3.57}$$

$$M_{\Sigma_2} \approx \beta v \sqrt{\lambda_2}\ , \tag{3.58}$$

and the mixing between the two states is generally small,

$$\alpha \approx \frac{\beta^2 \lambda_3}{\lambda_1 - \beta^2 \lambda_2}\ , \tag{3.59}$$

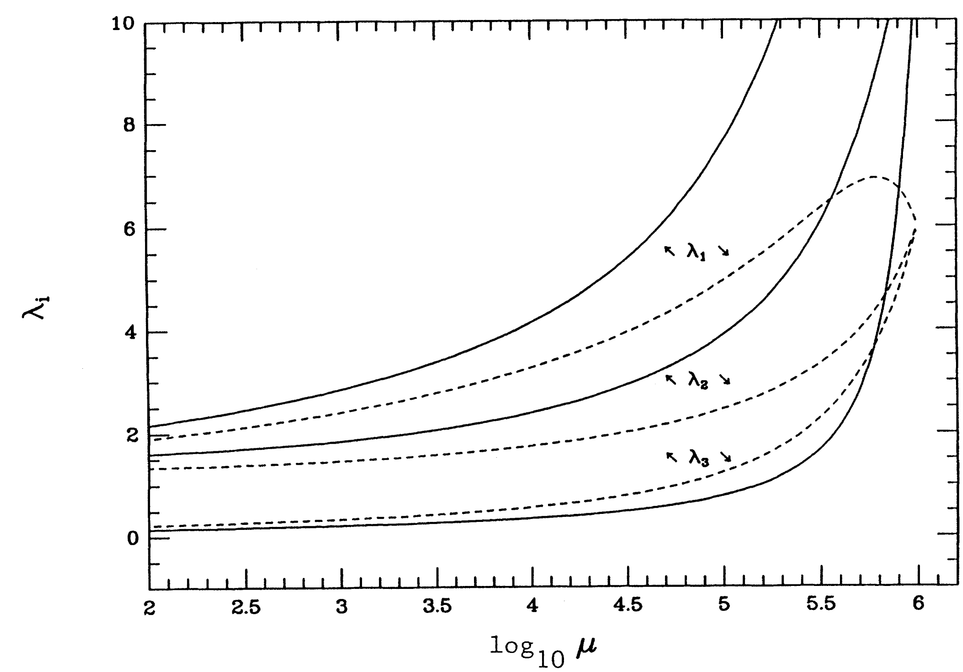

FIG. 7. Evolution of scalar quartic coupling constants with scale μ from initial values (solid lines) $g_i = 6$ and $\lambda_i = 12$ at $\mu = \Lambda = 10^6$ GeV; (dashed lines) $g_i = 2$ and $\lambda_i = 6$ at $\mu = \Lambda = 10^6$ GeV. The universality of the infrared fixed points is demonstrated. λ_3 is always driven small.

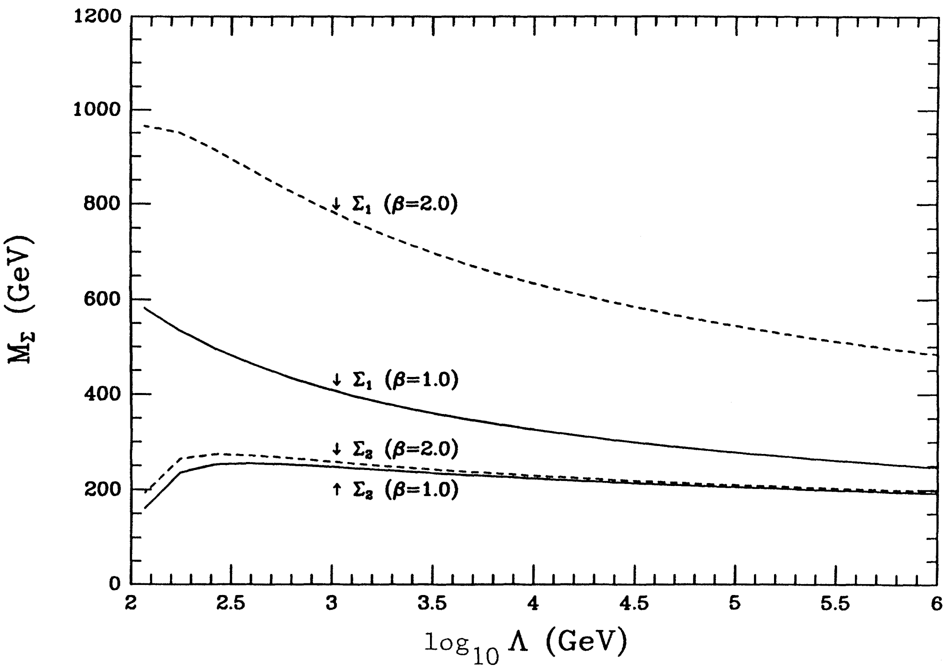

FIG. 8. Physical spin-zero boson masses (solid lines) as predicted in the model as a function of composite scale Λ, for (solid) $\beta=v_\Phi/v_H=1.0$, (dashed) $\beta=v_\Phi/v_H=1.0$.

with the exception of the "resonant" case when $\lambda_1-\beta^2\lambda_2\approx 0$.

In Fig. 8 we plot the masses of the physical scalars, M_{Σ_i}, as a function of the compositeness scale Λ. Here again we probe the sensitivity to the precise boundary conditions by choosing $g_i=6$ and $\lambda_i=12$ for $\mu=\Lambda$ (solid); $g_i=2$ and $\lambda_i=6$ as $\mu=\Lambda$ (dashed). The low-energy results are fairly universal until the RG "running time" becomes reduced for small Λ.

IV. CONCLUSIONS

We have given an analysis of the dynamical aspects of a low-energy theory in which the electroweak interactions are broken by condensates of fourth-generation quarks and leptons. Our model appeals to a seesaw mechanism in which the Majorana mass scale is generated by a right-handed neutrino condensate, and the Dirac masses of all neutrinos are assumed to be of the order their charged lepton counterparts. The seesaw mechanism is invoked principally to suppress the light neutrino masses, while the fourth-generation mass scale is chosen to be sufficiently heavy to evade the LEP-SLC neutrino counting limit. We view this as a natural mechanism for avoiding the LEP and SLC neutrino counting limits. We emphasize that in such a scheme there is an *upper limit* to the scale Λ of the new physics, as is evident from Figs. 3–5. Taking Λ too large brings the left-handed fourth-generation neutrino mass down, and $\Lambda \lesssim 10$ TeV is favored.

The neutrino phenomenology of such a model has not been discussed here in detail, but is expected to be fertile. This requires some further assumptions about mixing angles which we cannot predict in the model. Some of the results have been anticipated in Refs. 11 and 12 in which it is pointed out that the neutrino masses are expected to be near to their experimental upper limits. To avoid difficulties with cosmological limits it appears essential that heavy neutrinos decay, not to final states involving photons, but rather via the "invisible" modes involving the Majoron, e.g., $\nu'\to\nu+\chi$. This would appear to us, based upon simple estimates, to be the predominant mode for the Majoron decay constant in the range allowed for this model, $f\sim\Lambda$ (see also Ref. 13). Electroweak phenomenological constraints have also not been considered here in detail. In fact, the "ρ-parameter" constraint should be fairly restrictive, since the top-quark mass is already quite sizable. We have used the central value favored by global parameter analyses of $m_t=130$ GeV in this analysis. The 90% C.L. upper limit is of order ~ 192 GeV, so at this level we can probably tolerate a charged lepton of order $m_l \lesssim \sqrt{3\times(192^2-130^2)}\sim 240$ GeV, which is a comfortable upper limit in the present model, which predicts $m_{\text{lepton}}\sim 182$ GeV for $\Lambda=10^4$ GeV and $\beta=1.0$.

We note that the results presented here are somewhat more general than the specific model involving compositeness conditions which leads to them. These correspond roughly to the "triviality" bounds of the masses of fourth-generation leptons and quarks if the theory is considered to be valid up to the scale Λ. Indeed, these are the natural internal constraints on large neutrino masses in the standard model. If the standard model is a valid description up to some scale Λ, then the Dirac masses cannot be arbitrarily large. The essential idea is that no coupling constant of the standard-model Lagrangian can be permitted to diverge on a scale $\mu \leq \Lambda$. Moreover, if a vacuum expectation value giving rise to the Majorana masses is chosen to be near the weak scale, then there will be a triviality bound for the Majorana masses as well. These triviality bounds follow from the RG equations, and are related to RG fixed points and critical renormalization-group trajectories.[4]

Perhaps the most remarkable feature of this model is that Λ is bounded from above by the neutrino-counting limits. The $\Lambda \sim 1-10$ TeV scale is also encouraging for the discovery of a rich new dynamics in the not-so-distant (SSC?) future. This new dynamics must encompass the generation of all quark and lepton mass scales, so a model of this sort is most encouraging for eventually understanding the origin of quark and lepton mass within the next 20 years. We expect a number of other signatures that have not been discussed here, such as the occurrence of composite-vector-meson states, the analogues of the ρ, with masses of order Λ, etc. The model also suggests that neutrino phenomenology will be a rich and rewarding enterprise in future fixed-target experiments since the mass scales for the light neutrinos are tantalizingly close to their experimental upper limits.

The theoretical challenge is to construct the SBGHT model that most closely realizes the low-energy effective Lagrangian we have explored here. It is not clear that this is a simple task. For example, the issue of flavor-changing neutral processes must be faced. On the other hand, the phenomenological situation is likely to evolve rapidly over the next few years. While the simpler top condensate scheme is still potentially viable, we have proposed this alternative in the hopes that by lowering Λ, a more promising natural alternative may exist. The fourth generation is definitely not ruled out, the neutrino situation is perfectly natural and phenomenologically acceptable, and the fourth generation offers an obvious dynamical possibility for breaking the electroweak symmetries.

ACKNOWLEDGMENTS

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APPENDIX A: SCHWINGER-DYSON ANALYSIS

1. Gap equation

To treat the broken-symmetry phase of the model (2.1), it is convenient to rewrite it in terms of an auxiliary static scalar field Φ:

$$\mathcal{L}' = \bar{\nu}_{Rj} i\partial\!\!\!/ \nu_{Rj} - M_0^2 \Phi^\dagger \Phi + (\Phi \bar{\nu}^c_{Rj} \nu_{Rf} + \text{H.c.}) \ . \quad \text{(A1)}$$

The coefficient of the Yukawa term is fixed by appropriately scaling Φ. The field Φ has no kinetic term, so we can explicitly integrate out Φ to recover the model of Eq. (2.1), with

$$G_0 = \frac{1}{M_0^2} \ . \quad \text{(A2)}$$

In terms of $\mathcal{L}'$, the gap equation for the fermion propagator is obtained by minimizing the effective potential for Φ. This is equivalent to shifting Φ,

$$\Phi(x) = \phi(x) - \frac{m}{2} \ , \quad \text{(A3)}$$

and determining m by demanding that the sum of the tadpole diagrams with one external ϕ line vanish.[19] Note that, for nonzero m, the neutrino field has a Majorana mass term

$$-\frac{m}{2}(\bar{\nu}^c_{Rj}\nu_{Rj} + \text{H.c.}) \ . \quad \text{(A4)}$$

This shows that there is a condensate of the form (2.2).

To evaluate Feynman diagrams, we rewrite Eq. (A1) in terms of a Majorana field χ, defined by

$$\chi_j = \nu_{Rj} + \nu^c_{Rj} \ . \quad \text{(A5)}$$

Then

$$\mathcal{L}' = \tfrac{1}{2}\bar{\chi}_j i\partial\!\!\!/ \chi_j - M_0^2 \Phi^\dagger \Phi + (\Phi \chi_j P_R \chi_j + \text{H.c.}) \ , \quad \text{(A6)}$$

where $P_R = \frac{1}{2}(1+\gamma_5)$. The field χ satisfies the "Majorana condition"

$$\chi^c = \nu^c_R + (\nu^c_R)^c = \chi \ . \quad \text{(A7)}$$

The Feynman rules for Majorana fields have been given recently in the literature.[20]

The only two diagrams which contribute to the ϕ tadpole in the large-N limit are shown in Fig. 1. The one-loop diagram gives

$$\int \frac{d^4k}{(2\pi)^4} \left[-\frac{N}{2} \text{tr} \right] iP_R \frac{i}{k\!\!\!/ - m} = Nm \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2} \ . \quad \text{(A8)}$$

Demanding that this contribution cancel the tree-level contribution gives

$$\frac{iM_0^2 m}{2} + mN \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2-m^2} = 0 \ . \quad \text{(A9)}$$

For $m \neq 0$, we can write this as

$$\begin{aligned} \frac{1}{G_0} &= 2iN \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2-m^2} \\ &= \frac{N}{8\pi^2} \left[\Lambda^2 - m^2 \ln \frac{\Lambda^2}{m^2} \right] \ . \end{aligned} \quad \text{(A10)}$$

This is the gap equation for the theory defined by Eq. (2.1).

We see that in order to have $m \neq 0$, we require

$$G_0 > G_{\text{crit}} = \frac{8\pi^2}{N\Lambda^2} \ . \quad \text{(A11)}$$

If we want to maintain the hierarchy $m \ll \Lambda$, the gap equation shows that G_0 must be adjusted to be very close to $G_{\rm crit}$. (We note that in the large-N limit, there are no corrections to the neutrino propagator in the shifted theory, so that m is the physical mass of the right-handed neutrino.) In the formalism used here it is clear that this fine-tuning problem is *exactly* the same as the fine-tuning problem for scalar fields. We will see that all the quadratic divergences which appear subsequently can be canceled by imposing the gap equation. Thus, once the gap equation is fine-tuned, there is no further fine-tuning in the theory. This is the same situation as in scalar field theories in the broken-symmetry phase, where the quadratic divergences can be isolated in the minimization of the effective potential.

2. Collective modes

The auxilliary field ϕ was introduced above as a trick to simplify the calculations, but we will see that, in fact, ϕ is a physical propagating field at low energies. The signal for this is the appearance of poles in the two-point function of ϕ. These poles are physically manifested in right-handed neutrino scattering amplitudes, where they appear as resonances.

Note that under U(1)-flavor symmetry,

$$\phi \mapsto e^{-2i\theta}\phi \ . \tag{A12}$$

In terms of real and imaginary components of ϕ,

$$\phi = \frac{1}{\sqrt{2}}(\phi + i\chi) \ , \tag{A13}$$

we have, to first order in θ,

$$\phi \mapsto \phi \ , \tag{A14}$$

$$\chi \mapsto \chi - 2i\theta \ . \tag{A15}$$

We see that exciting the field χ is equivalent to performing a local U(1) transformation, suggesting that χ is the Nambu-Goldstone mode associated with broken-U(1) symmetry. We now show that this is indeed the case.

In the large-N limit, the self-energy of χ is given by the two diagrams of Fig. 2. Both diagrams give the same contribution, and their sum is

$$\begin{aligned} -i\Sigma_\chi(p) &= 2\int \frac{d^4k}{(2\pi)^4}\left[-\frac{N}{2}\mathrm{tr}\right]\left[-\frac{1}{\sqrt{2}}\gamma_5\right]\frac{i}{\not k - m}\left[-\frac{1}{\sqrt{2}}\gamma_5\right]\frac{i}{\not k + \not p - m} \\ &= -2N\int \frac{d^4k}{(2\pi)^4}\frac{k\cdot(k+p)-m^2}{(k^2-m^2)[(k+p)^2-m^2]} \ . \end{aligned} \tag{A16}$$

Shifting the integration momentum to isolate the quadratically divergent part of this expression, we have

$$-i\Sigma_\chi(p) = Np^2\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m^2)[(k+p)^2-m^2]} + N\int\frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2-m^2}+\frac{1}{(k+p)^2-m^2}\right] \ . \tag{A17}$$

Assuming $p^2 \ll \Lambda^2$, the last two terms can be rewritten using the gap equation (A10), and we get

$$\Sigma_\chi(p) = -p^2 A(p) - M_0^2 \ , \tag{A18}$$

where

$$\begin{aligned} A(p) &= -iN\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m^2)[(k+p)^2-m^2]} \\ &= \frac{N}{16\pi^2}\int_0^1 dx \ln\frac{\Lambda^2}{m^2-x(1-x)p^2} \ . \end{aligned} \tag{A19}$$

Note that the quadratic divergence in the self-energy has been completely canceled by imposing the gap equation. The exact χ propagator in the large-N limit is then

$$\Delta_\chi(p) = \frac{i}{-M_0^2 - \Sigma_\chi(p)} = \frac{iA^{-1}(p)}{p^2} \ . \tag{A20}$$

From (A19), we see that $A(p^2=0)\neq 0$, so $\Delta_\chi(p)$ has a pole at $p^2=0$. This shows that χ is a massless excitation and can be identified as the Nambu-Goldstone mode.

We can now repeat the same steps for ϕ. We obtain

$$\begin{aligned} -i\Sigma_\phi(p) &= 2\int\frac{d^4k}{(2\pi)^4}\left[-\frac{N}{2}\mathrm{tr}\right] \\ &\quad \times \frac{i}{\sqrt{2}}\frac{i}{\not k - m}\frac{i}{\sqrt{2}}\frac{i}{\not k + \not p - m} \\ &= i(p^2-4m^2)A(p) + iM_0^2 \ , \end{aligned} \tag{A21}$$

giving the ϕ propagator

$$\Delta_\phi(p) = \frac{iA^{-1}(p)}{p^2-4m^2} \ . \tag{A22}$$

We see that the ϕ mode has a mass $2m$. One might think that this is a loosely bound state of $\bar\nu_R\nu_R$, since it apparently has vanishing binding energy. However, we emphasize that this is *not* a nonrelativistic bound state, and normal intuition does not apply.

The results derived in this section are exact in the large-N limit, and are therefore completely equivalent to the more conventional bubble-sum treatment. However, we expect that there will be significant corrections to the large-N results for small N.

APPENDIX B: SPINOR CONVENTIONS

We follow the conventions of Bjorken and Drell,[21] with all fields viewed as operators, so that

$$\psi\chi = -\chi\psi \,, \tag{B1}$$

$$(\psi\chi)^\dagger = \chi^\dagger\psi^\dagger \,. \tag{B2}$$

The charge-conjugation matrix is given by

$$C = i\gamma^2\gamma^0 = -C^{-1} = -C^\dagger \tag{B3}$$

and satisfies the identity

$$C^\dagger\gamma_\mu C = -\gamma_\mu^T \,. \tag{B4}$$

Charge-conjugated spinors are defined by

$$\psi^c = C\bar{\psi}^T, \quad (\bar{\psi})^c = \psi^T C^\dagger \,. \tag{B5}$$

The phase of C has been chosen so that $(\psi^c)^c = \psi$. Note that the order of charge conjugation with respect to Dirac conjugation is important, since $\overline{(\psi^c)} = -(\bar{\psi})^c$. We will use the notation

$$\bar{\psi}^c \equiv \overline{(\psi^c)} \,. \tag{B6}$$

The chiral properties of charge-conjugate spinors are

$$(\psi^c)_L \equiv \tfrac{1}{2}(1-\gamma_5)\psi^c = (\psi_R)^c \,, \tag{B7}$$

$$(\psi^c)_R \equiv \tfrac{1}{2}(1+\gamma_5)\psi^c = (\psi_L)^c \,. \tag{B8}$$

We will use the notation

$$\psi_L^c \equiv (\psi_L)^c \,, \tag{B9}$$

$$\bar{\psi}_L^c \equiv \overline{(\psi_L^c)} \,, \tag{B10}$$

etc. The following identities are useful for rewriting Lagrangians containing charge-conjugated spinors:

$$\bar{\psi}^c\chi^c = \bar{\chi}\psi \,, \tag{B11}$$

$$\bar{\psi}^c\gamma_\mu\chi^c = -\bar{\chi}\gamma_\mu\psi \,. \tag{B12}$$

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