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COMMENTS

Comments are short papers which comment on papers of other authors previously published in the **Physical Review**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "Quark-meson coupling model for baryon wave functions and properties"

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We examine the Leonard-Gerace procedure for computing mesonic admixtures to baryonic wave functions.

Recently, Leonard and Gerace[1] (LG) reported a systematic study of the spectrum of low-lying baryons, motivated by the nonrelativistic quark model (NRQM) but including corrections associated with the coupling of the pseudo-Goldstone bosons π, K, and η. We are sympathetic with the authors' desire to understand how the phenomenological success of models such as those of Isgur and Karl[2] and Forsyth and Cutkosky[3] have been achieved without including pion degrees of freedom. Removing the effect of pseudoscalar-meson coupling to get "bare" masses that could then be used in a NRQM fit seems a sensible first approach to a complicated problem.

These authors claim to use a baryon-meson coupling interaction ($H_{\rm int}$) to mix pseudoscalar mesons into the physical baryon wave functions. Among their important results we note (i) the meson contributions to the baryon masses are typically 100 MeV or less, (ii) the ΔN mass splitting is not related to pion coupling, (iii) K and η mesons are as important in the baryon spectrum as pions, and (iv) there exist a number of extra low-energy resonances. Unfortunately we find significant problems with the procedures used to reach each of these conclusions. Because of the importance we attach to the authors' original aims we feel it is important to explain our objections clearly. We hope that this will serve to stimulate a further systematic investigation of the role of the pseudo-Goldstone bosons π, η, and K in hadronic spectroscopy.

We shall begin to illustrate our objections by treating the case of the nucleon in detail. LG correctly assert that the meson-baryon coupling will mix $N\pi$ and $\Delta\pi$ (and $K\Lambda$, $K\Sigma$, etc.) components into the wave function of the physical nucleon. However, as their Eq. (13) clearly shows, they do not get a Yukawa distribution. Instead the continuum of $N\pi$ states of different relative momentum is replaced by a single, effective πN channel with average energy $\bar{\omega}$ [Eq. (12), $\bar{\omega}=1301$ MeV for the $N\pi$ channel] and a Gaussian wave function ($\sim e^{-\beta^2 r^2/2}$ with range parameter β). The wave function for the $\Delta\pi$ component involves the *same* range parameter. Clearly this is incorrect. In first-order perturbation theory the momentum distributions of these components should be

$$\phi_{N\pi}^{(N)}(\mathbf{k}) \sim \frac{v_{NN}(k^2)}{(2\omega_k)} \frac{\boldsymbol{\sigma}\cdot\mathbf{k}}{\omega_k} \tag{1}$$

and

$$\phi_{\Delta\pi}^{(N)}(\mathbf{k}) \sim \frac{v_{N\Delta}(k^2)\boldsymbol{\sigma}\cdot\mathbf{k}}{(2\omega_k)(\omega_k + m_\Delta^{(0)} - m_N^{(0)})} , \tag{2}$$

where $v_{NN}(k^2)$ and $v_{N\Delta}(k^2)$ are the appropriate baryon-meson form factors. The latter has a much shorter range in coordinate space than the former, and both extend much further than a Gaussian at large r.

That these observations are not trivial is illustrated by the following. The β parameter is crucial in determining the energy shifts of the nucleon due to its coupling to mesons. But, LG determine β from the neutron charge radius. Now the dominant Clebsch-Gordan coefficients are those for $n\rightarrow p\pi^-$ (for the $N\pi$ channel) and $n\rightarrow\Delta^-\pi^+$ (for the $\Delta\pi$ channel). Clearly these two processes tend to cancel when we calculate $\langle r^2\rangle_n$, and this cancellation is overestimated if the $\Delta^-\pi^+$ wave function is (wrongly) given the same range as that for $p\pi^-$. This leads to a smaller value of β than would otherwise be required to fit $\langle r^2\rangle_n$. The fact that the large-r wave function is Gaussian rather than a Yukawa distribution also favors small values of β.

On the other hand, a small value of β suppresses pions of high momentum and leads to a much smaller (attractive) nucleon self-energy than one would otherwise expect. Indeed we have argued elsewhere[4,5] that because the πNN and $\pi N\Delta$ coupling constants are well known, and because one has quite strong constraints on the associated vertex functions, the N self-energy associated with its pion cloud is fairly model independent at about $(-300, -400)$ MeV.

This point deserves further explanation. Not too many calculations of hadronic spectra include big pion corrections. We argue that these are necessary. Since the coupling constants are known, the main uncertainty in computing such effects is the pion-baryon form factor. The use of typical ones from nucleon-nucleon potentials, e.g., Ref. 6, would lead to corrections more significant than -400 MeV. However, the proton's weak axial form factor is very closely related to the pion-baryon form factors in a huge class of models.[7] The observed axial form factor is relatively soft,[8] with the axial size not very different from the electromagnetic size. This indicates that the pion-baryon form factor is soft. In addition, muon-proton deep-inelastic scattering (DIS) provides relevant information.[9] This is because the incident photon can be absorbed by a pion, leading to a preponderence of $\bar{u}$ and $\bar{d}$ quarks over $\bar{s}$ quarks. (The effects of photon absorption by kaons are small.) Then $(\bar{u}+\bar{d})/2>\bar{s}$. But the distribution of antiquarks is believed to be relatively symmetric with respect to the flavor of the quarks. This limits the size of the pionic effects. If one uses a monopole parametrization the form factor is given by $v_{NN}(q)=(\Lambda^2-m_\pi^2)/(\Lambda^2-q^2)$, where m_π is the pion mass, and q is the pion four-momentum. Values of Λ of about 800 MeV are consistent with DIS and the axial form-factor data.[9] Such values lead to self-energy shifts on the order of $(-300,-400)$ MeV.

For these reasons we believe that the result claimed in Table III of LG, namely, -92 MeV, is untenable.

Next we consider item (ii), the $N\Delta$ mass splitting. The discussion of the Δ self-energy raises two questions of quite general importance. First there is the determination of the β parameter for the $\Delta\to N\pi$ vertex—that is, for the coupling to the open channel. The determination of β by LG is based entirely on the vertex $\Delta\to N\pi$. Their calculation is better than most because the renormalization of this vertex is included, whereas it is ofter ignored in other work. On the other hand, the imaginary contributions to the width through processes such as that shown in Fig. 1 are omitted. This figure shows an interference effect between a crossed Born and Δ term. This is complex due to the vanishing of the energy denominator associated with the vertical dashed line. The LG calculation of width does not include a renormalized $\Delta N\pi$ vertex but, the imaginary part of Fig. 1 is missing. For the Δ resonance such processes can account for as much as 10–20 % of the width.[4] Omitting them necessarily leads to errors in β of this order.

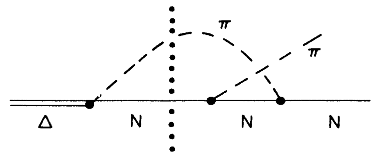

FIG. 1. An additional contribution to the width of the Δ is obtained from cross terms such as this. [The intermediate $N\pi$ state (vertical dots) can be on shell.]

Although this problem is bad enough, the second is much worse for resonant states with an open decay channel. That is, the real part of the shift in the Δ mass caused by its coupling to πN is determined by a principal-value integral.[4] This necessarily involves a cancellation from above and below the resonance and a consequently reduced downward shift. The shell-model approach used by LG is particularly unreliable for a case such as this. In particular, there is no principal-value integral, and the bare mass difference $m_\Delta^{(0)}-m_N^{(0)}=376$ MeV is near to $\varpi-m_N^0=478$ MeV (for the $N\pi$ component of the Δ wave function). Thus the attractive self-energy can be very large indeed. We believe that this explains the erroneous result in Table III of LG that the Δ mass is shifted down by more than that of the nucleon. In fact, our knowledge of πNN, $\pi N\Delta$, and $\pi\Delta\Delta$ coupling constants and form factors leads to the relatively model-independent conclusion that, between 100 and 200 MeV of the observed mass, splitting of N and Δ comes from the pion coupling.[4,5]

It is worthwhile to illustrate these remarks with an explicit example. Consider the dressed $\Delta, |\tilde{\Delta}\rangle$, as the solution of a two-channel problem—a bare state $|\Delta\rangle$ and a pion-nucleon state $|Nk\rangle$ (where k includes the isospin and momentum). In first-order perturbation theory

$$|\tilde{\Delta}\rangle=\sqrt{Z_\Delta}\left[|\Delta\rangle+\int d^3k\frac{|Nk\rangle\langle Nk|H_{\text{int}}|\Delta\rangle}{m_\Delta^{(0)}-(m^{(0)^2}+k^2)^{1/2}-(m_\pi^2+k^2)^{1/2}+i\varepsilon}\right]. \tag{3}$$

Thus the physical mass of the Δ is

$$m_\Delta=m_\Delta^{(0)}+\int d^3k\frac{|\langle Nk|H_{\text{int}}|\Delta\rangle|^2}{m_\Delta^{(0)}-(m_N^{(0)}+k^2)^{1/2}-(m_\pi^2+k^2)^{1/2}+i\varepsilon}, \tag{4}$$

to second order in H_{int}. We stress that the integral involves a principal-value piece and a δ-function piece (the width).

If instead we applied the method of LG to the same order the Δ would have two discrete components, the bare Δ and a single, discrete state $|N\pi\rangle$. Equation (3) is then replaced by

$$|\tilde{\Delta}\rangle=\sqrt{Z_\Delta}\left[|\Delta\rangle+\frac{\langle N\pi|H_{\text{int}}|\Delta\rangle}{m_\Delta^{(0)}-(m_N^{(0)^2}+\underline{k}^2)^{1/2}-(m_\pi^2+\underline{k}^2)^{1/2}}\right], \tag{5}$$

which is the first-order solution of the 2×2 matrix problem with Hamiltonian $\hat{H}$:

$$\hat{H}=\begin{bmatrix} m_{\Delta}^{(0)} & \langle \Delta|H_{\rm int}|N\pi\rangle \\ \langle N\pi|H_{\rm int}|\Delta\rangle & (m_N^{(0)^2}+\underline{k}^2)^{1/2}+(m_\pi^2+\underline{k}^2)^{1/2} \end{bmatrix} . \tag{6}$$

The underlines denote an average in the state $|N\pi\rangle$, and

$$(m_N^{(0)^2}+\underline{k}^2)^{1/2}+m_\pi^2+\underline{k}^2)^{1/2}$$

was called $\bar{\omega}$ above.

To illustrate the difference between the two procedures, we calculated the second-order correction to the real part of the Δ mass calculated from the principal value integral in Eq. (4). With the parameters of LG, the result is ten times smaller than the second-order correction which they would obtain (for $\bar{\omega}=m_N^{(0)}+478$ MeV).

We do not pretend that these comments make life any easier. One really needs to consider the P_{33} resonance (and any higher-energy state) as a coupled-channel scattering problem. This degree of effort is a strong deterrent to the systematic study of a wide range of particles. Even if one were to ignore background contributions (such as the Chew-Low contribution mentioned above) the self-energy contribution from an open channel must involve a principal-value integral which does not arise in the shell-model treatment.

This same criticism of the treatment of open channels leads us to question the conclusion [(iv) above] that there must be a large number of extra, low-lying resonances. It will require a much more sophisticated study to see whether this conclusion has any validity. Indeed, in the cases studied by LG no meson-baryon interaction was introduced other than the coupling to the particular bare state under study. In such a case the result can only be a dressed resonance and a discrete representation of the orthogonal continuum in each meson-baryon channel. There are *no* other resonance states.

We are much more sympathetic to the claim [item (iii) above] that the $\bar{K}$ may play a significant role in the wave function of the hyperons. For the $\Lambda(1116)$ for example, LG find an amplitude of -0.171 for it to look like $\bar{K}N$, compared with $0.197\Sigma\pi$, $0.172\Sigma^*\pi$, and 0.945 for a three-quark state. On the other hand, one must be very suspicious of the $S=-1$ resonances above 1600 MeV, where the bare mass difference $m_R^{(0)}-m_N^{(0)}$ is of order the mean $\bar{K}$ kinetic energy. There too a proper evaluation of the relevant principal-value integral could yield results quite different from the naive shell-model approach.

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[1]W. J. Leonard and W. J. Gerace, Phys. Rev. D **41**, 924 (1990).

[2]N. Isgur and G. Karl, Phys. Lett. **72B**, 109 (1977); Phys. Rev. D **18**, 4187 (1978); **19**, 2653 (1979); R. Koniuk and N. Isgur, *ibid.* **21**, 1868 (1979); **24**, 845 (1980); S. Capstick and N. Isgur, *ibid.* **34**, 2809 (1986).

[3]C. P. Forsyth and R. E. Cutkosky, Z. Phys. C **18**, 219 (1983).

[4]S. Théberge, A. W. Thomas, and G. A. Miller, Phys. Rev. D **22**, 2838 (1980); **23**, 2106(E) (1981).

[5]A. W. Thomas, Nucl. Phys. A (to be published).

[6]R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).

[7]H. Primakoff, in *Nuclear and Particle Physics at Intermediate Energies,* edited by J. B. Warren (Plenum, New York, 1976); P. A. M. Guichon, G. A. Miller, and A. W. Thomas, Phys. Lett. **124B**, 109 (1983); R. Tegen and W. Weise, Z. Phys. A **314**, 357 (1983).

[8]A. del Guerra *et al.*, Nucl. Phys. **B107**, 65 (1976); E. Amaldi, S. Fubini, and G. Furlan (Springer Tracts in Modern Physics, Vol. 83) (Springer, Berlin 1979); L. A. Ahrens *et al.*, Phys. Rev. D **35**, 785 (1987).

[9]A. W. Thomas, Phys. Lett. **126B** 97 (1983); L. L. Frankfurt, L. Mankiewicz, and M. I. Strikman, Z. Phys. A **334**, 343 (1989); S. Kumano, Phys. Rev. D. **43**, 59 (1991). The analysis of Frankfurt, Mankiewicz, and Strikman prefers somewhat softer form factors than the other two listed here, but Kumano has noted that more recent data allow Λ to be 800 MeV or more.