Higher-order moments of multiplicity distributions in high-energy nondiffractive pp and $p\bar{p}$ collisions and the quantum coherent production mechanism

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The physical mechanism of multihadron production in high-energy nondiffractive collisions is discussed based on the quantum coherent state. A remarkable agreement is obtained between the theoretical results and experimental data in respect of the high-order normalized moments at c.m. energies from 11.5 to 900 GeV for the full phase space. Predictions are made for Fermilab Tevatron, CERN Large Hadron Collider, and Superconducting Super Collider energies by using an empirical formula for $\langle n_{\rm ND} \rangle$ and our relation involving the parameter λ.

In recent years, an increasing interest in strong-interaction physics has been encouraged by the order-of-magnitude increase[1] of center-of-mass (c.m.) energy $\sqrt{s}$ due to the successful operation of the CERN S$p\bar{p}$S and Fermilab Tevatron colliders and the planned future operation of the CERN Large Hadron Collider (LHC) and the Superconducting Super Collider (SSC). The most significant feature in the vast majority of collision processes of two hadrons at high energy is the creation of a number of low-transverse-momentum hadrons that form rather complicated multihadron final states. Most physicists believe the theory of quantum chromodynamics (QCD) to be our best theoretical candidate for strong-interaction physics at the moment; within QCD hard processes with large momentum transfers can be successfully described in the perturbative calculus, but the unperturbative part, including the soft processes with small momentum transfers, still cannot be understood by the QCD. For this reason we need phenomenological analysis of experimental data on multihadron production in high-energy collisions.

Although the possibility of intermittent behavior of spectra of the hadrons produced in high-energy collision was investigated[2] as a new way to understand the dynamical fluctuations, it is too early to draw any definite conclusions at the present stage. There is plenty of work to be done before we shall be able to assess the real meaning of the intermittent phenomenon. While studying intermittency, we should pay some attention to the situation of the multiplicity distributions as a whole. It is obvious that the multiplicity distributions can be considered as the mostly readily obtainable and the simplest characteristics of multihadron final states. Multiplicity distributions contain much information on the multihadron production mechanism and from the phenomenological study of multiplicity distributions there were several models (see the excellent review by Carruthers and Shih[3]). In this paper we show that the higher-order moments of multiplicity distributions of the available nondiffractive pp and $p\bar{p}$ experimental data[4] in a broader energy region from $\sqrt{s}=11.5$ to 900 GeV can be described by the quantum coherent production mechanism and that predictions for higher energies can be made in the near future.

It is shown by experiment that most of the produced hadrons of the final states in high-energy hadron-hadron collisions are pions, which obey the Bose-Einstein statistics. Therefore, we can discuss the physical mechanism of multihadron production based on the quantum coherent states. As is well known, the best thing one can do is to measure the probability of finding particles in a state with minimal uncertainty, i.e., in a coherent state.

We recall that the S matrix is defined by

$$S=|\psi_{\rm in}\rangle\langle\psi_{\rm out}| \ , \tag{1}$$

which represents that the measured outgoing final state $|\psi_{\rm out}\rangle$ comes from the incoming initial state $|\psi_{\rm in}\rangle$ as

$$\rho_{\rm out}=S^\dagger\rho_{\rm in}S \ , \tag{2}$$

where $\rho_{\rm in}=|\psi_{\rm in}\rangle\langle\psi_{\rm in}|$ and $\rho_{\rm out}=|\psi_{\rm out}\rangle\langle\psi_{\rm out}|$ are density matrices of the incoming initial state and the outgoing final state, respectively. Although the dynamical representation of $\rho_{\rm out}$ might be more complicated than the S matrix, we can construct $\rho_{\rm out}$ according to the experimental data analysis and with the aid of the coherent-state description of quantum mechanics. By studying the multihadron final states that almost involve pions, we know that the best approximation is to adopt a set of boson-field operators, which is analogous to oscillator coherent states if we do not consider momentum and other degrees of freedom.

Consider a density matrix ρ represented by the quantum coherent state $|\xi\rangle$ as[5]

$$\rho=\int d^2\xi\,\phi(\xi)|\xi\rangle\langle\xi| \ , \tag{3}$$

where ξ is a complex variable and $\phi(\xi)$ is a weight function with real value. Expanding $|\xi\rangle$ in the basis of number representation and normalizing we have

$$|\xi\rangle=\exp(-\tfrac{1}{2}|\xi|^2)\sum_{n=0}^{\infty}\frac{\xi^n}{\sqrt{n!}}|n\rangle\ . \tag{4}$$

The distribution of number $n, P(n)$ is related to the density matrix as

$$P(n)=\langle n|\rho|n\rangle\ . \tag{5}$$

In general, the weight function $\phi(\xi)$ is taken as a Gaussian one,

$$\phi(\xi)=\frac{1}{\pi|\alpha|^2}\exp\left[-\frac{|\xi-\beta|^2}{|\alpha|^2}\right], \tag{6}$$

where α and β are complex parameters. Inserting Eqs. (4) and (6) into Eqs. (3) and (5), it leads to the distribution of n

$$P(n)=\frac{|\alpha|^{2n}}{(1+|\alpha|^2)^{n+1}}\exp\left[-\frac{|\beta|^2}{1+|\alpha|^2}\right]\times L_n\left[-\frac{|\beta|^2}{|\alpha|^2(1+|\alpha|^2)}\right], \tag{7}$$

where L_n is the Laguerre polynomial. For the convenience of the following discussion, we introduce a ratio of signal to noise, $\gamma=|\beta|^2/|\alpha|^2$.

We now return to the discussion on the multihadron production in high-energy hadron-hadron collisions. The physical picture is based on the following hypotheses, parts of which are very well known and have already been checked experimentally.

(i) Hadrons are spatially extended objects of many interior degrees of freedom.

(ii) In terms of the conventional quark-gluon picture,[6] hadrons are made out of valence quarks and gluons, and on the average about 35–50 % of the momentum of a high-energy hadron is carried by its valence quarks and the rest by the gluons (and/or sea-quark pairs).

(iii) At high energy the colliding objects, one of them named R and its counterpart named L, in nondiffractive hadron-hadron collision processes may go through each other. While going through and interacting with each other, they lose a considerable part of their energies and momenta.

(iv) While a part of this "lost energy" from a colliding object R (or from the counterpart L) will be brought by the flying valence quarks of R (or of L) and then decays into a number of hadrons in the fragmentation region (denoted by F), another part of this lost energy from R (or from L) will be detained due to the interactions between the gluons (and/or sea-quark pairs) in the colliding objects R and L and then materializes a number of hadrons in the central region (denoted by C).

(v) In fragmentation region F there are two hadron-producing sources, one from R and another from L (denoted by FR and FL, respectively), which were directly fragmented from the valence quarks of R and L and should strongly "keep their remembrance of the case before the collision," while in central region C, although there are also two hadron-producing sources, one from R and another from L (denoted by CR and CL, respectively), which "forget their history" after the materialization, because of the violent interactions among the gluons (and/or sea-quark pairs). Taking an approximation for simplicity, we assume that such four hadron-producing sources are independent.

We now calculate the higher moments of the multiplicity distributions and compare them with the existing experimental data.[4]

Let us first consider the contribution from the fragmentation region F. The produced hadrons in F stem out of sources FR and FL. As a result of fragmentation of valence quarks in colliding objects R and L, both of FR and FL should appear to be states with strong coherent character, which means that the ratio $\gamma\rightarrow\infty$. From Eq. (6), the weight function should be

$$\phi_\beta(\xi)=\delta(\xi-\beta)\ . \tag{8}$$

Inserting it into Eqs. (3) and (5), the distributions of the number of produced hadrons in sources FR and FL can be obtained and have the same form of the Poisson one as

$$P_\beta(n)=\frac{|\beta|^{2n}}{n!}\exp(-|\beta|^2)\ . \tag{9}$$

Because of the symmetry of pp (or $p\bar{p}$) collisions in their center-of-mass system, the mean values of the number of produced hadrons n_{FR} and n_{FL} in sources FR and FL are the same, i.e.,

$$\langle n_{FR}\rangle=\langle n_{FL}\rangle=|\beta|^2=\tfrac{1}{2}\langle n_F\rangle\ , \tag{10}$$

where $\langle n_F\rangle$ is the mean value of multiplicity n_F in the fragmentation region F. Therefore, the multiplicity distribution for region F is

$$P_F(n_F)=\frac{\langle n_F\rangle^{n_F}}{n_F!}\exp(-\langle n_F\rangle)\ , \tag{11}$$

which is followed by a trivial calculation with $n_F=n_{FR}+n_{FL}$.

Next, we consider the contribution from the central region C. Because the interactions in region C are quite violent and in confusion due to the complexity of exchange among the gluons (and/or sea-quark pairs), the states of the hadron-producing sources CR and CL are characteristic of incoherence (or chaos). This signifies that the ratio $\gamma\rightarrow 0$, and the corresponding weight function as

$$\phi_\alpha(\xi)=\frac{1}{\pi|\alpha|^2}\exp\left[-\frac{|\xi|^2}{|\alpha|^2}\right]. \tag{12}$$

Using Eqs. (3) and (5) we find the Bose-Einstein distribution,

$$P_\alpha(n)=\frac{|\alpha|^{2n}}{(1+|\alpha|^2)^{n+1}}\ . \tag{13}$$

As discussed above, the distributions of the number of the produced hadrons n_{CR} and n_{CL}, in sources CR and CL, respectively, have the same form as Eq. (13) and the same mean value

TABLE I. Mean multiplicities $\langle n_{\rm ND} \rangle$ and chaotic producing ratio λ in the energy region from 11.5 to 900 GeV. Data taken from Ref. 4.

$\sqrt{s}$ GeV	$\langle n \rangle$	λ
11.5	6.35±0.08	0.264
13.8	7.21±0.06	0.267
19.7	8.56±0.11	0.328
23.9	9.25±0.20	0.382
27.6	9.77±0.16	0.454
30.4	10.54±0.14	0.445
44.5	12.08±0.13	0.465
52.6	12.76±0.14	0.488
62.6	13.63±0.16	0.472
200.0	21.40±0.80	0.621
540.0	29.10±0.90	0.715
900.0	35.10±0.60	0.748

$$\langle n_{CR} \rangle = \langle n_{CL} \rangle = |\alpha|^2 = \tfrac{1}{2} \langle n_C \rangle \ , \tag{14}$$

where $\langle n_C \rangle$ is the mean value of multiplicity n_C in the fragmentation region C. It is known that in the case of k independent sources having the same distribution form as Eq. (13), the distribution is the negative-binomial one. For our case we have $k=2$ and the multiplicity distribution of n_C as

$$P_C(n_C) = \begin{bmatrix} n_C+1 \\ n_C \end{bmatrix} \left[\frac{\langle n_C \rangle}{\langle n_C \rangle + 2} \right]^{n_C} \left[\frac{2}{\langle n_C \rangle + 2} \right]^2 , \tag{15}$$

where $n_C = n_{CR} + n_{CL}$.

The observed multiplicity in the full phase space, $n_{\rm ND}$ (ND stands for nondiffractive collisions) is the sum of n_C and n_F and its distribution can be given by folding the distributions of n_C and n_F, i.e., Eqs. (11) and (15). The higher normalized moments of the multiplicity distribution in the full phase space is defined by

$$C_q = \frac{\langle n_{\rm ND}^q \rangle}{\langle n_{\rm ND} \rangle^q} \ , \tag{16}$$

where

$$\langle n_{\rm ND}^q \rangle = \sum_{i=0}^{q} \begin{bmatrix} q \\ i \end{bmatrix} \langle n_C^i \rangle \langle n_F^{q-i} \rangle \ . \tag{17}$$

By taking the derivation from Eqs. (11) and (15), it is easy to get

$$\langle n_F^i \rangle = \langle n_F \rangle^i + \sum_{j=1}^{i-1} A(i,j) \langle n_F^{i-j} \rangle \ , \tag{18}$$

and

$$\langle n_C^i \rangle = \frac{(i+1)!}{2^i} \langle n_C \rangle^i + \sum_{j=1}^{i-1} A(i,j) \langle n_C^{i-j} \rangle \ , \tag{19}$$

where the coefficient $A(i,j)$ is defined by

$$A(i,j) = (-1)^{j-1} \sum_{\substack{t_1, t_2, \ldots, t_j = 1 \\ t_1 < t_2 < \cdots < t_j}}^{i-1} t_1 t_2 \ldots t_j \ . \tag{20}$$

Let λ be the ratio of the mean multiplicities of the central region to the full phase space; then

$$\lambda = \frac{\langle n_C \rangle}{\langle n_{\rm ND} \rangle} \ . \tag{21}$$

It is quite evident that λ represents the contribution of the central region, i.e., of the chaotic production of had-

TABLE II. Higher-order moments, C_q ($q=2$–5), in the energy region of Serpukhov and Fermilab. Theoretical values given in the first row of each energy in Table I are obtained by taking data-fitting parameter λ; the second row of each energy are calculated by taking parameter λ from Eq. (22). Experimental data in brackets are taken from Ref. 4.

$\sqrt{s}$ (GeV)	C_2	C_3	C_4	C_5
11.5	1.1923	1.6274	2.4891	4.2073
	1.1928	1.6293	2.4944	4.2220
	(1.192±0.009)	(1.630±0.030)	(2.490±0.080)	(4.200±0.200)
13.8	1.1743	1.5666	2.3308	3.8209
	1.1925	1.6368	2.5366	4.3831
	(1.175±0.006)	(1.570±0.020)	(2.330±0.040)	(3.800±0.100)
19.7	1.1706	1.5620	2.3394	3.8920
	1.1938	1.6524	2.6071	4.6345
	(1.174±0.010)	(1.570±0.030)	(2.340±0.080)	(3.800±0.200)
23.9	1.1811	1.6064	2.4790	4.2947
	1.1950	1.6611	2.6430	4.7581
	(1.190±0.020)	(1.620±0.060)	(2.470±0.140)	(4.200±0.300)
27.6	1.2054	1.7051	2.7839	5.1805
	1.1962	1.6685	2.6716	4.8539
	(1.210±0.010)	(1.720±0.050)	(2.760±0.130)	(5.000±0.400)

TABLE III. Higher-order moments, C_q ($q=2-5$), in the energy region of the CERN ISR. Theoretical values given in the first row of each energy in Table I are obtained by taking the data-fitting parameter λ; the second row of each energy are calculated by taking parameter λ from Eq. (22). Experimental data in brackets are taken from Ref. 4.

$\sqrt{s}$ (GeV)	C_2	C_3	C_4	C_5
30.4	1.1939	1.6629	2.6627	4.8460
	1.2032	1.6998	2.7755	5.1737
	(1.200±0.010)	(1.680±0.030)	(2.640±0.100)	(4.600±0.300)
44.5	1.1909	1.6567	2.6564	4.8557
	1.2088	1.7278	2.8749	5.4954
	(1.200±0.010)	(1.670±0.030)	(2.630±0.100)	(4.600±0.300)
52.6	1.1974	1.6846	2.7460	5.1257
	1.2112	1.7395	2.9159	5.6278
	(1.210±0.010)	(1.700±0.030)	(2.700±0.090)	(4.800±0.300)
62.6	1.1848	1.6368	2.6058	4.7318
	1.2157	1.7599	2.9849	5.8464
	(1.200±0.010)	(1.670±0.030)	(2.600±0.080)	(4.400±0.200)

rons in high-energy hadron-hadron collisions. By fitting the available experimental data[4] at twelve c.m. energies $\sqrt{s}$ from 11.5 to 900 GeV, we obtain the corresponding values of λ which are shown in Table I. The second column in Table I represents data of the mean multiplicities $\langle n_{\mathrm{ND}} \rangle$ corresponding to each energy. One can see from Table I that the ratio λ increases with c.m. energy $\sqrt{s}$, which might indicate that the interactions among the gluons (and/or sea-quark pairs) will become stronger and the chaotic production of hadrons will increase as $\sqrt{s}$ rises. As a result of such an effect the contribution from the central region will be greater.[7]

Using Eqs. (16)–(21) and taking the values of $\langle n_{\mathrm{ND}} \rangle$ and λ in Table I as the parameters we can calculate the higher normalized moments C_q ($q=2-5$) for these twelve energies. The calculated results are collected in Table II for Serpukhov and Fermilab energies, in Table III for CERN ISR energies, and in Table IV for CERN $Sp\bar{p}S$ energies (in the first rows of each energy). The experimental data taken from Ref. 4 are also shown in Tables II–IV for comparison (in the third row of each energy). One can see that all calculated normalized moments are in excellent agreement with the observed data.

From Table I we find an interest relation between the mean multiplicity of the F region, $\langle n_F \rangle$, and the c.m. energy $\sqrt{s}$ as follows:

$$\langle n_F \rangle - (2+\epsilon) = \ln\sqrt{s} \ , \tag{22}$$

where the corrected parameter ϵ is 0.22. In terms of this relation we can directly calculate the value of λ for any c.m. energy $\sqrt{s}$ without the data fitting as an input. To repeat the calculation of the higher normalized moments we obtain a set of new results which are shown in Tables II–IV (in the second rows of each energy). They agree with the experimental data too.

For making predictions at Tevatron, LHC, and SSC energies we take the empirical formula from the experimental fit[8]

TABLE IV. Higher-order moments, C_q ($q=2-5$), in the energy region of the CERN $Sp\bar{p}S$. Theoretical values given in the first row of each energy in Table I are obtained by taking the data-fitting parameter λ; the second row of each energy are calculated by taking parameter λ from Eq. (22). Experimental data in brackets are taken from Ref. 4.

$\sqrt{s}$ (GeV)	C_2	C_3	C_4	C_5
200	1.2395	1.8676	3.3534	7.0409
	1.2571	1.9359	3.5880	7.7907
	(1.260±0.030)	(1.910±0.120)	(3.300±0.300)	(6.600±0.900)
540	1.2900	2.0802	4.0692	9.4063
	1.2846	2.0580	3.9937	9.1519
	(1.310±0.030)	(2.120±0.110)	(4.050±0.320)	(8.800±1.000)
900	1.3082	2.1587	4.3426	10.3519
	1.3045	2.1429	4.2879	10.1631
	(1.340±0.030)	(2.220±0.130)	(4.300±0.400)	(9.300±1.100)

TABLE V. Prediction of higher-order moments, C_q ($q=2-5$), in the energy region of the Tevatron, LHC, and SSC. Theoretical values are calculated by taking parameters $\langle n_{\rm ND}\rangle$ and λ from Eqs. (22) and (23).

$\sqrt{s}$ (TeV)	C_2	C_3	C_4	C_5
1.8	1.3864	2.4941	5.5454	14.7129
16.0	1.4196	2.6409	6.0987	16.8422
40.0	1.4642	2.8391	6.8623	19.8771

$$\langle n_{\rm ND}\rangle=(2.7\pm0.7)-(0.03\pm0.21)\ln s+(0.167\pm0.016)\ln^2 s \quad (23)$$

to estimate the mean multiplicities in the full phase space. Corresponding to the c.m. energies $\sqrt{s}=1.8$, 16, and 40 TeV we obtain $\langle n_{\rm ND}\rangle=39.78$, 64.72, and 77.07, respectively. Using the relation of Eq. (22) and the estimated mean multiplicities we estimate the values of $\lambda=0.756$, 0.816, and 0.834 for c.m. energies $\sqrt{s}=1.8$, 16, and 40 TeV, respectively. In Table V the predicted values of the higher normalized moments are shown.

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