High-energy photoproduction of jets

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We examine recent claims that, because of jet production, the cross section for the hadronic component of the photon-proton collisions at very high energies becomes much larger than the standard cross section based on vector-dominance models. If this were so, then the interpretation of cosmic-ray air showers associated with point sources might be very different because photon-induced showers would not be as muon poor as has previously been assumed. We show that although *inclusive* jet cross sections do become large, the *total* cross section does not increase much. It is possible that the photon-gluon-fusion contribution will increase the cross section by a factor of 2 or so. Parton-saturation effects come into play and prevent the cross section from rising more.

I. INTRODUCTION

Many recent experiments measuring high-energy cosmic rays have looked for cosmic rays associated with identifiable "point" sources (such as Cygnus X-3 and Hercules X-1).[1-5] One expects that extensive air showers associated with point sources are caused by photon primaries interacting with the atmosphere, and that photon-induced air showers are almost all electromagnetic. Consequently, the air showers from point sources should easily be distinguished from normal cosmic-ray air showers (due to hadrons) by containing relatively few muons. Note that the (purely electromagnetic) Bethe-Heitler cross section for photons on nitrogen is about 400 mb, while the conventional hadronic cross section is orders of magnitude less.

What cosmic-ray data there are from point sources appear to contradict this idea,[1,5] making it important to check whether the conventional ideas on photon-hadron interactions are correct. The interactions in question involve beam energies of the order of 1000 TeV, that is, center-of-mass energies of the order of a TeV. Although this energy is far beyond the range of accelerator experiments with photon beams, it is not beyond the range of energies of hadron colliders, where we know that normal QCD and QED are valid to sufficient accuracy for our purposes.

Although a naive extrapolation of low-energy physics might be incorrect, we certainly have sufficient knowledge to make valid predictions, since the dominant momentum transfers are low, and the interactions are those of QED and QCD. What is conceivable is that effects in QCD that are unimportant at lower energies become dominant at high energy. Predictions for photoproduction at high energies are also important for the DESY *ep* collider HERA.

Recently, Drees, Halzen, and Hikasa[6] published some calculations of cross sections for jet production in photon-hadron collisions. They claim that the production of jets of transverse energy above about a GeV becomes so common that the cross section for this hard-scattering process may push the total cross section far above the value conventionally assumed for the total photoproduction cross section. Gandhi *et al.*[7] have further developed this idea, and claim to take into account the unitarization that is necessary when the cross sections rise so rapidly.

In this paper we reexamine the issue of jet production in photon-hadron collisions. One of our conclusions is that QCD does not predict the rise in total cross section suggested by Drees, Halzen, and Hikasa. They calculate an inclusive cross section, and do not divide by a multiplicity factor. We show that this multiplicity must be large. Moreover, it is necessary to consider whether parton-saturation effects enter.

There are two hard subprocesses that are important: direct collisions of very-small-x gluons out of the hadron target with the photon, and collisions between gluons from the hadron with gluons of small x found inside the photon. There has been some confusion about the small-x part of the gluon distribution in a photon, and extrapolations of standard fits to small x give unphysically singular behavior. The Duke and Owens parametrization is a good example of this. Their gluon distribution behaves as $x^{-1.97}$, but should not be extrapolated from the region $x > 0.1$, where it is derived by fits to data and the Altarelli-Parisi evolution.

In Sec. II we shall quickly review the small-x properties of some recently used gluon distribution functions for the photon and proton. We will then use these distribution functions in Sec. III to study the hadronic behavior of the photon as it pertains to the high-energy production of jets in the γp interaction, while we will focus on the photon's pointlike nature in Sec. IV. We then discuss the possible relevance of saturation effects in high-energy γp interactions. With this in mind we proceed to examine some recent theoretical results in cosmic-ray photon physics with a special emphasis on multiplicity and on how the inclusive cross section reflects on the total cross section.

II. SMALL-x BEHAVIOR OF PARTON DISTRIBUTION FUNCTIONS

High-energy reactions such as those found in cosmic-ray physics have center-of-mass energies which can be of the order of a few TeV. So when we consider production of jets of a few GeV of transverse momentum, the calculation of the cross section involves very small fractional momenta for the initial partons for the hard scattering. So we must first discuss what is known about the $x \to 0$ behavior of the parton distributions.

Since the small x_{Bj} limit of deep-inelastic scattering is a Regge limit, a standard assumption when solving the Gribov-Lipatov-Altarelli-Parisi equations for the parton distributions is that the initial distributions—the parton distributions at some starting scale Q_0—are Pomeron dominated, that is, the number densities behave like $1/x$. There are several reasons why this is inappropriate.[8,9] Perhaps the simplest reason is that even leading-order evolution over a small range in Q makes the distributions much steeper, so that over several decades of x below about 10^{-2} the distributions are well parametrized by a power $1/x^J$, with J in the range $1.2-1.5$. This behavior leads to cross sections for the production of jets at a given transverse momentum that rise like a power law in s. One suggestion is that $J \approx 1.5$ is not unreasonable[10] as an approximation to the gluon distribution at the value of x and Q with which we are concerned.

Distributions that carry such a power-law behavior at small x will violate the Froissart bound at some point.[11] As explained in Ref. 11, the parton distributions must then be considered saturated, and ordinary hard-scattering calculations become invalid. Consequently, one should estimate where saturation effects become important; we will do this in a later section.

In the next two subsections, we will investigate what small-x behavior is appropriate for the gluon distribution in a proton, and then in a photon.

Now, as explained in Refs. 8 and 10, simple asymptotic formulas for the high-energy behavior of cross sections involving a hard scattering can be derived when the parton distributions have power-law behavior at small x. We will apply these ideas to the case of jet production in γp interactions. Such calculations enable us to see the dominant physics very easily, and to get reasonable estimates of cross sections quickly.

A. Parton distribution functions in the proton

One standard set of parton distributions in the proton which has a small-x behavior of $1/x$ for the initial distributions is due to Duke and Owens.[12] The parametrization form given by Duke and Owens for the gluon distribution is

$$x f_{g/p}(x,Q^2) = A x^a (1-x)^b (1+c_1 x + c_2 x^2 + c_3 x^3) , \quad (1)$$

where the scale dependence of the coefficients and exponents is given in Ref. 12. We note that the exponent a becomes significantly negative as we increase Q, reaching, for example, a value of -0.4 at $Q=9$ GeV. This steep behavior is valid as a fit over a limited range of x, but it illustrates the point made in Ref. 8, that $1/x$ behavior is unstable against evolution.

Another interesting example is given by the distributions of Glück, Godbole, and Reya,[13] which are dynamically generated from distributions that at a scale 0.25 GeV are pure valence (i.e., with no sea quarks or gluons). At a scale Q around 10 GeV, a log-log plot in Ref. 13 shows that the gluon distribution is about $1/x^{1.5}$ down to x's in the range we need to consider.

These power laws do not necessarily reflect the behavior of the distributions at infinitely small x, but they do represent an effective power law at small nonzero x. As we will see, these powers get reflected in the s dependence of the cross sections.

A simple parametrization $f_{g/p}(x_2) = C x_2^{-J}$ with $C \approx 0.7$ and $J = 1.5$ was suggested by one of us in Ref. 8. The value of J is assumed, and the value of C comes from requiring a standard $x \to 1$ behavior and that the gluons carry about 50% of the momentum of the proton. This parametrization gives minijet cross sections within a factor of 2 of the UA1 data, and is within a factor of 2 of the gluon distribution of Glück, Godbole, and Reya, to which we have just referred.

Recent fits of parton distributions to data from deep-inelastic scattering have generally allowed for the possibility that the small-x behavior is steeper than $1/x$.[14]

B. Parton distribution functions in the photon

Two sets of parton distributions in the photon, which have been used recently by Drees and Halzen, are those of Duke and Owens[15] and those of Drees and Grassie.[16] Since we will be focusing on the gluon content of the photon we present their gluon distributions.

The Duke and Owens parametrization is

$$f_{g/\gamma}(x,Q^2) = 0.194 \frac{\alpha}{2\pi} \ln(Q^2/\Lambda^2) x^{-1.97} (1-x)^{1.03} , \quad (2)$$

and the Drees and Grassie formula is

$$f_{g/\gamma}(x,Q^2) = \alpha A_G x^{B_G - 1} (1-x)^{C_G} , \quad (3)$$

where the parametrization of the Q^2 dependence of A_G, B_G, and C_G are given in Ref. 16.

Now the Duke and Owens parametrization is a parametrization of leading-logarithm photon distribution functions[17,18] that is only supposed to be valid for large x ($x > 0.1$). The power law $x^{-1.97}$ at small x is unphysical: this is a case where the leading-logarithm approximation and a correct analysis using the renormalization group, or the Altarelli-Parisi equation, disagree. The leading logarithm approximation gives a steep power that is entirely incorrect. Note that the power 1.97 is close to giving a divergence in $\int_0^1 x g(x)\, dx$, which is the momentum fraction carried by gluons.

The Drees and Grassie result is a better approximation, and we only consider the Duke and Owens parametrization in order to compare with the results of Drees and Halzen.

III. ASYMPTOTIC RESULTS FOR THE $gg \to gg, q\bar{q}$ SUBPROCESS

Jet production in photon-hadron collisions can be calculated by the usual factorization formula, just as in hadron-hadron scattering. The only difference is that the photon must be considered as a possible parton. That is, in the short-distance scattering, one must consider short-distance subprocesses that are induced by photon-gluon and photon-quark scattering, in addition to those induced by scattering of quarks and gluons. We will treat the purely quark and gluon processes in this section, and will treat the photon-induced subprocesses in the next section.

We wish to consider the asymptotic cross section for the production of jets of transverse momentum bigger than some $p_{T\min}$ when the center-of-mass energy $\sqrt{s}$ is much bigger than $p_{T\min}$. We will use the methods of Ref. 10. Because of the steepness of the gluon distributions at low x, the dominant subprocesses will involve gluons, but not quarks, in the initial state. Thus we will only compute the asymptotic contributions from the $gg \to gg, q\bar{q}$ subprocesses.

We start with the basic factorization formula:

$$\sigma_{\gamma p} = \int_0^{s_{\gamma p}/4} dp_T^2 \int_0^1 dx_2 \int_0^1 dx_1\, \theta(p_T - p_{T\min})\theta(x_1 x_2 s_{\gamma p} - 4p_T^2) f_{g/\gamma}(x_1) f_{g/p}(x_2) \frac{d\hat{\sigma}}{dp_T^2} . \tag{4}$$

We make a change of variable from x_1 to $z = 4p_T^2/\hat{s}$:

$$\sigma_{\gamma p} = \int_{p_{T\min}^2}^{s_{\gamma p}/4} dp_T^2 \int_{4p_T^2/s_{\gamma p}}^1 dx_2 \int_{4p_T^2/(x_2 s_{\gamma p})}^1 dz \left[\frac{4p_T^2}{z^2 x_2 s_{\gamma p}} \right] f_{g/\gamma}(x_1(z)) f_{g/p}(x_2) \frac{d\hat{\sigma}}{dp_T^2} , \tag{5}$$

where $x_1(z) = 4p_T^2/zx_2 s$.

As in Ref. 10 we consider distributions with a power-law behavior $1/x^J$, but here we will suppose that the power laws differ for the two distributions, $f_{g/\gamma} \propto x^{-J_1}$ and $f_{g/p} \propto x^{-J_2}$. Recall from Ref. 10 that the case $J_1 = J_2 = J$ gives a cross section $\sigma \propto s^{J-1}$. Relevant to our problem is when one J is significantly larger than the other. In this case the integral is driven by the distribution with the largest J value. This is most evident when we examine the γp cross section using the Duke and Owens gluon distribution for the proton and we extend our application of the Duke and Owens gluon distribution of the photon to regions of small x. Similarly, when we replace the photon distribution with that of Drees and Grassie, we also have $J_1 > J_2$ at scales of a few GeV. So we will consider only this case.

Since the integrals will almost entirely come from the region where $x_1 \ll 1$ and $p_T^2 \ll s_{\gamma p}$, the lower limits on the z and x_2 integrals may be replaced by 0, while the upper limit on the transverse momentum integral may be taken to infinity. The result is that the above triple integral now reads

$$\sigma_{\gamma p} \approx \int_{p_{T\min}^2}^{\infty} dp_T^2 \int_0^1 dx_2 \int_0^1 dz \left[\frac{4p_T^2}{x_2 s_{\gamma p} z^2} \right] f_{g/\gamma}(x_1(z)) f_{g/p}(x_2) \frac{d\hat{\sigma}}{dp_T^2} . \tag{6}$$

We write the differential cross section for the $2 \to 2$ parton subprocess as

$$d\hat{\sigma}/dp_T^2 = |M|^2/(16\pi\hat{s}^2 \cos\theta) , \tag{7}$$

where $\cos\theta = \sqrt{1 - 4p_T^2/\hat{s}}$, and $\hat{s} = x_1 x_2 s_{\gamma p}$ is the center-of-mass energy of the parton subprocess.

For $gg \to gg$ the amplitude squared is[19,20]

$$|M|^2 = 72\pi^2\alpha_s^2 \left[3 - \frac{3\hat{s}}{p_T^2} - \frac{p_T^2}{\hat{s}} + \frac{\hat{s}^2}{p_T^4} \right] , \tag{8}$$

while for $gg \to q\bar{q}$ it is

$$|M|^2 = 16\pi^2\alpha_s^2 \left[\frac{\hat{s}}{6p_T^2} - \frac{1}{3} - \frac{3}{8\hat{s}}(\hat{s} - 2p_T^2) \right] . \tag{9}$$

In the limit we are considering, the dominant contribution to the cross section is from the region of very small x_1, where the Duke and Owens parametrization of the distribution function may be taken as

$$f_{g/\gamma} = 0.194 \frac{\alpha}{2\pi} \ln(Q^2/\Lambda^2) x_1^{-J} , \tag{10}$$

with $J = 1.97$. For the Drees and Grassie distribution we have

$$f_{g/\gamma} = \alpha A_G x_1^{-J} , \tag{11}$$

where $J \approx 1.4$ at $Q^2 = p_T^2 = 4\ \text{GeV}^2$.

The asymptotic form for the $gg \to gg$ subprocess is therefore

$$\sigma_{gg}^{\gamma p}(\sqrt{s}\,)=\left[\frac{9\kappa_1}{8\pi s_{\gamma p}}\right]\int_{p_{T\min}^2}^{\infty} dp_T^2\, p_T^{-2}\int_0^1 dx_2\, x_2^{-1} f_{g/p}(x_2)\int_0^1 \frac{dz}{z^2}\left[\frac{4p_T^2}{x_2 s_{\gamma p} z}\right]^{-J}(1-z)^{-1/2}\left[1-\frac{3z}{4}+\frac{3z^2}{16}-\frac{z^3}{64}\right] . \tag{12}$$

For the Duke and Owens distribution we have

$$\kappa_1=0.194(8\pi\alpha\alpha_s^2)\ln(2p_{T\min}^2/\Lambda^2) \quad \text{and } J=1.97 , \tag{13}$$

while for the Drees and Grassie distribution we have

$$\kappa_1=16\pi^2\alpha\alpha_s^2 A_G \quad \text{and } J\approx 1.4 . \tag{14}$$

We have set the scale for the parton distribution functions to $p_{T\min}$.

Using the Duke and Owens proton distribution (set one) and including a contribution for each final-state jet yields the asymptotic result for the one-jet inclusive cross section for the photon-proton reaction via gluon fusion:

$$\tilde{\sigma}_{gg}^{\gamma p}=0.194\times(18\alpha\alpha_s^2)\ln\left[\frac{2p_{T\min}^2}{\Lambda^2}\right](s_{\gamma p}J)^{-1}\left[\frac{s_{\gamma p}}{4p_{T\min}^2}\right]^J I_2(I_z^{gg}+I_z^{q\bar q}) , \tag{15}$$

where

$$I_2=A\frac{\Gamma(J+a-1)\Gamma(b+1)}{\Gamma(J+a+b)}\left[1+c_1\frac{J+a-1}{(J+a+b)}+c_2\frac{(J+a-1)(J+a)}{(J+a+b)(J+a+b+1)} +c_3\frac{(J+a-1)(J+a)(J+a+1)}{(J+a+b)(J+a+b+1)(J+a+b+2)}\right] \tag{16}$$

and

$$I_z^{gg}=\sqrt{\pi}\frac{\Gamma(J-1)}{\Gamma(J-1/2)}\left[1-\frac{3(J-1)}{4(J-1/2)}+\frac{3J(J-1)}{16(J-1/2)(J+1/2)}-\frac{J(J-1)(J+1)}{64(J-1/2)(J+1/2)(J+3/2)}\right] . \tag{17}$$

In Eq. (16), A,a,b, and the c_i are the parameters in the gluon distribution in the proton, Eq. (1).

The contribution from $gg\to q\bar q$ is small. Using its cross-section formula for N quark flavors,

$$\frac{d\sigma_{q\bar q}}{dp_T^2}=\frac{N\pi\alpha_s^2}{\hat s^2\sqrt{1-4p_T^2/\hat s}}\left[\frac{\hat s}{p_T^2}-2\right]\left[\frac{1}{6}-\frac{3p_T^2}{8\hat s}\right] , \tag{18}$$

we find it has the same form as above except for a different value for I_z:

$$I_z^{q\bar q}=\frac{4N\sqrt{\pi}\Gamma(J)}{9\Gamma(J+1/2)}\left[\frac{1}{24}-\frac{17J}{384(J+1/2)} +\frac{3J(J+1)}{256(J+1/2)(J+3/2)}\right] . \tag{19}$$

In $I_z^{q\bar q}$ there is an overall factor of 2 so that the cross section covers the full polar range of 180°, and N represents the number of quark flavors. The values for this expression have been given in the tables.

The asymptotic approximations that we have made do not require that the simple power-law behavior for the parton distributions is good over the entire range of x down to zero, but only over that portion which is dominating for the particular values of the kinematic variables that we are considering.

IV. ASYMPTOTIC RESULT FOR THE POINTLIKE PHOTON CONTRIBUTION

In the previous section we derived the asymptotic contribution to the jet cross section given by the gluon-induced processes. We now consider the contribution of the photon-gluon-induced subprocesses. Since we are looking at center-of-mass energies on the order of a TeV and minijets with E_T's of a few GeV, our cross sections will be very much dominated by the behavior of the gluon distributions in the *proton* at low x; for the γg subprocess the range for the parton x's will run down to at least 10^{-4}.

As before, we know that at small x the gluon distribution is much bigger than the quark distributions, so that we can neglect the photon-quark subprocess.

The lowest-order $\gamma g\to q\bar q$ subprocess cross section can be derived from the photon-photon cross section,[21]

$$\frac{d\sigma}{d\hat t}(\gamma\gamma\to q\bar q)=\frac{2\pi\alpha^2}{\hat s^2}3e_q^4\left[\frac{\hat t}{\hat u}+\frac{\hat u}{\hat t}\right] , \tag{20}$$

by substituting $\alpha^2\to\alpha\alpha_s$ and $3e_q^4\to C_F e_q^2$ with $C_F=\frac{4}{3}$. We assume as before that the gluon distribution in the proton has the following asymptotic form at small x:

$$f_{g/p}(x)=Cx^{-J} . \tag{21}$$

It follows that the asymptotic contribution from this subprocess for flavor i to the cross section for $\gamma p \to 1$ jet$+X$ with $p_{T\text{jet}} > p_{T\text{min}}$ is

$$\sigma_{\rm PL}^{i} = \frac{8\pi^{3/2}}{3} C e_i^2 \alpha\alpha_s \frac{(J+1)\Gamma(J)}{J(2J+1)\Gamma(J+1/2)} \times \left[\frac{s_{\gamma p}}{4}\right]^{J-1} (p_{T\text{min}}^2)^{-J} . \quad (22)$$

For three quark flavors and $J=1$ we get a cross section independent of energy,

$$\sigma_{\rm PL} = \frac{64\pi C}{27} \alpha\alpha_s p_{T\text{min}}^{-2} , \quad (23)$$

while for $J=1.5$ we get

$$\sigma_{\rm PL} = \frac{5\pi^2 C}{27} \alpha\alpha_s \sqrt{s_{\gamma p}} p_{T\text{min}}^{-3} . \quad (24)$$

This case of a gluon distribution that has $J \equiv 1.5$ is maybe somewhat extreme, but it is not ruled out by current data. It does provide a zero-parameter calculation of minijets at UA1 valid to a factor of a few (see Ref. 8). We see that for the $J=1.5$ behavior for the gluon content of the proton that we get a $\sqrt{s_{\gamma p}}$ contribution to the cross section. Taking $\sqrt{s_{\gamma p}} = 1$ TeV, $p_{T\text{min}} = 2$ GeV, $\alpha_s = 0.3$, and $C = 0.7$ gives an inclusive singlet-jet cross section of about 136 μb, which is to be compared to the typical 100 μb total hadronic cross sections which have been seen at lower energies or the 200 μb cross sections which the vector-dominance model[22] would give.

Since the gluon's x is $s_{\gamma\text{glue}}/s_{\gamma p}$, we can get to very small x, and saturation effects can be very important. We will evaluate the effects in the next section and will see that saturation is beginning to be important at this energy and will prevent much of a further rise in $\sigma_{\rm PL}$ with $s_{\gamma p}$.

V. SATURATION

A. Saturation in the proton

It is experimentally known that approximately fifty percent of the proton's momentum is contained in gluons. Moreover, as one probes the proton at smaller and smaller distance scales the number of gluons in the proton at that scale increases. In fact, at sufficiently small x, the number of gluons is so high that a gluon initial particle for a hard scattering can no longer be treated as independent of the other partons. Thus the generalized impulse approximation that constitutes QCD factorization breaks down. We say that the partons in a proton have become saturated.

The above description is the one due to Gribov, Levin, and Ryskin.[11] They define a quantity which may be used to determine whether saturation has been reached:

$$W = \alpha_s(Q^2) \frac{x f_{g/p}(x,Q)}{Q^2 R^2} . \quad (25)$$

This represents the fraction of the area of a proton, as seen in a high-energy collision, that is occupied by gluons of about the specified value of x. Ordinarily factorization is valid only if $W \ll 1$. Saturation sets in as W increases to unity.

Note that quarks as well as gluons can participate in the saturation phenomena. However, it is the gluons that are dominant at small x.

Now, the value of W is clearly dependent on the parton distribution functions. The gluon distributions most commonly in use have an asymptotic behavior at low x of $G(x) \sim 1/x$, at the initial value $Q = Q_0$ used in the Altarelli-Parisi evolution. With this $1/x$ behavior, $xG(x)$ approaches a constant as x decreases to zero. However, if we use a steep distribution, such as the $f(x) = 0.7x^{-1.5}$ referred to earlier, W gets near 0.2 when $x \sim 10^{-4}$ [as appropriate for the numerical example we discussed after Eq. (24)] and $Q = \sqrt{5}$ GeV if we use $R = 1$ fm $= 5$ GeV^{-1}: Even the gluon distribution of Glück, Godbole, and Reya gives a value of $W = 0.16$ for $x = 10^{-4}$ and $Q = \sqrt{10}$ GeV. Thus parton saturation is liable to be important for the processes we are discussing.

B. Saturation in the photon

The same ideas can be applied to photon, provided that we take care to recognize that the photon is fundamentally an electromagnetic object that makes occasional transitions to an hadronic state.

Simple vector-dominance-model (VDM) estimates the probability that the photon is in an hadronic state to be

$$\frac{4\pi\alpha}{f_\rho^2} \approx \frac{1}{300} . \quad (26)$$

This is obtained from the coupling of the photon to a ρ meson, and is the probability that the photon is in a low-mass state. When we consider QCD, there is also the possibility of making a transition to a (virtual) high-mass state, and such probabilities may be calculated by suitable perturbative methods.

With the possibility of a very steep gluon distribution in the photon, it is important to find the correct form of the saturation condition, analogous to the condition $W \approx 1$ for a proton with W given by Eq. (25). The first idea is to use the same condition and the same formula. In that case the explicit factor of the electromagnetic α that is in $f_{g/\gamma}$ implies that saturation is not very relevant for the situation considered by Drees *et al.*

However let us consider the physics represented by the calculation of the distribution of gluons in a photon. The calculation is made by starting, typically, with some ansatz motivated by vector dominance. This is evolved by the Altarelli-Parisi formalism to higher Q. There is an inhomogeneous term in this equation that represents transitions from the bare photon state to a quark-antiquark state. Further evolution into gluons gives the characteristic features that distinguish the distributions in a photon from those in a proton.

These distributions should be thought of as the density of gluons, *given that we start with a quark-antiquark pair of some mass,* times the probability that we have the quark-antiquark pair:

$$x f_{g/\gamma} \propto \text{Prob}(q\bar{q} \text{ in } \gamma)\text{Prob}(\gamma \text{ in } q) . \quad (27)$$

So we should normalize W to the probability that the photon is in an hadronic state, and write

$$W_\gamma \approx \left[\frac{4\pi\alpha}{f_\rho^2}\right]^{-1} \alpha_s(Q^2)\frac{xf_{g/\gamma}(x,Q)}{Q^2R^2}$$
$$\approx 300\alpha_s(Q^2)\frac{xf_{g/\gamma}(x,Q)}{Q^2R^2} \ . \tag{28}$$

For the Duke and Owens gluon distribution of the photon we find at $Q^2=2p_{T\min}^2=8$ GeV^2 and at the minimum x at the center-of-mass energy of 1 TeV that $W_\gamma \approx 21 \gg 1$ when the radius is taken as comparable to the proton radius, $R=5$ GeV^{-1}. Since this value for W_γ is well above unity, saturation effects dominate, and the cross sections in Ref. 6 are large overestimates. The values become larger still if we use a smaller radius like that dictated by the mass of the ρ, $R^2 \approx 2$ GeV^{-2}, indicating that the saturation effects could be even more pronounced.

The Drees and Grassie distribution, which is given for scales as low as 1 GeV, gives a value of W_γ, under similar conditions, of 0.32. One notes that even this parametrization indicates saturation effects at small x.

C. Inclusive cross sections and unitarization

The large jet cross sections in hadron-hadron scattering do not lead to correspondingly large total cross sections: first there are multiple hard collisions and one must divide by a multiplicity factor, and then there is parton saturation, which limits the growth of the single jet cross section. Durand and Pi have an eikonal formalism that gives, for example, a total cross section of the form

$$\sigma_T = 2\pi \int db\, b \langle i|1-e^{-2\chi^R(b,s)}|i\rangle \ .$$

The inclusive jet calculation is put into the eikonal function χ. This model, as applied to hadron-hadron collisions appears to be reasonable.

Let us now examine the unitarized cross sections for photon-hadron scattering computed in Ref. 7. The same eikonal formula is used, and the eikonal function is derived from the soft cross section plus the jet cross section:

$$\chi^R = \tfrac{1}{2}A(b)(\sigma_{\text{soft}}+\sigma_{\text{QCD}}) \ .$$

Both these cross sections are proportional to α_{em}, and an expansion of the total cross section in powers of α_{em} reads

$$\sigma_T = 2\pi \int db\, b \langle i|2\chi - 2\chi^2 + \cdots |i\rangle \ .$$

The χ^2 term starts off the unitarization. Since it contains two powers of α_{em}, the unitarization does not become significant until the inclusive cross sections compensate the smallness of the extra power of α_{em}; this is when the jet cross section is comparable to the total hadron-hadron cross section, in the range of tens of millibarns upwards. This result can be seen in Fig. 2 of Ref. 7. The unitarization allows for the possibility of having two (or more) hard collisions in an event.

We disagree with this application of the eikonal model, and can demonstrate what a more correct formula should be. The graph in Fig. 1 is a contribution to the amplitude for two hard scatterings in a photon-hadron collision. The shaded bubbles represent low-mass physics associated with the photon and the hadron, and there are two gluonic ladders exchanged between them. This cross section has one power of α_{em}, rather than the two that the eikonal formula of Ref. 7 possesses, and is therefore much bigger. (The rest of the physics is comparable.) Consequently the contribution of our graph to the two-hard-scattering cross section is the dominant one. Therefore the unitarization in Ref. 7 severely overestimates total photoproduction cross sections when the jet cross sections are above the conventional value for the total cross section.

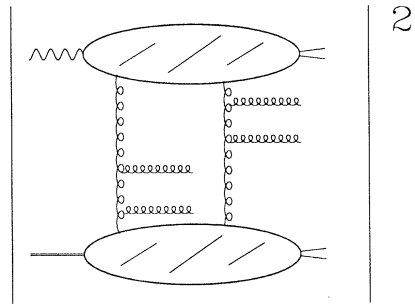

FIG. 1. A contribution to the amplitude for two hard scatterings in a photon-hadron collision.

Now the electromagnetic factors are the same as in the graphs for a single hard scattering (see Fig. 2). Therefore a more appropriate eikonal approximation is

$$\sigma_T = \frac{4\pi\alpha_{\text{em}}}{f_\rho^2} 2\pi \int db\, b\langle i|1-e^{-2\chi'^R(b,s)}|i\rangle \ ,$$

where the modified eikonal is

$$\chi'^R = \tfrac{1}{2}A(b)[\sigma_{\text{soft}}+\sigma_{\text{QCD}}]\frac{f_\rho^2}{4\pi\alpha_{\text{em}}} \ .$$

Corresponding ideas applied to the saturation issue produce Eq. (28). This constitutes a demonstration of Eq. (28). Of course, this is not a proof of the *precise* numbers

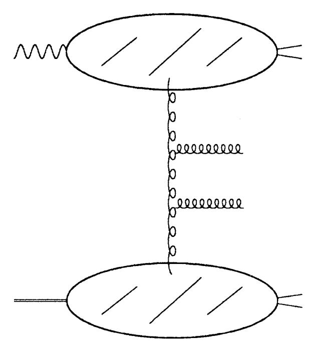

FIG. 2. A graph for a single hard scattering in a photon-hadron collision.

associated with the formula. What does matter and is correct, is the overall structure, in particular the places where the power of α_{em} appear.

VI. COSMIC-RAY PHOTON CROSS SECTIONS

Drees and Halzen explain the high muon content of cosmic-ray showers by supporting the view that the photon behaves rather uniquely at high energies.[23] They try to demonstrate this point by taking the existing parametrizations for the distribution functions in the photon and studying the photon-proton cross sections at energies up to 5000 GeV in the center of mass. The plots they show use the photon distributions from Duke and Owens,[15] Drees and Grassie,[16] and Storrow and Da Luz Vieira.

What they show are cross sections which increase dramatically with increasing center-of-mass energy. Are their results reasonable?

For the parton distributions in the *proton* we used the Duke and Owens distribution (set one). Computations show that numerical results do not vary much for the photon-proton reactions when the distributions of Glück, Hoffmann, and Reya[24] are used instead.

We have reproduced the results of Ref. 23. For simplicity, we restrict attention to the $gg \to gg$ and $gg \to q\bar{q}$ subprocesses. By using the asymptotic results, we can do the calculations very easily, and the results agree with a full calculation to within a factor of 2 even at moderate energies. The exact numerical computation uses the Duke and Owens distributions for the proton (set one) and for the photon with scales of p_T^2 and $2p_T^2$ respectively. Three quark flavors were taken and the strong coupling was taken in the one-loop order. Our results are shown in Tables I and II.

The discrepancy at low $\sqrt{s}$ of our partial cross sections with the total value is because at high x the subprocess of gluon-gluon fusion does not dominate, making it important to include the quark contributions. As we move to high $\sqrt{s}$ and therefore small x, we find the gluon process dominates the cross section and that the asymptotic calculations are good.

In Ref. 23 it is stated that the authors "have shown that the cross section for the production of two jets with $p_T > 2-3$ GeV strongly increases with photon energy and exceeds this "conventional" VDM cross section by more than an order of magnitude for $\sqrt{s} \geq 1$ TeV." The conclusion is drawn that the total photoproduction cross section is correspondingly high. We disagree strongly with this conclusion, as we will now explain.

TABLE I. Inclusive jet cross-section contribution from $gg \to gg, q\bar{q}$ using Duke and Owens distributions. Note $\alpha_s(2p_T^2) \approx 0.35$ when $p_T = p_{T\min} = 2$ GeV.

$E_{c.m.}$ (GeV)	$\tilde{\sigma}^{\gamma p}_{exact}$ (μb)	$\tilde{\sigma}^{\gamma p}_{asympt}$ (μb)	Ref. 23 (μb)
50	5.8	9.8	20
100	29	37	31
500	780	850	1000
1000	3000	3300	4000

TABLE II. Inclusive jet cross-section contribution from $gg \to gg, q\bar{q}$ using the Drees and Grassie form for the gluon content of the photon. Note $\alpha_s(p_T^2) \approx 0.43$ when $p_T = p_{T\min} = 2$ GeV.

$E_{c.m.}$ (GeV)	$\tilde{\sigma}^{\gamma p}_{exact}$ (μb)	$\tilde{\sigma}^{\gamma p}_{asympt}$ (μb)	Ref. 23 (μb)
50	5.3	43	13
100	20	73	30
500	180	260	250
1000	380	440	500
5000	2000	1500	2000

The calculation as performed above is for the the inclusive cross section for jet production, with the given minimum p_T. It is an inclusive cross section, because the cross section is summed over all final states containing the jet(s) that define the cross section. Now, the relevant quantity for the muon content of the cosmic-ray observations is the total hadronic part of the cross section. This is obtained from an inclusive cross section by dividing by an appropriate multiplicity. There is a minimum of two jets, so the jet multiplicity is at least two. Since at the small values of x we are discussing, there are many gluons, there can be many hard collisions occurring in parallel, and the multiplicity can be high.[11]

Drees *et al.*[6,23] do not do this. They even *multiply* by a factor of the hadronic multiplicity. Furthermore, as we have already observed, the distributions of gluons in the photon that they use to give their highest cross sections are those of Duke and Owens; these are not realistic at the small values of x that are needed, and they are strongly saturated. In saying they have reached millibarn cross sections, they have taken the results of their calculations at the extreme of their plots at $\sqrt{s} \approx 5$ TeV, where we find we are computing cross sections with photon fractional momenta on the order of 10^{-6}.

One can further criticize their conclusions by recalling the origin of the large gluon content of the photon at small x. It arises from the inhomogeneous term in the Altarelli-Parisi equations for the photon. As we observed earlier, we have a transition to a high-mass $q\bar{q}$ state, followed by gluon emission. After the collision the $q\bar{q}$ pair materializes in the final state. That part of the distribution that arises from low-mass states is something that is contained inside the vector meson in the VDM. This should not give rise to a cross section significantly above the conventional value, any more than the rapidly rising minijet cross section in hadron-hadron collisions impinges on the total cross section.

A quantity that directly estimates the extra contribution of the high-mass $q\bar{q}$ states to the total cross section is the contribution of the photon-gluon process, divided by two to compensate the multiplicity of the jets in the hard process. Now the resulting final state contains a quark or antiquark that has a large transverse momentum (on a hadronic scale), and that is in the forward direction. Therefore the final state is orthogonal to the final states from the VDM contribution, and we may add the contribution from photon-gluon fusion to the VDM cross section to estimate the total cross section. The value of this extra contribution was calculated in Sec. IV. At $\sqrt{s_{\gamma p}}=1$ TeV and $p_{T\min}=2$ GeV, it is about 68 μb, which gives a significant, but not dramatic, increase in the total cross section. Saturation effects are becoming significant under those conditions, so the true increase is less. The numerical value is, of course, very sensitive to the minimum p_T. As this cutoff is lowered, however, we first find saturation limits the cross section, and second, we get to a region of p_T where the quarks are in the nonperturbative regime. This region is already taken into account by the VDM. Saturation limits the new contribution to a factor of a few larger than the 68 μb just quoted.

VII. SUMMARY

High-energy cosmic-ray photons from point sources were expected to have atmospheric particle showers with few muons. However, experimental results from these high-energy cosmic-ray showers perhaps suggest otherwise. Conceivably, effects of QCD and QED that are insignificant at lower energy are becoming important in the new energy regime. At these high energies, where we are probing small-x physics, we have shown, with simple asymptotic formulas, how the steep small-x properties of the gluon content of the photon drive the gluon-gluon contribution to the jet cross section. This, however, does not significantly affect the total cross section. What does give a notable rise in the total cross section, but by less than a factor 2, is the photon-gluon-fusion process.

Confusion has existed with respect to the gluon distribution of the photon. The Duke and Owens photon distribution is a leading-log parametrization,[15] which was obtained by solving the relevant inhomogeneous Altarelli-Parisi equations using moments and then fitting simple parametrizations to the results.[18] The leading-log approximation does not reproduce leading-order evolution. The form of this equation at small x is $x^{-1.97}$, which is just 0.03 away in the exponent from leading to an infinite-momentum contribution from the gluon content of the photon. The Drees and Grassie distribution is more reasonable at small x since they included a VDM-inspired boundary condition to be able to extrapolate to lower x values. For $Q^2=p_{T\min}^2=4$ GeV^2 this distribution carries a dependence of about $x^{-1.4}$.

Both distributions, however, gave indications that saturation is being reached for high-energy γp reactions. Using the ideas stemming from Gribov, Levin, and Ryskin we have been able to demonstrate that saturation becomes relevant when considering γp reactions with $\sqrt{s_{\gamma p}}$ of the order of a few TeV and higher with transverse momenta down as low as 1–2 GeV. The result is that the theoretical cross section values being obtained by extrapolating these distribution functions to low x are high.

In our discussions we have focused on the total cross section rather than the inclusive cross section. This is because it is the ratio of the total hadronic part of cross section to the Bethe-Heitler cross section that determines the fraction of cosmic-ray air-shower events induced by photon that are hadronic in character. If this ratio stays small then most photon-induced air showers have to start out electromagnetic in nature. The rise in inclusive cross section affects the actual multiplicity and the detailed characteristics only of the small fraction of events that are induced by the hadronic component of the photon. It would be interesting to look for the extra hadronic component associated with the photon-gluon fusion process, since these appear to have a significant effect on the total cross section. Rather extreme values of parton x are involved, down to around $x=10^{-6}$. This implies that saturation effects will be important and cut off any dramatic rise in the cross section.

Care must be taken in applying these photon distribution functions when going to low-x values and low scales at high energies such as in high-energy cosmic-ray physics or in experiments at HERA.

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