Radiative weak decays $D^0 \to \bar{K}^{*0}\gamma$ and $D_s^+ \to \rho^+\gamma$

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In a quark model, modified to take into account the large recoil momentum, it is estimated that $B(D^0 \to \bar{K}^{*0}\gamma) = (0.86\times 10^{-3})\%$ and $B(D_s^+ \to \rho^+\gamma) = (2.1\times 10^{-3})\%$.

I. INTRODUCTION

One anticipates that with the construction of a τ-charm or a B factory it will become possible to measure the branching ratios of the rare decays of D and D_s^+ mesons. One such process is the radiative weak decay of D and D_s^+. The branching ratio for a Cabibbo-angle-favored radiative decay of D, say $D^0 \to \bar{K}^{*0}\gamma$, is naively expected to be $\sim \alpha B(D^0 \to \bar{K}^{*0}\rho^0)$, where α is the electromagnetic fine-structure constant, by a vector-meson-dominance argument. Since $B(D^0 \to \bar{K}^{*0}\rho^0)$ is known[1] to be $(1.9\pm 0.3\pm 0.7)\%$, we anticipate,[2] very roughly, $B(D^0 \to \bar{K}^{*0}\gamma) \sim 10^{-2}\%$. Note that at this level the radiative weak decays compete with the doubly Cabibbo-angle-suppressed hadronic rates. One expects, for example, that the doubly Cabibbo-angle-suppressed $D^0 \to K^{*0}\rho^0$ decay would have a branching ratio $B(D^0 \to K^{*0}\rho^0) = \tan^4\theta_C B(D^0 \to \bar{K}^{*0}\rho^0)$, where θ_C is the Cabibbo angle. This gives $B(D^0 \to K^{*0}\rho^0) \approx (0.6\times 10^{-2})\%$. In this paper, we have calculated the branching ratios for two radiative weak decays: $D^0 \to \bar{K}^{*0}\gamma$ and $D_s^+ \to \rho^+\gamma$.

Consider, first, the matrix element for a generic radiative weak decay of kind $P \to V\gamma$. The most general form for the S-matrix element, consistent with gauge invariance, is

$$S(P \to V\gamma) = \frac{1}{[(2\pi)^3 2P^0]^{1/2}} \frac{1}{[(2\pi)^3 2P'^0]^{1/2}} \times \frac{1}{[(2\pi)^3 2k^0]^{1/2}} \times (2\pi)^4 \delta^4(P - P' - k)(F^{\rm PC} + F^{\rm PV}) , \quad (1)$$

where

$$F^{\rm PC} = iA\,\epsilon_{\mu\nu\rho\sigma}\epsilon_V^{*\mu}\epsilon_\gamma^{*\nu}P^\rho k^\sigma \quad (2)$$

and

$$F^{\rm PV} = B\left[(\epsilon_V^* \cdot \epsilon_\gamma^*) - \left[\frac{2}{M^2 - M'^2}\right](\epsilon_\gamma^* \cdot P)(\epsilon_V^* \cdot k)\right] , \quad (3)$$

where P, P', and k are the four-momenta of the initial and the final mesons and the photon respectively. M and M' are the masses of the initial and the final mesons. ϵ_V^μ and ϵ_γ^ν are the vector-meson and the photon polarization vectors, respectively. $F^{\rm PC}$ and $F^{\rm PV}$ are the parity-conserving and parity-violating amplitudes. The parity-conserving amplitude involves P waves in the final state while the parity-violating amplitude involves S and D waves, which are related by gauge invariance. From (1)–(3), the decay rate is calculated to be

$$\Gamma(P \to V\gamma) = \frac{1}{4\pi}\left[|A|^2 k^3 + |B|^2 \frac{k}{M^2}\right] . \quad (4)$$

The calculation of the decay rate, thus, reduces to a calculation of the form factors A and B.

In this paper, we report a calculation of $D^0 \to \bar{K}^{*0}\gamma$ and $D_s^+ \to \rho^+\gamma$ decay rates in the framework of the quark model. Since the photon momentum is large $(k > m_u, m_d, m_s)$ in these decays, we have modified the quark model in a manner where the quark energy, instead of being approximated to the mass, is approximated by an average value. This allows an expansion in the parameter p/E. This "improvised" quark model is discussed in Sec. II. Results are discussed in Sec. III.

II. MODEL AND CALCULATION

We discuss $D^0 \to \bar{K}^{*0}\gamma$ in detail. Modifications leading to the results for $D_s^+ \to \rho^+\gamma$ are pointed out later.

The QCD-corrected effective four-fermion interaction leading to Cabibbo-angle-favored charm decays is[3]

$$H_W = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \left[\left[\frac{C_+ + C_-}{2}\right](\bar{\psi}_u \Gamma_\mu \psi_d)(\bar{\psi}_s \Gamma^\mu \psi_c) + \left[\frac{C_+ - C_-}{2}\right](\bar{\psi}_u \Gamma_\mu \psi_c)(\bar{\psi}_s \Gamma^\mu \psi_d)\right] , \quad (5)$$

where V_{ud} and V_{cs}^* are the Kobayashi-Maskawa mixing angles[4] and Γ_μ is $\gamma_\mu(1-\gamma_5)$. ψ_q is the fermion field for quark q. $(\bar{\psi}\psi)$ represents a color-singlet combination. C_+ and C_- are the QCD coefficients, given in the leading-log approximation by

$$C_\pm(\mu) = \left[\frac{\alpha_s(\mu^2)}{\alpha_s(m_W^2)}\right]^{d_\pm/2b} \quad (6)$$

with

$$d_- = -2d_+ = 8 \tag{7}$$

and

$$b = 11 - \tfrac{2}{3}N_f, \quad N_f = \text{number of flavors} . \tag{8}$$

μ, the mass scale, for charm decays is taken to be m_c and α_s is the strong fine-structure constant.

For $D^0 \to \bar{K}^{*0}\gamma$ decay, since the photon does not carry color, it is more convenient to rewrite H_W of (5) in the form

$$H_W = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* a_2 (\bar{\psi}_u \Gamma_\mu \psi_c)(\bar{\psi}_s \Gamma^\mu \psi_d) , \tag{9}$$

where

$$a_2 = \tfrac{1}{3}(2C_+ - C_-) . \tag{10}$$

Form (9) is obtained from (5) by a Fierz transformation of the first term of (5) in Dirac and color space. Rearranging colors suppresses the first term by a factor of 3.

Starting with (9) as the effective weak Hamiltonian and allowing the photon to be radiated by each of the four external legs in the Feynman diagram, the S matrix for $c(p_1) + \bar{u}(p_2) \to s(p_1') + \bar{d}(p_2') + \gamma(k)$, is given by

$$S(c + \bar{u} \to s + \bar{d} + \gamma) = \frac{1}{[(2\pi)^3 2k^0]^{1/2}} \frac{1}{(2\pi)^{3/2}} \left[\frac{m_s}{E_s}\right]^{1/2} \frac{1}{(2\pi)^{3/2}} \left[\frac{m_d}{E_{\bar{d}}}\right]^{1/2} \frac{1}{(2\pi)^{3/2}} \left[\frac{m_c}{E_c}\right]^{1/2}$$
$$\times \frac{1}{(2\pi)^{3/2}} \left[\frac{m_u}{E_{\bar{u}}}\right]^{1/2} (2\pi)^4 \delta^4(p_1 + p_2 - p_1' - p_2' - k)(\tilde{F}^{PC} + \tilde{F}^{PV}) , \tag{11}$$

where $\tilde{F}^{PC}$ and $\tilde{F}^{PV}$ are gauge-invariant amplitudes for the quarks on their mass shells. After a two-component reduction and in Coulomb gauge ($\epsilon^{*0} = 0$, $\boldsymbol{\epsilon}^* \cdot \mathbf{k} = 0$), $\tilde{F}^{PC}$ and $\tilde{F}^{PV}$ are given by (the calculation proceeds as in Refs. 5 and 6)

$$\tilde{F}^{PC} = i\frac{G_F e}{\sqrt{2}} V_{ud} V_{cs}^* a_2 \Lambda_s \Lambda_c \Lambda_{\bar{u}} \Lambda_{\bar{d}} \{ [(s^\dagger \boldsymbol{\sigma} \cdot \mathbf{k} d^{(-)})(u^{(-)\dagger} \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}^* c) - (s^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}^* d^{(-)})(u^{(-)\dagger} \boldsymbol{\sigma} \cdot \mathbf{k}\, c)] H(k)$$
$$+ i(\boldsymbol{\epsilon}^* \times \mathbf{k}) \cdot [(s^\dagger d^{(-)})(u^{(-)\dagger} \boldsymbol{\sigma} c) - (s^\dagger \boldsymbol{\sigma} d^{(-)})(u^{(-)\dagger} c)] G(k) \} \tag{12}$$

and

$$\tilde{F}^{PV} = i\frac{G_F e}{\sqrt{2}} V_{ud} V_{cs}^* a_2 \Lambda_s \Lambda_c \Lambda_{\bar{u}} \Lambda_{\bar{d}} \{ k [(s^\dagger d^{(-)})(u^{(-)\dagger} \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}^* c) - (s^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}^* d^{(-)})(u^{(-)\dagger} c)] H(k)$$
$$+ ik \boldsymbol{\epsilon}^* \cdot [(s^\dagger \boldsymbol{\sigma} d^{(-)}) \times (u^{(-)\dagger} \boldsymbol{\sigma} c)] G(k) \} , \tag{13}$$

where

$$\Lambda_{q,\bar{q}} = \left[\frac{E_{q,\bar{q}} + m_q}{2m_q}\right]^{1/2} ; \tag{14}$$

q stands for the appropriate quark flavor. s and c are the Pauli spinors for the appropriate flavors belonging to the positive-energy solutions and $d^{(-)}$ and $u^{(-)}$ are the Pauli spinors belonging to the negative-energy solutions for the appropriate flavor fields. $G(k)$ and $H(k)$ are the quark propagator factors with the external quarks on shell:

$$G(k) = \frac{Q_d}{2p_2' \cdot k} + \frac{Q_u}{2p_2 \cdot k} + \frac{Q_s}{2p_1' \cdot k} + \frac{Q_c}{2p_1 \cdot k} , \tag{15}$$

$$H(k) = \frac{Q_d}{2p_2' \cdot k} - \frac{Q_u}{2p_2 \cdot k} - \frac{Q_s}{2p_1' \cdot k} + \frac{Q_c}{2p_1 \cdot k} , \tag{16}$$

where we have written $G(k)$ and $H(k)$ in terms of the *quark* charges (in units of e).

The hadronic matrix element is now written in terms of the quark one as

$$S(D^0 \to \bar{K}^{*0}\gamma) = \int d^3\mathbf{p}_1' d^3\mathbf{p}_2' d^3\mathbf{p}_1 d^3\mathbf{p}_2 \psi_{\bar{K}^{*0}}^*(\mathbf{p}_1', \mathbf{p}_2')$$
$$\times \langle \bar{K}^{*0} | S(c + \bar{u}) \to s + \bar{d} + \gamma | D^0 \rangle$$
$$\times \psi_{D^0}(\mathbf{p}_1, \mathbf{p}_2) , \tag{17}$$

where $|D^0\rangle$ and $|\bar{K}^{*0}\rangle$ are the spin-flavor wave functions and ψ_{D^0} and $\psi_{\bar{K}^{*0}}$ the momentum-space wave functions[7]:

$$\psi_{D^0}(\mathbf{p}_1, \mathbf{p}_2) = \frac{1}{(\alpha\sqrt{\pi})^{3/2}} \delta^3(\mathbf{P} - \mathcal{P}) \exp\left[-\frac{\mathbf{p}^2}{2\alpha^2}\right] . \tag{18}$$

$\mathbf{p}_1$ and $\mathbf{p}_2$ are related to $\mathcal{P}$ and $\mathbf{p}$ by

$$\mathbf{p}_1 = f_c \mathcal{P} + \mathbf{p}, \quad \mathbf{p}_2 = f_u \mathcal{P} - \mathbf{p} \tag{19}$$

and

$$f_{c,u} = m_{c,u} / (m_c + m_u) . \tag{20}$$

$\psi_{\bar{K}^{*0}}$ is very similar and we assume that α is the same in both cases.

In carrying out the integrations in (17), we first approximate $\delta(p_1^0 + p_2^0 - p_1'^0 - p_2'^0 - k^0)$ by $\delta(P^0 - P'^0 - k^0)$, which is the "loose-binding" approximation. The other dependences on $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_1'$, $\mathbf{p}_2'$ are in the $\sqrt{m/E}$ factors,

$\Lambda_{q,\bar{q}}$, and in $G(k)$ and $H(k)$. The naive nonrelativistic-quark-model approach is to do a (p/m) expansion keeping only the lowest-order term. Since the final meson has a large recoil momentum, we believe that such an approximation may not be reliable. With a view to improve upon this approximation, we replace

$$E_{c,\bar{u}} \to \bar{E}_{c,\bar{u}} \equiv (\langle \mathbf{p}_{1,2}^2 \rangle + m_{c,u}^2)^{1/2} ,$$
$$E_{s,\bar{d}} \to \bar{E}_{s,\bar{d}} \equiv (\langle \mathbf{p}_{1,2}'^2 \rangle + m_{s,d}^2)^{1/2} , \tag{21}$$

where

$$\langle \mathbf{p}_{1,2}^2 \rangle = \frac{\int d^3\mathbf{p}_1 d^3\mathbf{p}_2 \psi_{D^0}^*(\mathbf{p}_1,\mathbf{p}_2)\mathbf{p}_{1,2}^2 \psi_{D^0}(\mathbf{p}_1,\mathbf{p}_2)}{\int d^3\mathbf{p}_1 d^3\mathbf{p}_2 \psi_{D^0}^*(\mathbf{p}_1,\mathbf{p}_2)\psi_{D^0}(\mathbf{p}_1,\mathbf{p}_2)} \tag{22}$$

and a similar definition for $\langle \mathbf{p}_{1,2}'^2 \rangle$. The expressions for the averages are

$$\bar{E}_{c,\bar{u}} = \left[f_{c,u}^2 \mathbf{P}^2 + \frac{3\alpha^2}{2} + m_{c,u}^2 \right]^{1/2} ,$$
$$\bar{E}_{s,\bar{d}} = \left[f_{s,d}'^2 \mathbf{P}'^2 + \frac{3\alpha^2}{2} + m_{s,d}^2 \right]^{1/2} , \tag{23}$$

where

$$f_{s,d}' = m_{s,d}/(m_s+m_d) . \tag{24}$$

We also approximate,[6] in $G(k)$ and $H(k)$,

$$\frac{1}{p\cdot k} = \frac{1}{Ek - \mathbf{p}\cdot\mathbf{k}} \cong \frac{1}{\bar{E}k} , \tag{25}$$

where $\bar{E}$ is the average energy of the quark or antiquark with three-momentum $\mathbf{p}$. The average energy has a dependence on meson momentum, as well as the cutoff α provided by the wave functions. Physically, therefore, one would expect this to be a better approximation than a (p/m) expansion.[6]

After these replacements, the only $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_1'$, $\mathbf{p}_2'$ dependence in $S(c+\bar{u} \to s+\bar{d}+\gamma)$ is in $\delta^3(\mathbf{p}_1+\mathbf{p}_2-\mathbf{p}_1'-\mathbf{p}_2'-\mathbf{k})$. The integration is readily carried out resulting in (in D^0 rest frame)

$$\begin{aligned} S(D^0 \to \bar{K}^{*0}\gamma) = {} & \frac{1}{[(2\pi)^3 2P^0]^{1/2}} \frac{1}{[(2\pi)^3 2P'^0]^{1/2}} \\ & \times \frac{1}{[(2\pi)^3 2k^0]^{1/2}} (2\pi)^4 \delta(M-P'^0-k^0) \\ & \times \delta^3(\mathbf{P}'+\mathbf{k}) N_{\bar{K}^{*0}}^* N_{D^0} \frac{\alpha^3}{\pi^{3/2}} \\ & \times \langle \bar{K}^{*0} | \tilde{F}^{\rm PC} + \tilde{F}^{\rm PV} | D^0 \rangle , \end{aligned} \tag{26}$$

where

$$N_{\bar{K}^{*0}}^* = \left[(2P'^0) \left[\frac{m_s}{\bar{E}_s} \right] \left[\frac{m_d}{\bar{E}_{\bar{d}}} \right] \right]^{1/2} , \tag{27}$$

$$N_{D^0} = \left[(2P^0) \left[\frac{m_c}{\bar{E}_c} \right] \left[\frac{m_u}{\bar{E}_{\bar{u}}} \right] \right]^{1/2} . \tag{28}$$

Comparing (26) with (1), we identify

$$F^{\rm PC(PV)} = N_{\bar{K}^{*0}}^* N_{D^0} \frac{\alpha^3}{\pi^{3/2}} \langle \bar{K}^{*0} | \tilde{F}^{\rm PC(PV)} | D^0 \rangle . \tag{29}$$

If, in the rest frame of D^0, we choose the photon to be moving in the negative z direction and $\bar{K}^{*0}$ and the photon have spin projections $\lambda_V = +1$ and $\lambda_\gamma = -1$, then, from (2) and (3),

$$F^{\rm PC} = -AMk \tag{30}$$

and

$$F^{\rm PV} = B . \tag{31}$$

Therefore, from (29)–(31),

$$A = -\frac{1}{Mk} N_{\bar{K}^{*0}}^* N_{D^0} \frac{\alpha^3}{\pi^{3/2}} \langle \bar{K}^{*0} | \tilde{F}^{\rm PC} | D^0 \rangle_{\substack{\mathbf{k}=(0,0,-k) \\ \lambda_V=+1 \\ \lambda_\gamma=-1}} \tag{32}$$

or

$$B = N_{\bar{K}^{*0}}^* N_{D^0} \frac{\alpha^3}{\pi^{3/2}} \langle \bar{K}^{*0} | \tilde{F}^{\rm PV} | D^0 \rangle_{\substack{k=(0,0,-k) \\ \lambda_V=+1 \\ \lambda_\gamma=-1}} . \tag{33}$$

Using (12) and (13), one can calculate the spin-flavor matrix elements in (32) and (33) in the specific helicity state. However, because of the presence of antiquarks, there are some subtleties involved in this calculation. The procedure is, however, well known.[8,9] We first write down $\tilde{F}^{\rm PC,PV}$ in the specific helicity state. Then we carry out a Fierz transformation in Pauli space to write (12) and (13) in a form where $s^\dagger$ is contracted with c and $u^{(-)\dagger}$ is contracted with $d^{(-)}$. We, then, convert $u^{(-)}$ and $d^{(-)}$ to positive-energy spinors; i.e., we convert negative-energy quark states into positive-energy antiquark states. This is done by a charge conjugation in Pauli space:

$$u^{(+)} = -i\sigma^2 u^{(-)} . \tag{34}$$

$\tilde{F}^{\rm PC,PV}$, then, become

$$\begin{aligned} \tilde{F}^{\rm PC} = i\frac{G_F e}{\sqrt{2}} V_{ud} V_{cs}^* a_2 \Lambda_s \Lambda_c \Lambda_{\bar{u}} \Lambda_{\bar{d}} k [& -H(k)(s^\dagger c)(u^{(+)\dagger}\sigma^- d^{(+)}) - H(k)(s^\dagger \sigma^+ c)(u^{(+)\dagger} d^{(+)}) \\ & - G(k)(s^\dagger \sigma^+ c)(u^{(+)\dagger}\sigma^3 d^{(+)}) + G(k)(s^\dagger \sigma^3 c)(u^{(+)\dagger}\sigma^- d^{(+)})] , \end{aligned} \tag{35}$$

$$\begin{aligned} \tilde{F}^{\rm PV} = i\frac{G_F e}{\sqrt{2}} V_{ud} V_{cs}^* a_2 \Lambda_s \Lambda_c \Lambda_{\bar{u}} \Lambda_{\bar{d}} k [& -H(k)(s^\dagger \sigma^+ c)(u^{(+)\dagger}\sigma^3 d^{(+)}) + H(k)(s^\dagger \sigma^3 c)(u^{(+)\dagger}\sigma^- d^{(+)}) \\ & - G(k)(s^\dagger c)(u^{(+)\dagger}\sigma^- d^{(+)}) - G(k)(s^\dagger \sigma^+ c)(u^{(+)\dagger} d^{(+)})] . \end{aligned} \tag{36}$$

Calculating the spin-flavor matrix elements in (32) and (33), we find

$$\begin{bmatrix} A \\ B \end{bmatrix} = i\frac{G_F e}{\sqrt{2}} V_{ud} V_{cs}^* a_2 N_{\bar{K}^{*0}}^* N_{D^0} \times \Lambda_s \Lambda_c \Lambda_{\bar{u}} \Lambda_{\bar{d}} \frac{2\alpha^3}{\pi^{3/2}} \begin{bmatrix} -G/M \\ kH \end{bmatrix} . \quad (37)$$

For numerical computations, we have used the constituent quark model parameters: $m_u = m_d = 0.35$ GeV, $m_s = 0.55$ GeV, $m_c = 1.5$ GeV, and $\alpha = 0.4$ GeV. We obtain (with[3] $a_2 = -0.5$)

$$\Gamma(D^0 \to \bar{K}^{*0}\gamma) = 1.32 \times 10^{-17} \text{ GeV} \quad (38)$$

which gives

$$B(D^0 \to \bar{K}^{*0}\gamma) = 0.86 \times 10^{-3}\% . \quad (39)$$

The calculation of the decay rate for $D_s^+ \to \rho^+\gamma$ proceeds in the same manner with the following substitutions: $D^0 \to D_s^+$, $\bar{K}^{*0} \to \rho^+$, $c \to c, u^{(\pm)} \to s^{(\pm)}$, $s \to u$, $d^{(\pm)} \to d^{(\pm)}$, $a_2 \to a_1$ $[=(2C_+ + C_-)/3]$. Also, the subscripts $c \to c$, $u \to s$ $(\bar{u} \to \bar{s})$, $s \to u$, and $d \to d$ $(\bar{d} \to \bar{d})$ in all places except in $V_{ud} V_{cs}^*$ and in (5), which remain unchanged. Using[3] $a_1 = 1.2$ and other parameters as in the case of $D^0 \to \bar{K}^{*0}\gamma$ decay, we obtain

$$\Gamma(D_s^+ \to \rho^+\gamma) = 3.2 \times 10^{-17} \text{ GeV} \quad (40)$$

and

$$B(D_s^+ \to \rho^+\gamma) = (2.1 \times 10^{-3})\% . \quad (41)$$

III. DISCUSSION

Our results in (39) and (41) indicate that the branching ratio for these Cabibbo-angle-favored radiative weak decays is roughly 10^{-3} times a typical two-body hadronic weak decay branching ratio. The two-body Cabibbo-angle-favored hadronic decays have branching ratios, typically, in the range 1–5 %. This result is also in conformity with the radiative weak decays in the baryon sector. For example, Λ and Σ^+ have branching ratios into exclusive two-body modes, typically, of the order of 50%. The radiative decay branching ratios are typically 0.1%, which is roughly a factor of 10^{-3} compared to the two-body exclusive hadronic branching ratios. The same is true of the theoretical expectation[6] for $B(\Lambda_c^+ \to \Sigma^+\gamma)$ which is predicted to be $\approx (3 \times 10^{-3})\%$. A typical two-body hadronic decay of Λ_c^+ has a branching ratio of $\approx 1\%$. The theoretical prediction is lower than this by a factor of 10^{-3}.

The branching ratio for $D_s^+ \to \rho^+\gamma$ is only a factor of 2 higher than that of $D^0 \to \bar{K}^{*0}\gamma$. One might have expected an enhancement of roughly $(a_1/a_2)^2 = 6$. However, the charge contents of D_s^+ and D^0 being different, the factors $G(k)$ and $H(k)$ are also different. This eventually results in an enhancement of only a factor of 2 for $B(D_s^+ \to \rho^+\gamma)$ over that for $B(D^0 \to \bar{K}^{*0}\gamma)$.

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