Heavy-top-quark decay into a W boson and a photon (gluon)

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We consider the decay of a heavy top quark into a W boson and a hard photon (gluon). Our results for the differential decay rate for these two processes are remarkably simple and have been independently checked. We obtain a branching ratio of 0.33% for $bW^+\gamma$.

Recently we have considered the radiative decay $W^-\rightarrow b\bar{t}\gamma$ for massive quarks.[1] This process is sensitive to the magnetic moment of the W boson and can be used as a test of the standard model (SM). The magnetic moment of the W is given by $\kappa=1$ in the SM. Radiation amplitude zeros[2] (RAZ's) occur for $\kappa=1$ only, thus providing a sensitive test of the SM. It now appears likely, however, that the top-quark mass is very large[3,4] $m_t>M_W+m_b$ and thus one has to consider the decay $t\rightarrow bW^+\gamma$ for massive quarks. As will be seen, this process exhibits the factorization which occurs when RAZ's are present. However, no RAZ's are present in the physical region for the radiative t decay because of the presence of opposite-sign charges. Nevertheless, this process is sensitive to the $WW\gamma$ trilinear coupling and κ, and thus can be used as a test of the sm.

Let us consider the general process

$$q_t(P)\rightarrow q_i(p_1)+W(p_2)+\gamma(k) \tag{1}$$

where M, m_i, and M_W are the masses of the top quark q_t, q_i, and W, respectively, and P, p_1, p_2, and k are the respective four-momenta. The fermion electric charges are Q_i for q_i and Q_j for q_t. The fermions may be quarks or leptons either known or to be discovered, and the W may be the standard $W^\pm$ or a heavy W such as W_R in the left-right-symmetric model.[5] We use the standard-model $WW\gamma$ coupling which corresponds to $\kappa=1$ for the W. Our results are equally valid for W_L ($V-A$ coupling) or W_R ($V+A$ coupling).

We will choose to work in the following variables:

$$X=\frac{2(p_1+p_2)\cdot k}{M^2}\ , \tag{2}$$

$$Y=\frac{(p_1-p_2)\cdot k}{(p_1+p_2)\cdot k}\ . \tag{3}$$

These are a generalization of the variables used by Samuel and Tupper.[6] In the center-of-mass frame, X is simply the scaled photon energy:

$$X=2E_\gamma/M\ . \tag{4}$$

The lowest-order partial differential decay rate is remarkably simple in these variables:

$$\frac{1}{M}\frac{\partial^2\Gamma}{\partial X\,\partial Y}=\frac{\alpha^2|K_{ij}|^2}{32\pi\sin^2\theta_W}\frac{[Q_j(1+Y)-2Q_i]^2}{(1-Y^2)}\times\left[\frac{1}{X}\left[\frac{(1-\mu_1^2)^2}{\mu_2^2}-2\mu_2^2+1+\mu_1^2\right]\Omega+\frac{X[4+(1+Y)^2]}{2(1-Y)}+\frac{X(1-Y)}{2}\left[\frac{(1+\mu_1^2)}{2\mu_2^2}\right]\right]\ , \tag{5}$$

where

$$\Omega=1-X-2\left[\frac{\mu_1^2}{1+Y}+\frac{\mu_2^2}{1-Y}\right] \tag{6}$$

and

$$\mu_1=\frac{m_i}{M_t},\quad \mu_2=\frac{M_W}{M_t}\ , \tag{7}$$

and K_{ij} are the Kobayashi-Maskawa matrix elements. This result has been obtained in several different ways: Eqs. (5)–(7) have been calculated by hand, by a computer calculation from first principles using REDUCE, and by a computer calculation using REDUCE, by crossing from radiative W decay.

These calculations all agree, thus providing two independent checks on our result.[7] One can see explicitly the factorization in Eq. (5). The zero factor is $Z=Q_j(1+Y)-2Q_i$.

To obtain the result for $t\rightarrow bW^+g$ one needs only to replace α^2 in Eq. (5) by $\alpha\alpha_sC_F$ (C_F is the color factor which we take to be $C_F=\frac{4}{3}$) and choose the charges $Q_i=Q_j=1$.

The constraint on X and Y to lie in the phase space is simply

$$\Omega\geq 0\ . \tag{8}$$

The extreme values of X are given by

$$0\leq X\leq 1-(\mu_1+\mu_2)^2 \tag{9}$$

and, for a given fixed X, the Y limits are

$$\frac{\Delta-\lambda(X)}{1-X} \leq Y \leq \frac{\Delta+\lambda(X)}{1-X} , \qquad (10)$$

where

$$\lambda(X)=[(1-X)^2-2(1-X)\epsilon+\Delta^2]^{1/2} \qquad (11)$$

and

$$\epsilon=\mu_1^2+\mu_2^2, \quad \Delta=\mu_1^2-\mu_2^2 . \qquad (12)$$

Some representative results are shown in Figs. 1–7. Figures 1–3 show $(1/M)\partial^2\Gamma/\partial X\,\partial Y$ vs Y for various values of the charges and the masses. Figure 1 is for the decay $t\to bW^+\gamma$ with $M_t=200$ GeV, $m_i=5$ GeV, $M_W=81$ GeV, $Q_i=-\frac{1}{3}$, and $Q_j=\frac{2}{3}$ $(X=0.3)$. In Fig. 2 everything is unchanged except that $M_t=150$ GeV (and $X=0.2$). Figure 3 illustrates the heavy-lepton decay $L^+\to\bar{\nu}W^+\nu$ with $M_L=200$ GeV, $\mu_1=0.0005$, $\mu_2=0.405$, $Q_i=0$, and $Q_j=1$ $(X=0.3)$. In this case, since we do not have unlike-sign charges, there is a RAZ at $Y=-1$, provided $m_\nu=0$. This is as expected from previous work.[8] Figure 3 shows the approximate RAZ at $Y=-1$, which follows from Eq. (5). From Eq. (10) it can be seen that $Y=-1$ is in the physical region provided $m_\nu=0$.

In Figs. 4–7 we present the results of a Monte Carlo generation of events. Here in Figs. 4–6 we apply the cuts $X\geq X_{\text{cut}}=0.1$ and an angle cut, $|\cos\theta|\leq C_0=0.9$ where θ is the angle between the photon and the fermion in the t-quark center of mass. In Fig. 7 we do not need to apply the $\cos\theta$ cut. Figure 4 shows the distribution of events for $t\to bW^+\gamma$ with $M_t=100$ GeV, $Q_j=\frac{2}{3}$, and $Q_i=-\frac{1}{3}$. In Fig. 5 everything remains the same, but the top-quark mass is increased to $M_t=200$ GeV. Figure 6 shows the distribution of events for $t\to bW^+g$ with $M_t=100$ GeV and $Q_i=Q_j=1$. Finally, in Fig. 7 we give the results for the heavy-lepton decay $L^+\to\bar{\nu}W^+\gamma$ with $M_L=200$ GeV, $\mu_1=5\times10^{-4}$, $\mu_2=0.405$, $Q_i=0$, and $Q_j=1$. The

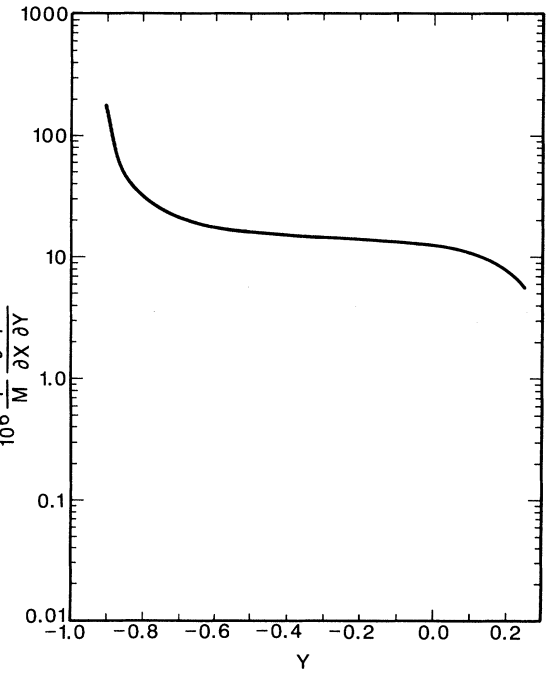

FIG. 2. $(1/M)/\partial^2\Gamma/\partial X\partial Y$ vs Y for $t\to bW^+\gamma$ with $M_t=150$ GeV, $\mu_1=0.0333$, $\mu_2=0.54$, $Q_i=-\frac{1}{3}$, and $Q_j=\frac{2}{3}$ $(X=0.2)$.

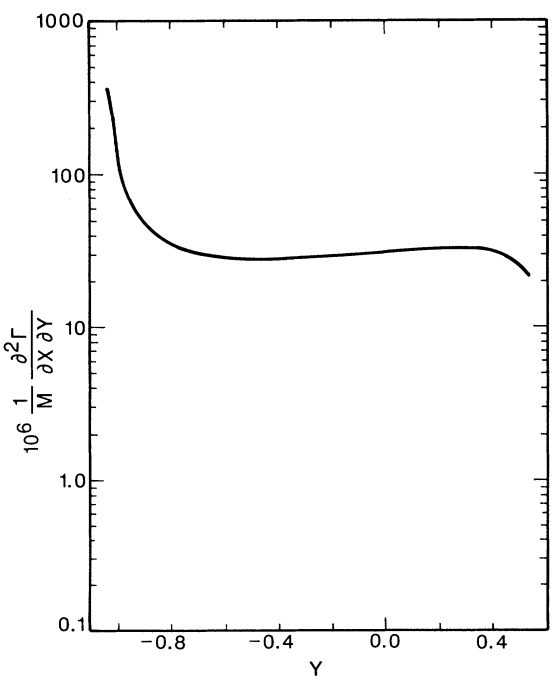

FIG. 1. $(1/M)/\partial^2\Gamma/\partial X\,\partial Y$ vs Y for $t\to bW^+\gamma$ with $M_t=200$ GeV, $\mu_1=0.025$, $\mu_2=0.405$, $Q_i=-\frac{1}{3}$, and $Q_{ij}=\frac{2}{3}$ $(X=0.3)$.

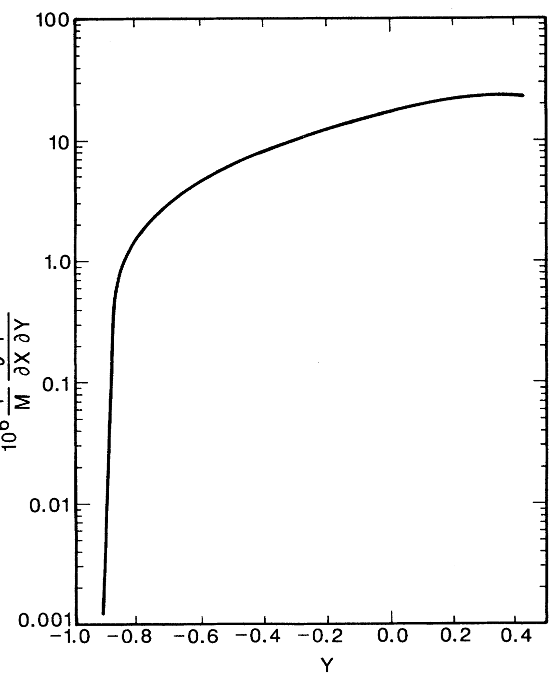

FIG. 3. $(1/M)/\partial^2\Gamma/\partial X\partial Y$ vs Y for $L^+\to\bar{\nu}W^+\gamma$ with $M_L=200$ GeV, $\mu_1=0.0005$, $\mu_2=0.405$, $Q_i=0$, and $Q_j=1$ $(X=0.3)$.

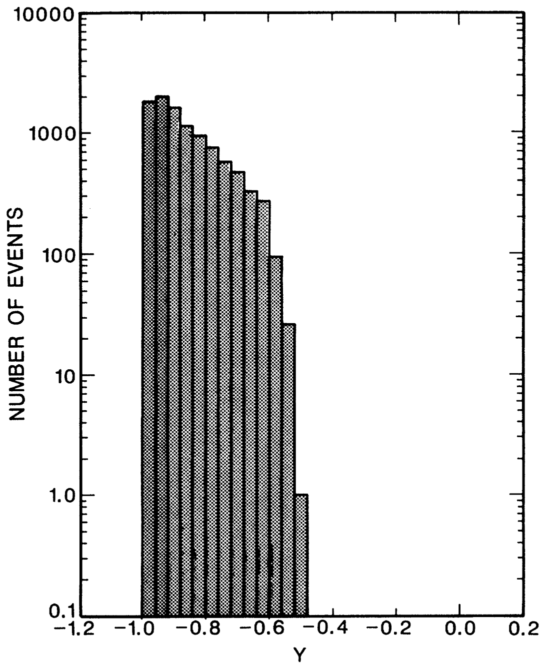

FIG. 4. Monte Carlo events generated for $t \rightarrow bW^{+}\gamma$. $X_{\rm cut}=0.1$, $|\cos\theta| \leq 0.9$, $M_t=100$ GeV, $\mu_1=0.05$, $\mu_2=0.81$, $Q_i=-\frac{1}{3}$, and $Q_j=\frac{2}{3}$. The total number of events generated is 10 000.

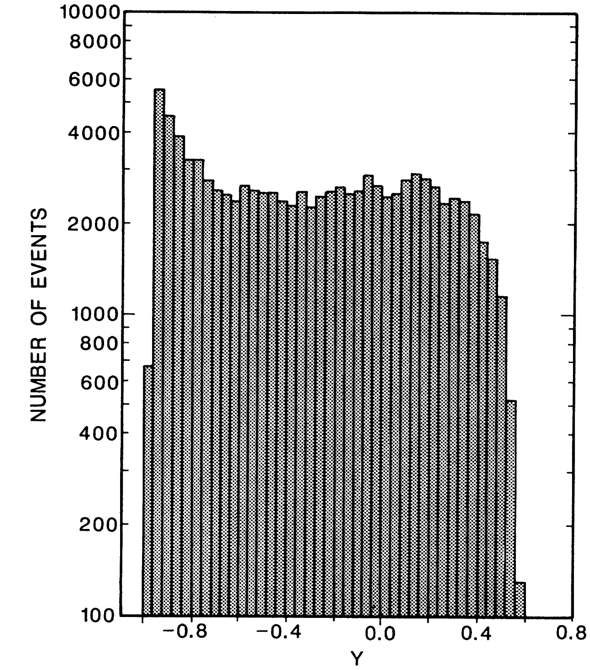

FIG. 6. Monte Carlo events generated for $t \rightarrow bW^{+}g$. $X_{\rm cut}=0.1$, $|\cos\theta \leq 0.9|$, $M_t=100$ GeV, $\mu_1=0.05$, $\mu_2=0.81$, and $Q_i=Q_j=1$. The total number of events generated is 16 280.

RAZ at $Y=-1$ is evident.

We wish to emphasize that our results have more general applicability than the specific cases given in Figs. 1–7. They can be used in other situations where fermion masses cannot be neglected. These include heavy W (W_L or W_R) which occur in L-R-symmetric models and other models. Our results are also applicable to decays involving possible fourth-generation quarks and leptons.

Although a fourth generation with a light neutrino has been ruled out by experiments at the CERN $e^{+}e^{-}$ collider LEP,[9] our results are completely general and could be used for a fourth generation with a heavy neutrino.

Finally, we would like to comment on the experimental situation. First, of course, the top quark or the heavy lepton must be discovered. Then these radiative decays can provide a test of the $WW\gamma$ trilinear coupling and the SM, which requires $\kappa=1$. As a representative example, if we integrate the results of Fig. 5 over Y, we obtain a partial width $\Gamma=7.86$ MeV. Combining this with the expected total width of the t quark, $\Gamma_{\rm total}=2.37$ GeV (for

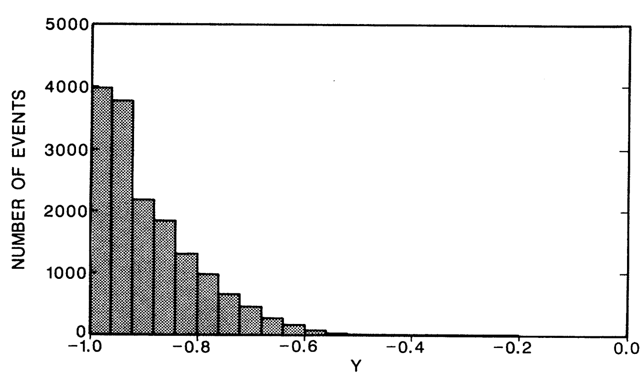

FIG. 5. Monte Carlo events generated for $t \rightarrow bW^{+}\gamma$. $X_{\rm cut}=0.1$, $|\cos\theta| \leq 0.9$, $M_t=200$ GeV, $\mu_1=0.025$, $\mu_2=0.405$, $Q_i=-\frac{1}{3}$, and $Q_j=\frac{2}{3}$. The total number of events generated is 10 000.

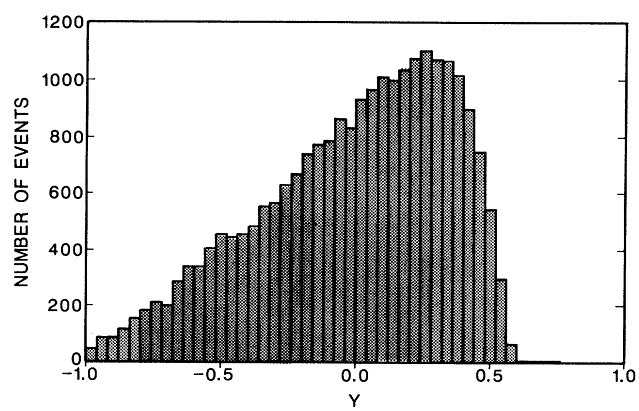

FIG. 7. Monte Carlo events generated for $L^{+} \rightarrow \bar{\nu}_L W^{+}\gamma$. $X_{\rm cut}=0.1$, no θ cut, $M_L=200$ GeV, $\mu_1=5\times10^{-4}$, $\mu_2=0.405$, $Q_i=0$, and $Q_j=1$. The total number of events generated is 23 556.

TABLE I. Branching ratios (B) for $t \to bW^+\gamma$ for various cuts and M. $\Gamma_0 = \Gamma(t \to bW)$ and $B = \Gamma/\Gamma_0$.

M (GeV)	X_{cut}	C_0	Γ_0 (MeV)	Γ (MeV)	B (%)
100	0.1	0.9	87	0.023	0.026
100	0.2	0.7	87	0.0025	0.0029
200	0.1	0.9	2372	7.86	0.33
200	0.2	0.7	2372	3.13	0.13

TABLE II. Branching ratios (B) for $t \to bW^+g$ for various cuts and M. $\Gamma_0 = \Gamma(t \to bW)$ and $B = \Gamma/\Gamma_0$.

M (GeV)	X_{cut}	C_0	Γ (MeV)	Γ (MeV)	B (%)
100	0.1	0.9	87	2.25	2.58
100	0.2	0.7	87	0.25	0.29
200	0.1	0.9	2372	298	12.6
200	0.2	0.7	2372	95	4.0

$M_t = 200$ GeV), we obtain a branching fraction for $t \to bW^+\gamma$ of 0.33%. At the Superconducting Super Collider, where one expects 10^8 t quarks/yr, one would get $3.3 \times 10^5 bW\gamma$ events/yr, making this experiment clearly quite feasible. Our results, however, are *strongly dependent on the cuts used.* For the $bW\gamma$ decay the branching ratio varies considerably as is shown in Table I. The results for the bWg decay are shown in Table II.

We have recently been in contact with both Couture and Stange.[10,11] Our results now agree with both groups. This includes both our formulas Eqs. (5), (6), and (7) and our branching ratios for both the radiative decay and the gluon decay.

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