Gauge-fixing conditions in canonical quantization of solitary-wave classical solutions

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We develop a canonical quantization formalism of solitons that can eliminate all infrared divergences associated with the zero-frequency translational mode. The gauge-fixing condition on the field fluctuation and the constraint on its conjugate momentum are chosen such that undesired admixture of divergent components in the continuum meson wave functions are excluded in the normal-mode expansion of the field variables. A systematic perturbation expansion is made in the coupling parameter g. The quantization is performed in terms of the normal modes of the g^0-order Hamiltonian. The resulting Hamiltonian possesses a meson-soliton linear coupling of order g and a quadratic coupling of order g^0. We show that the sum of the first-order meson-soliton scattering amplitude coming from the quadratic interaction and the second-order amplitude induced by the linear interaction exactly reproduces the Born term. We also show by explicit calculations that in the conventional gauge the Born term is attributed to completely different origins. We find that a correction due to the soliton recoil plays an important role in the conventional gauge. Another possible form of the canonical transformation is also investigated.

I. INTRODUCTION

The quantum mechanics of solitary classical solutions of nonlinear field theories was studied by Dashen, Hasslacher, and Neveu[1] using WKB method, and by Goldstone and Jackiw[2] using the Kerman-Klein method. At the same period of time, a considerable body of literature appeared concerning quantization of classical solutions.[3–7] Gervais, Jevicki, and Sakita[3] developed the path-integral collective-coordinate method, and Tomboulis[4] derived the same result (except for the reordering term) in the canonical operator formulation. In these studies a classical solution is interpreted as an extended particle, the soliton, and the position of the soliton is added as an extra degree of freedom. A subsidiary condition is imposed to preserve the number of degrees of freedom of the original Lagrangian. The system is an interacting system of a particle and a field with constraints. A gauge-theoretical approach due to Dirac[8] is widely used for quantizing constrained Hamiltonian systems, and is developed by Hosoya and Kikkawa[7] and Gervais, Jevicki, and Sakita[9] for soliton quantization. In gauge theory we have to fix a specific gauge so that physical observables can be computed explicitly. Although an arbitrary choice of gauge should lead to a consistent description of the system, almost all the authors choose a gauge-fixing condition so that the quantum fluctuation χ is orthogonal to the zero-frequency eigenfunction ϕ_c' (ϕ_c is the classical solution and a prime denotes differentiation with respect to x):

$$\int dx\, \phi_c'(x)\chi(x,t)=0 \; . \tag{1.1}$$

It is also a common practice to define the conjugate momentum Π_T that is subject to the constraint

$$\int dx\, \phi_c'(x)\Pi_T(x,t)=0 \; . \tag{1.2}$$

As a result, the zero mode is excluded from the eigenfunction expansion of the fluctuation and its conjugate momentum.

Recently Verschelde[10,11] has proposed an alternative gauge-fixing condition, a nonrigid gauge condition, for a description of rotating Skyrmions.[12] The present author[13] developed the nonrigid quantization method for the translational motion of solitons in two spacetime dimensions. In these investigations, a linear gauge-fixing condition

$$Q=\int dx\, f(x)\chi(x,t)=0 \tag{1.3}$$

is imposed on the fluctuation, where f is a real function of x. In contrast with the conventional gauge condition (1.1) in which f is identified with ϕ_c', f contains all components from a mathematically complete set of meson wave functions $\{\psi_n\}$, solutions of the free field equation

$$f(x)=\sum_n \zeta_n \psi_n(x) \; . \tag{1.4}$$

On the other hand, the constraint on Π_T is chosen to be the same as (1.2). Namely, in the nonrigid quantization the field fluctuation and its conjugate momentum are not put on an equal footing. From the physical point of view, the nonsymmetrical treatment of the field variables is undesirable. The gauge condition (1.3) is motivated by the fact that the continuum meson wave function $\psi_k(x)$ in the limit of the meson energy $\omega_k=0$ is dominated by the term proportional to the zero mode:

$$\psi_k(x)\cong\frac{\zeta_k^*}{M_0}\phi_c'(x) \; , \tag{1.5}$$

where $M_0=\int dx\, \phi_c'^2$ is the soliton mass. The coefficient ζ_k is given by the Fourier transform of the zero mode,[13] and has a double pole at $\omega_k=0$ [see Eq. (5.2) below]. The admixture of the divergent component proportional ϕ_c' in the meson wave function is unavoidable since all eigenfunctions, solutions to a Schrödinger-like equation, are inevitably coupled to each other. A meson in the continuum propagates making virtual transitions to intermediate states that include the zero-frequency state. The infrared singularity in the meson propagator is reflected in the meson wave functions. The gauge condition (1.3) is devised to eliminate such divergences from the quantum fluctuation χ. In contrast with this, the constraint on Π_T removes the zero mode but does not affect the singular terms in the continuum wave functions. In a recent paper,[14] we have proposed a new canonical transformation that makes both χ and Π_T orthogonal to f, and therefore allows us to eliminate all divergences associated with the zero mode in a symmetrical way. In the present paper we describe our method in full detail.

In Sec. II we present our canonical transformation. In Sec. III, the Hamiltonian is quantized and a systematic perturbative expansion is developed. In Sec. IV, corrections brought about by the soliton recoil are discussed. In Sec. V, meson scattering off solitons is studied. In Sec. VI, a comparison with the standard quantization method is made. In Sec. VII, another possible form of the symmetric quantization is investigated. A brief summary is given in Sec. VIII.

II. CANONICAL TRANSFORMATION

We consider a generic two-dimensional field theory of a single scalar field Φ characterized by a Lagrangian density

$$\mathcal{L}=\tfrac{1}{2}(\dot{\Phi}^2-\Phi'^2)-U(\Phi) \tag{2.1}$$

with the field potential U. The Hamiltonian is given by

$$H=\tfrac{1}{2}\int dx(\Pi^2+\Phi'^2)+\int dx\, U(\Phi), \tag{2.2}$$

where $\Pi=\dot{\Phi}$ is the canonical momentum conjugate to Φ, satisfying the Poisson-brackets relation

$$\{\Phi(x,t),\Pi(y,t)\}_{\rm PB}=\delta(x-y)\ . \tag{2.3}$$

We introduce an additional variable X as a collective coordinate and consider a canonical transformation from the original canonical variable Φ to X and fluctuation χ:

$$\Phi(x,t)=\phi_c(x-X)+\chi(x-X,t)\ . \tag{2.4}$$

The Q condition (1.3) is imposed on χ but for the moment we leave f unspecified and proceed with the general development. The canonical momentum conjugate X,

$$P=\dot{X}\int dx\,\Phi'^2-\int dx\,\dot{\chi}\Phi'\ , \tag{2.5}$$

satisfies the Poisson-brackets relation

$$\{X,P\}_{\rm PB}=1\ . \tag{2.6}$$

The conjugate momentum P should appear such that the constraint

$$F=P+\int dx\,\Pi\Phi'=0 \tag{2.7}$$

is satisfied. Under the Poisson-brackets relation (2.3), the gauge-fixing condition (1.3) does not commute with F but

$$\{Q,F\}_{\rm PB}=\int dx\, f\Phi'\equiv M_\chi\ . \tag{2.8}$$

The constraint $F=0$ is quadratic in fields but in principle constraints which are nonlinear in field variables can be linearized by a change of variables. In the previous papers,[10,11,13] Π_T is defined by

$$\Pi=\Pi_T-\frac{1}{M_\chi}f\left[P+\int dx\,\Pi_T\chi'\right] \tag{2.9}$$

so that F is reduced to $\int dx\,\phi_c'\Pi_T$. Then, as in the standard quantization formalism, Π_T is expanded in terms of the complete set $\{\psi_n\}$ and $\psi_0=\phi_c'/\sqrt{M_0}$ is discarded. However, as remarked in the Introduction, continuum eigenfunctions are plagued with undesired contamination of the zero-frequency eigenfunction. The constraint $\int dx\,\phi_c'\Pi_T=0$ cannot remove the singularity in the continuum meson wave functions. Instead of (2.9), we want a canonical transformation that would lead to the constraint

$$F=\int dx\, f(x)\Pi_T(x,t)=0\ . \tag{2.10}$$

For this purpose, we follow Gervais, Jevicki, and Sakita[9] and assume that Π is decomposed into Π_T and the remainder that is proportional to f,

$$\Pi=\Pi_T-Af\ , \tag{2.11}$$

and demand that F becomes $\int dx\, f\Pi_T$. Then A is uniquely determined to give

$$\Pi=\Pi_T-\frac{1}{M_\chi}f\left[P+\int dx\,\Pi_T(\Phi'-f)\right]\ . \tag{2.12}$$

In this way the canonical momentum Π becomes the sum of two components that are orthogonal to each other, whereas the two components in (2.9) are nonorthogonal. Gervais *et al.*, however, identified f with ϕ_c' and did not exploit this transformation any further.

Under the constraint $\int dx\, f\Pi_T=0$, Eq. (2.12) becomes

$$\Pi=\Pi_T-\frac{1}{M_\chi}f\left[p+\int dx\,\phi_c'\Pi_T\right]\ , \tag{2.13}$$

where

$$p=P+\int dx\,\Pi_T\chi' \tag{2.14}$$

is the soliton momentum operator. Equation (2.13) can also be derived using the standard procedure: From (2.5) we obtain

$$\dot{X}=\frac{1}{M_0+\xi_0}\left[P+\int dx(\dot{\chi}-\dot{X}\chi')\Phi'\right]\ , \tag{2.15}$$

where

$$\xi_0=\int dx\,\phi_c'\chi'\ . \tag{2.16}$$

On the other hand, from $\Pi=\dot{\chi}-\dot{X}\Phi'=\Pi_T-Af$ it follows that

$$\dot{\chi}-\dot{X}\chi'=\Pi_T+\dot{X}\phi_c'-Af\ . \tag{2.17}$$

Substituting this into (2.15) determines A and yields (2.13). There exists another form of the canonical transformation that brings F into $\int dx\, f\Pi_T$. Such a possibility will be discussed in Sec. VII.

Under the Poisson-brackets relation

$$\{\chi(x,t),\Pi_T(y,t)\}_{\rm PB}=\delta(x-y)\ , \tag{2.18}$$

the constraint $Q=0$ is second class since

$$\{Q,F\}_{\rm PB}=\int dx\, f^2(x)\equiv M \tag{2.19}$$

is nonvanishing. The second-class constraint can be eliminated by altering the original Poisson-brackets relation as the Dirac brackets:

$$\{\chi(x,t),\Pi_T(y,t)\}_{\rm DB}=\delta(x-y)-\frac{1}{M}f(x)f(y)\ . \tag{2.20}$$

The Hamiltonian (2.2) is written in terms of new canonical variables as

$$H=M_0+\frac{M}{2M_\chi^2}\left[p+\int dx\,\phi_c'\Pi_T\right]^2 +\tfrac{1}{2}\int dx(\Pi_T^2+\chi'^2)+\int dx\, U(\chi,\phi_c)\ , \tag{2.21}$$

where

$$U(\chi,\phi_c)=U(\phi_c+\chi)-\chi U'(\phi_c)-U(\phi_c)\ . \tag{2.22}$$

The equations of motion for χ and Π_T become

$$\dot{\chi}=\{\chi,H\}_{\rm DB}=\Pi+\frac{M}{M_\chi^2}\Phi'\left[p+\int dx\,\phi_c'\Pi_T\right] \tag{2.23}$$

and

$$\begin{aligned}\dot{\Pi}_T&=\{\Pi_T,H\}_{\rm DB}\\ &=\chi''-U'(\chi,\phi_c)+\frac{M}{M_\chi^2}\Pi'\left[p+\int dx\,\phi_c'\Pi_T\right]\\ &\quad-\frac{1}{M}f\int dx\, f\left[\chi''-U'(\chi,\phi_c)+\frac{M}{M_\chi^2}\Pi'\left[p+\int dx\,\phi_c'\Pi_T\right]\right]\ ,\end{aligned} \tag{2.24}$$

where

$$\Phi'=\phi_c'+\chi'\ , \tag{2.25}$$

$$\Pi'=\Pi_T'-\frac{1}{M_\chi}f'\left[p+\int dx\,\phi_c'\Pi_T\right]\ , \tag{2.26}$$

and

$$U'(\chi,\phi_c)=U'(\phi_c+\chi)-U'(\phi_c)\ . \tag{2.27}$$

The equations of motion for X and P are

$$\dot{X}=\{X,H\}_{\rm DB}=\frac{M}{M_\chi^2}\left[p+\int dx\,\phi_c'\Pi_T\right]\ , \tag{2.28}$$

$$\dot{P}=\{P,H\}_{\rm DB}=0\ . \tag{2.29}$$

We can write (2.28) using $\int dx\, f\Pi_T=0$ as

$$\dot{X}=-\frac{1}{M_\chi}\int dx\, f\Pi\ . \tag{2.30}$$

From Eqs. (2.23) and (2.28), we obtain $\dot{\chi}$ in the form

$$\dot{\chi}=\Pi+\dot{X}\Phi'\ , \tag{2.31}$$

which is the direct consequence of the differentiation of Φ with respect to time, $\dot{\Phi}=\dot{\chi}-\dot{X}\Phi'$. It is straightforward to prove that

$$\dot{F}=\{F,H\}_{\rm DB}=0\ , \tag{2.32}$$

$$\dot{Q}=\{Q,H\}_{\rm DB}=0\ , \tag{2.33}$$

so the subsidiary conditions are independent of time. The Hamiltonian (2.21) is not manifestly relativistic (even the Galilean invariance is violated unless $f=\phi_c'$). In Appendix A, we show that the equations of motion (2.23) and (2.24) have the classical solution that has the correct Lorentz-invariant form.

III. QUANTIZATION

Quantization of the classical Hamiltonian (2.21) can be achieved by replacing the classical Dirac brackets by quantum commutators

$$i[P,X]=1 \tag{3.1}$$

and

$$i[\Pi_T(x,t),\chi(y,t)]=\delta(x-y)-\frac{1}{M}f(x)f(y). \tag{3.2}$$

It is straightforward to show that Eqs. (3.1) and (3.2) lead to the commutation relation of the original field variables:

$$i[\Pi(x,t),\Phi(y,t)]=\delta(x-y)\ . \tag{3.3}$$

Because of the nonlinear nature of the canonical transformation (2.13), we have to be careful about operator ordering. Two order prescriptions, from Tomboulis[4] and Gervais and Jevicki,[15] give the same result. Following Tomboulis we symmetrize the noncommuting factors in (2.13):

$$\Pi=\Pi_T-\tfrac{1}{2}\left[fM_\chi^{-1}\left[p+\int dx\,\phi_c'\Pi_T\right]+\left[p+\int dx\,\phi_c'\Pi_T\right]M_\chi^{-1}f\right]\ . \tag{3.4}$$

Inserting Eqs. (2.4) and (3.4) into the Hamiltonian, we write H in terms of new quantum variables as

$$H=M_0+\tfrac{1}{8}M\left\{p+\int dx\,\phi_c'\Pi_T,M_\chi^{-1}\right\}^2 +\tfrac{1}{2}\int dx(\Pi_T^2+\chi'^2)+\int dx\, U(\chi,\phi_c)-\tfrac{1}{8}M_\chi^{-2}\int dx\, f'^2\ . \tag{3.5}$$

Because of the constraint $\int dx\, f\Pi_T=0$, a meson-soliton linear coupling proportional to $\int dx\, f\Pi_T$ disappears, whereas $\int dx\, \phi_c'\Pi_T$ becomes nonvanishing.

We expand the Hamiltonian in powers of fluctuations. To do this, we note that the normalization of f is at our disposal. We fix it by

$$\int dx\, f\phi_c' = M_0 \ . \tag{3.6}$$

Then

$$M_\chi = M_0 + \xi \tag{3.7}$$

with

$$\xi = \int dx\, f\chi' \ . \tag{3.8}$$

In the conventional gauge with $f=\phi_c'$, ξ is reduced to ξ_0 as is defined by (2.16). Retaining terms up to quadratic in fields, we get

$$H = H_0 + H_{\rm I} + H_{\rm II} \ , \tag{3.9}$$

where

$$H_0 = M_0 + \frac{M}{2M_0^2}p^2 + \int dx\,[\tfrac{1}{2}\Pi_T^2 + \tfrac{1}{2}\chi'^2 + \tfrac{1}{2}\chi^2 U''(\phi_c)] \tag{3.10}$$

is the Hamiltonian of a soliton and that of the free meson field, while $H_{\rm I}$ and $H_{\rm II}$ are the interaction Hamiltonians that are linear and quadratic in field operators, respectively. They are explicitly given by

$$H_{\rm I} = \frac{M}{2M_0^2}\left\{p, \int dx\, \phi_c'\Pi_T\right\} - \frac{M}{4M_0^3}\{p,\{p,\xi\}\} \tag{3.11}$$

and

$$\begin{aligned} H_{\rm II} = {} & \frac{M}{2M_0^2}\left[\int dx\, \phi_c'\Pi_T\right]^2 - \frac{M}{4M_0^3}\left\{\int dx\, \phi_c'\Pi_T, \{p,\xi\}\right\} \\ & - \frac{M}{4M_0^3}\left\{p, \left\{\int dx\, \phi_c'\Pi_T, \xi\right\}\right\} \\ & + \frac{M}{8M_0^4}\{p,\xi\}^2 + \frac{M}{4M_0^4}\{p,\{p,\xi^2\}\} \ . \end{aligned} \tag{3.12}$$

The last term in Eq. (3.5), the reordering term, is also expanded as

$$\begin{aligned} -\tfrac{1}{8}M_\chi^{-2}\int dx\, f'^2 \\ = -\frac{1}{8M_0^2}\int dx\, f'^2\left[1 - \frac{2\xi}{M_0} + \frac{3\xi^2}{M_0^2} - \cdots\right] . \end{aligned} \tag{3.13}$$

A systematic perturbation expansion is carried out by the field potential $U(g;\Phi)$ scaling with respect to the coupling constant g as

$$U(g;\Phi) = \frac{1}{g^2}U(1;g\Phi). \tag{3.14}$$

One sees that the classical solution is $O(g^{-1})$ and the classical action associated with this solution is of $O(g^{-2})$. We thus see that g^2 plays the role of the Planck constant, and $M_0 = O(g^{-2})$, $M = O(g^{-2})$, and $\xi = O(g^{-1})$. Considering quantum fluctuation around the classical solution leads to a perturbation expansion in powers of g. We obtain a meson-solution linear coupling of $O(g)$,

$$H_{\rm I} \cong \frac{M}{2M_0^2}\left\{p, \int dx\, \phi_c'\Pi_T\right\} \tag{3.15}$$

and a quadratic interaction of order 1:

$$H_{\rm II} \cong \frac{M}{2M_0^2}\left[\int dx\, \phi_c'\Pi_T\right]^2 . \tag{3.16}$$

The neglected terms are smaller than the leading-order terms by $O(g^2)$. The other higher-order interactions are trivially obtained. The cubic and quartic interactions are given by

$$H_{\rm III} \cong -\frac{M}{4M_0^3}\left\{\int dx\, \phi_c'\Pi_T, \left\{\int dx\, \phi_c'\Pi_T, \xi\right\}\right\} \tag{3.17}$$

and

$$\begin{aligned} H_{\rm IV} \cong {} & \frac{M}{4M_0^4}\left\{\int dx\, \phi_c'\Pi_T, \left\{\int dx\, \phi_c'\Pi_T, \xi^2\right\}\right\} \\ & + \frac{M}{8M_0^4}\left\{\int dx\, \phi_c'\Pi_T, \xi\right\}^2 . \end{aligned} \tag{3.18}$$

A. Normal-mode expansion

The quadratic part of order 1 in H_0 yields the equations of motion

$$\dot\chi = \Pi_T \ , \tag{3.19}$$

$$\dot\Pi_T = \chi'' - U''(\phi_c)\chi - \frac{1}{M}f\int dx\, f[\chi'' - U''(\phi_c)\chi] \ . \tag{3.20}$$

Eliminating Π_T we obtain the free field equation

$$\ddot\chi + K\chi - \frac{1}{M}f\langle f|K|\chi\rangle = 0 \ . \tag{3.21}$$

Here $K = -d^2/dx^2 + U''(\phi_c)$ and

$$\langle f|K|\chi\rangle = \int dx\, fK\chi \ . \tag{3.22}$$

The eigenvalue equation to be solved for the normal mode $\chi = \chi_n e^{-i\omega_n t}$ becomes

$$K\chi_n - \frac{1}{M}f\langle f|K|\chi_n\rangle = \omega_n^2\chi_n \ . \tag{3.23}$$

Multiplying Eq. (3.23) by f and integrating, it is easy to prove that χ_n (except for χ_0) is orthogonal to f:

$$\langle f|\chi_n\rangle = \int dx\, f\chi_n = 0 \ , \tag{3.24}$$

as is required. We define the projection operator onto the space orthogonal to f:

$$\mathcal{P} = 1 - \frac{1}{M}|f\rangle\langle f| , \quad \mathcal{P}^2 = \mathcal{P} \ . \tag{3.25}$$

Then the solutions ($n \neq 0$) to (3.23) are those of

$$\mathcal{P}K\mathcal{P}\chi_n = \omega_n^2 \chi_n , \tag{3.26}$$

where we have used the orthogonality condition of χ_n to f, namely, $\mathcal{P}\chi_n = \chi_n$. In the strong-coupling theory, Parmentola[16] discussed a similar equation. This equation has a zero-frequency solution f. After normalization it becomes

$$\chi_0(x) = \frac{1}{\sqrt{M}} f(x) . \tag{3.27}$$

The completeness relation for the set $\{\chi_n\}$ of solutions to the free field equation reads

$$\sum_n{}' \chi_n(x)\chi_n^*(y) = \delta(x-y) - \frac{1}{M} f(x) f(y) , \tag{3.28}$$

where the primed sum is to omit the zero mode.

The zero-frequency solution χ_0 is a localized state: Since we have chosen the normalization of f as in Eq. (3.6), ζ_0 is given by $\sqrt{M_0}$, i.e.,

$$f = \phi_c' + \sum_n{}' \zeta_n \psi_n . \tag{3.29}$$

All the results of the standard quantization formalism can be recovered by putting $\zeta_n = 0$ ($n \neq 0$). By making use of the form of ζ_n derived in the previous paper,[13]

$$\zeta_n = -\int dx\, e^{-inx} \phi_c' = -\langle n | \phi_c' \rangle , \tag{3.30}$$

we obtain, at $x = \pm\infty$,

$$\sum_n{}' \zeta_n \psi_n \to \sum_n{}' \zeta_n e^{inx} = -\phi_c' , \tag{3.31}$$

so that f and χ_0 fall off rapidly at large x. As a result, the continuum eigenfunction χ_k obeys asymptotically the same equation as the normal mode ψ_k, $K\psi_k = \omega_k^2 \psi_k$, without the orthogonality-condition term. Therefore χ_k is written in terms of ψ_k as

$$\chi_k = \psi_k - \frac{1}{M(\omega_k^2 - K)} f \langle f | K | \chi_k \rangle . \tag{3.32}$$

From the orthogonality of χ_k to f, we obtain

$$\langle f | K | \chi_k \rangle = \frac{M \zeta_k^*}{\langle f | 1/(\omega_k^2 - K) | f \rangle} , \tag{3.33}$$

where we have used

$$\langle f | \psi_k \rangle = \zeta_k^* \tag{3.34}$$

which follows from the orthonormal condition $\langle \psi_n | \psi_m \rangle = \delta_{nm}$. (We use the box normalization. See Appendix B.) Inserting (3.33) into (3.32) leads to

$$\chi_k = \psi_k - \frac{1}{\omega_k^2 - K} f \frac{\zeta_k^*}{\langle f | 1/(\omega_k^2 - K) | f \rangle} . \tag{3.35}$$

Furthermore we have

$$\langle \phi_c' | \chi_k \rangle = -\frac{M_0 \zeta_k^*}{\omega_k^2 \langle f | 1/(\omega_k^2 - K) | f \rangle} . \tag{3.36}$$

In the neighborhood of $\omega_k = 0$, double-pole terms dominate,

$$\frac{1}{\omega_k^2 - K} f = \frac{1}{\omega_k^2} \phi_c' + \sum_n{}' \frac{\zeta_n}{\omega_k^2 - \omega_n^2} \psi_n \cong \frac{1}{\omega_k^2} \phi_c' \tag{3.37}$$

and

$$\left\langle f \left| \frac{1}{\omega_k^2 - K} \right| f \right\rangle = \frac{1}{\omega_k^2} M_0 + \sum_n{}' \frac{|\zeta_n|^2}{\omega_k^2 - \omega_n^2} \cong \frac{1}{\omega_k^2} M_0 , \tag{3.38}$$

and hence

$$\chi_k \cong \psi_k - \frac{\zeta_k^*}{M_0} \phi_c' \tag{3.39}$$

which is orthogonal to f and free from the divergence associated with the zero mode. Moreover, one finds

$$\langle \phi_c' | \chi_k \rangle \cong -\zeta_k^* \tag{3.40}$$

which also follows from replacing χ_k by e^{ikx}:

$$\langle \phi_c' | \chi_k \rangle \cong \int dx\, e^{ikx} \phi_c' . \tag{3.41}$$

Equation (3.40) is valid only in the limit of $\omega_k = 0$. By taking the limit $\omega_k \to \infty$, we gain $-(M_0/M)\zeta_k^*$. The completeness property (3.28) implies a sum rule

$$\sum_n{}' |\langle \phi_c' | \chi_n \rangle|^2 = M_0 - \frac{M_0^2}{M} = \frac{M_0}{M} \sum_n{}' |\zeta_n|^2 , \tag{3.42}$$

where we have used

$$M - M_0 = \sum_n{}' |\zeta_n|^2 . \tag{3.43}$$

Clearly, (3.40) violates this sum rule by the factor M_0/M. To be consistent with (3.42), we approximate the overlap integral as

$$\langle \phi_c' | \chi_k \rangle \cong -\left[\frac{M_0}{M} \right]^{1/2} \zeta_k^* \tag{3.44}$$

which will be of much avail in evaluating matrix elements of the interaction Hamiltonians. It is noted that $M = 2M_0$ since

$$\sum_n{}' |\zeta_n|^2 = \sum_n \langle \phi_c' | n \rangle \langle n | \phi_c' \rangle = M_0 , \tag{3.45}$$

where the completeness of the plane-wave states was used. To clarify the physical meaning, we retain M unless stated otherwise.

We now expand the field variables as

$$\chi(x,t) = \sum_k{}' \frac{1}{\sqrt{2\omega_k}} [a_k \chi_k(x) e^{-i\omega_k t} + a_k^\dagger \chi_k^*(x) e^{i\omega_k t}] \tag{3.46}$$

and

$$\Pi_T(x,t) = -i \sum_k{}' \frac{\omega_k}{\sqrt{2\omega_k}} [a_k \chi_k(x) e^{-i\omega_k t} - a_k^\dagger \chi_k^*(x) e^{i\omega_k t}] , \tag{3.47}$$

so that χ satisfies the gauge-fixing condition (1.3) and Π_T satisfies the constraint (2.10). It is straightforward to check that the conjugate variables satisfy the canonical commutation relation (2.20) if we assume the usual commutation relations

$$[a_k,a_{k'}^\dagger]=\delta_{kk'}\ ,\qquad [a_k,a_{k'}]=[a_k^\dagger,a_{k'}^\dagger]=0\ . \tag{3.48}$$

Corresponding to "in" and "out" scattering solutions, there exist "in" and "out" creation and annihilation operators related by the S matrix: namely,

$$a_{k,\mathrm{in}}=\sum_{k'}S_{kk'}a_{k',\mathrm{out}}\ ,\qquad a_{k,\mathrm{out}}^\dagger=\sum_{k'}S_{kk'}^*a_{k',\mathrm{in}}^\dagger\ . \tag{3.49}$$

Either of these two can be used in the normal-mode expansion. Throughout this paper, we drop the "in" or "out" label, and $\pm i\epsilon$ in the meson propagator.

B. Interaction matrix elements

Both of the interactions H_I and H_II contain $\int dx\,\phi_c'\Pi_T$ which is nonzero owing to the condition $\int dx\,f\Pi_T=0$, or

$$\int dx\,\phi_c'\Pi_T=-\sum_n{}'\zeta_n\int dx\,\psi_n\Pi_T\ . \tag{3.50}$$

This has nonvanishing matrix elements for meson emission,

$$\left\langle k,p-k\left|\int dx\,\phi_c'\Pi_T\right|p\right\rangle=-i\frac{\omega_k}{\sqrt{2\omega_k}}\left[\frac{M_0}{M}\right]^{1/2}\zeta_k \tag{3.51}$$

and, for meson absorption,

$$\left\langle p+k\left|\int dx\,\phi_c'\Pi_T\right|k,p\right\rangle=i\frac{\omega_k}{\sqrt{2\omega_k}}\left[\frac{M_0}{M}\right]^{1/2}\zeta_k^*\ , \tag{3.52}$$

where we have used (3.44). The soliton momentum operator p can be replaced with the c number since we have the matrix elements

$$\left\langle p\left|P+\int dx\,\Pi_T\chi'\right|p\right\rangle=p+i\sum_n{}'\langle\chi_n|\chi_n'\rangle=p\ , \tag{3.53}$$

$$\left\langle k,p\left|P+\int dx\,\Pi_T\chi'\right|k,p\right\rangle=p+k+i\langle\chi_k|\chi_k'\rangle\cong p\ . \tag{3.54}$$

Here we have used the fact that the wave function χ_k is infrared finite, and we can make an approximate $\langle\chi_k|\chi_k'\rangle\cong ik$. The matrix element of H_I between the meson-soliton state and the soliton state can be evaluated using (3.51) and (3.52) as

$$\langle k,p-k|H_\mathrm{I}|p\rangle=-i\frac{\omega_k}{\sqrt{2\omega_k}}\frac{M(2p-k)}{2M_0^2}\left[\frac{M_0}{M}\right]^{1/2}\zeta_k\ , \tag{3.55}$$

$$\langle p+k|H_\mathrm{I}|k,p\rangle=i\frac{\omega_k}{\sqrt{2\omega_k}}\frac{M(2p+k)}{2M_0^2}\left[\frac{M_0}{M}\right]^{1/2}\zeta_k^*\ . \tag{3.56}$$

For computing matrix elements of H_II, we insert one-soliton states and meson-soliton states between $(\int dx\,\phi_c'\Pi_T)^2$ to obtain

$$\begin{aligned}\langle k',p'|H_\mathrm{II}|k,p\rangle=&\frac{M}{2M_0^2}\left\langle k',p'\left|\int dx\,\phi_c'\Pi_T\right|P\right\rangle\left\langle P\left|\int dx\,\phi_c'\Pi_T\right|k,p\right\rangle\\&+\frac{M}{2M_0^2}\left\langle p'\left|\int dx\,\phi_c'\Pi_T\right|k,P'\right\rangle\left\langle k',P'\left|\int dx\,\phi_c'\Pi_T\right|p\right\rangle,\end{aligned} \tag{3.57}$$

where $P=p+k=p'+k'$ and $P'=p-k'=p'-k$ are the momenta of intermediate solitons. Substituting (3.51) and (3.52) leads to

$$\langle k',p'|H_\mathrm{II}|k,p\rangle=\frac{1}{M_0}\frac{\omega_{k'}}{\sqrt{2\omega_k}}\zeta_{k'}\frac{\omega_k}{\sqrt{2\omega_k}}\zeta_k^*\ . \tag{3.58}$$

For computing matrix elements of H_III and H_IV, we need matrix elements of ξ:

$$\langle k,p-k|\xi|p\rangle=-\frac{1}{\sqrt{2\omega_k}}\langle\chi_k|f'\rangle\ . \tag{3.59}$$

The overlap integral in the right-hand side consists of two parts:

$$\langle\chi_k|\phi_c''\rangle\cong-ik\zeta_k \tag{3.60}$$

and

$$\sum_n{}'\zeta_n\langle\chi_k|\psi_n'\rangle\cong ik\zeta_k\left[1-\frac{1}{M_0}|\zeta_k|^2\right] \tag{3.61}$$

so that

$$\langle k,p-k|\xi|p\rangle\cong-\frac{1}{\sqrt{2\omega_k}}ik\zeta_k\frac{1}{M_0}|\zeta_k|^2\ . \tag{3.62}$$

As the first example of perturbative calculations, we compute the self-energy of the soliton in the presence of one spectator meson. We calculate the second-order con-

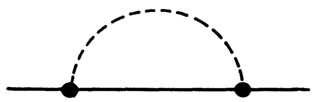

FIG. 1. Time-ordered diagram for the second quantum correction to the soliton energy. The solid line and the dashed line represent soliton and meson, respectively, while the black dots represent the meson-soliton linear interaction $H_{\rm I}$.

tribution to the soliton energy, as indicated in Fig. 1, and retain only a term proportional to p^2 to find a correction to the soliton kinetic energy:

$$\Sigma^{(2)}(p)=-\frac{M^2}{2M_0^4}p^2\sum_n{}'|\langle \phi_c'|\chi_n\rangle|^2=-\frac{M}{2M_0^3}p^2\sum_n{}'|\zeta_n|^2 . \tag{3.63}$$

We have performed the exact sum using the completeness relation (3.42). Note that the identical result follows from the approximate matrix element (3.44). Since $H_{\rm II}$ is of order 1, higher-order graphs that involve both $H_{\rm I}$ and $H_{\rm II}$ contribute to the soliton energy of $O(g^2)$. Among third-order graphs, those indicated in Figs. 2(a)–2(c) contribute the kinetic-energy corrections, each of them being predicted as

$$\Sigma_a^{(3)}(p)=\frac{M}{4M_0^4}p^2\sum_n{}'|\zeta_n|^2\sum_m{}'|\zeta_m|^2 , \tag{3.64}$$

$$\Sigma_b^{(3)}(p)=\frac{M}{4M_0^4}p^2\sum_n{}'\sum_m{}'\frac{\omega_n}{\omega_n+\omega_m}|\zeta_n|^2|\zeta_m|^2 , \tag{3.65}$$

$$\Sigma_c^{(3)}(p)=\frac{M}{4M_0^4}p^2\sum_n{}'\sum_m{}'\frac{\omega_m}{\omega_n+\omega_m}|\zeta_n|^2|\zeta_m|^2 . \tag{3.66}$$

The graph indicated in Fig. 2(d) gives the contribution

$$\Sigma_d^{(3)}(p)=\frac{M}{4M_0^4}p^2\sum_n{}'\frac{1}{\omega_n}|\zeta_n|^2\sum_m{}'\omega_m|\zeta_m|^2 , \tag{3.67}$$

which is exactly canceled by wave-function renormalization. The final result of the third-order energy reads

$$\Sigma^{(3)}(p)=\frac{M}{2M_0^4}p^2\sum_n{}'|\zeta_n|^2\sum_m{}'|\zeta_m|^2. \tag{3.68}$$

We continue the computation of higher-order graphs that can be performed much more easily with Feynman diagrams. The Nth-order correction is given by

$$\Sigma^{(N)}(p)=\frac{M}{2M_0^2}p^2\left[-\frac{1}{M_0}\sum_n{}'|\zeta_n|^2\right]^{N-1} . \tag{3.69}$$

Summing up the corrections to all orders, we eventually find the kinetic-energy correction

$$\Sigma(p)=\Sigma^{(2)}(p)+\Sigma^{(3)}(p)+\cdots=-\frac{1}{2M_0^2}p^2\sum_n{}'|\zeta_n|^2 , \tag{3.70}$$

where we have used (3.43). In Eq. (3.10), the dynamical

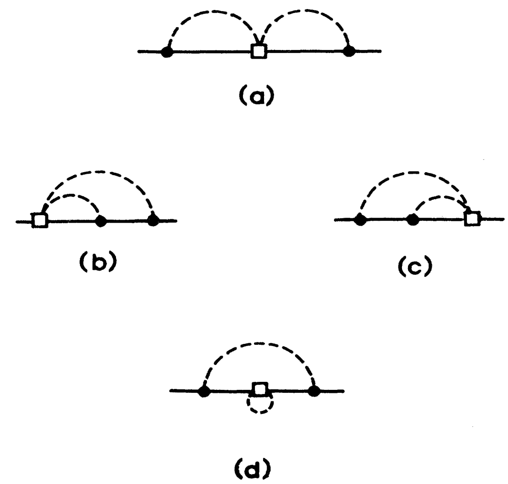

FIG. 2. Time-ordered diagrams for the third quantum corrections to the soliton energy. The open squares represent the meson-soliton quadratic interaction $H_{\rm II}$.

mass of the collective motion differs from the static one. The transitional invariance of the theory in the no-meson sector is proved in Appendix A. For the case of one meson present, the discrepancy can be resolved by the self-energy correction which is added to the collective kinetic energy to obtain

$$E_p=M_0+\frac{M}{2M_0^2}p^2+\Sigma(p)=M_0+\frac{p^2}{2M_0} . \tag{3.71}$$

In this way, the translational invariance is restored in the one-meson one-soliton sector.

IV. RECOIL CORRECTION

We have defined the basis functions $\{\chi_n\}$ by the free field equation in lowest order in g. Since the free Hamiltonian contains the term of $O(g^2)$, there arises a recoil correction of $O(g^2)$. Inclusion of the soliton kinetic-energy term leads to the equations of motion

$$\dot{\chi}=\Pi_T+\frac{M}{2M_0^2}\left\{p,\chi'-\frac{1}{M}f\int dx\, f\chi'\right\} , \tag{4.1}$$

$$\dot{\Pi}_T=\chi''-U''(\phi_c)\chi-\frac{1}{M}f\int dx\, f[\chi''-U''(\phi_c)\chi] + \frac{M}{2M_0^2}\left\{p,\Pi_T'-\frac{1}{M}f\int dx\, f\Pi_T'\right\} . \tag{4.2}$$

The soliton momentum p is a conserved quantity in lowest order: Using the equations of motion (3.19) and (3.20) we have

$$\dot{p}=\int dx(\chi'\ddot{\chi}-\dot{\chi}\dot{\chi}') . \tag{4.3}$$

Substituting the eigenfunction expansion of χ,we get

$$\dot{p}=-\sum_k{}'\omega_k\langle \chi_k|\chi_k'\rangle=0 . \tag{4.4}$$

With the aid of $\dot{p}=0$ in the correction terms in (4.1) and (4.2), we can eliminate Π_T to obtain

$$K\chi_k - \frac{1}{M} f\langle f|K|\chi_k\rangle + v_{\rm rec}\chi_k = \omega_k^2\chi_k \ , \tag{4.5}$$

where

$$v_{\rm rec}\chi_k \equiv 2i\omega_k \frac{M}{M_0^2} p \left[\chi_k' - \frac{1}{M} f\langle f|\chi_k'\rangle \right] . \tag{4.6}$$

It is easy to find that lowest-order perturbation theory predicts the energy shift

$$\langle \chi_k|v_{\rm rec}|\chi_k\rangle = 2i\omega_k \frac{M}{M_0^2} p \langle \chi_k|\chi_k'\rangle \cong -2\omega_k \frac{M}{M_0^2} pk \tag{4.7}$$

which coincides with that resulting from the Klein-Gordon operator $-\partial^2/\partial t^2 - \Omega^2$,

$$\left[\frac{M}{2M_0^2} p'^2 - \omega_k - \frac{M}{2M_0^2} p^2 \right]^2 - \omega_k^2 \cong -\omega_k \frac{M}{M_0^2}(p'+p)k \ , \tag{4.8}$$

with p' and p being the momenta of the soliton before and after meson absorption [because of anticommutation, $2p$ in (4.7) should be interpreted as $p'+p$]. The recoil correction in the present quantization scheme does not entail any important modification. Especially, the matrix element $\langle \phi_c'|\chi_k\rangle$ is unaffected by the recoil to leading order.

The situation is different in the conventional gauge. The recoil term gives rise to the interaction (cf., Jacobs[17])

$$v_{\rm rec}\psi_k \equiv 2i\omega_k \frac{p}{M_0} \left[\psi_k' - \frac{1}{M_0} \phi_c' \langle \phi_x'|\psi_k'\rangle \right] . \tag{4.9}$$

This leads to the shift

$$\langle \psi_k|v_{\rm rec}|\psi_k\rangle = 2i\omega_k \frac{p}{M_0} \langle \psi_k|\psi_k'\rangle \ . \tag{4.10}$$

In Appendix B we have derived

$$\langle \psi_k|\psi_k'\rangle \cong ik - ik\frac{1}{M_0}|\zeta_k|^2 \tag{4.11}$$

in the limit of $\omega_k=0$. Inserted into (4.10), the first term gives the energy difference similar to (4.7) and the second term gives rise to the meson-soliton scattering amplitude

$$T_{\rm rec} = 2\omega_k \frac{pk}{M_0^2}|\zeta_k|^2 \ . \tag{4.12}$$

The significance of this term will be made clear in Sec. VI.

V. MESON-SOLITON SCATTERING

With the systematic perturbation theory developed in the preceding section, we now calculate the meson-soliton scattering amplitude. From (3.58) we find the first-order amplitude

$$T^{(1)} = \frac{1}{M_0}\omega_k^2|\zeta_k|^2 = \frac{k^2|u(k)|^2}{M_0\omega_k^2} \ , \tag{5.1}$$

where we have used

$$\zeta_k = ik\frac{u(k)}{\omega_k^2} \ , \tag{5.2}$$

as derived in the previous paper.[13] Here $u(k)$ is the field form factor defined by the Fourier transform of the source function for the classical meson field:[2]

$$u(k) = -\int dx\, e^{-ikx}\Omega^2\phi_c(x) \ , \tag{5.3}$$

where $\Omega^2 = -d^2/dx^2 + \mu^2$ and μ is the meson mass. Next we compute the second-order term induced by $H_{\rm I}$:

$$T^{(2)} = \frac{|\sqrt{2\omega_k}\langle p+k|H_{\rm I}|k,p\rangle|^2}{\omega_k - E_{p+k} + E_p} - \frac{|\sqrt{2\omega_k}\langle k,p-k|H_{\rm I}|p\rangle|^2}{\omega_k - E_p + E_{p-k}} \ . \tag{5.4}$$

Using (3.55) and (3.56), we obtain

$$T^{(2)} = \left[\frac{(2p+k)^2}{\omega_k - E_{p+k} + E_p} - \frac{(2p-k)^2}{\omega_k - E_p + E_{p-k}} \right] \frac{1}{4M_0^2}\omega_k^2|\zeta_k|^2 + \left[\frac{(2p+k)^2}{\omega_k - E_{p+k} + E_p} - \frac{(2p-k)^2}{\omega_k - E_p + E_{p-k}} \right] \frac{1}{4M_0^3}\omega_k^2|\zeta_k|^2 {\sum_n}'|\zeta_n|^2 \ . \tag{5.5}$$

The second term produces the fourth-order scattering amplitude generated by the meson-soliton vertex u. With the aid of (5.2), we write the first term as

$$T^{(2)} = \left[\frac{(E_{p+k}-E_p)^2}{\omega_k - E_{p+k} + E_p} - \frac{(E_p - E_{p-k})^2}{\omega_k - E_p + E_{p-k}} \right] \frac{|u(k)|^2}{\omega_k^2} . \tag{5.6}$$

Collecting from $T^{(1)}$ and $T^{(2)}$ all terms that contain u twice, we arrive at the result

$$\left[\frac{1}{\omega_k - E_{p+k} + E_p} - \frac{1}{\omega_k - E_p + E_{p-k}} \right] |u(k)|^2 \ , \tag{5.7}$$

which exactly reproduces the direct and crossed Born terms generated by the meson-soliton vertex u.

In the previous paper,[13] it is shown that the T matrix coming from the background scattering,

$$t_\psi = \langle k|v|\psi_k\rangle \ , \tag{5.8}$$

has the zero-mode contribution, where $v = U''(\phi_c) - \mu^2$

and ψ_n is the solution of the equation

$$K\psi_k=(\Omega^2+v)\psi_k=\omega_k^2\psi_k \ . \tag{5.9}$$

In fact, t_ψ can be written as

$$t_\psi=\langle k|v|k\rangle+\sum_n\frac{|\langle k|v|\psi_n\rangle|^2}{\omega_k^2-\omega_n^2} \ . \tag{5.10}$$

Therefore, in the limit of $\omega_k=0$, t_ψ is dominated by the $n=0$ mode contribution

$$t_\psi\cong\frac{|\langle k|v|\psi_0\rangle|^2}{\omega_k^2}=\frac{1}{M_0}\omega_k^2|\zeta_k|^2\equiv T_0 \ , \tag{5.11}$$

where we have used the Schrödinger-like equation (5.9):

$$\langle k|v|\psi_0\rangle=-\langle k|\Omega^2|\psi_0\rangle=\frac{1}{\sqrt{M_0}}\omega_k^2\zeta_k \ . \tag{5.12}$$

In the symmetric quantization approach we have developed, the zero mode does not contribute to scattering. The scattering amplitude t_χ obtained from (3.23) is given by

$$t_\chi=\langle k|v|\chi_k\rangle-\frac{1}{M}\langle k|f\rangle\langle f|K|\chi_k\rangle \ . \tag{5.13}$$

Inserting (3.32) into the first term leads to

$$t_\chi=t_\psi-\frac{1}{M}\left\langle k\left|v\frac{1}{\omega_k^2-K}\right|f\right\rangle\langle f|K|\chi_k\rangle$$
$$-\frac{1}{M}\langle k|f\rangle\langle f|K|\chi_k\rangle \ . \tag{5.14}$$

Since ψ_k is given by the Schrödinger equation (5.9), one finds

$$\langle\psi_k|f\rangle=\langle k|f\rangle+\left\langle k\left|v\frac{1}{\omega_k^2-K}\right|f\right\rangle \ . \tag{5.15}$$

Substituting this into (5.14) leads to

$$t_\chi=t_\psi-\frac{1}{M}\langle\psi_k|f\rangle\langle f|K|\chi_k\rangle=t_\psi-\frac{|\zeta_k|^2}{\langle f|1/(\omega_k^2-K)|f\rangle} \ , \tag{5.16}$$

where we have used $\langle\psi_k|f\rangle=\zeta_k$ and (3.33). Taking the limit $\omega_k=0$, we have (3.38), whereby the last term in Eq. (5.16) becomes

$$-\frac{|\zeta_k|^2}{\langle f|1/(\omega_k^2-K)|f\rangle}\cong-\frac{1}{M_0}\omega_k^2|\zeta_k|^2 \ . \tag{5.17}$$

Consequently, the zero-mode contribution in t_ψ is exactly canceled out.

In the previous nonsymmetric quantization method,[10,11,13] the canonical variables satisfy the commutation relation

$$i[\Pi_T(x,t),\chi(y,t)]=\delta(x-y)-\frac{1}{M_0}f(x)\phi_c'(y) \ . \tag{5.18}$$

The equations of motion are

$$\dot\chi=\Pi_T-\frac{1}{M_0}\phi_c'\int dx\, f\Pi_T \ , \tag{5.19}$$

$$\dot\Pi_T=\chi''-U''(\phi_c)\chi \ , \tag{5.20}$$

so that the free field equation becomes

$$\ddot\chi+K\chi+\frac{1}{M_0}\phi_c'\langle f|K|\chi\rangle=0 \ . \tag{5.21}$$

The normal mode is given by the equation

$$K\chi_n-\frac{1}{M_0}\phi_c'\langle f|K|\chi_n\rangle=\omega_n^2\chi_n \ , \tag{5.22}$$

which has the zero-frequency solution $\chi_0=\psi_0$ and the solutions that are orthogonal to f. This equation can be solved using $K\phi_c'=0$ as

$$\chi_k=\psi_k-\frac{1}{M_0\omega_k^2}\phi_c'\langle f|K|\chi_k\rangle \ . \tag{5.23}$$

From $\langle f|\chi_k\rangle=0$ it follows that

$$\langle f|K|\chi_k\rangle=\omega_k^2\zeta_k^* \ . \tag{5.24}$$

Consequently we obtain

$$\chi_k=\psi_k-\frac{\zeta_k^*}{M_0}\phi_c' \ . \tag{5.25}$$

One can prove directly that χ_k satisfies the equation (5.22). It is also immediately seen that χ_k satisfies the completeness property

$$\sum_n{}'\psi_n(x)\chi_n^*(y)=\delta(x-y)-\frac{1}{M_0}f(x)\phi_c'(y) \ . \tag{5.26}$$

The T matrix is calculated from

$$t_\chi=\langle k|v|\chi_k\rangle-\frac{1}{M_0}\langle k|\phi_c'\rangle\langle f|K|\chi_k\rangle$$
$$=\langle k|v|\chi_k\rangle+\frac{1}{M_0}\omega_k^2|\zeta_k|^2 \ . \tag{5.27}$$

Substituting (5.25) yields

$$t_\chi=t_\psi-\frac{\zeta_k^*}{M_0}\langle k|v|\phi_c'\rangle+\frac{1}{M_0}\omega_k^2|\zeta_k|^2 \ . \tag{5.28}$$

Owing to the identity $\langle k|v|\phi_c'\rangle=\omega_k^2\zeta_k$ the last two terms cancel each other, and we obtain $t_\chi=t_\psi$. The zero mode contributes to scattering in the nonsymmetric quantization.

VI. CONVENTIONAL GAUGE

Although gauge independence of physical observables is guaranteed, no one has succeeded in showing by explicit calculations the existence of the Born term in the conventional gauge.[18] In this section we give a rigorous derivation of the Born term and make clear the relationship with the nonconventional gauge condition we have developed. The quantized Hamiltonian is obtained from (3.5) by putting $f=\phi_c'$, $\int dx\,\phi_c'\Pi_T=0$, and $M_\chi=M_0+\xi_0$:

$$H=M_0+\tfrac{1}{8}M_0\{p,(M_0+\xi_0)^{-1}\}^2+\tfrac{1}{2}\int dx(\Pi_T^2+\chi'^2)$$
$$+\int dx\ U(\chi,\phi_c)-\tfrac{1}{8}(M_0+\xi_0)^{-2}\int dx\ \phi_c''^2\ , \quad (6.1)$$

where ξ_0 is defined by (2.16). Up to terms quadratic in fields,

$$H=M_0+\frac{p^2}{2M_0}+\int dx[\tfrac{1}{2}\Pi_T^2+\tfrac{1}{2}\chi'^2+\tfrac{1}{2}\chi^2U''(\phi_c)]$$
$$+H_{\rm I}+H_{\rm II}\ . \quad (6.2)$$

The translational invariance of the obtained Hamiltonian is manifest up to $O(g^2)$, namely, to nonrelativistic order and there is no need of calculating the self-energy correction to get the correct soliton energy. This corresponds to the fact that the meson-soliton linear coupling starts from the term of $O(g^3)$:

$$H_{\rm I}=-\frac{1}{4M_0^2}\{p,\{p,\xi_0\}\}\ . \quad (6.3)$$

On the other hand, the quadratic interaction becomes

$$H_{\rm II}=\frac{1}{8M_0^3}\{p,\xi_0\}^2+\frac{1}{4M_0^3}\{p,\{p,\xi_0^2\}\}\ . \quad (6.4)$$

The computation of the meson-soliton scattering amplitude proceeds as follows: Both of the interactions $H_{\rm I}$ and $H_{\rm II}$ involve ξ_0 which has the matrix element

$$\langle k,p-k|\xi_0|p\rangle=-i\frac{1}{\sqrt{2\omega_k}}\langle\psi_k|\phi_c''\rangle\ . \quad (6.5)$$

Taking the limit $\omega_k=0$, we can use (3.39) to calculate the overlap integral

$$\langle\psi_k|\phi_c''\rangle\cong\langle\chi_k|\phi_c''\rangle\cong-ik\zeta_k\ . \quad (6.6)$$

The quadratic interaction $H_{\rm II}$ has the matrix element

$$\langle k,p|H_{\rm II}|k,p\rangle=\frac{1}{2\omega_k}\frac{k^2(12p^2+k^2)}{4M_0^3}|\zeta_k|^2 \quad (6.7)$$

so that the first-order scattering amplitude induced by $H_{\rm II}$ is given by

$$T^{(1)}=[(E_{p+k}-E_p)^3-(E_p-E_{p-k})^3]\frac{|u(k)|^2}{\omega_k^4}\ . \quad (6.8)$$

The second-order amplitude induced by the linear interaction $H_{\rm I}$ is evaluated using the matrix elements

$$\langle k,p-k|H_{\rm I}|p\rangle=-i\frac{1}{\sqrt{2\omega_k}}\frac{k(2p-k)^2}{4M_0^2}\zeta_k \quad (6.9)$$

and

$$\langle p-k|H_{\rm I}|k,p\rangle=i\frac{1}{\sqrt{2\omega_k}}\frac{k(2p+k)^2}{4M_0^2}\zeta_k^*\ . \quad (6.10)$$

Substituting (5.2) we find

$$T^{(2)}=\left[\frac{(E_{p+k}-E_p)^4}{\omega_k-E_{p+k}+E_p}-\frac{(E_p-E_{p-k})^4}{\omega_k-E_p+E_{p-k}}\right]\frac{|u(k)|^2}{\omega_k^4}. \quad (6.11)$$

As we have shown in the preceding section, the zero-mode part of the background scattering contributes to the amplitude by the amount

$$T_0=\frac{k^2|u(k)|^2}{M_0\omega_k^2}=[(E_{p+k}-E_p)-(E_p-E_{p-k})]\frac{|u(k)|^2}{\omega_k^2}\ . \quad (6.12)$$

Another important distinction between the conventional gauge and the present approach is the recoil correction. As shown in Sec. IV, the recoil term produces the scattering amplitude $T_{\rm rec}$ as given by (4.12):

$$T_{\rm rec}=\frac{2pk^3|u(k)|^2}{M_0^2\omega_k^4}$$
$$=[(E_{p+k}-E_p)^2-(E_p-E_{p-k})^2]\frac{|u(k)|^2}{\omega_k^3}\ . \quad (6.13)$$

The sum of $T^{(1)}$, $T^{(2)}$, T_0, and $T_{\rm rec}$ recovers the full Born term. We have to be aware that the computation of $T^{(1)}$ and $T^{(2)}$ as well as $T_{\rm rec}$ is valid only to order quadratic in ζ_k. In evaluating $T^{(1)}$ and $T^{(2)}$, we have replaced the soliton momentum operator with a c number but its matrix element has an additional term (see Appendix B),

$$\left\langle k,p\left|P+\int dx\ \Pi_T\chi'\right|k,p\right\rangle=p+k+i\langle\psi_k|\psi_k'\rangle$$
$$\cong p-ik\frac{1}{M_0}|\zeta_k|^2\ . \quad (6.14)$$

VII. ANOTHER FORM OF CANONICAL TRANSFORMATION

To be complete, we inquire another possible form of the canonical transformation that linearizes F as $\int dx\ f\Pi_T$, and at the same time has the same form as the conventional gauge:

$$\Pi=\Pi_T-A\phi_c'\ . \quad (7.1)$$

The condition $P+\int dx\ \Pi_T=\int dx\ f\Pi_T$ fixes A and gives

$$\Pi=\Pi_T-\frac{1}{M_0+\xi_0}\phi_c'\left[P+\int dx\ \Pi_T(\Phi'-f)\right]\ . \quad (7.2)$$

Under the constraint $\int dx\ f\Pi_T=0$,

$$\Pi=\Pi_T-\frac{1}{M_0+\xi_0}\phi_c'\left[p+\int dx\ \phi_c'\Pi_T\right]\ . \quad (7.3)$$

This is different from the standard canonical transformation only in the presence of $\int dx\ \phi_c'\Pi_T$ which does not vanish because of the condition $\int dx\ f\Pi_T=0$. The Hamiltonian is given by

$$H=M_0+\tfrac{1}{8}M_0\left\{p+\int dx\ \phi_c'\Pi_T,(M_0+\xi_0)^{-1}\right\}^2$$
$$-\tfrac{1}{4}M_0\left\{\int dx\ \phi_c'\Pi_T,\left\{p+\int dx\ \phi_c'\Pi_T,(M_0+\xi_0)^{-1}\right\}\right\}$$
$$+\tfrac{1}{2}\int dx(\Pi_T^2+\chi'^2)+\int dx\ U(\chi,\phi_c)$$
$$-\tfrac{1}{8}(M_0+\xi_0)^{-2}\int dx\ \phi_c''^2\ . \quad (7.4)$$

If we put $\int dx\, \phi_c' \Pi_T = 0$, we regain (6.1). Expanding H as before in terms of the field variables, one sees that the linear coupling of $O(g)$ is canceled out so that the interaction $H_{\rm I}$ is unchanged, whereas the quadratic interaction has additional terms

$$\Delta H_{\rm II} = -\frac{1}{2M_0}\left[\int dx\, \phi_c' \Pi_T\right]^2 - \frac{1}{4M_0^2}\left\{p, \left\{\int dx\, \phi_c' \Pi_T, \xi_0\right\}\right\} . \quad (7.5)$$

The interaction of order 1 emerges again but with the opposite sign to (3.16). Since the free-field equation is identical with (3.23), the quantization can be performed by its eigenfunctions $\{\chi_n\}$ rather than $\{\psi_n\}$.

We now repeat the computation of the meson-soliton scattering amplitude. The first-order term induced by $H_{\rm II}$ and the second-order term induced by $H_{\rm I}$ are the same as in the conventional gauge because the matrix element

$$\langle k, p-k|\xi_0|p\rangle = -i\frac{1}{\sqrt{2\omega_k}}\langle \chi_k|\phi_c''\rangle \quad (7.6)$$

is the same to leading order. The contributions from $\Delta H_{\rm II}$ consist of two parts: From the order-1 term containing $(\int dx\, \phi_c' \Pi_T)^2$ we obtain the correction to the first-order amplitude:

$$\Delta T_a^{(1)} = -\frac{k^2|u(k)|^2}{M_0\omega_k^2} . \quad (7.7)$$

From the $O(g^2)$ term that contains $\int dx\, \phi_c' \Pi_T$ we find

$$\Delta T_b^{(1)} = 2\omega_k \frac{pk}{M_0^2}|\xi_k|^2 \quad (7.8)$$

which coincides with the recoil-correction term in the standard gauge. If $\Delta T_a^{(1)}$ had the opposite sign, the sum of $T^{(1)}$, $T^{(2)}$, $\Delta T_a^{(1)}$, and $\Delta T_b^{(1)}$ would have led to the Born term. We recall that the background scattering described by χ_k does not contribute to the Born term.

The minus sign in $\Delta T_a^{(1)}$ prevents us from deducing the Born term with lowest-order perturbative calculations. Yet, the full scattering amplitude has a relation with the Born term. To see this, we sum up the sequence of Feynman diagrams, as indicated in Fig. 3, that contain repetitions of the order-1 quadratic interaction and which therefore give rise to poles in the integration over one-meson intermediate states. The sum can be performed exactly with the result

$$\Delta T_a = \Delta T_a^{(1)} + \Delta T_a^{(2)} + \Delta T_a^{(3)} + \cdots = \frac{k^2|u(k)|^2}{M_0\omega_k^2}\left[-1+\frac{\mathcal{J}(\omega_k)}{M_0}\right]^{-1} , \quad (7.9)$$

where

$$\mathcal{J}(\omega_k) = \sum_n{}' \frac{\omega_n^2|\xi_n|^2}{\omega_n^2-\omega_k^2} . \quad (7.10)$$

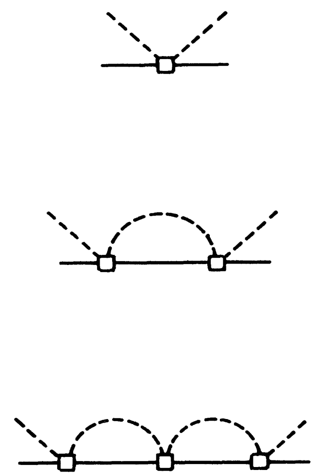

FIG. 3. Feynman diagrams for meson-soliton scattering induced by the quadratic interaction.

With the relation (3.45) we find

$$\mathcal{J}(\omega_k) = M_0 + \omega_k^2 \sum_n{}' \frac{|\xi_n|^2}{\omega_n^2-\omega_k^2} . \quad (7.11)$$

Substituting this into (7.9) yields

$$\Delta T_a = \frac{k^2|u(k)|^2}{\omega_k^4}\left[\sum_n{}' \frac{|\xi_n|^2}{\omega_n^2-\omega_k^2}\right]^{-1} . \quad (7.12)$$

Recall that ω_k^2 in the denominator has $+i\epsilon$ implicitly. In the point-source limit, we take $u(k)=$const and carry out the integration:

$$\Delta T_a = \frac{2k^2}{\mu - \dfrac{\omega_k^2}{2\mu} + i|k|} . \quad (7.13)$$

For the case of pseudoscalar coupling, we take $u(k)=\text{const}\times k$ to obtain

$$\Delta T_a = \frac{2k^2}{\mu + \dfrac{\mu\omega_k^2}{2k^2} + i|k|} \quad (7.14)$$

which agrees with (7.13) in the neighborhood of $\omega_k=0$.

With the use of (5.2), we have

$$\mathcal{J}(\omega_k) = \sum_n \frac{n^2|u(n)|^2}{\omega_n^2(\omega_n^2-\omega_k^2)} , \quad (7.15)$$

which can be decomposed as

$$\mathcal{J}(\omega_k) = I(0) + \frac{k^2}{\omega_k^2}[I(\omega_k)-I(0)] , \quad (7.16)$$

where

$$I(\omega_k) = \sum_n \frac{|u(n)|^2}{\omega_n^2-\omega_k^2} . \quad (7.17)$$

We renormalize the soliton mass as $M_r = M_0 - I(0)$. Then ΔT_a becomes

$$\Delta T_a = \frac{k^2|u(k)|^2}{M_r \omega_k^2} \left[-1 + \frac{k^2 J(\omega_k)}{M_r} \right]^{-1} , \tag{7.18}$$

where

$$J(\omega_k) = \frac{1}{\omega_k^2} [I(\omega_k) - I(0)] . \tag{7.19}$$

If we ignore -1 in the denominator in Eq. (7.18), we obtain

$$\Delta T_a \cong \frac{|u(k)|^2}{\omega_k^2 J(\omega)} = \frac{|u(k)|^2}{I(\omega_k) - I(0)} \tag{7.20}$$

in complete agreement with the celebrated result of the strong-coupling theory.[19–23] The integral (7.17) is convergent without $u(k)$. For the point source of scalar coupling,

$$\Delta T_a \cong -\frac{2i\mu|k|}{\mu + i|k|} . \tag{7.21}$$

This coincides with (7.13) near $\omega_k = 0$ except for $-\omega_k^2/2\mu$ in the denominator. In soliton models, the mass M_0 and the coupling u always appear in the form of the product $(1/\sqrt{M_0})u$ of order 1. In the strong-coupling theory, in which the target mass and the coupling are taken to be large, however, the two parameters are varied independently. The neglect of -1 that lead to (7.21) corresponds to taking the limit $u \to \infty$ while M_r is kept finite.

We compare this with the consequence of the multiple scattering caused by $H_{\rm II}$ we have derived in Sec. III. The sum of the diagrams in Fig. 3 leads to

$$T = \frac{k^2|u(k)|^2}{M_0 \omega_k^2} \left[+1 + \frac{\mathcal{J}(\omega_k)}{M_0} \right]^{-1} \tag{7.22}$$

which has the point-source limit for scalar coupling,

$$T = \frac{2k^2}{\mu + \dfrac{\omega_k^2}{2\mu} + i|k|} , \tag{7.23}$$

and, for pseudoscalar coupling,

$$T = \frac{2k^2}{\mu - \dfrac{\mu\omega_k^2}{2k^2} + i|k|} . \tag{7.24}$$

With the modified $M_r = M_0 + I(0)$, T is cast in the form

$$T = \frac{k^2|u(k)|^2}{M_r \omega_k^2} \left[+1 + \frac{k^2 J(\omega_k)}{M_r} \right]^{-1} . \tag{7.25}$$

Ignoring $+1$ in the denominator leads to (7.20). Either of the order-1 quadratic interactions with different signs leads to the same strong-coupling limit. Since $H_{\rm II}$ in Sec. III has the matrix element that coincides with the Born term to leading order, the multiple scattering caused by $H_{\rm II}$ is that of the Born term. It is thus confirmed that the Hamiltonian $\Delta H_{\rm II}$ has the same strong-coupling limit as the repetitions of the Born term. The approximate coincidence of the sequential sum of the Born terms and the strong-coupling theory is clarified by Goebel.[24]

If we apply the strong-coupling theory to a one-dimensional scalar field, the scattering consists of transmission

$$t = -\frac{2i\mu|k|}{\mu + i|k|} \tag{7.26}$$

and reflection

$$r = \frac{2i\mu|k|}{\mu + i|k|} \tag{7.27}$$

with equal strengths, i.e., the incident meson flux is transmitted and reflected with equal probabilities. The T matrices satisfy the one-meson unitarity relations

$$\mathrm{Im} t = -\frac{1}{4|k|}(|t|^2 + |r|^2) , \tag{7.28}$$

$$\mathrm{Im} r = -\frac{1}{4|k|}(t^* r + r^* t) . \tag{7.29}$$

We have seen that the order-1 quantum Hamiltonians derived in this paper predict these results with an appropriate limiting procedure (the T matrix for reflection is given by

$$-\frac{k^2 u(k)^2}{M_0 \omega_k^2} \left[\pm 1 + \frac{\mathcal{J}(\omega_k)}{M_0} \right]^{-1} \tag{7.30}$$

which leads to (7.27) in the strong-coupling limit). We compare this with the background scattering amplitude of the sine-Gordon equation

$$t = -\frac{4i\mu|k|}{\mu + i|k|} . \tag{7.31}$$

Despite the apparent similarity to the result of the strong-coupling theory, their physical contents are different. Equation (7.31) describes pure transmission,[25] and there is no reflected wave ($r = 0$).

VIII. SUMMARY

We have presented a new form of canonical quantization of solitary-wave classical solutions in two space-time dimensions. The gauge-fixing condition and the constraint imposed on the field fluctuation and its conjugate momentum are treated symmetrically in such a way that the infrared-divergent terms contained in the meson wave functions are eliminated. The resulting Hamiltonian possesses a linear meson-soliton interaction of $O(g)$ and a quadratic interaction of order 1. The emergence of these interactions is the most important consequence of the present quantization formalism.

We showed that the sum of the first-order amplitude of the quadratic interaction and the second-order amplitude of the linear interaction exactly reproduces the Born term of the meson-soliton scattering amplitude. The scattering amplitude produced by the quadratic interaction is found to be identical with the zero-mode contribution to meson scattering off the soliton background. In the strong-coupling limit, we[26] have shown that the zero-mode term of the background scattering coincides with the classical-field contribution to scattering.[3,27,28] In the present approach, we have defined the normal modes that are free from infrared divergences so that the background scattering does not develop the double pole at $\omega_k=0$, and hence does not contribute to the Born term, while in the previous nonsymmetrical approach[11,13] the full Born term is explained as a sum of the zero-mode term from the background scattering and the second-order term induced by the linear coupling proportional to $\int dx\, f\Pi_T$. One thus sees that whether the background scattering contributes to the Born term is highly dependent on the choice of gauge. There has been a controversy about the origin of the meson-soliton Yukawa coupling.[29-32] It is argued that the classical part, and therefore the zero-mode part, is unlikely to contribute to quantum scattering.[33] In the present quantization method, the Born term is produced fully quantum mechanically by the meson-soliton interaction Hamiltonians $H_{\rm I}$ and $H_{\rm II}$.

To demonstrate the advantageous features of the present quantization formalism, we investigated the meson-soliton scattering in the conventional gauge. The Born term is deduced from the first-order amplitude of the $O(g^4)$ quadratic interaction, the second-order amplitude of the $O(g^3)$ linear interaction, the zero-mode contribution of order 1, and the recoil correction of $O(g^2)$. Compared to the present formalism, the derivation is indirect and the Born term emerges from various origins. The gauge invariance of physical scattering amplitudes is ensured by gauge theory, and it is not surprising to find that the Born term is deduced from different origins in different gauges. The present approach is advantageous in that only quantum fluctuations are responsible for meson scattering and yet reproduce the Born term with the classical mass and the classical vertex function. It is also very effective for computing higher-order scattering amplitudes. We have investigated another form of the canonical transformation that makes both χ and Π_T orthogonal to f, but it turned out that the relationship with the Born term is only through a very remote way and it is not suitable for perturbative calculations.

APPENDIX A

Gervais, Jevicki, and Sakita[3] recovered, in the conventional gauge, the relativistic form for the soliton energy by summing all tree graphs. The Lorentz invariance is guaranteed by the gauge-theoretical formalism, and it should not depend on the choice of gauge. Here we check that independently of the gauge function f, (2.21) has the classical soliton solution which has the correct Lorentz-invariant form. To see this, we look for a time-independent solution for the equations of motion. Namely, we seek for a solution of the form

$$\Phi(x,t)=\varphi_0(x-X)\ , \tag{A1}$$

$$\Pi(x,t)=\pi_0(x-X)\ , \tag{A2}$$

which depends on time only through X, i.e., $\dot{\chi}=0$ and $\dot{\Pi}_T=0$. From (2.23) and (2.24) with (2.28) inserted, we obtain

$$\pi_0=-\dot{X}\varphi_0'\ , \tag{A3}$$

$$\chi''-U'(\chi,\phi_c)+\dot{X}\pi_0'=0\ . \tag{A4}$$

At this stage the gauge function f disappears so that the Lorentz-invariant classical solution follows immediately. In fact, eliminating π_0, we find the equation for φ_0,

$$\varphi_0''-U'(\varphi_0)-\dot{X}^2\varphi_0''=0\ , \tag{A5}$$

which has the solution

$$\varphi_0(x-X)=\phi_c\left[\frac{x-X}{\sqrt{1-\dot{X}^2}}\right]\ . \tag{A6}$$

Multiplying (A3) by φ_0' and integrating we find the well-known relation

$$\dot{X}=\frac{P}{\int dx\,\varphi_0'^2}=\frac{P}{M_0}\sqrt{1-\dot{X}^2}\ . \tag{A7}$$

The Lorentz-invariant form for the soliton energy follows from $E_0=M_0\sqrt{1-\dot{X}^2}$. It should be emphasized that the result is independent of the gauge-fixing condition.

APPENDIX B

In the limit of vanishing ω_k, we have shown that the normal modes χ_k and ψ_k are related by (3.39), or

$$\chi_k\sim\psi_k-\frac{\zeta_k^*}{\sqrt{L}M_0}\phi_c'\ , \tag{B1}$$

where L is the length of the box (in the text we have dropped the normalization factor $1/\sqrt{L}$ for notational simplicity.) We note that (B1) entails a modification of normalization:

$$\langle\chi_k|\chi_k\rangle\sim\langle\psi_k|\psi_k\rangle+\frac{|\zeta_k|^2}{LM_0}\ . \tag{B2}$$

For finite ω_k, the extra term of $O(L^{-1})$ can be ignored in the large-L limit but for vanishing ω_k, it makes a difference. We insert a normalization constant N_k and write

$$\chi_k\sim\frac{1}{N_k}\left[\psi_k-\frac{\zeta_k^*}{\sqrt{L}M_0}\phi_c'\right]\ . \tag{B3}$$

Normalizing χ_k and ψ_k to unity, we find

$$N_k^2\sim 1+\frac{|\zeta_k|^2}{LM_0}\ . \tag{B4}$$

We now calculate using (B3) the integral

$$\langle\psi_k|\psi_k'\rangle\sim N_k^2\langle\chi_k|\chi_k'\rangle+\frac{N_k\zeta_k^*}{\sqrt{L}M_0}\langle\chi_k|\phi_c''\rangle+\frac{N_k\zeta_k}{\sqrt{L}M_0}\langle\phi_c'|\chi_k'\rangle\ . \tag{B5}$$

We have seen that in evaluating overlap integrals between

χ_k and localized functions such as ϕ_c', we can approximate $\chi_k \sim (1/\sqrt{L})e^{ikx}$. We find

$$\langle \chi_k | \phi_c'' \rangle \sim -ik \frac{1}{\sqrt{L}} \zeta_k^* , \tag{B6}$$

$$\langle \phi_c' | \chi_k' \rangle \sim -ik \frac{1}{\sqrt{L}} \zeta_k . \tag{B7}$$

Making use of these results and Eq. (B4), we get

$$\langle \psi_k | \psi_k' \rangle \sim \langle \chi_k | \chi_k' \rangle + \frac{|\zeta_k|^2}{LM_0} (\langle \chi_k | \chi_k' \rangle - 2ikN_k) . \tag{B8}$$

Since $\langle \chi_k | \chi_k' \rangle = ik + O(L^{-1})$, we are led to the result

$$\langle \psi_k | \psi_k' \rangle \sim \langle \chi_k | \chi_k' \rangle - ik \frac{|\zeta_k|^2}{LM_0} + O(L^{-2}) . \tag{B9}$$

The overlap integral can be calculated directly using explicit forms for ψ_k in specific models. The extra term in (B9), however, cannot be obtained from the resulting integral which is of th form $(L\omega_k^2)^{-1}$ rather than $(L\omega_k^4)^{-1}$. The reason for this is the following: The behavior of $\psi_k(x)$ in the vicinity of $\omega_k = 0$,

$$\psi_k(x) \sim \frac{\zeta_k^*}{\sqrt{L M_0}} \phi_c'(x) , \tag{B10}$$

was deduced from its Fourier transform

$$\psi_k(p) = \delta_{pk} + \frac{1}{\omega_k^2 - \omega_p^2} \langle p | t | k \rangle \tag{B11}$$

by retaining the infrared-divergent term in the half-off-shell T matrix

$$\langle p | t | k \rangle \sim \omega_p^2 \psi_0(p) \psi_0^*(k) = -\omega_p^2 \frac{\zeta_k^*}{LM_0} \int dx \, e^{-ipx} \phi_c'(x) . \tag{B12}$$

The limit (B10) into the unphysical region of k cannot be reached directly from the wave function $\psi_k(x)$ given in the physical region. One should follow the steps as we have taken.

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