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Analysis of inflation driven by a scalar field and a curvature-squared term

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The evolution of an isotropic cosmological model in the $R+R^2$ theory of gravity with a coherent massive scalar field is studied analytically and numerically. For certain initial conditions, the model goes through a superinflationary stage ($\dot{H}>0$) followed by usual subinflation ($\dot{H}<0$). Double inflation consisting of two inflationary stages divided by a period of power-law expansion $a(t)\propto t^{2/3}$ is possible if the scalar particle mass is small compared to the mass of the scalaron (scalar graviton). The spectrum of adiabatic perturbations generated from vacuum quantum fluctuations is investigated. It is quasiflat (though not necessarily varying monotonically with scale) in the case of single inflation; for double inflation, it has a characteristic step.

I. INTRODUCTION

It is well known that both vacuum-polarization effects[1] and a scalar field[2–5] can lead to an inflationary stage in the early Universe (for a recent review, see Ref. 6). Further, it was argued that the combined action of these effects may lead to two consecutive inflationary stages (double inflation;[7–9] for the generalization to the multiscalar case, see Ref. 10). To elucidate these questions further we shall discuss here a cosmological model including vacuum-polarization effects which are described by an additional R^2 term in the gravitational action and a coherent massive scalar field. We shall follow the general approach of Ref. 6 but choose a different potential for the scalar field ($m^2\phi^2$ instead of $\lambda\phi^4/4$). Thus we start with the Lagrangian

$$L=\frac{1}{16\pi G}(-R+\alpha R^2)+\tfrac{1}{2}(\phi_{,\mu}\phi^{,\mu}-m^2\phi^2)\ , \qquad (1)$$

where α is a positive coupling constant, R denotes the Ricci scalar, and ϕ the scalar field. We use units where the speed of light $c=1$ and the Planck constant $\hbar=1$; i.e., the gravitational constant G is equal to the Planck length squared: $G=l_{\rm Pl}^2$. The coupling parameter α and the mass m are assumed to satisfy the conditions $\alpha\gg l_{\rm Pl}^2$, $ml_{\rm Pl}\ll 1$.

The question of whether or not two consecutive inflationary stages exist is crucial for understanding the spectrum of perturbations generated during inflation. In the case of two consecutive inflationary stages separated by an intermediate stage of power-law expansion a break in the spectrum is expected. However, in model (1) the case of two really disconnected inflationary stages takes place only if the mass of the scalar particles is small in comparison with the inverse coupling constant, $m^2\ll 1/6\alpha$, and if vacuum polarization dominates initially; i.e., the energy density of the scalar field is small compared to the Hubble parameter during the first part of inflation. Otherwise the combined action of the scalar field and vacuum polarization leads to a single quasi de Sitter stage with a quasiflat perturbation spectrum.

At first, we study spatially flat homogeneous isotropic cosmological models, for which a generalized Friedmann equation can be obtained. It is a second-order differential equation for the Hubble parameter $H(t)=\dot{a}/a$, where the overdot denotes a time derivative and $a(t)$ is the scale factor (that does not explicitly enter into equations for spatially flat models). General expressions for super- and subinflationary stages arising in model (1) are presented in Sec. II. The process of relaxation to the de Sitter (inflationary) stage is studied in Sec. III. Different possible paths of the general evolution of the curve $\dot{H}(H)$ in the H-$\dot{H}$ plane obtained by numerical calculations are discussed in Secs. IV and V.

If there exist two consecutive inflationary stages characteristic for R^2 inflation and scalar field inflation, the transition between them may be accompanied by rapid oscillations of the Hubble parameter. This behavior is investigated in Sec. VI. In Sec. VII we present equations for small inhomogeneous perturbations on the homogeneous background and study their solutions. Section VIII contains conclusions and discussions of initial conditions for the model.

The variation of the Lagrangian (1) leads to the field equations for the gravitational field and the scalar field. With the metric

$$ds^2=dt^2-a^2(t)\delta_{ik}dx^i dx^k \qquad (2)$$

of the spatially flat Friedmann universe we obtain the field equation in terms of the Hubble parameter and its derivatives and the scalar field and its derivatives. It is convenient to introduce the dimensionless quantity $z^2=4\pi G\phi^2/3$, which is related to the number of e-folds during scalar-field-driven inflation [cf. Eq. (9)]. Then the 00 component of the gravitational field equations takes the form

$$H^2+\frac{1}{M^2}(2H\ddot{H}-\dot{H}^2+6H^2\dot{H})=\dot{z}^2+m^2z^2\ , \qquad (3)$$

where the mass M of the scalarons—scalar gravitons corresponding to the R^2 term in the Lagrangian—is defined by $M^2=1/(6\alpha)$; $Ml_{\rm Pl}\ll 1$. A second equation of interest is the trace equation

$$\dot H+2H^2+\frac{1}{M^2}(H^{(3)}+7H\ddot H+4\dot H^2+12H^2\dot H)=-\dot z^2+2m^2z^2\ , \quad (4)$$

where $H^{(3)}$ denotes the third time derivative. Equation (3) is the first integral of Eq. (4). The field equation for the scalar field reads

$$\ddot z+3H\dot z+m^2z=0\ . \quad (5)$$

It is convenient to follow the behavior of a solution of Eqs. (3) and (5) not in the complete phase space $(H,\dot H,z,\dot z)$, but projected into the H-$\dot H$ plane. To accomplish this we rewrite Eq. (3) in the form

$$\frac{d\dot H}{dH}=\frac{\dot H}{2H}-3H-\frac{M^2}{2H\dot H}(H^2-\dot z^2-m^2z^2)\ . \quad (6)$$

Inflationary stages driven solely by the R^2 term or the massive scalar field are characterized by the conditions $|\dot H_{\rm inf}|\ll H^2$, $\dot H_{V\text{-inf}}=-M^2/6$,[11] and $\dot H_{S\text{-inf}}=-m^2/3$,[12] respectively. In order to compare the roles of the R^2 term and the scalar field within the combined scenario (1) we shall discuss the two cases $m^2\gg M^2$ and $m^2\ll M^2$, i.e., the cases of heavy scalar particles and light scalar particles.

To investigate Eqs. (3) and (5), we have to fix the initial values H_0, $\dot H_0$, z_0, and $\dot z_0$. Since we are mainly interested in the existence of inflationary stages, the quantity H_0^2 should be large enough to ensure the existence of such stages, i.e., $H_0^2\gg\max(m^2/3,M^2/6)$. In the case of the flat Friedmann model (2), all solutions (except for a set of zero measure) begin or terminate in a singularity. As was shown in Ref. 7, the most general behavior of solutions of the combined model near the singularity is $a(t)\propto|t|^{1/2}$ (the fact that a different scalar potential is used in our paper makes no change). We shall assume $\dot H_0<0$, $|\dot H_0|\sim H_0^2$; i.e., we start in the phase of decelerated power-law expansion from the singularity. We take $\dot z_0=0$ as an initial condition for numerical calculations, but a kinetic term would not change substantially our results. Concerning the initial condition for the scalar field itself, three cases are of particular interest. For $mz_0\ll H_0$, vacuum polarization dominates over matter. In the Einsteinian case $mz_0=H_0$, higher-order gravity terms vanish initially. Finally, if $mz_0\gg H_0$, the Einstein tensor may be neglected as far as $H\gg M$, and then the theory reduces to a case of pure fourth-order gravity with the scalar field energy-momentum tensor as a source.

II. INFLATIONARY STAGE IN THE COMBINED MODEL

If the space-time (2) goes through the quasi de Sitter (inflationary) stage and the scalar field is in the slow-rolling regime, i.e., $|\dot H|\ll H^2$ and $|\ddot\phi|\ll H|\dot\phi|$, Eqs. (3)–(5) simplify and can be integrated for an arbitrary $V(\phi)$ (see Ref. 7). In our case we have

$$H^2(6\dot H/M^2+1)=m^2z^2\ ,$$
$$3H\dot z+m^2z=0\ . \quad (7)$$

It is clear that $\dot z<0$; thus, the scalar field may only decrease with time (we consider the region $z>0$). However, it follows from Eq. (7) that

$$\frac{dH^2}{dz}=\frac{M^2}{m^2z}(H^2-m^2z^2)\ . \quad (8)$$

Therefore, we have "superinflation" with $\dot H>0$ (following the terminology of Ref. 13) if $H<mz$ and usual subinflation with $\dot H<0$ in the opposite case. It should be mentioned that superinflationary regimes appeared already in the inflationary model proposed in Ref. 1 [they corresponded to the separatrix emerging to the right from the point (1,0) in Fig. 1 of Ref. 1. However, in that model they all terminate in a singularity with an infinite scalar factor and curvature and have no "graceful exit" to the region of low curvature. Thus, model (1) (and its generalization to the case of arbitrary power-law potential of the scalar field) presents the first example of an inflationary model having both super- and subinflationary regimes and possessing a smooth transition to the power-law Friedmann expansion [with $a(t)\propto t^{2/3}$, see below].

The solution of Eq. (7) has the following parametrical form where the role of the parameter is played by the scalar field itself ($M\neq m\sqrt{2}$):

$$H^2=H_1^2\left[\frac{z}{z_1}\right]^{M^2/m^2}-\frac{M^2z^2}{2-M^2/m^2}\left[1-\left[\frac{z}{z_1}\right]^{M^2/m^2-2}\right],$$
$$t-t_1=-3\int_{z_1}^{z}dz\frac{H(z)}{m^2z}\ , \quad (9)$$
$$\ln(a/a_1)=3\frac{z_1^2-z^2}{2}+3\frac{H_1^2-H^2}{M^2}\ .$$

Here a_1, H_1, and z_1 denote the values of a, H, and z at the moment $t=t_1$ when inflation begins ($H_1\gg M,m;z_1\gg 1$). For the special case $M=m\sqrt{2}$ the first equation in (9) should be replaced by

$$H^2=H_1^2\left[\frac{z}{z_1}\right]^2-M^2z^2\ln\left[\frac{z}{z_1}\right]\ . \quad (10)$$

As was pointed out in Refs. 7 and 10, the last expression in Eq. (9) looks like a sum of two independent contributions from scalar field inflation and R^2 inflation. However, this is a formal analogy only, because $H(t)$ and $z(t)$ are interrelated and cannot be considered as independent entities. For example, the second term in the expression for $\ln(a/a_1)$ is negative during superinflation but the first term makes the sum positive.

Let us now compare the lengths in $\ln a$ of the super- and subinflationary parts of the expansion of the Universe. Superinflation arises if $H_1<mz_1$. It ends at the moment $t=t_s$ when $\dot H_s=0$ and $H_s=mz_s$. It is straightforward to obtain from Eq. (9) that

$$z_s = z_1 \times \begin{cases} \left[\dfrac{M^2}{2m^2} + \dfrac{H_1^2}{m^2 z_1^2} \left[1 - \dfrac{M^2}{2m^2} \right] \right]^{1/(2-M^2/m^2)}, & M \neq m\sqrt{2}, \\ \exp\left[-\dfrac{1}{2} \left[1 - \dfrac{H_1^2}{m^2 z_1^2} \right] \right], & M = m\sqrt{2}. \end{cases} \quad (11)$$

For a given M/m, the quantity $I = \ln(a_s/a_1)/\ln(a_f/a_1)$ [where a_f is the scale factor at the end of inflation and $a_s = a(t_s)$] approaches its maximum

$$I_{\max} = 1 - \left[1 + \frac{2m^2}{M^2} \right] \left[\frac{M^2}{2m^2} \right]^{1/(1-M^2/2m^2)} \quad (12)$$

for $H_1 \to 0$. Actually, $I \simeq I_{\max}$ if

$$H_1 \ll z_1 \min(m, M) . \quad (13)$$

We remark that H grows significantly during superinflation [i.e., $H(t_s) \gg H_1$] only if inequality (13) is satisfied. Considered as a function of M/m, $I_{\max}$ has its maximum equal to $1-2/e \simeq 0.264$ at $M = m\sqrt{2}$, and it is small for both large and small values of M/m [$I_{\max} \simeq \gamma \ln(1/\gamma)$ for $\gamma = M^2/(2m^2) \ll 1$, $I_{\max} \simeq \ln\gamma/\gamma$ for $\gamma \gg 1$]. Thus, we come to the conclusions, first, that superinflation is always followed by subinflation in our model and, second, that the duration of superinflation (measured in terms of $\ln a$) is no more than 26% of the total duration of inflation.

III. RELAXATION TO THE DE SITTER (INFLATIONARY) STAGE

In this section we investigate the process of relaxation of the metric (2) from the initial conditions described in Sec. I to the de Sitter (inflationary) phase of its evolution analytically using some simplifying approximations. This will help us to understand the results of exact numerical solution of Eqs. (3) and (5) presented in Secs. IV and V.

First, let us assume that $H \gg M, m$ and that the scalar field energy density can be neglected at all. Then Eq. (3) simplifies to

$$2H\ddot{H} - \dot{H}^2 + 6H^2\dot{H} = 0 , \quad (14)$$

that corresponds to the case of the pure R^2 gravity. The first integral of Eq. (14) has the form

$$\dot{H} = 2\sqrt{H}(H_1^{3/2} - H^{3/2}), \quad H_1^{3/2} = \left| H_0^{3/2} + \frac{\dot{H}_0}{2\sqrt{H_0}} \right| . \quad (15)$$

Equation (15) gives the expression for evolution paths in the H-$\dot{H}$ plane. It can be easily integrated further to get the corresponding $a(t)$. We do not display it here because isotropic homogeneous solutions for pure fourth-order gravity have already been studied (see, e.g., Ref. 14).

The H-$\dot{H}$ diagram for this case is shown in Fig. 1. All solutions with $\dot{H}_0 \neq -2H_0^2$ terminate in the de Sitter stage of exponential expansion with $H = H_1$. Therefore, H_1 defined in Eq. (15) just coincides with H_1 introduced in the preceding section, and we can use the second of Eqs. (15) to obtain an approximate estimate of H at the beginning of inflation in the exact solution from initial data at $t = t_0$. The characteristic time of relaxation to the de Sitter stage is also of the order of H_1^{-1}. The de Sitter stage is reached without change of sign of $\dot{H}$ if $\dot{H}_0 > -2H_0^2$, and in the opposite case $a(t)$ passes through an inflection point where $H = \dot{H} = 0$.

The approximation (14) is not sufficient for the investigation of the slow evolution of H during inflation. So, as a next approximation, we assume that $H \gg m$ and $z = z_0 = \text{const} \gg 1$ but the scalar field energy density cannot be neglected. Then, instead of Eq. (3), we get

$$2H\ddot{H} - \dot{H}^2 + 6H^2\dot{H} = M^2(m^2 z_0^2 - H^2) . \quad (16)$$

This is the case of the $R + R^2$ gravity with a positive cosmological constant. In terms of H and $\dot{H}$,

$$\frac{d\dot{H}}{dH} = \frac{M^2(m^2 z_0^2 - H^2)}{2H\dot{H}} - 3H + \frac{\dot{H}}{2H} . \quad (17)$$

The phase diagram of this equation is presented in Fig. 2. The detailed structure near the point $(mz_0, 0)$ depends on M/mz_0; the point $(mz_0, 0)$ is a stable node with two negative eigenvalues $\lambda_{1,2} = -\frac{3}{2}mz_0 \pm (\frac{9}{4}m^2 z_0^2 - M^2)^{1/2}$ if $M < \frac{3}{2}mz_0$, and it is an attractive focus for $M > \frac{3}{2}mz_0$. All curves terminate in this point (i.e., in the pure de Sitter stage with $H = mz_0 = \text{const}$). However, before that they may pass either through the subinflationary stage with $\dot{H} = -M^2/6$ characteristic of R^2 inflation if H_1 defined according to Eq. (15) is much larger than mz_0 and

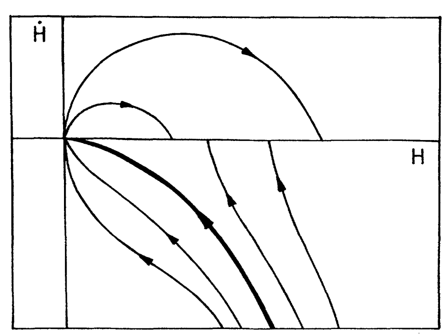

FIG. 1. Phase diagram of Eq. (14). The separatrix $\dot{H} = -2H^2$ [$a(t) \propto \sqrt{t}$] is shown by the heavy line.

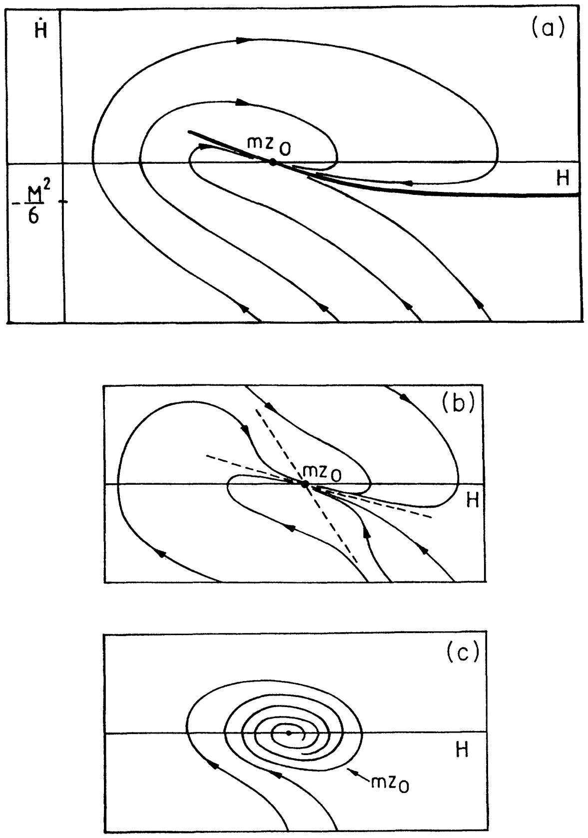

FIG. 2. (a) Phase diagram of Eq. (17). The inflationary asymptote $\dot{H}=M^2(m^2z_0^2/H^2-1)/6$ is shown by the heavy line. (b) Detailed structure near the point $(mz_0,0)$ in the case $M<\frac{3}{2}mz_0$. Separatrices $\dot{H}=\lambda_{1,2}(H-mz_0)$ are shown by dashed lines. (c) Structure near the point $(mz_0,0)$ for $M>\frac{3}{2}mz_0$.

M, or through a superinflationary stage with

$$\dot{H}=\frac{M^2m^2z_0^2}{6H^2}\ ,$$
$$H=(M^2m^2z_0^2t/2)^{1/3}\ , \tag{18}$$
$$a(t)\propto \exp(\text{const}\times t^{4/3})\ ,$$

if $Mmz_0 \ll H_1^2 \ll m^2z_0^2$ (thus, the latter regime can appear only if $M\ll mz_0$). It is worthwhile to note that Eq. (18) yields also the generic asymptote of expansion for the pure R^2 gravity with positive cosmological term. [That corresponds to the limit $mz_0\to\infty$ in Eq. (16).]

Furthermore, there may be a break of inflation between the subinflationary stage and the pure de Sitter stage with $H=mz_0$ driven by the scalar field if $M\gg mz_0$, and then we have double inflation. (This case was first considered in Ref. 15.) The superinflationary stage (18), if it exists at all, always smoothly matches the pure de Sitter stage without a break of inflation.

Finally, let us consider the case $m\gg H\gg M$. Then the scalar field oscillates with frequency $\omega=m$ (as in flat space-time), $z\propto a^{-3/2}\cos(mt+\text{const})$, and the dimensionless energy density $\dot{z}^2+m^2z^2=Ca^{-3}$, $C>0$. Neglecting the Einstein term, we obtain, from Eq. (3)

$$2H\ddot{H}-\dot{H}^2+6H^2\dot{H}=CM^2a^{-3}\ . \tag{19}$$

Thus, this approximation corresponds to the pure R^2 gravity with a source in the form of dustlike matter. Multiplying Eq. (19) by a^3 and differentiating, we get

$$\dot{H}\frac{d}{dH}(2H\ddot{H}-\dot{H}^2+6H^2\dot{H})+3H(2H\ddot{H}-\dot{H}^2+6H^2\dot{H})=0\ . \tag{20}$$

With the definitions $\gamma=\dot{H}/H^2$ and $\chi=\ln H$, Eq. (20) can be rewritten in the form

$$\gamma\frac{d^2\gamma}{d\chi^2}+\left[\frac{d\gamma}{d\chi}\right]^2+(7\gamma+6)\frac{d\gamma}{d\chi}+6\gamma^2+\tfrac{33}{2}\gamma+9=0\ . \tag{21}$$

This equation can be reduced to the first-order one by denoting $\delta=d\gamma/d\chi$ and changing the independent variable:

$$\frac{d\delta}{d\gamma}=-\frac{6\gamma^2+33\gamma/2+9}{\delta\gamma}-\frac{\delta+7\gamma+6}{\gamma}\ . \tag{22}$$

The phase diagram of this equation is given in Fig. 3. The region $\gamma(\delta+3+\frac{3}{2}\gamma)<0$ corresponding to $C<0$ is unphysical. For $C>0$, all curves go out of the point $(-2,0)$ [or $a(t)\propto\sqrt{t}$], tend to the point $(0,-\infty)$, and reemerge from the point $(0,+\infty)$ to the first quadrant [the latter jump corresponds to the regular transition of $a(t)$ through the point where $\dot{H}=0$]. In the end, all

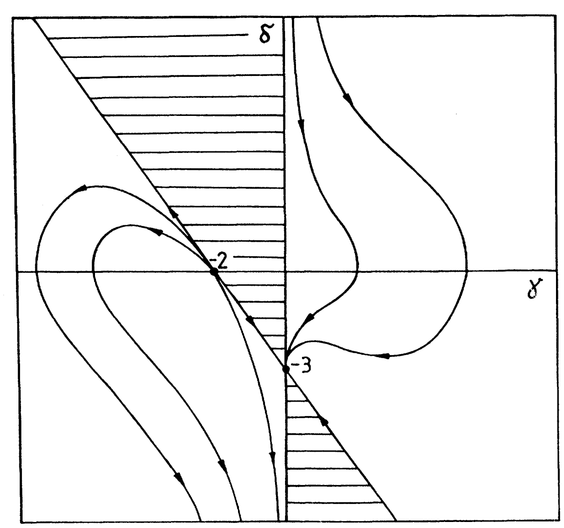

FIG. 3. Phase diagram of Eq. (22). The unphysical region $\gamma(\delta+3+\frac{3}{2}\gamma)<0$ is dashed. The curves lying on its boundary correspond to the pure R^2 gravity $(C=0)$. The point $(-2,0)$ is $a(t)\propto\sqrt{t}$; the point $(0,-3)$ is the de Sitter solution with an arbitrary H.

curves approach the point $(0,-3)$—the de Sitter solution with some constant H. In this case the de Sitter solution is approached from the side $\dot{H}>0$ for all curves in the H-$\dot{H}$ plane.

IV. HEAVY SCALAR PARTICLES

For heavy scalar particles the characteristic value of $\dot{H}$ for the scalar-field-driven inflation is much larger than that of the inflation driven by vacuum-polarization effects, i.e., $|\dot{H}_{S\text{-inf}}| \gg |\dot{H}_{V\text{-inf}}|$. First, let us consider an inflationary period driven by vacuum-polarization effects. It is characterized by the condition $H^2 \gg |\dot{H}_{V\text{-inf}}| \simeq M^2/6$. In that case Eq. (3) reduces to $H^2(1+6\dot{H}/M^2)\simeq m^2z^2$, which obviously can be satisfied only if $m^2z^2 \ll H^2$. From the equation of motion it follows that the scalar field decreases linearly during R^2 inflation. Note that the inflation can only start if the energy density of the scalar field ($\simeq m^2z^2$) is smaller than the Hubble parameter squared.

On the other hand a scalar-field-driven inflationary period would be characterized by the condition $H^2=m^2z^2 \gg |\dot{H}_{S\text{-inf}}|=m^2/3$, $\dot{z}^2 \ll m^2z^2$. In that case Eq. (3) would reduce to $H^2(1-2m^2/M^2)\simeq m^2z^2$ which obviously cannot be satisfied. Indeed, this case leads to a rapidly changing scalar field $\dot{z}^2=2/9M^2 \gg m^{-2}$. Therefore, for heavy scalar particles the vacuum-polarization effects prevent generally the typical pure scalar-field-driven inflation. Obviously, this means that in this case two separate inflationary stages are impossible. Of course, inflation driven by the combined action of the scalar field and the vacuum polarization as described in Sec. II can take place.

Let us now investigate numerically the behavior of the curves $\dot{H}(H)$ in the H-$\dot{H}$ plane. Generally, for the initial values under consideration the vacuum-polarization-driven inflationary stage $\dot{H}_{V\text{-inf}}=-M^2/6$ acts as an attractor for all $\dot{H}(H)$. The relaxation to the inflationary stage is already described in Sec. III. Figures 4 and 5 show examples for the direct relaxation to the R^2 inflation in the vacuum case and for the relaxation to a superinflationary stage arising because of the condition

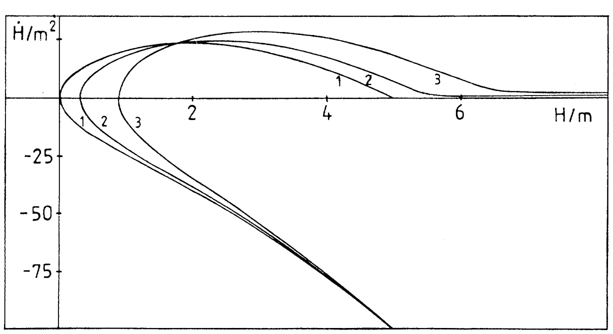

FIG. 4. H-$\dot{H}$ diagram in the case of heavy scalar particles $m/M=3.7$ with fixed initial values $H_0=5m$ and $\dot{H}_0=-100m^2$, vacuum (curve 1), $z_0=50$ (curve 2), $z_0=100$ (curve 3); curves 2 and 3 show superinflation.

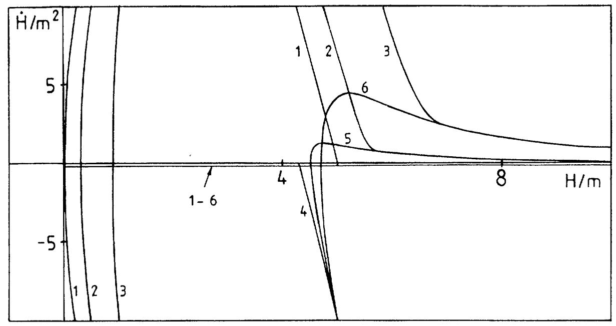

FIG. 5. H-$\dot{H}$ diagram in the case of heavy scalar particles $m/M=3.7$; initial values: curves 1–3 the same as Fig. 4 and $H_0=5m$, $\dot{H}_0=-10m^2$, vacuum (curve 4), $z_0=50$ (curve 5), $z_0=100$ (curve 6); curves 2, 3, 5, and 6 show superinflation.

$mz_0 \gg H_0$. Then the preinflationary stage is quite similar to that shown in Fig. 1. In particularly, Eq. (15) gives an estimate of H_1: $H_1=5m$ for Fig. 4 and $H_1\simeq 4.3m$ for the curves 4–6 in Fig. 5. Also, Eq. (11) can be used to estimate $H_s=mz_s$ at the turnover point (e.g., at the end of superinflation). For Figs. 4 and 5 that gives $H_s/m\simeq 12$ for $z_0=50$ and $H_s/m\simeq 22$ for $z_0=100$. The agreement between the simplified description of the superinflationary regime given by Eq. (18) and the results of numerical calculations is also reasonable.

Note that during superinflation z is not constant now [as compared to the approximation (16)]. Also $\dot{H}$ does not remain constant but $\dot{H}\simeq -M^2(1-m^2z^2/H^2)/6$ is a slowly decreasing function. Contrary to the vacuum models with negative coupling constant α (i.e., tachyonic gravitons), this leads to a natural end of the superinflationary stage [cosmological vacuum models corresponding to the Lagrangian (1) and $\alpha<0$ have been considered in Ref. 16. For $z_1 \gg H_1/M$ a long superinflationary stage takes place. It can be described approximately by $\dot{H}=M^2m^2z_1^2/(6H^2)-m^2/3$, from which one obtains the maximum $H_s\simeq Mz_1/\sqrt{2}$. At the turnover point of trajectories in the H-$\dot{H}$ plane, the scalar field and the Hubble parameter are related by $z_s\simeq H_s/m$. Therefore, during superinflation the scalar field decreases by the amount $z_s/z_1\simeq M/(\sqrt{2}m)$. The solution remains near H_s until z/z_1 falls down to a small value $(M^2/m^2)\ln(z_1/z)\simeq 1$. After that, H begins to decrease and the solution quickly approaches the inflationary regime driven by the R^2 term: $\dot{H}_{V\text{-inf}}\simeq -M^2/6$. This is shown in Fig. 5.

V. LIGHT SCALAR PARTICLES

For light scalar particles the characteristic value of $\dot{H}$ for the scalar-field-driven inflation is much smaller than that of the inflation driven by vacuum-polarization effects, i.e., $|\dot{H}_{S\text{-inf}}| \ll |\dot{H}_{V\text{-inf}}|$. First let us consider a scalar-field-driven inflationary period described by $H^2=m^2z^2 \gg |\dot{H}_{S\text{-inf}}|=m^2/3$, $\dot{z}^2 \ll m^2z^2$. Equation (3) reduces to $H^2(1-2m^2/M^2)\simeq m^2z^2$, which can be satisfied, in principle, because $m^2/M^2 \ll 1$. That means that scalar-field-driven inflation can start if H and mz have the same magnitude and are large in comparison to m.

On the other hand, in the case of inflation driven by vacuum-polarization effects ($\dot{H}_{V\text{-inf}}=-M^2/6$) Eq. (3) reduces to $H^2(1+6\dot{H}/M^2)\simeq m^2z^2$, which can be satisfied if $m^2z^2 \ll H^2$. Since the scalar field with $\dot{z}^2=m^4z^2/(9H^2)\ll m^2/9$ decreases during this stage much more slowly than the Hubble parameter the inflation ends if H and mz are of the same magnitude. At this moment a transition to scalar-field-driven inflation occurs.

Let us first consider the case $mz_0 \gg H_0$ where vacuum-polarization effects initially dominate classical gravity. Then relaxation to superinflation takes place. This behavior is quite similar to that described in the preceding section (see. Figs. 4 and 5, curves 2, 3, 5, and 6). During this superinflationary stage H increases and z decreases, so that the superinflation ends if $H \simeq mz$. At this moment $\dot{H}(H)$ turns into the lower half plane and scalar-field-driven inflation starts.

Let us now investigate the opposite case if $mz_0 \ll H_0$ where vacuum-polarization effects initially dominate over matter. As discussed in Sec. III the further evolution of the model depends crucially on whether the value of $\dot{H}_0$ is greater or smaller than $-2H_0^2$. This follows from the fact that the radiation-dominated universe ($\dot{H}=-2H^2$) is an exact solution in $R+R^2$ gravity, and $\dot{H}=-2H^2$ is also an exact vacuum solution in pure fourth-order gravity. If $|\dot{H}_0|<2H_0^2$ the curve $\dot{H}(H)$ tends directly to the R^2 inflation, during which H and z decrease and get the same magnitude as argued above, so that a transition to scalar-field-driven inflation occurs (Fig. 6, curves 1–4).

If $|\dot{H}_0|>2H_0^2$, the curve turns into the domain of accelerated expansion and at H_1 it returns to R^2 inflation in the lower half plane. This behavior is described by Eqs. (14) and (15). Indeed, the scalar field remains constant and small. In the vacuum case the maximum value H_1 is reached, and then the curve goes over into R^2 inflation (Fig. 7, curve 1). If the initially scalar field mz_0 is less than the maximum Hubble parameter H_1 defined by the vacuum model then the curve $\dot{H}(H)$ turns directly to the lower half plane, i.e., $H_s=H_1$. During the following R^2

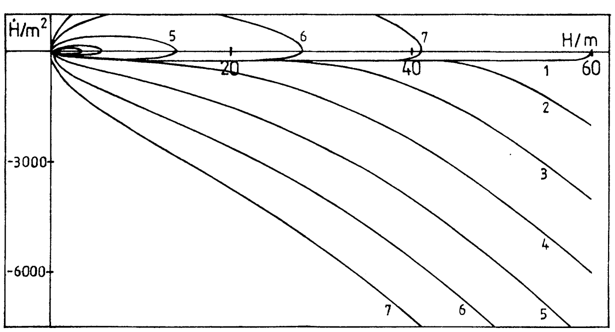

FIG. 6. H-$\dot{H}$ diagram in the case of light scalar particles $m/M=0.024$ for fixed initial values $H_0=60m$ and $z_0=2$, $\dot{H}_0=0$ (curve 1), $\dot{H}_0=-2000m^2$ (curve 2), $\dot{H}_0=-4000m^2$ (curve 3), $\dot{H}_0=-6000m^2$ (curve 4), $\dot{H}_0=-8000m^2$ (curve 5), $\dot{H}_0=-10\,000m^2$ (curve 6), $\dot{H}_0=-12\,000m^2$ (curve 7).

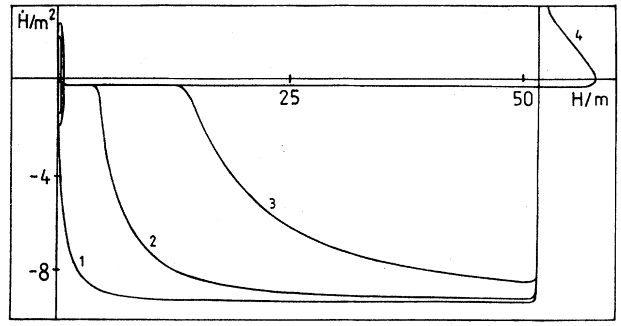

FIG. 7. H-$\dot{H}$ diagram in the case of light scalar particles $m/M=0.13$ for fixed initial values $H_0=25m$ and $\dot{H}_0=-5000m^2$, vacuum (curve 1), $z_0=5$ (curve 2), $z_0=15$ (curve 3), $z_0=60$ (curve 4); curves and 2 and 3 show double inflation with smooth transition; curve 4 shows the transition from superinflation to scalar-field-driven inflation.

inflation H and mz decrease and become of the same order of magnitude (Fig. 7, curves 2 and 3). On the other hand, if mz_0 is greater than H_1, a period of superinflation occurs during which H increases and mz decreases until they have the same magnitude at H_s. Then the curve turns to the lower half plane and ordinary scalar-field-driven inflation starts. Therefore, $H_s>H_1$, where H_1 is given by Eq. (15) (Fig. 7, curve 4; cf. also Fig. 5, curves 2, 3, 5, and 6). The question of whether or not two consecutive inflationary stages exist depends on the ratio of z_0 to H_1 and, therefore, it depends indirectly on the initial values H_0 and $\dot{H}_0$.

Finally let us compare these results with the behavior of singularity-free models.[6] In these closed models Planck-sized initial values were assumed for the scale factor and the energy density of the scalar field. Therefore, $H_1\simeq 1/a_0$ and the scalar field has approximately the magnitude of the Hubble parameter (i.e., $H_1\simeq mz_0=H_s$, the condition valid in the Einsteinian case) so that scalar-field-driven inflation starts. Consequently, in such singularity-free models there do not exist two consecutive inflationary stages. In general, also in singularity-free cosmological models double inflation can be the source of a nonflat perturbation spectrum.[17]

VI. THE TRANSITION FROM R^2 INFLATION TO SCALAR-FIELD-DRIVEN INFLATION

As already mentioned above the R^2 inflation acts as an attractor for most of the solutions. We shall now consider in more detail the end of this inflationary stage. In the vacuum case R^2 inflation is characterized by the condition $\dot{H}_{V\text{-inf}}=-M^2/6$ and it ends if $H^2\simeq|\dot{H}_{V\text{-inf}}|$. At this moment oscillations of the type $H\propto M[1-\sin(Mt)]/(6+3Mt/2)$ occur, where $\dot{H}(t)$ passes the origin. From Eq. (3) one can see that the existence of a coherent scalar field leads to an earlier breaking off of the inflationary stage. Namely if $m^2z^2>|\dot{H}_{V\text{-inf}}|$ the inflation already breaks off if $H\simeq mz$ (cf. Fig. 7; note that $z\simeq z_0$). In this case the transition to the scalar-field-driven inflation occurs smoothly. On the other hand if $m^2z^2 < |\dot{H}_{V\text{-inf}}|$ the transition is so rapid that oscillations around the scalar-field-driven inflationary stage occur (Figs. 8 and 9). From Eqs. (3) and (4) one finds

$$\dot{H} + (1/M^2)(H^{(3)} + 3H\ddot{H} + 6\dot{H}^2) = -3\dot{z}^2 . \tag{23}$$

During the scalar-field-driven inflationary stage one has $\dot{H}_{S\text{-inf}} = -3\dot{z}^3$ and with $\dot{H} = \dot{H}_{S\text{-inf}} + y$ Eq. (23) reduces to an equation for damped oscillations of the quantity y:

$$\ddot{y} + 3H\dot{y} + M^2 y = 0 , \tag{24}$$

where the term $6(y + \dot{H}_{S\text{-inf}})^2$ has been neglected in comparison with M^2y, and H can be treated in first approximation as a constant of the order z_0. Therefore, if initially the scalar field is small ($z_0 \ll M^2/m^2$), during the transition damped oscillations around the scalar-field-driven inflation occur. The oscillations are similar to those at the end of the inflation driven by vacuum-polarization effects in the vacuum model, but in our case after these oscillations there follows a long period of inflation only if $z_0 \gg 1$. During the second inflation the length scales of all perturbations which may originate during this transition increase. In the case of single inflation, perturbations from the reheating period correspond now to characteristic lengths of a few meters. In the case considered here, perturbations stemming from the transition between two inflationary stages would acquire much larger scales due to the second inflation; i.e., they may influence the perturbation spectrum responsible for galaxy formation.

If on the other hand $z_0^2 \gg M^2/m^2$ then y is aperiodical and the transition is smooth (Fig. 7, curves 2 and 3). In this case during the transition from one inflationary stage to another, no additional perturbations can be expected, but one should see only the break between the two flat spectra coming from the different inflationary stages (cf. Fig. 12).

Finally, it should be mentioned that the small oscillations of a and R considered in this section decay exponentially with a characteristic time $\tau \sim (gM^3)^{-1}$ when the effect of creation of matter-field quanta by these oscillations is taken into account (see Refs. 1 and 18–20).

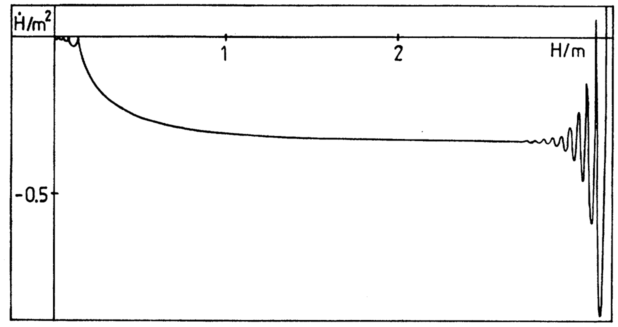

FIG. 9. The same as Fig. 8 but the inner part of the curve. Since $z_0 = 3.5$ the scalar field inflation is only short, and at the end oscillations start again, but here because of scalar field oscillations at the transition to the Friedmann stage.

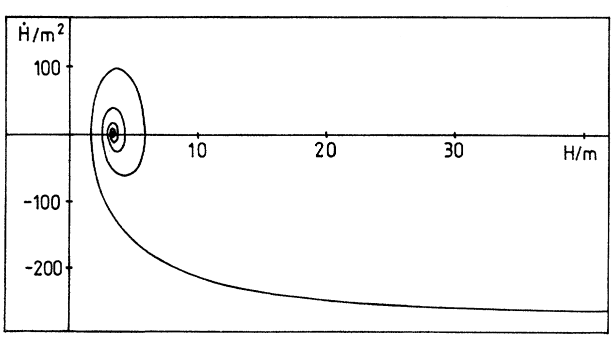

FIG. 8. H-$\dot{H}$ diagram in the case of light scalar particles $m/M = 0.024$ for a wide class of initial values H_0 and $\dot{H}_0$ leading to R^2 inflation and $z_0 = 3.5$; the curve shows the transition from R^2 inflation to scalar-field-driven inflation with damped oscillations.

VII. PERTURBATIONS

In the inflationary models adiabatic density perturbations arise from quantum fluctuations during the de Sitter stage. The first quantitatively correct expressions for the adiabatic perturbations generated at the inflationary stage were presented in Refs. 21–23 for scalar-field-driven inflation and in Ref. 11 for inflation driven by vacuum polarization. For further consideration of adiabatic perturbations in the $(R + R^2)$ theory see Refs. 24–26.

We shall now take into account small scalar perturbations of the background metric (1) in the notation used in Ref. 25:

$$ds^2 = (1 + 2\Phi)dt^2 - a^2(t)(1 - 2\Psi)\delta_{ik}dx^i dx^k , \tag{25}$$

where the invariant potentials are the same, up to the sign, as those given by Bardeen[27] ($\Phi = \Phi_A, \Psi = -\Phi_H$). We denote the perturbations of the scalar field by $\delta z = z - z^{(0)}$, and the perturbations of the Ricci scalar by $\delta R = R - R^{(0)}$, where $z^{(0)}$ and $R^{(0)}$ are the solutions of the background equations discussed in the preceding sections. We decompose the perturbations and the potentials in plane waves [$\delta z = \delta z_k \exp(ikr)$, etc.]. By straightforward linearization (cf. Ref. 25 for the case of single inflation) of the trace equation

$$\Box R + M^2 R = 6M^2(z_{,\alpha}z^{,\alpha} - 2m^2z^2) , \tag{26}$$

one obtains

$$\delta\ddot{R} + 3H\delta\dot{R} + (k^2/a^2 + M^2)\delta R$$
$$= (2\Phi M^2)[-R + 6(\dot{z}^2 - 2m^2z^2)] + (\dot{\Phi} + 3\dot{\Psi})\dot{R}$$
$$+ 12M^2(\dot{z}\delta\dot{z} - \dot{z}^2\Phi - 2m^2 z\delta z) , \tag{27}$$

where the indices for the k modes of the perturbations are omitted and δR is given by

$$\delta R = 6\ddot{\Psi} + 2(k^2/a^2)(2\Psi - \Phi) + 12(\dot{H} + 2H^2)\Phi + 6H\dot{\Phi} + 24H\dot{\Psi} \ . \tag{28}$$

From the $(i \neq k)$ component of the field equation one has a connection between the two potentials Φ and Ψ:

$$\Phi = \Psi - \delta R / (R - 3M^2) \ . \tag{29}$$

The equation of motion of the scalar field leads to

$$\delta\ddot{z} + 3H\delta\dot{z} + (k^2/a^2 + m^2)\delta z = (\dot{\Phi} + 3\dot{\Psi})\dot{z} - 2m^2 z\Phi \ . \tag{30}$$

From the $0i$ component one obtains additionally a first integral

$$(1 - R/3M^2)(\dot{\Psi} + H\Phi) + (\delta\dot{R} - \Phi\dot{R} - H\delta R)/6M^2 = 3\dot{z}\delta z \ . \tag{31}$$

We are especially interested in background models with two consecutive inflationary stages, i.e., in a transition from typical R^2 inflation to scalar field inflation, which was described in Secs. V and VI. Therefore, in this section we generally assumed $m < M$ and initial conditions which lead at first to R^2 inflation ($H_0^2 \gg |\dot{H}_0| = |\dot{H}_{V\text{-inf}}|$, $mz_0 \ll H_0$).

To fix the initial values of the perturbations let us consider the perturbation equations (27)–(30) in more detail. With the relations (29) and (31), Eq. (27) can be rewritten as a fourth-order equation for the potential Φ with one first integral (31). To solve it together with the second-order perturbation equation of the scalar field, one has to fix five initial values.

On the other hand the inevitable initial quantum fluctuations of the scalar quantities δR and $\delta\phi$ at the quasi de Sitter stage provide the initial values for the perturbations. To this end let us consider the quantum fluctuations of a scalar quantity f in the de Sitter space which satisfy the equation

$$(\Box + \mu^2) f = 0 \ , \tag{32}$$

where μ denotes the mass term. Decomposing f into modes

$$f = (2\pi)^{-3/2} \int d^3k [\hat{a}_k f_k(\eta) e^{-ikr} + \hat{a}_k^\dagger f_k^*(\eta) e^{ikr}] \ , \tag{33}$$

where $\hat{a}_k^\dagger$ and $\hat{a}_k$ are the usual creation and annihilation operators and η is the conformal time $d\eta = dt/a(t)$, one finds the solution of the k mode:

$$f_k = -\tfrac{1}{2}(\pi\eta)^{1/2} H\eta H_\nu^{(2)}(k\eta) \ . \tag{34}$$

The index of the Hankel function $H_\nu^{(2)}$ is $\nu^2 = \frac{9}{4} - \mu/H^2$, where H is the constant Hubble parameter in de Sitter space. The short- and long-wave limits of Eq. (34) are

$$f_k = (2k)^{-1/2} H\eta e^{-ik\eta}, \quad k\eta \to \infty \ , \tag{35}$$

$$f_k = Hk^{-1}(2k)^{-1/2}, \quad k\eta \to 0 \ , \tag{36}$$

respectively, where additionally $\mu^2 \ll H^2$, i.e., $\nu = \frac{3}{2}$ is assumed. These considerations remain valid in the quasi de Sitter stage of the inflationary cosmology, where H is a slowly decreasing function. The limits (35) and (36) correspond to the two limiting cases of a physical wavelength much smaller than the horizon ($\lambda \ll H^{-1}$) and much larger than the horizon ($\lambda \gg H^{-1}$). Since $\lambda = a/k$ is an exponentially increasing function the transition from one limit to the other is very rapid.

Let us now return to the problem of the initial values of the perturbations δz of the scalar field and δR of the Ricci scalar. Assuming that they arise from quantum fluctuations one obtains in the short-wave limit damped oscillations with amplitudes

$$\sqrt{4\pi G/6k}\ /a$$

for δz and

$$\sqrt{192\pi G(1 - 3M^2/R)/2k}\, MH/a$$

for δR. In the short-wave limit Eqs. (27) and (30) decouple. Therefore, the phases are arbitrary. There remains an additional initial condition. From Eq. (29) one can see that the potentials Φ and Ψ oscillate in the same way as δR. Then in the short-wave limit from Eq. (28) it follows $\Phi = -\Psi$; otherwise δR would be damped proportional a^{-2} in contradiction to our assumption. That means the perturbations are conformally flat. Of course, in the linear perturbation equations all perturbations could be multiplied by a constant factor without changing the results, but the relation $\delta z_0/\delta R_0$ may influence the further evolution.

Let us first consider adiabatic perturbations generated during the inflationary regime (9) generated by the mutual action of the scalar field and the R^2 term with no break of inflation. For this case, the result was already presented in Ref. 10:

$$k^{3/2} h_k = \sqrt{24\pi G}\, H(c_1 z + c_2 H/M) \ , \tag{37}$$

where $h(\mathbf{k}) = (2\pi)^{-3/2} \int h(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}} d^3x$, $k = |\mathbf{k}|$, c_1 and c_2 are two independent Gaussian random variables with zero average and unit dispersion, and the quantities z and H in the right-hand side are taken at the moment of the first horizon crossing ($k = aH$) at the inflationary stage. The quantity $h(\mathbf{r})$ is the metric perturbation characterizing a growing adiabatic mode in the "ultrasynchroneous" gauge where

$$ds^2 = dt^2 - a^2(t)[1 + h(\mathbf{r})]\delta_{ik} dx^i dx^k \tag{38}$$

for $k \ll aH$ irrespective of the structure of the matter energy-momentum tensor. The relation of h to the quantities Φ and Ψ in the longitudinal gauge is given by the expression

$$\Phi = -\Psi = -\tfrac{1}{2}\left[1 - \frac{\dot{a}}{a^2}\int_0^t a\, dt\right] h \ , \tag{39}$$

valid whenever matter pressure perturbations are diagonal [$\delta p_{ik} \propto \delta_{ik}$ and $v_s k/(aH) \ll 1, v_s^2 = dp/d\epsilon$]. For $p = 0$ and $a(t) \propto t^{2/3}$, $\Phi = -\Psi = -\frac{3}{10}h$.

Let $\sigma(k)$ denote the rms value of $k^{3/2} h_k$. Then

$$\sigma^2=24\pi GH^2(z^2+H^2/M^2)|_{k=aH} . \tag{40}$$

Using Eqs. (8) and (9), we obtain

$$\frac{d\ln\sigma}{d\ln k}=\frac{d\ln\sigma}{d\ln a}\left[1+\frac{d\ln H}{d\ln a}\right]^{-1}\simeq\frac{1}{2\sigma^2}\frac{d\sigma^2}{dz^2}\frac{dz^2}{d\ln a}$$
$$=-\frac{H^2+\frac{1}{2}M^2z^2(1-m^2z^2/H^2)}{3H^2(z^2+H^2/M^2)} . \tag{41}$$

It follows from the condition $|\dot{H}|\ll H^2$ at the stage (7)–(9) that $|d\ln\sigma/d\ln k|\ll 1$. Thus, the spectrum of adiabatic perturbations is approximately flat (or, of the Harrison-Zeldovich type). However, in contrast with the cases of inflation driven solely by the scalar field or the R^2 term, it is not monotoneous. $d\ln\sigma/d\ln k$ is always positive at the subinflationary stage (in particular, at $t=t_s$ when $H_s=mz_s$ and at the end of inflation), but it may be negative during a part of the superinflationary stage if $H_1^2/z_1^2+\frac{1}{2}M^2(1-m^2z_1^2/H_1^2)<0$.

If $k=k_m$ is the point where $\sigma(k)$ has a maximum and a_m, H_m, and z_m are the values of a, H, and z at the moment $t=t_m$ when $k_m=a_mH_m$, then

$$\frac{H_m^2}{z_m^2}=\frac{1}{4}M^2(-1+\sqrt{1+8m^2/M^2}) . \tag{42}$$

It follows from Eq. (9) that

$$z_m=z_1\times\begin{cases}\left[\dfrac{1+\frac{1}{4}(2-M^2/m^2)(\sqrt{1+8m^2/M^2}-1)}{1+(2-M^2/m^2)H_1^2/(M^2z_1^2)}\right]^{-1/(2-M^2/m^2)} , & M\neq m\sqrt{2},\\ \exp[-\frac{1}{4}(\sqrt{5}-1)+H_1^2/(M^2z_1^2)], & M=m\sqrt{2}.\end{cases} \tag{43}$$

For a given M/m, the quantity $Q=\ln(a_m/a_1)/\ln(a_f/a_1)$ approaches its maximum

$$Q_{\max}=\begin{cases}1-\dfrac{1+\sqrt{1+8m^2/M^2}}{2}\left[1+\dfrac{(2-M^2/m^2)(\sqrt{1+8m^2/M^2}-1)}{4}\right]^{-1/(1-M^2/2m^2)} , & M\neq m\sqrt{2},\\ 1-\frac{1}{2}(\sqrt{5}+1)\exp[-\frac{1}{2}(\sqrt{5}-1)]\simeq 0.128, & M=m\sqrt{2},\end{cases} \tag{44}$$

for $H_1\to 0$ ($H_1\ll Mz_1$). Considered as a function of M/m, $Q_{\max}$ is maximal and equal to 0.134 at $M\simeq 1.98m$. Thus, the scale $\lambda_m=a/k_m$ is far beyond the present-day cosmological horizon if $\ln(a_f/a_1)>80$.

Now we turn to the case of double inflation with the power-law intermediate period between the two stages of inflation described in Sec. VI. Let us consider a perturbation leaving the horizon during the first inflationary stage. After crossing the horizon it remains at a nearly constant value. These values have a Gaussian distribution with $\langle\delta\phi^2\rangle=H^2/2k^3$ and $\langle\delta R^2\rangle=192\pi GM^2H^4/2k^3$.[11] This leads to the well-known quasiflat spectrum of perturbations coming from the first inflationary stage (and also for perturbations leaving the horizon during the second inflationary stage). Note that the phase between the two perturbations δR and δz also influences the result. A phase shift of $\pi/2$ ($\delta\dot{R}_0=0,\delta z_0=0$) gives a perturbation amplitude half as large as in the case $\delta\dot{R}_0=\delta\dot{z}_0=0$. An interesting effect comes into play if one considers perturbations in the short-wave limit during the transition period, i.e., perturbations with wavelengths smaller than the horizon at that time. Then in Eq. (27) the right-hand side cannot be neglected, but it acts as a source amplifying the oscillations of δR. This amplification concerns only the behavior of the δR perturbations (and consequently the behavior of the potentials $\Phi=-\Psi$) and not the scalar field perturbations δz which are damped also during the transition period up to crossing the horizon. Figure 10 shows the behavior of the scalar perturbations of the metric during this period. Already before leaving the horizon the relation $\Phi=-\Psi$ is not valid.

To obtain a spectrum of the perturbations crossing the horizon during the first and second inflation and during the transition period between them we have chosen parameters ($m/M=0.032, z_0=6.2$) which lead to a long transition period with oscillations of the Hubble parameter, as was demonstrated in Figs. 8 and 9. The perturba-

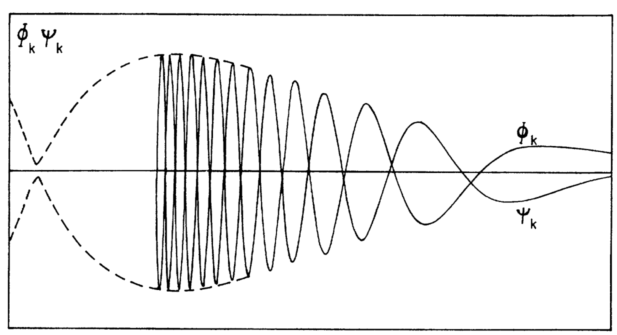

FIG. 10. The behavior of the k modes of the scalar perturbation of the metric $\Phi(t)$ and $\Psi(t)$ during the transition from R^2 inflation to scalar field inflation (for a wavelength within the horizon).

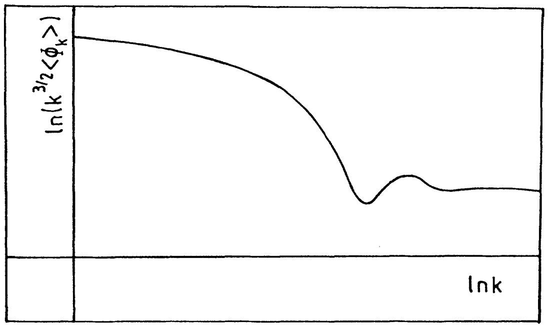

FIG. 11. Spectrum of the scalar perturbation at the end of the inflationary stage for wave numbers k corresponding to horizon crossing between 60 and 50 e-foldings before the end of inflation ($m/M=0.032$).

tions δR and δz are in phase ($\delta\dot{R}_0=\delta\dot{z}_0=0$) and their magnitudes are $\delta R_0/\delta z_0 \simeq 12M$. Figures 11 and 12 show the logarithm of the rms value of the potential multiplied by $k^{3/2}$ as a function of the logarithm of the wave number for two different values of the coupling parameter α, i.e., for different masses M. The parameter α adopted in Fig. 11 corresponds to a transition between both inflationary stages accompanied by rapid oscillations of the Hubble parameter, whereas Fig. 12 corresponds to a smooth transition between the inflationary stages. Between the characteristic almost flat parts of the spectrum corresponding to horizon crossing within one of the inflationary stages one sees a smooth transition from the larger perturbations for long wavelengths to the smaller ones for short wavelengths. In the case described in Fig. 11 this transition shows characteristic oscillations for perturbations crossing the horizon at the end of the transition period.

VIII. CONCLUSIONS

We investigated the evolution of the isotropic cosmological model and small scalar perturbations on its background in the $(R+R^2)$ theory of gravity with a massive scalar field. This combined model possesses the intermediate quasi de Sitter (inflationary) asymptote (9) that has an attractor property for a nonzero measure of all solutions. The characteristic relaxation time to this de Sitter stage is of the order of H_1^{-1}, the inverse Hubble parameter at its beginning. The de Sitter (inflationary) regime may begin with the superinflationary expansion ($\dot{H}>0$) necessarily followed by the subinflationary one ($\dot{H}<0$). The existence of double inflation, i.e., two separate de Sitter stages with an intermediate period of power-law expansion $a(t)\propto t^{2/3}$ with small superimposed oscillations, is possible, too, if $m \ll M$.

The spectrum of adiabatic metric perturbations generated from quantum fluctuations of the scalar field ϕ and the scalar curvature R is quasiflat: $|d\ln|k^{3/2}h_k|/d\ln k| \ll 1$ (i.e., of the Harrison-Zeldovich type) in the case of a single inflationary stage. The quantity $d\ln|k^{3/2}h_k|/d\ln k$ is always negative for sufficiently small wavelengths, but it may change the sign for those large wavelengths which first crossed the horizon during the superinflationary stage. However, both the superinflationary region ($\dot{H}>0$ at the first horizon crossing) and the region where $d\ln|k^{3/2}h_k|/d\ln k>0$ typically correspond to scales much exceeding the present-day cosmological horizon. In the most interesting case of double inflation, the perturbation spectrum has a characteristic step (with increase of the amplitude to the direction of large λ). Small superimposed oscillations in the spectrum appear in this case, too. In general, nonflat fluctuation spectra can result from the combined action of different scalar fields with complicated potentials,[28] but also double inflation in sixth-order gravity[29] may be a source of nonflat spectra. A spectrum with more power on small k values could explain why one observes in the Universe on large scales more structure than expected for the scale-invariant Zeldovich spectrum. Indeed, some recent observational results as the great attractor[30] or the cluster-cluster correlation function[31,32] indicate extra power in the spectrum on large scales.

Let us now discuss the question of whether it is possible to obtain "naturally" the break in the spectrum at the right place (e.g., at about 100 Mpc) and with right amplitudes on small and large scales in our model. As already mentioned in our model the place of the break depends only on the initial value of the scalar field. Thus, we have to choose

$$\frac{\phi_1^2}{m_{\rm Pl}^2} \simeq \frac{1}{2\pi}\ln 10^{27} \simeq 10, \quad z_1^2 \simeq 42 , \qquad (45)$$

where $m_{\rm Pl}=G^{-1}$. The rather natural way to produce such a value of the scalar field is the following. Let us use the idea first proposed in Ref. 7 and assume that $\phi=z=0$ initially. Then, Gaussian quantum fluctuations of the scalar field are generated during the first part of inflation (that is driven solely by the R^2 term in this case). The rms value of z at the end of the first part of inflation depends on H_1—the value of H at the beginning of inflation. We shall find, below, what value of H_1 is necessary to obtain the correct typical value of z_1 given in Eq. (45). Note that in Ref. 33 a low probability of double inflation and an extreme fine-tuning of parameters is de-

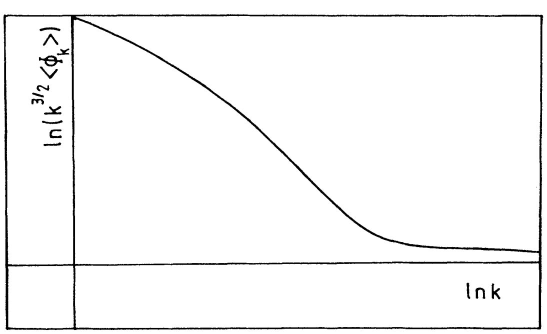

FIG. 12. The same as Fig. 11 but with $m/M=0.078$. (Note that the scale of the ordinate is different from that in Fig. 11.)

rived. But these results depend crucially upon the assumption that the first inflationary stage starts at the Planck density.

In the one-loop approximation the dispersion of the fluctuations of the scalar field $u=\langle z^2\rangle$ is given by (see, e.g., Ref. 22)

$$\frac{du}{d\ln a}=\frac{GH^2}{3\pi}-\frac{2}{3}\frac{m^2u}{H^2}, \quad H^2=H_1^2-\tfrac{1}{3}M^2\ln a \ . \quad (46)$$

For light scalar particles $m \ll M$ one finds

$$u\simeq\frac{G}{2M^2}(H_1^4-H^4)\to\frac{H_1^4}{2M^2m_{\rm Pl}^2} \ . \quad (47)$$

The limit is valid for $H \ll H_1$; note that Eq. (47) contains M^2 instead of m^2 as it would in the case of an equilibrium ("thermal") state of a massive scalar field in the de Sitter background.

To have the right amplitude at the scale of 10 Mpc one has to choose massive scalar particles with $m\simeq 10^{-6}m_{\rm Pl}$. The ratio Δ of the amplitude of the perturbations spectrum at small and large scales is given by

$$\Delta=\frac{M}{mz_1}\simeq\frac{M}{6.5m} \ . \quad (48)$$

Any spectral distortions of the primordial fluctuation spectrum are strongly restricted by the observed high isotropy of the cosmic background radiation. In our case, from the large-scale $\Delta T/T$ measurements one finds $\Delta<5$. Taking $\Delta=3$, this implies $M\simeq 20m\simeq 2\times 20^{14}$ GeV. Thus, $H_1\simeq 0.014m_{\rm Pl}$. The fact that H_1 appears to be significantly less than the Planck value explains why our conclusions upon the viability of double inflation differ essentially from Ref. 33. Such a value of H_1 might not be unnatural if an exact self-consistent de Sitter stage with this curvature exists (e.g., in the spirit of Ref. 1).

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