Interference effects in the Schwinger pair-production mechanism

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Interference effects in pair production by external fields via the Schwinger mechanism are investigated. Electric fields consisting of two time-separated pulses are shown to be capable of producing resonant behavior in the spectra of produced particles as a result of the interference between the pair-production amplitudes of the two individual pulses. No *a priori* assumptions about the length of the time interval between pulses relative to the individual pulse durations are made; the interference feature is seen to exist even when the interval is comparable to the pulse widths. Several numerical examples are examined.

I. INTRODUCTION

Pair production from a static electric field was originally studied by Sauter, who carried out the calculation of vacuum tunneling.[1] An explicit expression for the pair-production rate was obtained from the imaginary part of the effective photon Lagrangian by Schwinger.[2] He determined that the rate could not be represented by an analytic function of the electric field, thus explicitly displaying the nonperturbative nature of the problem. This problem has since been investigated through various nonperturbative techniques by numerous authors.[3–6] The Schwinger pair-production mechanism and its extensions continue to play a very important role in a number of physical situations of current interest. Examples of such situations are electron-positron pair production in the low-energy (near Coulomb barrier) collisions of heavy ions,[7] measurements of beam luminosity and determinations of beam stability in a relativistic heavy-ion collider,[8] energy deposition and quark-gluon-plasma formation in ultrarelativistic heavy-ion collisions,[9] and particle production in Friedmann-Robertson-Walker expanding cosmologies.[10]

In this article we want to focus on one particular aspect of pair production in background fields, namely, interference effects which can lead to sharp resonances in the spectra of the produced pairs. Cornwall and Tiktopoulos have shown that[3] strong classical electric fields with certain kinds of space-time variation can create fermion pairs which are strongly resonant in energy and momentum. By explicitly solving the Dirac equation for a step-function vector field, they have demonstrated that rotation by an odd multiple of π of a free-particle solution to a free-antiparticle solution (and vice versa) yields the desired resonant behavior. For more general time variations of the electric field, they work the problem "backwards," i.e., by finding a smooth rotation of antiparticle to particle and constructing from that rotation the required smooth vector potential. In this article we wish to provide a prescription for solving the problem "forwards," i.e., starting from the vector potential. A qualitative discussion of how a resonant behavior in the number density as a function of energy might arise is given in Sec. II. In Sec. III such a resonant behavior is quantitatively demonstrated by numerically evaluating the time-evolution operator of the system. Section IV contains a brief discussion of our results and conclusions.

II. RESONANT BEHAVIOR IN PAIR PRODUCTION BY A TIME-DEPENDENT, HOMOGENEOUS FIELD

In this section we wish to illustrate resonant behavior in pair production by a time-dependent, homogeneous electric field $E(t)$, which is fixed in direction. We first summarize some of the results of Ref. 6. The Hamiltonian describing pair production in such a field is given by

$$H=\int d^3\mathbf{k}\left[2\left[k_0-eA_i\frac{k_i}{k_0}\right]J_0(k)-\frac{e\mu A_i}{k_0}[J_+(k)+J_-(k)]\right], \tag{1}$$

where no sum over i is implied,

$$\mu=(k_0^2-k_i^2)^{1/2},$$

and the operators

$$J_+(k)=\frac{m}{\mu}\sum_{\alpha,\beta}b_\alpha^\dagger(k)d_\beta^\dagger(\tilde{k})\bar{u}_\alpha(k)\gamma_i v_\beta(\tilde{k}), \tag{2a}$$

$$J_-(k)=[J_+(k)]^\dagger, \tag{2b}$$

$$J_0(k)=\tfrac{1}{2}\sum_\alpha[b_\alpha^\dagger(k)b_\alpha(k)-d_\alpha(\tilde{k})d_\alpha^\dagger(\tilde{k})] \tag{2c}$$

generate an infinite number (one for each k) of SU(2) algebras in the $j=1$ representation, all commuting with each other. In the above equations the symbol k denotes the four-vector $(k_0,\mathbf{k})$ and $\tilde{k}$ denotes $(k_0,-\mathbf{k})$ and i is the direction of the electric field. The evolution operator, to be determined by solving the equation

$$i\frac{\partial\hat{U}}{\partial t}=H\hat{U}\ , \tag{3}$$

is an element of the SU(2) group. Although this Hamiltonian is realized in a $j=1$ representation, because of the symmetry properties, it is sufficient to solve Eq. (3) in the simplest irreducible (i.e., spinor) representation. In Ref. 6, it was demonstrated that the pair-creation process is an indication of the deviations from adiabaticity. Rewriting Eq. (3) in the spinor representation and in the basis of adiabatic eigenstates yields

$$i\frac{d}{dt}\begin{bmatrix}a_1\\a_2\end{bmatrix}=\begin{bmatrix}\epsilon(k) & i\mu E/2\epsilon^2\\ -i\mu E/2\epsilon^2 & -\epsilon(k)\end{bmatrix}\begin{bmatrix}a_1\\a_2\end{bmatrix}\ , \tag{4}$$

where

$$\epsilon(k)=\{\mu^2+[k_i-eA_i(t)]^2\}^{1/2}\ . \tag{5a}$$

In the above equations the gauge

$$A_0=0,\quad A_i(t)=-\int_{-\infty}^{t}E_i(t')dt' \tag{5b}$$

is adopted. The evolution operator can be completely determined by solving Eq. (4). (For further details the reader is referred to Ref. 6). Equation (4) needs to be solved with the initial condition

$$\lim_{t\to-\infty}a_2(k,t)=0\ . \tag{6}$$

Note also that the relation $|a_1|^2+|a_2|^2=1$ is satisfied at all times.

As shown in Ref. 6, the quantities which describe pair production can be written in terms of the function $|c_2(k)|=|\lim_{t\to+\infty}a_2(k,t)|$. Then the probability of creating fermion pairs per unit volume per unit time is given by

$$\omega=\frac{1}{T(2\pi)^3}\sum_{n=1}^{\infty}\frac{1}{n}\int d^3\mathbf{k}|c_2(k)|^{2n}\ . \tag{7}$$

Defining P_n to be the probability of creating n pairs *with any momenta,* one can write the probability generating function

$$\sum_n P_n\lambda^n=\exp\left[\frac{2V}{(2\pi)^3}\int d^3\mathbf{k}\ln[1+(\lambda-1)|c_2(k)|^2]\right]\ , \tag{8}$$

from which the average number of pairs created is calculated to be

$$\langle n\rangle=\frac{2V}{(2\pi)^3}\int d^3\mathbf{k}|c_2(k)|^2\ . \tag{9}$$

We want to investigate the resonant behavior of the quantity $d\langle n\rangle/dW$, the number of particles produced with individual energies between W and $W+dW$. The final energy of each of the particles in the created pair is given by[6]

$$W=\lim_{t\to+\infty}\epsilon(k,t)\ .$$

Choosing the z axis along the direction of the electric field, we get

$$\frac{d\langle n\rangle}{dW}=\frac{2V}{(2\pi)^2}W\sqrt{W^2-m^2}\times\int_{-1}^{+1}d(\cos\theta)|c_2(W,\cos\theta)|^2\ , \tag{10}$$

which shows that the resonant behavior observed in the number of particles produced per unit volume within a given energy range is directly related to the resonant behavior of $c_2(k)$.

At this point, the resonant behavior in $d\langle n\rangle/dW$ could be demonstrated by numerically solving for $c_2(k)$ for various choices of the vector potential and using Eq. (10) to generate the resulting spectra. We postpone the numerical calculations until the following section and instead investigate some general features of the interference effect. We begin by examining the solutions of Eq. (4). Initially, before the electric field is turned on, $a_1=1$ and $a_2=0$. The quantity a_2 will increase most significantly when either $\epsilon(k,t)$ attains its minimum value for a given k_i, i.e., when the condition $k_i=eA_i(t)$ is satisfied, or when $E(t)$ (the off-diagonal term) attains its maximum value. For a monotonically increasing $A(t)$ there can be only one such "level-crossing" time t_R for each k_i. [The time t_R represents an avoided level crossing in the Hamiltonian of Eq. (4); see Ref. 6.] Note that even if the magnitude of the electric field decreases, $A(t)$ could still increase; an example would be a singly pulsed electric field. On the other hand if $A(t)$ fluctuates in time then there may be more than one "crossing." We can gain some insight into the question of how resonances appear in $d\langle n\rangle/dW$ by considering the special case where $A(t)$ remains almost constant for a sufficiently long time (as we quantify below) between two such points.

In the following discussion we assume that there are two "level-crossing" times for a particular value of k_i; the generalization to three or more crossings is straightforward. Taking

$$T>t_2>t_1>-T\ ,$$

we assume that the field is turned on (off) when $t=-T$ $(+T)$; the first crossing occurs when $t_1>t>-T$ and the second crossing occurs when $T>t>t_2$ with $A(t)$ being approximately constant for $t_2>t>t_1$ as depicted in Fig. 1. If there were only the single first (second) crossing alone, then the effect would be to change the initial values of $a_1=1$ and $a_2=0$ to the final values $a_1^{(1)}$ and $a_2^{(1)}$ ($a_1^{(2)}$ and $a_2^{(2)}$), respectively. When these two level crossings occur together as shown in Fig. 1, the values of a_1 and a_2 at time T are given by

$$\begin{bmatrix}a_1(T)\\a_2(T)\end{bmatrix}=\begin{bmatrix}a_1^{(2)} & -a_2^{*(2)}\\ a_2^{(2)} & a_1^{*(2)}\end{bmatrix}\hat{U}(t_2,t_1)\begin{bmatrix}a_1^{(1)} & -a_2^{*(1)}\\ a_2^{(1)} & a_1^{*(1)}\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\ , \tag{11}$$

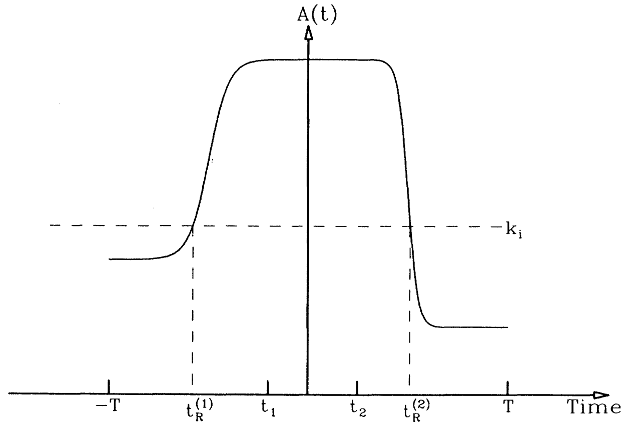

FIG. 1. An example of a vector potential which corresponds to the type of electric field configuration considered in the text.

where the first matrix describes the evolution of the system for $t_2 \le t \le T$, $\hat{U}(t_2,t_1)$ for $t_1 \le t \le t_2$, and the final matrix for $-T \le t \le t_1$.

During the time interval $t_1 \le t \le t_2$, $E(t)$ is very small since $A(t)$ is changing very slowly. This allows Eq. (4) to be solved by treating the off-diagonal terms as a perturbation. To first order in $E(t)$ we find

$$\hat{U}(t_2,t_1)=\begin{bmatrix} e^{-iQ(t_2,t_1)} & I(t_2,t_1)e^{-iQ(t_2,t_1)} \\ -I^*(t_2,t_1)e^{iQ(t_2,t_1)} & e^{iQ(t_2,t_1)} \end{bmatrix}, \tag{12}$$

where we defined

$$Q(t,t_1)=\int_{t_1}^{t}\epsilon(t')dt' \tag{13a}$$

and

$$I(t,t_1)=\frac{\mu e}{2}\int_{t_1}^{t}\frac{E(t')}{\epsilon^2(t')}e^{2iQ(t',t_1)}dt' . \tag{13b}$$

In the following we ignore the slow variation of the vector potential for $t_2 > t > t_1$. Substituting Eq. (12) into Eq. (11) we get

$$|c_2| \approx |a_2^{(2)}a_1^{(1)}e^{-iQ(t_2,t_1)}+a_1^{*(2)}a_2^{(1)}e^{iQ(t_2,t_1)}| , \tag{14}$$

where $Q(t_2,t_1)=(t_2-t_1)\epsilon$. Although the reader would already observe the oscillatory behavior in Eq. (14), we would like to provide simple expressions for the quantities $a_1^{(1)},\ldots$ appearing in the above equation. These quantities can be conveniently calculated in the Magnus approximation.

In order to calculate the evolution operator $\mathcal{U}(\tau)$ for the Hamiltonian in Eq. (4) we write

$$\mathcal{U}(\tau_2,\tau_1)=\mathcal{U}_0(\tau_2,\tau_1)\mathcal{U}_1(\tau_2,\tau_1) , \tag{15a}$$

where

$$\mathcal{U}_0(\tau_2,\tau_1)=\begin{bmatrix} e^{-iQ(\tau_2,\tau_1)} & 0 \\ 0 & e^{iQ(\tau_2,\tau_1)} \end{bmatrix} \tag{15b}$$

and $\mathcal{U}_1(\tau,\tau_1)$ satisfies the equation

$$i\frac{\partial}{\partial\tau}\mathcal{U}_1=H_1\mathcal{U}_1 , \tag{16a}$$

where

$$H_1=\frac{\mu e E}{2\epsilon^2}\begin{bmatrix} 0 & ie^{2iQ(\tau,\tau_1)} \\ -ie^{-2iQ(\tau,\tau_1)} & 0 \end{bmatrix} . \tag{16b}$$

When the electric field is well localized in time, the Magnus method[11] provides a very good approximation as we describe below. In this approximation $\mathcal{U}_1$ is given by

$$\mathcal{U}_1(\tau_2,\tau_1)=\exp[A(\tau_2,\tau_1)] , \tag{17a}$$

where

$$A(\tau_2,\tau_1)=A_1(\tau_2,\tau_1)+A_2(\tau_2,\tau_1)+\cdots , \tag{17b}$$

with

$$A_1(\tau_2,\tau_1)=-i\int_{\tau_1}^{\tau_2}H_1(t)dt , \tag{17c}$$

$$A_2(\tau_2,\tau_1)=\tfrac{1}{2}(i)^2\int_{\tau_1}^{\tau_2}dt\int_{\tau_1}^{t}dt'[H_1(t),H_1(t')] , \tag{17d}$$

and so on. The evolution operator in this expansion remains unitary even if the series in Eq. (17b) is truncated. The commutator in Eq. (17d) can easily be calculated yielding

$$[H_1(t),H_1(t')]=i\frac{(\mu e)^2}{2}\frac{E(t)E(t')}{\epsilon^2(t)\epsilon^2(t')} \times \sin Q(t,t')\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} . \tag{18}$$

One observes that for a well-localized (in time) electric field $A_2(\tau_2,\tau_1)$ is very small. (For the δ-function electric field investigated in Ref. 3, the above commutator vanishes.) Hence for such situations it is sufficient to truncate the series after the first term. We obtain

$$\mathcal{U}(\tau_2,\tau_1)=\begin{bmatrix} a_1 & -a_2^* \\ a_2 & a_1^* \end{bmatrix} , \tag{19}$$

where

$$a_1=e^{-iQ(\tau_2,\tau_1)}\cos|I(\tau_2,\tau_1)| \tag{20a}$$

and

$$a_2=-e^{iQ(\tau_2,\tau_1)}e^{-i\phi(\tau_2,\tau_1)}\sin|I(\tau_2,\tau_1)| , \tag{20b}$$

with

$$e^{-i\phi(\tau_2,\tau_1)}=\frac{I^*(\tau_2,\tau_1)}{|I(\tau_2,\tau_1)|} . \tag{20c}$$

Note that the above results are exact for a δ-function electric field. Finally inserting Eqs. (20) into Eq. (14) after some algebra we get

$$|c_2|^2=\cos^2\mathcal{Q}\sin^2[|I(t_1,-T)|+|I(T,t_2)|]$$
$$+\sin^2\mathcal{Q}\sin^2[|I(t_1-T)|-|I(T,t_2)|] , \qquad (21a)$$

where

$$\mathcal{Q}=Q(t_2,-T)+\tfrac{1}{2}\phi(T,t_2)-\tfrac{1}{2}\phi(t_1,-T) . \qquad (21b)$$

It is instructive to look at various limits of the expression given in Eq. (21a). First let us consider the situation where the initial and final pulses (electric fields) have identical shapes. For definiteness, we consider the case where $A(t)$ monotonically increases as time changes from $-T$ to T although similar arguments can be made for a monotonically decreasing $A(t)$. In this case, $A(t)$ will be smaller for the first electric field pulse than for the second one. Hence for lower values of k_i (smaller values of t_R) the first pulse will cause level crossings, but the second one will not $(I(t_1,-T)\neq0, I(T,t_2)=0)$. On the other hand for larger values of k_i (larger values of t_R) the first pulse will not cause level crossings, but the second one will $(I(t_1,-T)=0, I(T,t_2)\neq0)$. Inserting those values in Eqs. (20) and (21) it follows that one has a double-humped spectrum of produced particles: one hump at lower energies corresponding to the first pulse and another at higher energies corresponding to the second one. Since effectively only one of the pulses will be producing particles at a given energy, there will be no interference pattern.

Next we consider the case where the strengths of the initial and the final electric fields are the same, but their directions opposite (the example considered in Ref. 3). We will assume that $A(t)$ increases near the first pulse and decreases near the second one, but the arguments also hold for the alternate choice. Hence, both pulses can produce particles with a given energy and one gets

$$I(t_1,-T)=-I(T,t_2) \qquad (22a)$$

(the sign change is due to the opposite signs of the two pulses) or

$$\phi(T,t_2)-\phi(t_1,-T)=\pi , \qquad (22b)$$

which yields

$$|c_2|^2=\sin^2Q(t_2,-T)\sin^2(2|I|) . \qquad (22c)$$

In this case for a δ-function electric field the integral in Eq. (13b) can be easily carried out yielding

$$|c_2|^2\sim\sin^2[W(t_2-t_1)]\sin^2\left[\frac{\mu eE_0}{W^2}\right] , \qquad (23)$$

which, except for kinematic factors, is the same result as obtained in Ref. 3. Hence the number of particles per energy interval oscillates (first sine function) with a decreasing magnitude (second sine function) as the energy increases. Note that, in general, for such an oscillation to occur it is not even necessary to have the electric field zero over the region $t_2>t>t_1$. A sufficient condition is that the quantity $\mathcal{Q}$ of Eq. (21b) changes significantly (several times π) as the energy of the produced pair increases. It is even possible in principle to take $t_1=t_2$ by considering two pulses rather different in strength so that the phases $\phi(T,t_2)$ and $\phi(t_1,-T)$ differ appreciably as W increases. An alternative interpretation for this situation was given in Ref. 12, where two oppositely directed pulses are shown to act like two Fabry-Perot time mirrors between which the electron wave is reflected back and forth in time.

III. NUMERICAL ANALYSIS

In this section we quantify the qualitative arguments given in the preceding section by numerically solving Eq. (4). We first wish to emphasize that in the case where only a single pulse acts, there is no resonant behavior observed in the number density of the produced particles. This can easily be seen from Eq. (14) by substituting zero for either $a_2^{(1)}$ or $a_2^{(2)}$. To illustrate this, we consider the case where the electric field $E(t)=E_0\text{sech}^2\alpha t$ as shown in Fig. 2(a). For this field it is possible to evaluate $c_2(k)$ using the semiclassical approach developed in Ref. 5. Using the result given in Eq. (4.9) of Ref. 6 one can also perform the integration in Eq. (10) to obtain

$$\frac{d\langle n\rangle}{V\,dW}\sim\frac{W\alpha^3}{eE_0}\{[1-y(1)]e^{y(1)}-[1-y(-1)]e^{y(-1)}\}$$
$$\times\exp\left[\frac{\pi}{\alpha}\left[\frac{2\pi eE_0}{\alpha}-W\right]\right] , \qquad (24)$$

where

$$y(x)=-\frac{\pi}{\alpha}\left[\left[\sqrt{W^2-m^2}\frac{2xeE_0}{\alpha}\right]^2+m^2\right]^{1/2} . \qquad (25)$$

The resulting number density is plotted in Fig. 2(b), and as expected does not show any resonant behavior. The salient features of this figure are easy to understand. As stated previously, the most significant increase in $a_2(k,t)$ occurs when the off-diagonal components of the matrix in Eq. (4) are maximized. This is equivalent to the conditions that the most favorable longitudinal momentum be given by $k_i=eA(t_{\max})$ where $t_{\max}$ is the time where the electric field is a maximum and that the perpendicular momentum be zero (since then $\epsilon^2=\mu^2=m^2+k_\perp^2$). According to Ref. 6, the final momentum along the direction of the field is then given by

$$k_i^{(f)}=eA(t_{\max})-eA(\infty) , \qquad (26)$$

which leads to a most favorable particle energy of

$$W=\sqrt{m^2+[eA(t_{\max})-eA(\infty)]^2} . \qquad (27)$$

From Fig. 2(a), it is seen that $eA(t_{\max})=0$ which leads to a predicted peak in the number density at $W=\sqrt{10}\,m$, in fair agreement with Fig. 2(b). The width of $d\langle n\rangle/dW$ can also be qualitatively estimated from the maximum possible momentum which can be transferred from the field to the produced particles. This is roughly given by $|eA(-\infty)-eA(\infty)|$. In this particular example, a width of $\Delta W=\sqrt{37}m$ is predicted. This is also in fair agreement with Fig. 2(b), although the kinematic cutoff at low energy tends to skew the distribution.

As a second example we consider two pulses of the form $E(t)=E_0\mathrm{sech}^2\alpha t$ and $E(t)=-E_0\mathrm{sech}^2\alpha(t-t_0)$ suitably connected to provide a zero field region in between as depicted in Fig. 3(a). The resulting number density, evaluated by integrating Eq. (4) numerically, is shown in Fig. 3(b). This electric field configuration, which we qualitatively discussed in the preceding section [cf. Eq. (23)], can be considered as a generalization of the

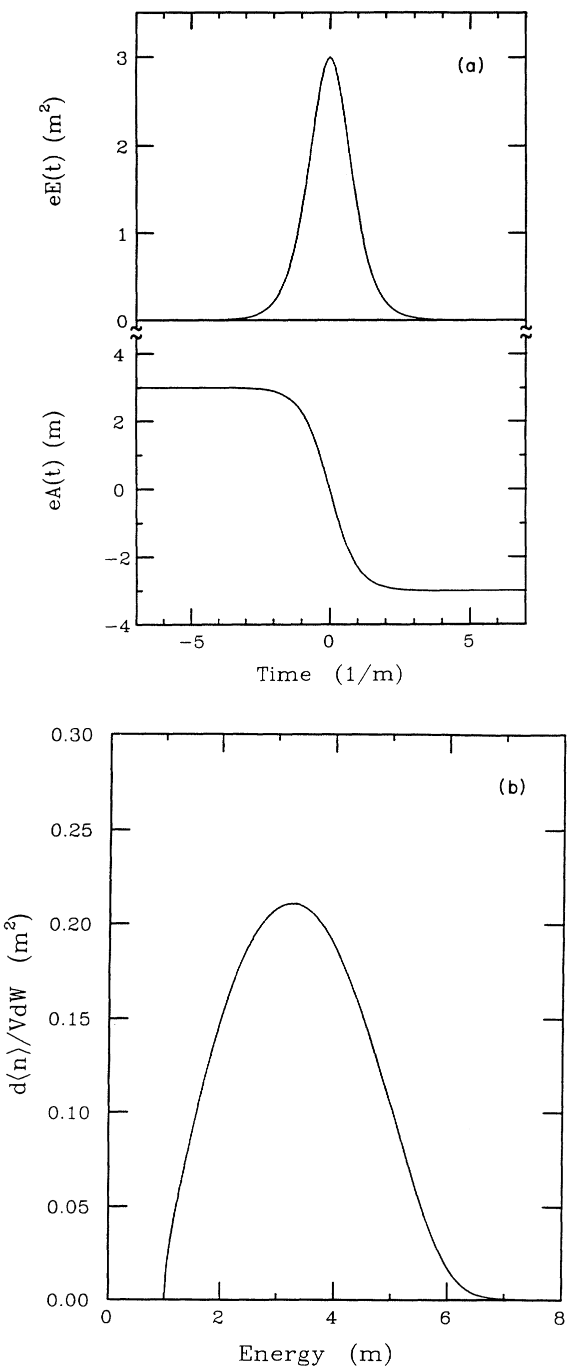

FIG. 2. (a) Upper panel: the single pulsed electric field $E(t)=E_0\mathrm{sech}^2\alpha t$. All the quantities are given in terms of the mass of the produced fermions. In this figure $eE_0=3m$ and $\alpha=m$. Lower panel: the corresponding vector potential. (b) The number density of produced particles as a function of energy for the electric field shown in (a).

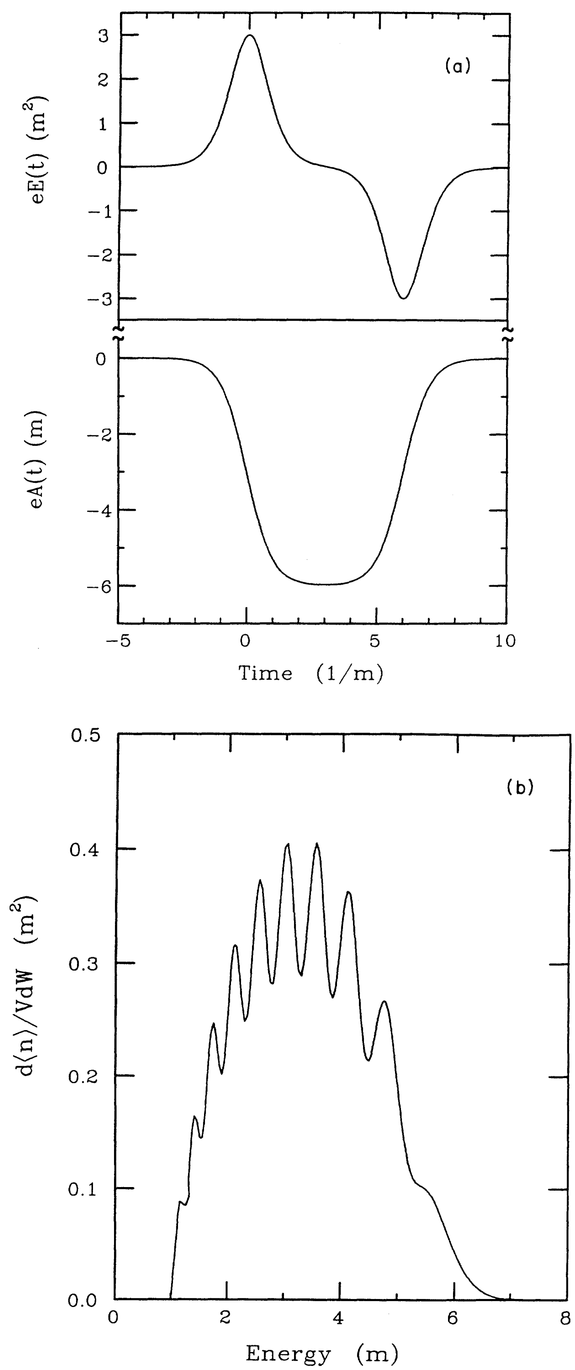

FIG. 3. (a) Upper panel: the doubly pulsed electric field $E(t)=E_0\mathrm{sech}^2\alpha t-E_0\mathrm{sech}^2\alpha(t-t_0)$. Lower panel: the corresponding vector potential. The values of eE_0 and α are the same as those used in Fig. 2(a) while $t_0=6m^{-1}$. (b) The number density of produced particles as a function of energy for the electric field shown in (a).

two δ-function electric field configurations investigated in Ref. 3. As expected, a resonant behavior in the spectra is observed. Note that for a resonant behavior to occur, one does not need to introduce two time scales which differ by several orders of magnitude: the time delay between the two pulses can be comparable to the width of each individual pulse. The number of peaks depends on two quantities: the highest possible energy of the produced particles (which depends on the change in the vector potential as discussed in the preceding paragraph) and the period of the oscillation [which is determined by the distance between the two pulses; cf. Eq. (23)]. In general, the larger the separation between the two pulses, the smaller the period of the oscillation is. To illustrate this we plot in Fig. 4 the number density of the particles produced by an electric field configuration similar to that in Fig. 3(a), but with twice the distance between the electric pulses. One observes that the number of peaks nearly doubles as expected from Eq. (23).

As we discussed in the preceding section, it is necessary to take two pulses with opposite directions in order to produce an interference effect. As an example, we consider an electric field similar to that given in Fig. 3(a), but with both pulses in the same direction as shown in Fig. 5(a). Figure 5(b) shows the resulting number density of produced particles as a function of energy. As can be seen from the figure, no resonant behavior is present, but instead a double-humped structure is obtained. This is as

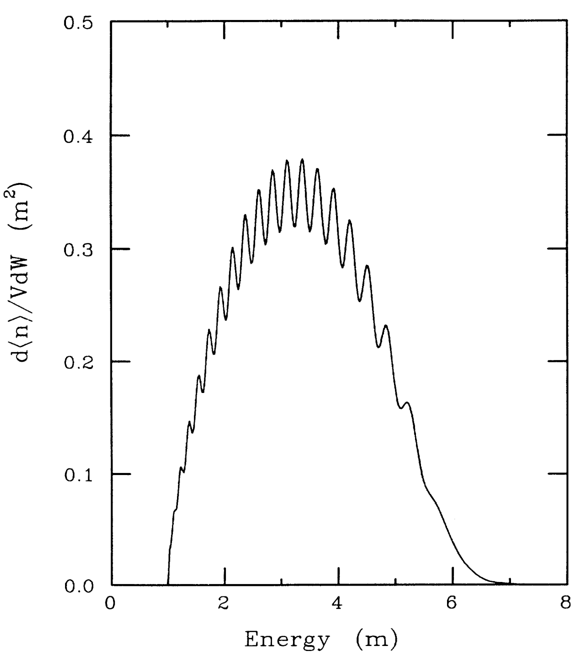

FIG. 4. The number density of produced particles as a function of energy for an electric field similar to that given in Fig. 3(a), but with twice the distance ($t_0 = 12m^{-1}$) between the pulses.

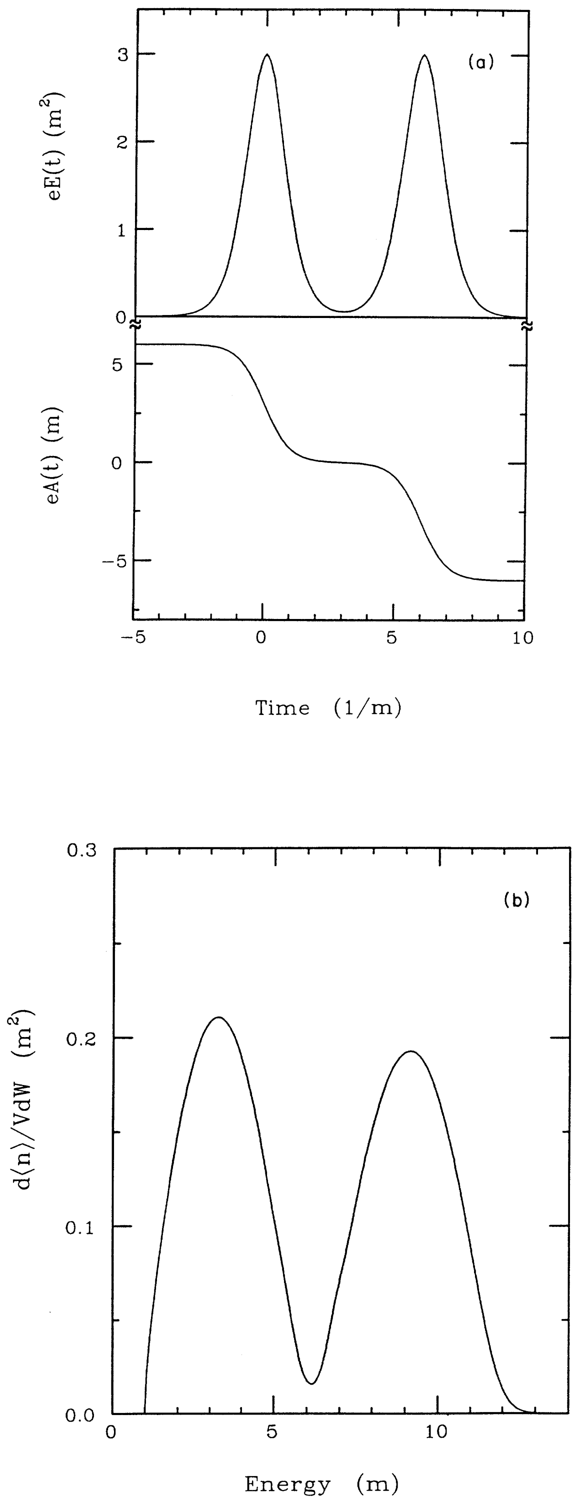

FIG. 5. (a) Upper panel: the doubly pulsed electric field $E(t) = E_0 \text{sech}^2 \alpha t + E_0 \text{sech}^2 \alpha (t - t_0)$. Lower panel: the corresponding vector potential. The values of eE_0, α, and t_0 are the same as those used in Fig. 3(a), but here the electric field pulses have the same direction. (b) The number density of produced particles as a function of energy for the electric field shown in (a).

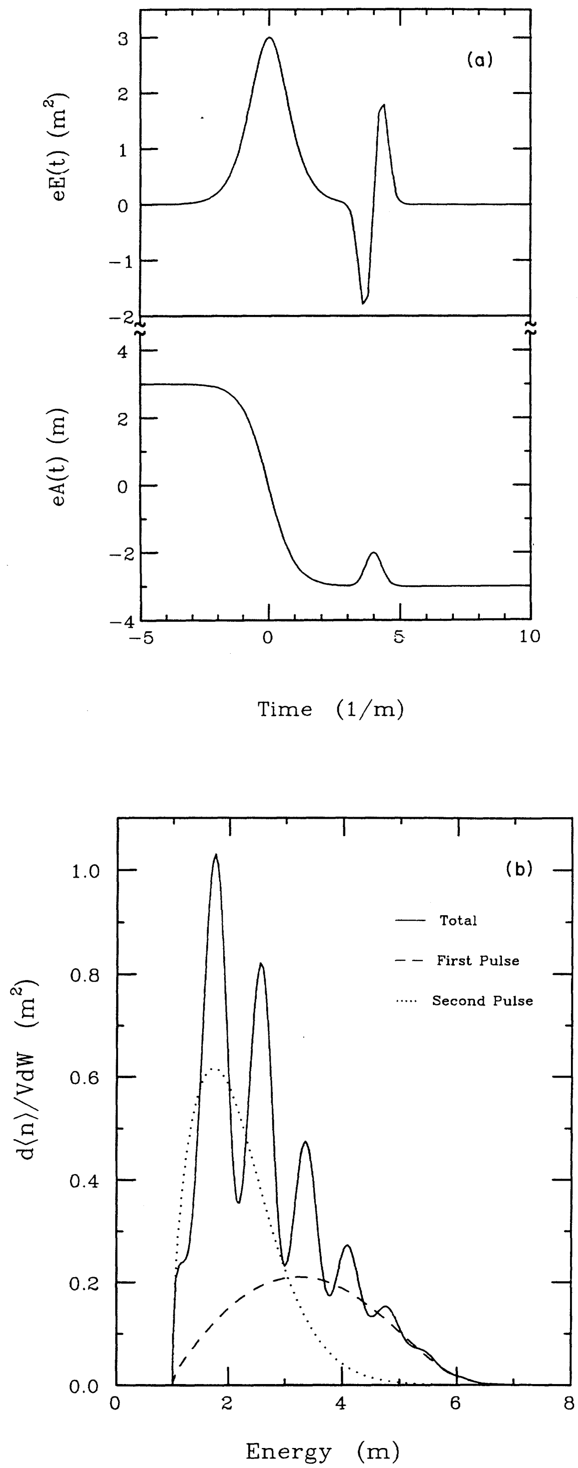

FIG. 6. (a) Upper panel: the combination of the electric field used in Fig. 2(a) and that obtained from a Gaussian vector potential of the form $A(t)=\exp[-5(t-t_0)^2]$ where $t_0=4m^{-1}$. Lower panel: the corresponding vector potential. (b) The number density of produced particles as a function of energy for the electric field given in (a) (the solid line). The dashed and dotted lines are the number densities corresponding to only the first and only the second pulses, respectively.

we qualitatively predicted in the preceding section.

As a final example we consider the combination of an electric field of the type $E(t)=E_0\text{sech}^2\alpha t$ and another one obtained from a Gaussian vector potential [Fig. 6(a)]. The resulting number density is plotted as a function of the energy in Fig. 6(b) (the solid line). To illustrate the interference effect in this figure we also plot the number densities obtained when the first (the dashed line) or second (the dotted line) electric field acts alone. As expected [cf. Eq. (14)], one needs two pulses to obtain an interference pattern.

IV. DISCUSSION AND CONCLUSIONS

In this paper we discussed both qualitatively and quantitatively the conditions under which the number of particles produced in an external field with energies between W and $W+dW$ would exhibit a resonant behavior as a function of W. An electric field consisting of two pulses separated by a time interval comparable to the widths of individual pulses yields such a behavior as a result of the interference between the pair-production amplitudes of the two pulses. It is also possible to understand the quantitative features of the spectra such as the number of the peaks.

The reader at this point might ask if there is a connection between our work and the recent experimental results[13] on electron-positron pair production in heavy-ion collisions, showing multiple resonances in the singles and coincidence spectra. Indeed various theoretical explanations of those events were given invoking interference effects among different amplitudes.[14] We feel that the effects we investigated here typify the wealth of possibilities of interesting phenomena in nonperturbative pair production by external fields. As such, they merit a study on their own. Interference effects in pair production can also play a role in other physical situations[8–10] in addition to heavy-ion physics near the Coulomb barrier. For this reason we defer the discussion of the possible connection between this interference phenomenon and the heavy-ion experiments. We nevertheless note that such a connection would be hard to establish unless one takes into account the spatial dependence of the electric field as well.

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