# ARTICLES

## Nonsymmetric gravitation theories and local Lorentz invariance

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We analyze the motion and internal structure of test bodies in theories of gravity which couple the antisymmetric part of a nonsymmetric-tensor gravitational field to the electromagnetic field. We establish that such theories necessarily violate the Einstein equivalence principle by breaking local Lorentz invariance. We also show how atomic-physics experiments designed to test the isotropy of space can be used to test these nonsymmetric theories of gravity. We comment briefly on the constraint such experiments are capable of imposing on Moffat's nonsymmetric gravitation theory, the prototype for the theories of gravity we study.

### I. INTRODUCTION

General relativity and other metric theories of gravity are able to offer their beautiful, purely geometrical accounts of gravitation only because they exhibit symmetries defined by the Einstein equivalence principle. This principle states that the outcomes of nongravitational test experiments performed within a local, freely falling frame are independent of the frame's location in and velocity through a gravitational field.[1] The symmetries this defines are referred to as local position invariance and local Lorentz invariance.[2] Theories of gravity which break either of these symmetries fail to attribute a unique operational geometry to spacetime. In such nonmetric theories the "geometry" revealed by local measurements of distance and time depends upon the nature of the rulers and clocks used to make the measurements.

Moffat's nonsymmetric gravitation theory[3] (NGT) is the prototype for a diverse class of Lagrangian-based nonmetric theories.[4] Studied extensively during the past decade, these theories feature a nonsymmetric-tensor gravitational field. Their nonmetric character is a consequence of direct gravitational couplings between matter and the antisymmetric part of the nonsymmetric-tensor field. Such couplings break local position and local Lorentz invariance. In a previous paper[5] we analyzed the effect a coupling between the electromagnetic field and the antisymmetric part of a nonsymmetric gravitational field has on the propagation of light and showed how measurements of gravitational light deflection and the gravitational propagation delay can be used to test nonsymmetric theories of gravity. In this paper we analyze effects such a coupling has on the motion and internal structure of test bodies in a nonsymmetric gravitational field and show how atomic-physics experiments designed to test the isotropy of space[6–8] can be used to test nonsymmetric theories of gravity.

Mann[9] *et al.* give a general form for the action that governs the dynamics of charged test particles and electromagnetic fields in nonsymmetric theories of gravity:

$$I=-\sum_a m_a\int dt(-g_{\mu\nu}v_a^\mu v_a^\nu)^{1/2}+\sum_a e_a\int dt\, v_a^\mu A_\mu+I_{\rm em}\ , \tag{1}$$

where $m_a$, $e_a$, and $v_a\equiv dx_a/dt$ are the rest mass, charge, and ordinary velocity ($v_a^0=1$) of particle $a$ and where $A_\mu$ is the electromagnetic vector potential. The electromagnetic action $I_{\rm em}$ is

$$I_{\rm em}=-\frac{1}{16\pi}\int d^4x\sqrt{-g}\,\mathcal{F}g^{\mu\alpha}g^{\nu\beta}\times[ZF_{\mu\nu}F_{\alpha\beta}+(1-Z)F_{\alpha\nu}F_{\mu\beta}+YF_{\mu\alpha}F_{\nu\beta}]\ , \tag{2}$$

where the electromagnetic field tensor $F_{\mu\nu}$ is related to the vector potential in the usual way: $F_{\mu\nu}\equiv A_{\nu,\mu}-A_{\mu,\nu}$. The matrix $g^{\mu\nu}$ is the inverse of the nonsymmetric graviational field $g_{\mu\nu}$ defined by $g^{\mu\alpha}g_{\nu\alpha}=g^{\alpha\mu}g_{\alpha\nu}=\delta^\mu{}_\nu$. The symbols $Y$ and $Z$ denote constants while $\mathcal{F}$ is a scalar function whose value is unity when $g_{[\mu\nu]}=0$ implying that $\mathcal{F}=\mathcal{F}(\sqrt{-g}/\sqrt{-\gamma})$ where $g\equiv\det g_{\mu\nu}$ and $\gamma\equiv\det g_{(\mu\nu)}$.

The coupling between the electromagnetic field and the antisymmetric part of the nonsymmetric gravitational field, the coupling that breaks local position and local Lorentz invariance, is more apparent when we insert a representation of the field of the Sun or the Earth into the action (1). For our purposes a static, spherically symmetric approximation to these fields is adequate. These symmetries imply the existence of an "isotropic" coordinate system centered on the field's source in which the symmetric part of the field takes the form $g_{00}=-T(r)$, $g_{(0i)}=0$, and $g_{(ij)}=H(r)\delta_{ij}$, with $r\equiv|\mathbf{x}|$. In the following analysis we treat explicitly theories such as Moffat's NGT which have the property that the representation of the antisymmetric part of a static, spherically symmetric field in these "isotropic" coordinates has the special form $g_{[0i]}=L(r)n_i$ and $g_{[ij]}\equiv 0$, where $n_i\equiv x_i/r$. The precise nature of the effects on test-body motion and internal structure that we derive reflect this special form of the nonsymmetric gravitational field, but analysis employing a general representation reveals analogous effects.

Defining electric and magnetic fields via $F_{j0}\equiv E_j$ and $F_{jk}\equiv\epsilon_{jkl}B_l$ and employing our special representation of a static, spherically symmetric gravitational field we can cast the action (1) into the form

$$I=-\sum_a m_a\int dt(T-Hv_a^2)^{1/2}+\sum_a e_a\int dt\, v_a^\mu A_\mu+I_{\text{em}}\ , \tag{3}$$

with

$$I_{\text{em}}=\frac{1}{8\pi}\int d^4x\left[\epsilon\mathbf{E}\cdot\mathbf{E}+X\epsilon\alpha(\hat{\mathbf{n}}\cdot\mathbf{E})^2-\frac{1}{\mu}\mathbf{B}\cdot\mathbf{B}+\frac{\Omega}{\mu}(\hat{\mathbf{n}}\cdot\mathbf{B})^2\right]\ , \tag{4}$$

where

$$\epsilon\equiv\mathcal{F}\left[\frac{H}{T}\right]^{1/2}\left[1-\frac{L^2}{TH}\right]^{-1/2},$$

$$\mu\equiv\mathcal{F}^{-1}\left[\frac{H}{T}\right]^{1/2}\left[1-\frac{L^2}{TH}\right]^{1/2},$$

$$\alpha\equiv\frac{2L^2}{TH}\left[1-\frac{L^2}{TH}\right]^{-1},$$

$$\Omega\equiv\frac{L^2}{TH}\ ,$$

and where $X\equiv 1-Y-Z$.

Apart from the term proportional to $(\hat{\mathbf{n}}\cdot\mathbf{B})^2$, the action (3) with $X=0$ is the action of the $TH\epsilon\mu$ formalism.[10] Noting this and arguing that the $(\hat{\mathbf{n}}\cdot\mathbf{B})^2$ term can be set aside, Will[11] concludes that minimally coupled nonsymmetric theories of gravity, theories with $Y=0$, $Z=1$, and $\mathcal{F}\equiv 1$, violate the Einstein equivalence principle because they predict that a test body's gravitational acceleration depends upon its internal electrostatic structure. In its original formulation,[3] Moffat's NGT is a minimally coupled theory. Mann[9] *et al.* note that nonsymmetric theories with $X=0$ and $\mathcal{F}=\sqrt{-g}/\sqrt{-\gamma}$ possess an action (3) which has a structure characteristic of metric theories of gravity[10] except for the term proportional to $(\hat{\mathbf{n}}\cdot\mathbf{B})^2$. Setting aside this term as Will did, they conclude that these nonsymmetric theories predict that a test body's gravitational acceleration has no structure-dependent component proportional to the body's electrostatic self-energy.

In this paper we begin to analyze the effect that the $(\hat{\mathbf{n}}\cdot\mathbf{B})^2$ term set aside in these earlier papers has on the motion and structure of test bodies. Since no choice of the constants $Y$ and $Z$ or of the function $\mathcal{F}$ can eliminate this term from the action (3), its presence is an unavoidable consequence of coupling the electromagnetic field to the antisymmetric part of a nonsymmetric gravitational field. We find its effects particularly interesting because the term singles out the rest frame of the nonsymmetric gravitational field's source as a preferred frame of reference and, so, breaks local Lorentz invariance. Coupling the electromagnetic field to the antisymmetric part of a nonsymmetric gravitational field is a mechanism for breaking this symmetry that cannot be represented in the $TH\epsilon\mu$ formalism, a formalism designed to encompass the structure of earlier nonmetric theories such as that of Belinfante and Swihart.[12] The consequences of the $(\hat{\mathbf{n}}\cdot\mathbf{B})^2$ coupling represent new gravitational physics for experiment to explore.

In Sec. II we apply a general theory governing local Lorentz noninvariance in Lagrangian-based theories of gravity[13,14] to calculate effects of the $(\hat{\mathbf{n}}\cdot\mathbf{B})^2$ coupling on the motion and internal structure of atoms, neutral test bodies composed of charged particles. Working to leading (electrostatic) order in the relativistic expansion of an atom's internal structure, we compute the *anomalous inertial mass tensor* associated with the atom in each of its internal states. This tensor determines the structure-dependent component of a test body's gravitational acceleration that arises from the breakdown of local Lorentz invariance.[13] The $(\hat{\mathbf{n}}\cdot\mathbf{B})^2$ coupling does generate a structure-dependent component proportional to the body's electrostatic self-energy, a component overlooked by previous analyses.

The anomalous inertial mass tensor associated with an atom in a particular internal state also determines the effect that motion of the atom through a nonsymmetric gravitational field has on the state's energy.[13,14] Since the ticking rates of atomic clocks are governed by transitions between pairs of states, the anomalous inertial mass tensors associated with the states involved in a particular transition determine the effect motion through a nonsymmetric field has on the ticking rate of a clock governed by that transition. In other words, they determine the time-dilation factor a local observer at rest in the nonsymmetric field would associate with that type of clock. The presence of the anomalous inertial mass tensors causes this factor to vary from one type of clock to another, the characteristic symptom of Lorentz noninvariance.[13,14]

The energy emitted in a transition between states of an atom moving through a nonsymmetric gravitational field can depend upon the orientation of the atom's spin relative to its velocity through the field and relative to the vector $\hat{\mathbf{n}}$ defined by the structure of the field. Atomic-physics experiments designed to test the isotropy of space[6–8] are extraordinarily sensitive to orientation dependence of the energies of certain ground-state hyperfine transitions and, so, can be used to test nonsymmetric theories of gravity. In Sec. III we show that because of the $(\hat{\mathbf{n}}\cdot\mathbf{B})^2$ coupling the energies of hyperfine transitions in atoms whose nuclei possess an electric quadrupole moment depend sensitively on the orientation of their spins relative to a vector perpendicular to the plane of the ecliptic. This dependence is a consequence of the Earth's orbital motion through the preferred frame singled out by a nonsymmetric solar gravitational field. We comment briefly on the limit atomic anisotropy experiments are capable of imposing on the magnitude of such energy dependence and on the significance of this limit for Moffat's NGT.

## II. ANOMALOUS INERTIAL MASS TENSORS

In this section we analyze the behavior of neutral test bodies composed of charged particles, specifically atoms, which move slowly through a static, spherically symmetric nonsymmetric-tensor gravitational field. We are interested in effects due to the term in the action (3) proportional to $(\hat{\mathbf{n}}\cdot\mathbf{B})^2$ which breaks local Lorentz invariance. To isolate these effects we study nonsymmetric theories having $X=0$ and $\mathcal{J}=\sqrt{-g}/\sqrt{-\gamma}$. In these theories the action (3) departs from metric form[10] only because of the $(\hat{\mathbf{n}}\cdot\mathbf{B})^2$ term

$$I=-\sum_a m_a\int dt(T-Hv_a^2)^{1/2}+\sum_a e_a\int dt\, v_a^\mu A_\mu+I_{\text{em}}\ , \tag{5}$$

where

$$I_{\text{em}}=\frac{1}{8\pi}\int d^4x\left[\epsilon E^2-\frac{1}{\mu}[B^2-\Omega(\hat{\mathbf{n}}\cdot\mathbf{B})^2]\right]\ , \tag{6}$$

with $\epsilon=\mu=(H/T)^{1/2}$.

For our purposes, an atom is moving slowly through a nonsymmetric gravitational field when it remains within a region small on the scale associated with variations of the gravitational potentials $T$, $H$, and $\Omega$ for periods long in comparison to the time scale for the internal motions of the atom's constituent particles. Under these conditions, a local approximation to the action (5) is adequate. Within a neighborhood of some event $\mathcal{P}_0$ initially near the atom, we rescale coordinates, charges, and electromagnetic potentials[2] to obtain the local action

$$I=-\sum_a m_a\int dt(1-v_a^2)^{1/2}+\sum_a e_a\int dt\, v_a^\mu A_\mu+I_{\text{em}}\ , \tag{7}$$

with

$$I_{\text{em}}=\frac{1}{8\pi}\int d^4x\{E^2-[B^2-\Omega_0(\hat{\mathbf{n}}_0\cdot\mathbf{B})^2]\}\ , \tag{8}$$

where $\Omega_0$ and $\hat{\mathbf{n}}_0$ denote the values of $\Omega$ and $\hat{\mathbf{n}}$ at the event $\mathcal{P}_0$. We refer to the local frame of rescaled coordinates as the preferred frame of reference because it is at rest in the "isotropic" coordinate system in which the action (5) is represented and, so, at rest with respect to the source of the nonsymmetric gravitational field. Note that we employ "relativistic" coordinates in this frame. Velocities are dimensionless and the speed of light is of order unity. The local action (7) governs the structure of an atom while it remains in the neighborhood of $\mathcal{P}_0$. The behavior of atoms which eventually leave this neighborhood can often be followed by employing an adiabatic approximation in which the value of $\sqrt{\Omega_0}\hat{\mathbf{n}}_0$ and the scaling of local coordinates, charges, and electromagnetic potential are slowly varied.

Consider an atom that moves with velocity $\boldsymbol{\beta}$ through the preferred frame. Its internal structure is most easily analyzed in a local frame in which the atom is at rest. We define such a frame by means of a standard Lorentz transformation from the preferred frame. We obtain a convenient representation of the local action (7) in this moving frame by specifying that the fields $\mathbf{A}$, $\mathbf{E}$, and $\mathbf{B}$ transform via the corresponding Lorentz transformation laws for vector and electromagnetic fields. The resulting action is, to $O(\beta^2)$ and displaying only those terms needed to analyze the atom's electrostatic internal structure,

$$I\approx-\sum_a m_a\int dt(1-\tfrac{1}{2}v_a^2)-\sum_a e_a\int dt\,\phi+\frac{1}{8\pi}\int d^4x(E^2+\Omega_0|\boldsymbol{\beta}\cdot\hat{\mathbf{n}}_0\times\mathbf{E}|^2)\ , \tag{9}$$

where $\phi\equiv-A_0$ is the electrostatic potential.

This representation of the action is convenient because it is the standard action of special-relativistic electrodynamics, truncated to account only for electrostatics, plus a single term that stems from the $(\hat{\mathbf{n}}_0\cdot\mathbf{B})^2$ term in the action (7), a term proportional to the dimensionless parameter $\Omega_0$ which specifies the degree to which local Lorentz invariance is broken in the neighborhood of $\mathcal{P}_0$. Since viable nonsymmetric theories of gravity assign values far smaller than unit to $\Omega_0$ throughout the solar system, we can compute effects of the term in the action (9) that breaks local Lorentz invariance via a perturbative analysis about the familiar and well-behaved $\Omega_0=0$ limit.

The desirable form of the action (9) is a consequence of both the form of the preferred-frame action (7) and our use of conventional Lorentz coordinate and field transformations to define the moving-frame action. Note, however, that only formal properties of the Lorentz transformations are exploited. When local Lorentz invariance is broken (i.e., $\Omega_0\neq0$), the moving-frame coordinates do not have the universal operational significance that we would associate with them in special relativity.[14] The relationship between the moving-frame coordinates and coordinates a local observer would construct by making measurements of distance and time becomes clear only once we have used the action (9) to analyze systems that the observer could use as rulers or clocks.

The Hamiltonian that governs the electrostatic structure of an atom at rest in the moving frame is

$$H=\sum_a\left[m_a+\frac{p_a^2}{2m_a}+\frac{e_a}{2}\phi(\mathbf{x}_a)\right]\ . \tag{10}$$

Its form is a consequence of the action (9), as is the form of the electrostatic field equation that determines $\phi$,

$$\nabla^2\phi+\Omega_0(\boldsymbol{\beta}\times\hat{\mathbf{n}}_0)^j(\boldsymbol{\beta}\times\hat{\mathbf{n}}_0)^k\nabla^j\nabla^k\phi=-4\pi\rho\ , \tag{11}$$

where $\rho=\sum_a e_a\delta(\mathbf{x}-\mathbf{x}_a)$ is the charge density.

Inserting the solution of this field equation into Eq.

(10), we obtain an expression for the Hamiltonian governing the atom's structure that involves only the coordinates and momenta of its constituent particles

$$H=\sum_{a}\left[m_a+\frac{p_a^2}{2m_a}\right]+U_C\ , \tag{12}$$

where, to $O(\Omega_0)$, the electrostatic or Coulomb potential energy is

$$U_C=\sum_{a,b}{}' \frac{e_a e_b}{2|\mathbf{x}_{ab}|}\left[1-\frac{\Omega_0}{2}|\boldsymbol{\beta}\times\hat{\mathbf{n}}_0|^2+\frac{\Omega_0}{2}\frac{|(\boldsymbol{\beta}\times\hat{\mathbf{n}}_0)\cdot\mathbf{x}_{ab}|^2}{|\mathbf{x}_{ab}|^2}\right]. \tag{13}$$

The prime on the summation symbol is a reminder to exclude self-interaction terms as usual.

The expectation value of the Hamiltonian (12) in an unperturbed $\Omega_0=0$ state is an estimate of the state's energy expressed in units consistent with the unit of time in the atom's rest frame. It is accurate to $O(\Omega_0)O(\beta^2)O(v^2)$, where $v$ is a speed typical of particle motion within the atom. The general analysis presented in the Appendix of Ref. 13 relates this moving-frame coordinate energy to the energy a local observer at rest in the moving frame would measure. The dependence of this locally measured energy on $\boldsymbol{\beta}$, the atom's velocity relative to the source of the nonsymmetric gravitational field, is a direct violation of local Lorentz invariance.

Equation (A59) of Ref. 13 establishes the relationship between the moving-frame coordinate energy of a state and the anomalous inertial mass tensor associated with an atom in that state. From the structure of the Hamiltonian (12), we conclude that

$$\delta M_I^{jk}=\frac{\partial^2}{\partial\beta^i\partial\beta^k}\langle U_C\rangle \tag{14}$$

is the electrostatic contribution to this anomalous inertial mass tensor. Expressed in terms of the state's electrostatic self-energy tensor

$$\Omega_{\rm ES}^{jk}\equiv\sum_{a,b}{}'\left\langle\frac{e_a e_b x_{ab}^j x_{ab}^k}{2|\mathbf{x}_{ab}|^3}\right\rangle \tag{15}$$

this becomes

$$\delta M_I^{jk}=\Omega_0(\delta^{jk}-n_0^j n_0^k)\Omega_{\rm ES}-\Omega_0\epsilon^{jac}\epsilon^{kbd}n_0^a n_0^b\Omega_{\rm ES}^{cd}\ , \tag{16}$$

where the electrostatic self-energy $\Omega_{\rm ES}$ is the trace of the self-energy tensor (15).

The structure of this expression for the anomalous inertial mass tensor differs markedly from the expression that emerges from the $TH\epsilon\mu$ formalism.[13] Notice, for example, that the expression above involves the vector $\hat{\mathbf{n}}$, defined by the antisymmetric part of the nonsymmetric gravitational field, as well as reflecting the internal structure of the atom's state. As a result, even spherically symmetric atomic states possess anisotropic anomalous inertial mass tensors. The mechanisms for breaking local Lorentz invariance encompassed by the $TH\epsilon\mu$ formalism associate anisotropic anomalous inertial mass tensors only with aspherical atomic states.

The breakdown of local Lorentz invariance gives rise to a structure-dependent contribution to the gravitational acceleration of a freely falling body,[13]

$$\delta a^i=-\frac{\delta M_I^{ij}}{M_R}g^j\ , \tag{17}$$

where $\mathbf{g}$ is the gradient of the Newtonian potential and where $M_R$ is the conventional rest mass of the atom. Present constraints on Moffat's NGT force the magnitude of $\Omega_0$ to be less than $10^{-18}$ in the Earth's neighborhood. Consequently, in this theory the acceleration anomaly implied by Eqs. (16) and (17) is far too small to have been detected in experimental tests of the weak equivalence principle.[2] Other effects of the anomalous inertial mass tensors (16) which NGT attributes to atoms are accessible to experiment, however, as we demonstrate in the next section.

## III. TESTING NONSYMMETRIC THEORIES WITH ATOMIC CLOCKS

The recent experiments of Prestage[6] *et al.*, Lamoreaux[7] *et al.*, and Chupp[8] *et al.* are each distinct and remarkably precise realizations of a simple scheme for testing the isotropy of space. In essence, each experiment monitors the relative frequency of a selected pair of ground-state hyperfine transitions in atoms immersed in a laboratory magnetic field. One transition occurs in atoms whose nuclei possess electric quadrupole moments while the other occurs in atoms whose nuclei do not, that is in atoms whose nuclei have angular momentum less than one. The experiments are designed to detect any variation of the relative frequency of the selected transitions as the Earth rotates changing the spatial orientation of the laboratory magnetic field and, thus, of the emitting atoms. No significant variation is observed.

The outcome of such experiments have been used to impose limits on the degree to which local Lorentz invariance could be broken by theories of gravity encompassed by the $TH\epsilon\mu$ formalism.[10] In these theories the Earth's velocity relative to the cosmic-microwave-background radiation, $\mathbf{V}$, can be singled out as a preferred direction in space and the frequencies of hyperfine transitions in atoms whose nuclei possess an electric quadrupole moment become extremely sensitive to changes in atomic orientation relative to $\mathbf{V}$; see Ref. 12.

The outcome of atomic anisotropy experiments can also be used to impose limits on the degree to which local Lorentz invariance could be broken by nonsymmetric theories of gravity of the type analyzed in the preceding section. We show that in these theories the frequencies of hyperfine transitions in atoms whose nuclei possess an electric quadrupole moment are extremely sensitive to changes in atomic orientation relative to a vector directed toward the north ecliptic pole. The Earth's orbital motion through the Sun's gravitational field singles out this preferred direction in space.

For an atom moving with the Earth's velocity $\boldsymbol{\beta}$ through the solar field, expectation values of the Hamiltonian (12) provide estimates of the energies of the atom's states. In terms of the anomalous inertial mass tensor associated with the atom in a state of interest, the $\boldsymbol{\beta}$-dependent portion of this coordinate energy is

$$\delta E = -\tfrac{1}{2}\delta M_I^{ij}\beta^i\beta^j \ . \tag{18}$$

The $\boldsymbol{\beta}$ dependence of the coordinate-independent relative frequency monitored in an atomic anisotropy experiment results from the combined effect of the energy shifts (18) suffered by each of the four states involved in the experiment's hyperfine transitions.

Since these transitions connect hyperfine states having essentially the same electrostatic binding energy, only that part of the energy shift (18) stemming from the trace-free part of the electrostatic self-energy tensor (15) is important. Effectively, the displacements (18) of such states are

$$\begin{aligned}\delta E &= \frac{\Omega_0}{2}\epsilon^{jac}\epsilon^{kbd}n_0^a n_0^b(\Omega_{\rm ES}^{cd}-\tfrac{1}{3}\Omega_{\rm ES}\delta^{cd})\beta^j\beta^k \\ &= \frac{\Omega_0}{2}\beta^2(\Omega_{\rm ES}^{cd}-\tfrac{1}{3}\Omega_{\rm ES}\delta^{cd})N_0^c N_0^d \ ,\end{aligned} \tag{19}$$

where $\mathbf{N}_0 = \hat{\mathbf{n}}_0 \times \hat{\boldsymbol{\beta}}$ is a vector directed toward the north ecliptic pole. If we neglect the slight eccentricity of the Earth's orbit, $\mathbf{N}_0$ is a unit vector.

This expression is the analogue of Eq. (1) in Ref. 5. Note that the nucleus provides the dominant contribution to an atom's electrostatic self-energy tensor. Inserting the estimate of the trace-free part of the nuclear electrostatic self-energy tensor employed in Ref. 5, we find that

$$\delta E = \frac{\Omega_0}{12}\beta^2\left[\frac{3m_I^2 - I(I-1)}{I(2I-1)}\right]\frac{(Z-1)e^2Q}{R^3}P_2(\cos\theta_N) \ , \tag{20}$$

where $I$ denotes the nuclear spin and $m_I$ the magnetic quantum number of a hyperfine level. The symbols $Z$, $Q$, and $R$ denote the nuclear charge, quadrupole moment, and charge radius. The symbol $\theta_N$ represents the angle between $\mathbf{N}_0$ and the spin quantization axis defined by the laboratory magnetic field.

Equation (20) implies a quadrupolar dependence of the relative frequency monitored by an atomic anisotropy experiment on the angle between the laboratory magnetic field and $\mathbf{N}_0$. The Earth's rotation turns this orientation dependence into time dependence with a period tied to that of the sidereal day.

The factor $\Omega_0\beta^2$ in Eqs. (19) and (20) is determined by an atom's gravitational environment. Comparison of Eq. (19) with Eq. (1) in Ref. 5 reveals a correspondence between the factor $\Omega_0\beta^2$ and the factor $(1-T_0\epsilon_0\mu_0/H_0)V^2$ encountered in the interpretation of atomic anisotropy experiments in the context of the $TH\epsilon\mu$ formalism. In that context, the experiment of Lamoreaux[7] *et al.* constrains the factor $(1-T_0\epsilon_0\mu_0/H_0)V^2$ to be less than $6\times10^{-28}$. Since $V^2 \approx 10^{-6}$ and $\beta^2 \approx 10^{-8}$, we might naively expect an experiment of comparable precision to impose the constraint $\Omega_0 < 6\times10^{-20}$ on nonsymmetric theories of gravity. Actually, such an experiment would impose a constraint weaker by a factor 2, $\Omega_0 < 1.2\times10^{-19}$, because a fixed laboratory magnetic field cannot be oriented so that the Earth's rotation drives the function $P_2(\cos\theta_N)$ in Eq. (20) through more than half its range.

According to Moffat's NGT, in the neighborhood of the Earth the Sun contributes $\Omega_0 = l^4/R^4$, where $l^2$ is the charge of the Sun associated with NGT's fermion-number current and $R$ is the radius of the Earth's orbit. Moffat[15] favors the value $l=3230$ km which implies $\Omega_0 \approx 2\times10^{-19}$. In contrast, an experimental limit $\Omega_0 < 1.2\times10^{-19}$ would imply $l <$ 2850 km. Note that because NGT is so finely tuned, even a small reduction in the allowed range of values for $l$ challenges the theory.[16] Proposals to improve the accuracy of atomic anisotropy experiments[6-8] suggest that a new experiment undertaken specifically to test NGT is likely to push the limit on $l$ for the Sun well below 2850 km.

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