Vector-meson dominance, one-loop-order quark graphs, and the chiral limit

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We study six quark loop graphs involving the ρ meson coupling to quarks. In the chiral limit these graphs justify most aspects of the vector-dominance model.

The vector-dominance model (VDM) of the 1960s consists of four phenomenological observations.

(i) Universality of the ρ couplings to mesons, nucleons, and even leptons:[1]

$$g_{\rho\pi\pi}=g_{\rho NN}=g_\rho \ . \tag{1}$$

Present data[2] in fact require $g^2_{\rho\pi\pi}/4\pi \approx 2.9$ from $\Gamma(\rho\to\pi\pi)\approx 149$ MeV and $g^2_\rho/4\pi\approx 2.0$ from $\Gamma(\rho\to e^+e^-)\approx 6.8$ keV. The agreement in (1) is even better because the $\rho\bar{e}e$ coupling $g^2_\rho/4\pi$ is narrow-width corrected from 2.0 up to 2.4.

(ii) The ρ-photon analogy with γ-ρ vector vertex

$$\langle 0|V^{em}_\mu|\rho^0\rangle=\frac{em^2_\rho}{g_\rho}\epsilon_\mu \tag{2}$$

transforms a photon to a ρ^0 meson, such that the radiative $\rho\to\pi\gamma$ rate relative to the $\pi^0\to\gamma\gamma$ rate now predicts[3]

$$g_\rho/e=2F_{\rho\pi\gamma}/F_{\pi\gamma\gamma}\approx 17.8 \ , \tag{3}$$

due to the revised measured rate[2] $\Gamma(\rho\pi\gamma)\approx 67$ keV. In fact, the universal ρ couplings in (1) require this ratio to be close to (3) for $e^2=4\pi\alpha$, namely $g_\rho/e\approx 16.6-18.2$ and the VDM appears justified.

(iii) The VDM combined with current algebra further suggests the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation[4] accurate to the 10% level:

$$m^2_\rho=2g^2_\rho f^2_\pi \ , \tag{4}$$

where $f_\pi\approx 93$ MeV is the pion decay constant. However, this somewhat *ad hoc* connection between VDM and current algebra has yet to be rigorously justified.

(iv) The SU(3) extension of the VDM to ω and ϕ vector mesons in (i)–(iii) is also accurate within the 10% level.

In this paper we seek to unify the above VDM statements (i)–(iv) by computing various constituent quark loops and working in the chiral limit (CL). In this limit we shall always employ constant point coupling $g=g_{\pi qq}$ in the Goldberger-Treiman relation (GTR) at the quark level (with[5] $g_A=1$) $g=m_{\rm qk}/f_\pi$, where $m_{\rm qk}$ is the (nonrunning) quark mass in constituent quark loops. This holds in the linear σ model (LSM) at the quark level[6] or in the four-fermion Nambu–Jona-Lasinio (NJL) model.[7]

Consider first the quark loop for the pion decay constant f_π depicted in Fig. 1, which is proportional to $gm_{\rm qk}$. Since $f_\pi=m_{\rm qk}/g$ from the quark level GTR, one power of $m_{\rm qk}\neq 0$ cancels out leading to the log-divergent "gap equation" in such constituent-quark (or LSM-NJL) models.[6,7] In the chiral limit $q_\pi\to 0$, the resulting gap equation becomes[6]

$$1=-i4N_c g^2\int\frac{đ^4p}{(p^2-m^2_{\rm qk})^2} \ , \tag{5}$$

where $đ^4p=d^4p/(2\pi)^4$. In Eq. (5), N_c represents the number of colors traversing quark loops, and we have worked in the CL $q_\pi\to 0$ to simplify the momentum structure of (5). For $m_{\rm qk}=M_N/3\approx 313$ MeV and $f_\pi\approx 90$ MeV in the CL, the point coupling $g=m_{\rm qk}/f_\pi\approx 3.5$ substituted into (5) determines the implied UV cutoff in this log-divergent gap equation to be $\Lambda\sim 700$ MeV, reasonably close to the scalar σ mass.[5] This cutoff scale also holds for the NJL model.[8]

To proceed to vector currents, we take the point coupling of ρ to quarks as $\frac{1}{2}g_\rho\bar{\psi}\gamma_\mu\bar{\tau}\psi$. Then the $\rho\to\pi\pi$ meson *VPP* coupling $g_{\rho\pi\pi}\Delta_\mu$ (where $\Delta=k'-k$ is the difference of outgoing pion four-momenta) can be computed from the two constituent quark loops of Fig. 2. The particular (symmetric) choice of momentum routings in Fig. 2 is known to generate no artificial surface terms for $2k'=q+\Delta$, $2k=q-\Delta$, $q=k'+k$, and $q\cdot\Delta=0$ with propagator denominators

$$D_\pm=[(p+\tfrac{1}{2}q)^2-m^2_{\rm qk}][(p-\tfrac{1}{2}q)^2-m^2_{\rm qk}]\times[p^2+\tfrac{1}{4}\Delta^2-m^2_{\rm qk}\pm\Delta\cdot p] \ .$$

Then the trace terms of Fig. 2 are of the form

$$\frac{T_\mu(k',k)}{D_-}-\frac{T_\mu(k,k')}{D_+}=\frac{(p^2+\frac{1}{4}\Delta^2-m^2_{\rm qk})(p^2-\frac{1}{4}q^2-m^2_{\rm qk})\Delta_\mu}{(p^2+\frac{1}{4}q^2-m^2_{\rm qk}+p\cdot q)(p^2+\frac{1}{4}q^2-m^2_{\rm qk}-p\cdot q)(p^2+\frac{1}{4}\Delta^2-m^2_{\rm qk}-\Delta\cdot p)(p^2+\frac{1}{4}\Delta^2-m^2_{\rm qk}+\Delta\cdot p)} \ . \tag{6}$$

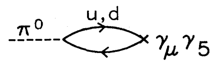

FIG. 1. Quark loop for the axial-vector transition $\langle 0|A_\mu^3|\pi^0\rangle = if_\pi q_\mu$.

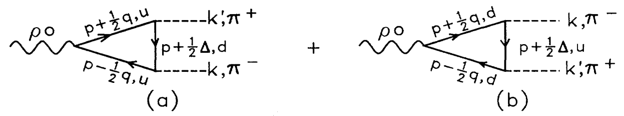

FIG. 2. Quark loops for the coupling constant $g_{\rho\pi\pi}$.

It is important to appreciate that the relative antisymmetrization between Fig. 2(a) and 2(b) cancels the otherwise linear divergent parts of the quark triangle graphs. After isolating the $\Delta_\mu = (k'-k)_\mu$ covariant in $g_{\rho\pi\pi}\Delta_\mu$ and in (6), the soft chiral-limiting ($k,k' \to 0$) value of the $\rho\pi\pi$ coupling constant is found from standard Feynman rules:

$$g_{\rho\pi\pi} \to -i\tfrac{1}{2}g_\rho(\sqrt{2}g)^2 4N_c \int d^4p\,(p^2 - m_{\mathrm{qk}}^2)^{-2} \ . \tag{7}$$

This result (7) is general to any choice of the SU(2) charge states of ρ and π. At the (constituent) quark (LSM or NJL) level we can now merge together Figs. 1 and 2 in the CL. More specifically, substituting the log-divergent gap equation (5) into the loop-graph result (7) for $g_{\rho\pi\pi}$ (also log divergent) we immediately deduce $g_{\rho\pi\pi} \to g_\rho$ in the CL, which is part of the VDM universality statement (1).

Once VDM universality is a consequence of the chiral limit (the $q^2 \to 0$ VDM limit also follows from the soft CL $k \to 0$, $k' \to 0$ by momentum conservation $q = k' + k$), the standard KSRF relation (4) must also hold. In particular, $g_{\rho\pi\pi} \to g_\rho$ by (1) and likewise $g_{\rho NN} \to g_\rho$ and then the VDM requires the low-energy isospin-odd πN scattering amplitude to be $\nu^{-1}F_{\pi N}^{(-)} \approx g_\rho^2/m_\rho^2$. But the soft CL along with the slightly stronger statement of chiral symmetry automatically requires current algebra (CA) partial conservation of the axial-vector current (PCAC) to hold. The latter CA-PCAC assumption is known to imply that the above low-energy πN amplitude is $\nu^{-1}F_{\pi N}^{(-)} \approx 1/2f_\pi^2$. Thus, chiral symmetry [which includes the CL and (1)] along with the (chiral-symmetric) linkage[9] of the above VDM and CA-PCAC expressions for $\nu^{-1}F_{\pi N}^{(-)}$ directly predict the KSRF relation (4), which is the VDM statement (iii).

Apart from VDM statements (i) and (iii), we can also justify the VDM statement (ii) to one-loop order in the quark model in the CL. In Fig. 3 we display the *PVV* loop graphs corresponding to $\pi^0 \to \gamma\gamma$ and $\rho \to \pi\gamma$ decays. The ratio of these one-loop-level diagrams just reproduces the $2F_{\rho\pi\gamma}/F_{\pi\gamma\gamma}$ amplitude ratio of Ref. 3 according to the ρ-γ g_ρ/e analogy in Eq. (2). Of course the actual one-loop level graph of Fig. 3(a) is Steinberger's version[10] of the Adler-Bell-Jackiw (ABJ) anomaly in the CL,[11] but with quarks instead of nucleons traversing the triangle with[5] $g_A = 1$ and $g_{\pi qq} = m_{\mathrm{qk}}/f_\pi$ is the GTR at the quark level. Using the latter in Fig. 3(a) gives the ABJ amplitude $|F_{\pi\gamma\gamma}| = (\alpha/\pi f_\pi) \approx 0.025$ GeV^{-1}, within 2% of the observed $\pi^0 \to 2\gamma$ amplitude.[2] Now since $|F_{\rho\pi\gamma}| \approx 0.222$ GeV^{-1} from experiment[2] and from $\Gamma_{\rho\pi\gamma} = k^3F_{\rho\pi\gamma}^2/12\pi$, the numerical ρ-γ analogy (3) follows. In short, the VDM ρ-γ analogy replacing the electromagnetic coupling $e\gamma_\mu$ by the strong vector coupling $g_\rho\gamma_\mu$ to quarks clearly corresponds to the ratio of the CL quark triangle graphs in Fig. 3.

Moreover, the new measurements[2] of $\omega \to \pi\gamma$ and $\omega \to e\bar{e}$ allow one to extend the ρ-γ analogy to ω-γ according to VDM statement (iv). Specifically the decay rate $\Gamma(\omega \to \pi\gamma) \approx 717$ keV predicts

$$g_\omega/e = 2F_{\omega\pi\gamma}/F_{\pi\gamma\gamma} \approx 56.3 \pm 1.7 \ , \tag{8}$$

in excellent agreement with $g_\omega/e = 56.3 \pm 1.2$ obtained from the new width[2] $\Gamma(\omega \to e\bar{e}) \approx 0.6$ keV substituted in $\Gamma_{\omega ee} = \frac{1}{3}\alpha^2(g_\omega^2/4\pi)^{-1}m_\omega$.

Finally the loop diagram of Fig. 4 is the ρ-meson transition amplitude $\langle 0|V_\mu^{em}|\rho^0\rangle$ at the one-loop quark level. This amplitude is given by

$$\frac{m_\rho^2}{g_\rho}\epsilon_\mu = -i\frac{g_\rho}{2}4N_c\epsilon^\nu\frac{1}{(2\pi)^4}\int d^4p\,\frac{2p_\mu p_\nu + k_\nu p_\mu + k_\mu p_\nu + g_{\mu\nu}(m_{\mathrm{qk}}^2 - p^2 - p\cdot k)}{[(p+k)^2 - m_{\mathrm{qk}}^2](p^2 - m_{\mathrm{qk}}^2)} \ , \tag{9}$$

where m_{qk} is the (CL) constituent quark mass. The structure of (9) is quite similar to Fig. 1 and the consequent gap equation (5) except that (9) is quadratically divergent as the dimensionless UV cutoff $\Lambda \to \infty$. As such, the value of (9) depends on the surface terms in the $\Lambda \to \infty$ limit when one combines the denominators in (9). However, with Λ being finite, surface terms do not exist.

FIG. 3. Quark triangle graphs for the radiative transitions (a) $\pi^0 \to \gamma\gamma$ and (b) $\rho \to \pi\gamma$.

We then calculate (9) on the ρ mass shell $k^2 = m_\rho^2$ using the standard Feynman procedure. Keeping Λ as a parameter, (9) becomes

$$\frac{m_\rho^2}{g_\rho^2(k^2 = m_\rho^2)} = \frac{2N_c m_{\mathrm{qk}}^2}{16\pi^2}f(\Lambda) \ , \tag{10}$$

where $f(\Lambda)$ is the integral in (9) with $k^2 = m_\rho^2 \simeq 5.1m_{\mathrm{qk}}^2$. The integral for $f(\Lambda)$ is quite tedious and lengthy. In-

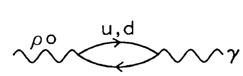

FIG. 4. Quark loop for the vector transition $\langle 0|V_\mu^3|\rho^0\rangle = (m_\rho^2/g_\rho)\epsilon_\mu$.

stead we show its numerical evaluation in Fig. 5. For the invariant cutoff $\Lambda^2 \approx (700\ \mathrm{MeV}/m_{\mathrm{qk}})^2 \approx 5$ as already found in (5), $f(\Lambda) \simeq 2.4$ and we find $g_\rho(m_\rho^2) \simeq 7.5$ from Eq. (10). In the σ-model range $\Lambda^2 = 3-5$, the value g_ρ shifts by only 6% (i.e., $g_\rho \simeq 7.0-7.5$). Such a cutoff scale also appears to hold[8] for the quadratically divergent NJL four-fermion model. This range of values for g_ρ is also compatible with the result $g_\rho \sim 2\pi \sim 6.3$ in the large-N_c limit.[12] We believe it significant that the above one-loop mass-shell value $g_\rho \sim 7$ is near the $\rho e\bar{e}$ and large-N_c values $g_\rho \sim 5$ to 6, and that all these g_ρ estimates are close to the universality coupling $g_{\rho\pi\pi} \approx 6.0$ as obtained from the ρ width. Thus, Fig. 4 with a cutoff consistent with the CL gap equation (5) also appears compatible with the VDM.

With hindsight our *chiral-limiting* quark loop analysis, while employing the GTR $f_\pi g = m_{\mathrm{qk}}$ and slightly simplifying the momentum structure of (3), (5), and(7), is primarily needed together with chiral symmetry to derive the KSRF relation (4). In fact, the soft momentum limit $q_K \to 0$ also links the radiative decay $K^* \to K\gamma$ [analogous to Fig. 3(b)] amplitude ratio[13] $R = |F_{K^{*0}K^0\gamma}/F_{K^{*+}K^+\gamma}|_{q_K \to 0} \approx 1.51$ to the VDM ratio[14] $R_{q_{K^*}^2=0} = 2(1-\delta/2) \approx 1.5$ for the usual constituent quark-mass ratio $(m_s/\hat{m}) = 1+\delta \approx 1.5$. Both the chiral-limiting and VDM ratios above are close to the observed $K^* \to K\gamma$ ratio[2] $R = 1.53 \pm 0.11$.

However, the slightly weaker massless pion assumption $m_\pi^2 = 0$ (rather than $q_\pi \to 0$) is all that is required to derive VDM universality (1). More specifically, replacing the ρ^0 in Fig. 2 by an off-shell photon with squared invariant momentum k^2 and also crossing the outgoing π^- to an incoming π^+, the gauge-invariant pion current $F_\pi(k^2)\,(q'_\pi + q_\pi)_\mu$ is generated by the quark-loop graph when $m_\pi^2 = 0$, with the pion form factor[15]

$$F_\pi(k^2) = -i4N_c g^2 \int_0^1 dx \int d^4p\,[p^2 - \hat{m}^2 + x(1-x)k^2]^{-2} . \qquad (11)$$

To derive (11) a rerouting of the loop momentum p is needed to eliminate the linearly divergent surface term,[15] which is the analog of the antisymmetrization in (6). Then at $k^2 = 0$, charge conservation means that (11) becomes

$$F_\pi(0) = 1 = -i4N_c g^2 \int d^4p\,(p^2 - \hat{m}^2)^{-2} . \qquad (12)$$

The latter is equivalent to our gap equation (5) derived from the CL GTR, only now the nonstrange constituent mass $\hat{m}$ in (11) and (12) is the chiral-broken extension of m_{qk} in (5). Consequently, the VDM universality (7) or $g_{\rho\pi\pi} = g_\rho$ is now a consequence of charge conservation $[F_\pi(0) = 1]$ and is true in any theory with a vector gauge invariance (as is well known). However, the latter is true in our quark-loop model only if $m_\pi^2 = 0$.

One final (the sixth) connection between VDM and the CL quark loop model is the pion (π^+) charge radius. The VDM value is $r_{\pi^+} = \sqrt{6}/m_\rho \approx 0.63$ fm, very near the experimental charge radius[16] $r_{\pi^+} \approx 0.66$ fm. For the CL quark-loop graph of Fig. 2, differentiating the pion form factor (11) immediately leads to[15]

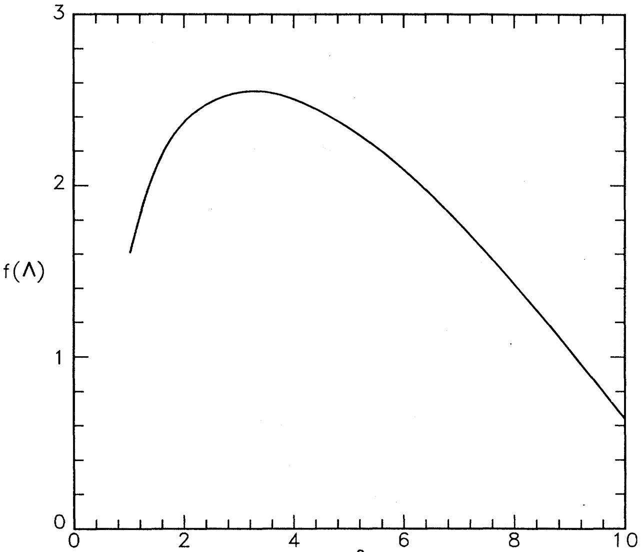

FIG. 5. Plot of integral in Eq. (9) versus the dimensionless cutoff Λ.

$$\langle r_{\pi^+}^2 \rangle = 6 \left. \frac{dF_\pi(k^2)}{dk^2} \right|_{k^2=0} = \frac{N_c}{4\pi^2} \frac{g^2}{m_{\mathrm{qk}}^2} = \frac{3}{(2\pi f_\pi)^2} \approx (0.60\ \mathrm{fm})^2 , \qquad (13)$$

for $N_c = 3$ and the CL value[17] $f_\pi \approx 90$ MeV. In fact, this successful quark-loop result (13) was first derived eleven years ago[18] and extended to kaon charge radii.[18,19] The latter approach, however, did not link the r_{π^+} calculation to VDM universality (1).

In conclusion, we have shown that the six chiral-limiting quark graphs for f_π, $\rho \to 2\pi$, $\gamma \to \pi\pi$, $\pi^0 \to 2\gamma$, $\rho \to \pi\gamma$, and $\rho^0 \to \gamma$ transitions lead to six relations linked to the VDM: (i) $g_{\rho\pi\pi} = g_\rho$ universality, (ii) ρ-γ analogy for $g_\rho/e \approx 18$, (iii) KSRF $m_\rho^2 = 2g_\rho^2 f_\pi^2$, (iv) SU(3) extension to ω and ϕ, (v) $K^* \to K\gamma$ ratio, and (vi) π^+ charge radius r_π^+ and extensions to K charge radii.

Other approaches using Lagrangians instead of CL quark loops also recover some of the above VDM results. A hidden local symmetry scheme[20] obtains the KSRF relation for a parameter value $a = 2$, while the physics of the anomalous $\gamma\pi\pi\pi$ vertex is shown[21] to be consistent with KSRF. This KSRF result apparently is countered by a "vector limit" approach.[22] Finally a superconductivity-type effective Lagrangian has been employed[23] to simulate many properties of the VDM. From our perspective, however, only the CL quark loop scheme described in this paper appears to recover all six (experimentally observed) aspects of the VDM without the introduction of extra parameters.

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