PHYSICAL REVIEW D VOLUME 43, NUMBER 7 1 APRIL 1991

BRIEF REPORTS

Brief Reports are short papers which report on completed research or are addenda to papers previously published in the **Physical Review**. *A Brief Report may be no longer than four printed pages and must be accompanied by an abstract.*

Cluster cascading in the geometrical branching model

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(Received 24 August 1990)

The geometrical branching model is further developed in a direction that allows the branching part to be implemented by a Monte Carlo scheme of cascading cluster decays. A universal mass distribution is adopted for the determination of cluster masses. Good agreements with data on multiplicity distribution and average multiplicity are achieved.

Multiparticle production at low p_T in high-energy hadronic collisions has been described by a large number of models, all of which can reproduce the gross features very well.[1] The geometrical branching model[2] (GBM) is one among them, which combines the geometrical properties of hadronic collisions[3,4] with the stochastic properties of particle production.[5] After the suggestion by Bialas and Peschanski[6] to investigate intermittency in multiplicity fluctuations in various scales of resolution, experimental data on normalized factorial moments have revealed power-law behavior in leptonic, hadronic, and nuclear processes.[7] Since random cascade processes can lead to intermittency, the GBM is well poised to account for the observed effect in small rapidity intervals. However, before a Monte Carlo code can be developed to demonstrate intermittency, there is one intermediate step that must be taken to render the GBM suitable for such considerations. This is, the branching process, which was summarized by the Furry distribution previously,[2] must now be implemented by a specific scheme of successive branching. It is the aim of this Brief Report to accomplish this limited objective.

Our basic input will be the massive cluster decays in cascading. This mechanism in itself is, of course, not new. Ochs and Wosiek[8] have already shown that it can lead to intermittency. Our concern at this point is not so much intermittency as the formulation of GBM in such a way that successive cluster decay can be amalgamated with the Glauber-Gribov approach[4,9,10] to high-energy collisions. Our immediate aim is to show that a sensible cluster cascading scheme can be described to reproduce the multiplicity distribution in the whole rapidity space. The subject of fluctuations in small rapidity intervals is deferred to a future investigation.

Let us first recall the eikonal description of σ_{in}, which is related by the Abramovski-Gribov-Kancheli (AGK) cutting rule[9] to the eikonal function $\Omega(b,s)$ for the elastic amplitude by[10]

$$\sigma_{\text{in}} = \int d^2b \sum_{\mu=1}^{\infty} \frac{1}{\mu!} [2\Omega(b,s)]^{\mu} e^{-2\Omega(b,s)} , \tag{1}$$

where the summand is the contribution from μ-cut Pomerons. To determine the multiplicity distribution P_n, it is necessary to specify what a cut Pomeron is, for which only model descriptions have been given. For example, in the dual-parton model[11] (DPM), which is formulated in the momentum space, the lowest-order contribution to a cut Pomeron consists of two chains stretched between quark-diquark pairs. In the GBM we have represented the multiplicity distribution of each cut Pomeron by the Furry distribution[2] $F_{n_j}^{k_j}$, whose self-reproducing property gives rise to an effective $F_n^{k(\mu)}$ for a μ-cut Pomeron contribution, where $k(\mu)$ is a parameter denoting the number of initial branching clusters. Thus, if we use $\pi_{\mu}(b)$ to signify the summand in Eq. (1), we have[2]

$$P_n = \frac{1}{\sigma_{\text{in}}} \int d^2b \sum_{\mu=1}^{\infty} \pi_{\mu}(b) F_n^{k(\mu)} . \tag{2}$$

By adjusting $k(\mu)$ it has been possible to fit the data on

P_n throughout the CERN ISR energy range and then with minijet contributions in the CERN SPS collider range also.

As an initial attempt to combine geometrical and stochastic properties of multiparticle production, it was sensible to let the Furry distribution represent approximately the effects of branching so that we could judge quickly whether we were aiming in the right direction. Now we begin a second-level consideration in which the branching process is to be treated explicitly in a Monte Carlo simulation with energy-momentum conservation applied at each step. There are two issues to be addressed. First, how many initial clusters are there and what are their masses? Second, how does a cluster decay? Since these questions are central to our improved treatment, let us discuss them at some length separately.

Since the average number of cut Pomerons at each impact parameter b is specified by the probability $\pi_\mu(b)$, we know that μ can vary on the average from 1 to a large value at small b. If a massive cluster is associated with each cut Pomeron, then impact-parameter smearing results in an effective distribution of initial clusters for each collision. This picture is different from, and perhaps complementary to, the two currently fashionable models: the DPM (Ref. 11) and the FRITIOF models,[12] which have no explicit impact-parameter smearing, but either have two chains with varying lengths or have two excited strings with varying effective masses. In our model we use $\pi_\mu(b)$ and stochasticity to determine the number of initial clusters. More specifically, our procedure is to first convert Eq. (1) into an integral over the scaled impact parameter $R=b/b_0$, where $b_0=(\sigma_{\rm in}/\pi)^{1/2}$, as in Ref. 2. Then we rewrite Eq. (2) in the form

$$P_n=\int_0^\infty dR^2\sum_{\mu=1}^{\infty}\pi_\mu(R)B_n^\mu\ , \tag{3}$$

$$\pi_\mu(R)=\frac{1}{\mu!}[2\Omega(R)]^\mu e^{-2\Omega(R)}\ , \tag{4}$$

where B_n^μ is a new branching distribution that we now attempt to determine by Monte Carlo calculation. For every value of R, we generate a value for μ according to the probability $\pi_\mu(R)$. We then have μ initial clusters, each having c.m. energy E_i, whose value is specified by a randomly chosen variable x_i according to $E_i=x_i\sqrt{s}$, satisfying $0<x_i<1$ and $\sum_{i=1}^{\mu}x_i=1$. These clusters undergo successive binary decays in a manner to be described below, and the distribution B_n^μ can then be calculated at the end of the branching process. Note that this procedure automatically gives rise to diffractivelike processes at large R, since $\mu=1$ is the only important contribution to Eq. (3) in this case, and with $x_1=1$ one-cut Pomeron gives rise to a two-cluster fragmentation process. At smaller R higher μ values also become important, and so the sharing of $\sqrt{s}$ among the μ initial clusters forces the decay particles to populate the central region, resulting in a central plateau for the single-particle inclusive distribution. Momentum conservation is imposed in each cut Pomeron when the clusters undergo cascading.

The second issue of major importance is the nature of cluster decay. In a soft hadronic process no high virtuality is associated with any constituents. The valence quarks that carry nearly half the incident momentum do not interact strongly with soft partons because of the large separation in rapidity; they hadronize into the leading particles in the fragmentation region. A description of the hadronization in the central region that does not give any preeminent role to any individual partons would consider the interacting partons collectively in terms of s-channel clusters that evolve in a self-similar way. This point of view has received some support from the recent interest in intermittency, which is essentially the phenomenology of self-similarity. As mentioned in the previous paragraph, the leading cluster and the centrally produced ones are all included in our approach, since for every R, μ is summed over all values, the $\mu=1$ term contributing principally to the fragmentation region.

Since we want to describe the cluster decay by a self-similar process, we recall the statistical boost-trap model of Hagedorn,[13] who has for many years emphasized the picture of fireball within fireballs in a self-consistent description. Indeed, it was Hagedorn who first pointed out that the mass spectrum of hadrons is an exponentially growing one: e^{m/T_0}, where T_0 is the Hagedorn temperature.[14] The distribution of cluster masses at temperature T is

$$\rho(m)\sim m^\alpha e^{(1/T_0-1/T)m}\ , \tag{5}$$

where $T<T_0$ and we shall consider T to be a constant in

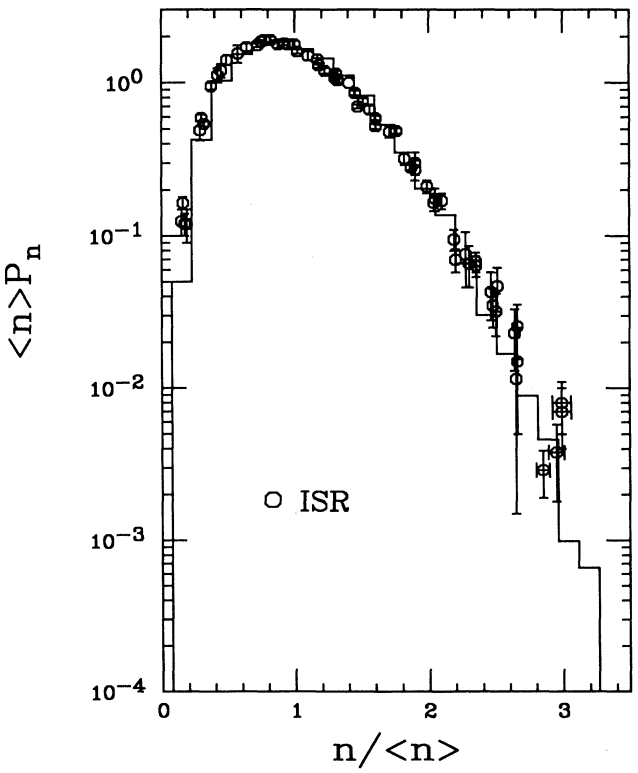

FIG. 1. KNO plot of the calculated result (in histogram) as compared to the data, taken from Ref. 15.

the hadronic interaction. We adopt the general ideas of Hagedorn as hints on our cluster decays and use

$$\rho(m) \sim m^{\alpha} e^{-\beta m} , \tag{6}$$

as the universal mass distribution of clusters produced at all stages of the successive decays. In Eq. (6), α and β are parameters in our model to be adjusted to fit the data.

Specifically, our procedure is the following. For each set of values of μ and E_i, $i=1,\ldots,\mu$, generated in our Monte Carlo simulation, we let the initial cluster in the ith-cut Pomeron decay into two clusters whose masses m_1 and m_2 are determined randomly according to Eq. (6). Energy-momentum conservation then fixes their longitudinal momenta, their transverse momenta being neglected in this investigation. We then go to the rest frames of the daughter clusters and repeat the decay procedure, always using Eq. (6) to determine the subsequent cluster masses. The decay sequence is terminated when the mass reaches a value below $2m_{\pi}$. In this way we determine B_n^{μ} in Eq. (3). Because we have not considered the charge and flavors of the produced hadrons, we take all the final particles to be pions and regard the number of charged pions as $\frac{2}{3}$ of the total produced particles.

We have found that by choosing $\alpha=0.1$ and $\beta=0.01$ GeV^{-1} we obtain approximately Koba-Nielsen-Olesen (KNO) scaling for pp collision in the ISR energy region. The resultant multiplicity distribution agrees well with the data,[15] as shown in Fig. 1. The normalized moments $C_q=\langle n^q\rangle/\langle n\rangle^q$ are shown in Fig. 2 as functions of $\sqrt{s}$. The average multiplicity $\langle n\rangle$, shown in Fig. 3, is also in good agreement with the data.[15] At higher energies minijet production must also be taken into consideration, and so P_n is expected to get broader.[2]

We have not fine tuned the parameters to yield the best fit because our present calculation ignores the transverse momenta of clusters. We are encouraged by our results, since they indicate that the GBM, when implemented by cascade cluster decays in one dimension, can well describe the global data on P_n and $\langle n\rangle$. This is an improvement on our previous results, since we could not calculate $\langle n\rangle$ before. More importantly, we now have a realistic scheme of branching, which will facilitate further development into a comprehensive code. For that we need to improve on our consideration of the transverse degrees of freedom, the production of resonances, the inclusion of charges and flavors, etc. After those improvements are made, we shall then be able to calculate more reliably the rapidity distribution and the associated quantities related to intermittency. The fact that this

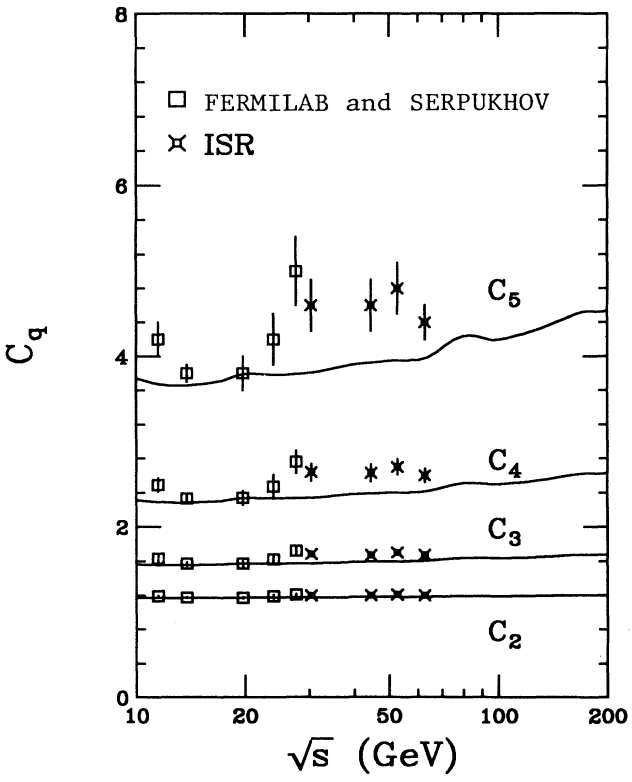

FIG. 2. Normalized moments of multiplicity distributions at various energies. The data are taken from Ref. 15.

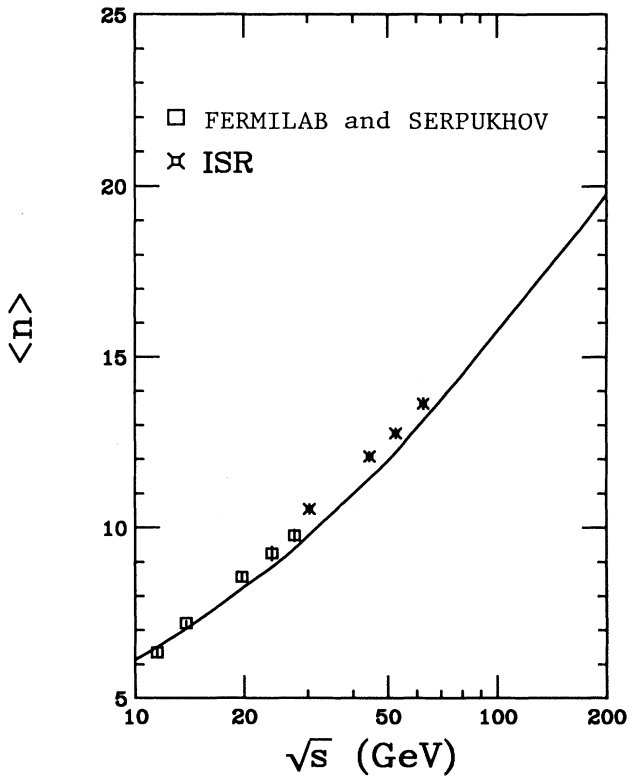

FIG. 3. Comparison of calculated average multiplicity (solid line) with the data, taken from Ref. 15.

model treats the geometrical features of hadronic collisions properly and contains a self-similar scheme of particle production makes us feel quite hopeful that this approach will render a successful description of the soft interaction of hadrons.

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contracts Nos. DE-AC03-76SF00098 and DE-FG06-85ER-4022.

[1]Most of the models are described in review articles in *Hadronic Multiparticle Production,* edited by P. Carruthers and J. Rafelski (Advanced Series on Directions in High Energy Physics, Vol. 2) (World Scientific, Singapore, 1988).

[2]W. R. Chen and R. C. Hwa, Phys. Rev. D **36**, 760 (1987); R. C. Hwa, *ibid.* **37**, 1830 (1988); X. N. Wang and R. C. Hwa, *ibid.* **39**, 187 (1989); W. R. Chen and R. C. Hwa, *ibid.* **39**, 179 (1989); W. R. Chen, R. C. Hwa, and X. N. Wang, *ibid.* **38**, 3394 (1988); X. N. Wang, *ibid.* **43**, 104 (1991).

[3]T. T. Chou and C. N. Yang, Phys. Rev. Lett. **20**, 1213 (1968); Phys. Rev. **170**, 1591 (1968); L. Durand and R. Lipes, Phys. Rev. Lett. **20**, 637 (1968).

[4]R. J. Glauber, in *Lectures in Theoretical Physics,* edited by W. E. Brittin and L. G. Danham (Interscience, New York, 1959), Vol. I; R. J. Glauber and G. Mathiae, Nucl. Phys. **B21**, 135 (1970).

[5]See Hwa (Ref. 1).

[6]A. Bialas and R. Peschanski, Nucl. Phys. **B273**, 703 (1986).

[7]For reviews, see experimental papers in *Multiparticle Dynamics–Festschrift for Léon Van Hove,* proceedings, La Thuile, Italy, 1989, edited by A. Giovannini and W. Kittel (World Scientific, Singapore, 1989); B. Buschbeck and P. Lipa, Mod. Phys. Lett. A **4**, 1871 (1989).

[8]W. Ochs and J. Wosiek, Phys. Lett. B **214**, 617 (1988).

[9]V. A. Abramovski, V. V. Gribov, and O. V. Kancheli, Yad. Fiz. **18**, 595 (1973) [Sov. J. Nucl. Phys. **18**, 308 (1974)].

[10]V. Anisovich, M. N. Kobrinsky, J. Nyiri, and Yu. M. Shabelski, *Quark Model and High Energy Collisions* (World Scientific, Singapore, 1985); V. M. Braun and Yu. M. Shabelski, Int. J. Mod. Phys. A **3**, 2417 (1988).

[11]A. Capella and J. Tran Thanh Van, Z. Phys. C **10**, 249 (1981); see also A. Capella *et al.*, in *Hadronic Multiparticle Production* (Ref. 1).

[12]B. Andersson, G. Gustafson, and B. Nilsson-Almqvist, Nucl. Phys. **B281**, 289 (1987).

[13]R. Hagedorn, Nuovo Cimento Suppl. **3**, 147 (1965); Riv. Nuovo Cimento **6**, 1 (1983).

[14]R. Hagedorn, Astron. Astrophys. **5**, 184 (1970).

[15]G. J. Alner *et al.*, Phys. Lett. **160B**, 193 (1985); **167B**, 476 (1986).