Anomalous magnetic moment of the muon arising from the extensions of the supersymmetric standard model based on left-right symmetry

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(Received 10 September 1990)

Extensions of the supersymmetric standard model to $SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$ and to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with Higgs triplets are considered. Calculations of all possible contributions to the anomalous magnetic moment of the muon are made and the resulting constraints on the masses of supersymmetric partners are examined in detail.

I. INTRODUCTION

Supersymmetric models are inevitably used to examine the muon anomaly. Although the experimental measurements are not quite as accurate as in the electron case, the contributions of supersymmetry are considerably larger and indeed are likely to be observable in the new generation of $g-2$ experiments presently begun.

In the present paper the muon anomalous magnetic moment and the constraints it imposes on the supersymmetric partner masses are examined in the context of an extension of the standard supersymmetric model to contain the left-right model $[SU(2)_L \times SU(2)_R]$.

The paper is organized as follows. In Sec. II the rationale for using locally broken supersymmetry (supergravity) to solve the gauge hierarchy problem, the reason for considering the left-right extension of the standard model, and a full description of the particle content and Lagrangian are given. In Sec. III the mass mixing matrices for particles used in the model are developed. In Sec. V the calculation of $(g-2)_\mu$ and the resulting constraints[1] on the masses of supersymmetric partners are presented. Conclusions and prospects are found in Sec. V.

II. DESCRIPTION OF THE MODEL

There is no doubt that the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge model is an extremely successful theory reproducing all the previously known features of electroweak theory and predicting new ones, since confirmed. Successful as it is, there are many reasons to believe that this theory is not complete.

The scalar sector of the standard model is the least tested part of the model. The Higgs particles have several nice properties, among which is the ability to possess a nonvanishing vacuum expectation value (VEV) without breaking Lorentz invariance. But their masses are subject to quadratic divergences in perturbation theory which would push them to orders of the Planck mass, unless the perturbation theory series would cancel to 26 decimal places. [This is known as the gauge hierarchy problem (GHP).]

As a plausible way to incorporate gravity into unified models, but also to deal with the gauge hierarchy problem, the model proposed in this paper incorporates supersymmetry. Supersymmetry resolves the GHP by including a bosonic-fermionic partner for every fermion and/or boson in the nonsupersymmetric theory. Boson and fermion loops give contributions of opposite signs to the mass of the Higgs boson. In the unbroken supersymmetry limit, bosonic and fermionic partners have equal masses and couple with the same strengths, and so their corresponding perturbation theory series cancel exactly. In broken supersymmetry the superpartners no longer have the same mass, but the divergences are more "friendly" and the naturalness problem does not reemerge.

Exact supersymmetry would require the mass of the scalar partner of the electron (selectron) to be degenerate in mass with the electron. Such a light particle should have been observed by now. Moreover, as shown by Fayet,[2] exact supersymmetry would also mean that the anomalous magnetic moment of all the leptons would be identically zero. This occurs because every loop in the calculation of the magnetic moment has a counterpart loop composed of opposite-type particles to cancel them. As shown by Dimopoulos and Georgi,[3] global breaking of supersymmetry is unacceptable for three reasons.

(1) Such a model still contains light scalar particles, which should have been observed by now; in particular, a scalar partner of one of the quarks (squark) must always be lighter than the lightest up or down quark.

(2) In global supersymmetry the vacuum energy is an order parameter. Breaking occurs if $E_{\text{vac}} \neq 0$, implying a nonzero cosmological constant.

(3) As shown by Hawking *et al.*,[4] there remain quadratic divergences in the calculation of the Higgs-boson mass because of the couplings of the Higgs boson to gravitons. Such a coupling as seen in Fig. 1 yields a contribution of

$$\delta m_H^2|_{\text{gravity}} = O(\Lambda^4/m_{\text{Pl}}^2) , \tag{2.1}$$

where the Planck mass $m_{\text{Pl}} = (2.8\times 10^{18})/\sqrt{8\pi}$ GeV, in the absence of a cutoff $\Lambda = m_{\text{Pl}}$ and

$$\delta m_H^2|_{\text{gravity}} \approx (10^{19}\ \text{GeV})^2 .$$

In broken local supersymmetry, usually referred to as supergravity, a spin-$\frac{3}{2}$ partner to the graviton (gravitino)

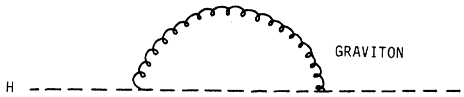


FIG. 1. Quadratic divergences coming from interaction of the Higgs particle with gravitons.

naturally occurs. The contribution to the Higgs-boson mass arising from the coupling of the graviton to the Higgs boson cancels the contribution from the gravitino.[5]

Consider then a supersymmetric extension of the standard model. The most general superpotential has the form

$$\begin{aligned} W = {} & h_u Q H_u U^c + h_d Q H_d D^c + h_e L H_d E^c \\ & + \mu_1 H_u H_d + \mu_2 H_u L + \not{l}_{pqr} Q_r L_q D_r^c \\ & + \tilde{h}_{[p,q]r} L_p L_q E_r^c + \lambda_1 E^c H_d H_d + \lambda_{[p,q]r} U_p^c D_q D_r^c \ . \end{aligned} \tag{2.2}$$

The last term violates baryon-number symmetry and corresponds to a rapid proton decay. The next to last four terms violate lepton number. If the coefficients μ_2, $\not{l}_{pqr}$, $\tilde{h}_{[p,q]r}$, λ_1, and $\lambda_{[p,q]r}$ are set to zero, the terms cannot be regenerated at the tree level because of the nonrenormalization theorem of supersymmetric field theory. Setting the coefficients to zero corresponds to the "standard" supersymmetric model.[6] Nevertheless, the theory is unsatisfactory as there is no theoretical justification for setting the coefficients to zero.

The simplest possible extension of the supersymmetric standard model in which no baryon-number- or lepton-number-violating terms are present *a priori* is based upon the gauge group $SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$. Calculations of the anomalous moment of the muon and neutral-current constraints in this model have been considered by Frank and Kalman.[7] Having introduced a partial right-handed symmetry $[U(1)_{I_{3R}}]$, it is interesting to consider the full left-right-symmetry extension of the standard model.

The original motivation for the introduction of left-right- (LR-) symmetric models based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ was to provide a possible mechanism for parity violation in weak interactions. In this framework the weak interaction respects all space-time symmetries, as do the strong, electromagnetic, and gravitational interactions. The asymmetry observed in nature at low energies is then attributed to the noninvariance of the vacuum under parity symmetry.[8] A bonus of this approach is that it reproduces all the features of $SU(2)_L \times U(1)_Y$ at low energies.

There are other important reasons for considering this kind of LR model. Foremost among them is the question of the neutrino mass:[9] If the neutrino has a mass, then this class of model becomes the most natural framework in which to work. In addition, if it turns out that quarks and leptons are themselves the results of a more fundamental substructure, and that the forces operating at the substructure level are similar to QCD,[10] then there are strong arguments which point to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ as the weak-interaction symmetry, rather than $SU(2)_L \times U(1)_Y$. The $B-L$ quantum number[11] (baryon number minus lepton number) is the only anomaly-free quantum number left ungauged in the standard model, a fact which seems to suggest a deeper symmetry structure. By replacing the gauge generator $U(1)_Y$, which has no physical significance, with $U(1)_{B-L}$, all the generators of the theory acquire a physical meaning.

Another compelling reason to consider LR models is found in *CP* violation. In the Kobayashi-Maskawa (KM) parametrization of generation mixing, for three generations, all *CP* violations are dependent on only one parameter δ_{KM} (the KM phase) and there is no hint as to why the observed *CP* violation has milliweak strength. The LR model can give rise to *CP* violation for only two generations and can account for its strength by relating it to the suppression of $V+A$ currents.[12]

Proceeding to study the supersymmetric left-right model, the particle content of the model is given in Table I. The Higgs sector of the theory consists of two bidoublets

$$\Phi_u(\tfrac{1}{2},\tfrac{1}{2},0) \ , \quad \Phi_d(\tfrac{1}{2},\tfrac{1}{2},0) \tag{2.3}$$

and four triplets

$$\begin{aligned} &\Delta_L(1,0,2) \ \text{ and } \ \Delta_R(0,1,2) \ , \\ &\delta_L(1,0,-2) \ \text{ and } \ \delta_R(0,1,-2) \ . \end{aligned} \tag{2.4}$$

Supersymmetry is responsible for the doubling in the number of Higgs fields; Φ_u and Φ_d are needed in order to give masses to both the up and down quarks, and δ_L and δ_R, with *B-L* quantum number -2, are introduced to cancel the anomalies in the fermionic sector that would otherwise occur. The gauge fields consist of an $SU(2)_R$ triplet $\mathbf{W}_R^\mu$, an $SU(2)_L$ triplet $\mathbf{W}_L^\mu$, and a $U(1)_{B-L}$ singlet V_μ. The gauge coupling constants are g_L, g_R, and g_V. The model is constructed in such a way that, before symmetry breaking, it contains three gauge symmetries and a discrete parity symmetry, i.e., $g_L = g_R$. The breaking of symmetry is accomplished in three stages:[13]

$$\begin{aligned} & SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \\ & \qquad \xrightarrow[M_P]{} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \ , \\ & SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow[M_{W_R}]{} SU(2)_L \times U(1)_Y \ , \\ & SU(2)_L \times U(1)_Y \xrightarrow[M_{W_L}]{} U(1)_{\text{em}} \ . \end{aligned} \tag{2.5}$$

At the first stage only the parity symmetry is broken (M_P is the mass scale at which this breaking occurs; no gauge boson of that mass is produced). This results in $g_L \neq g_R$, and leaves $\mathbf{W}_L$ and $\mathbf{W}_R$ massless. The second stage breaks $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$ and is achieved through $\langle \Delta_R \rangle \neq 0$. The Higgs multiplets can be chosen in such a way that the parity symmetry and $SU(2)_R$ are broken at the same scale, i.e., $M_P = M_{W_R}$. The final stage

TABLE I. Quantum numbers of particles.

Field	Component fields	$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ quantum number			Name
Matter					
Q_L	$\begin{bmatrix} u \\ d \end{bmatrix}_L$	$\frac{1}{2}$	0	$\frac{1}{3}$	left−handed up quark left−handed down quark
Q_R	$\begin{bmatrix} u \\ d \end{bmatrix}_R$	0	$\frac{1}{2}$	$\frac{1}{3}$	right−handed up quark right−handed down quark
L_L	$\begin{bmatrix} \nu \\ e \end{bmatrix}_L$	$\frac{1}{2}$	0	−1	left−handed neutrino left−handed electron
L_R	$\begin{bmatrix} \nu \\ e \end{bmatrix}_R$	0	$\frac{1}{2}$	−1	right−handed neutrino right−handed electron
$\tilde{Q}_L$	$\begin{bmatrix} \tilde{u} \\ \tilde{d} \end{bmatrix}_L$	$\frac{1}{2}$	0	$\frac{1}{3}$	left−handed up squark left−handed down squark
$\tilde{Q}_R$	$\begin{bmatrix} \tilde{u} \\ \tilde{d} \end{bmatrix}_R$	0	$\frac{1}{2}$	$\frac{1}{3}$	right−handed up squark right−handed down squark
$\tilde{L}_L$	$\begin{bmatrix} \tilde{\nu} \\ \tilde{e} \end{bmatrix}_L$	$\frac{1}{2}$	0	−1	left−handed *s* neutrino left−handed *s* electron
$\tilde{L}_R$	$\begin{bmatrix} \tilde{\nu} \\ \tilde{e} \end{bmatrix}_R$	0	$\frac{1}{2}$	−1	right−handed *s* neutrino right−handed *s* electron
Gauge					
W_L	W_L^+, W_L^-, W_L^0	triplet	singlet	singlet	gauge boson
W_R	W_R^+, W_R^-, W_R^0	singlet	triplet	singlet	gauge boson
V	V	singlet	singlet	singlet	gauge boson
λ_L	$\lambda_L^+, \lambda_L^-, \lambda_L^0$	triplet	singlet	singlet	gaugino
λ_R	$\lambda_R^+, \lambda_R^-, \lambda_R^0$	singlet	triplet	singlet	gaugino
λ_V	λ_V	singlet	singlet	singlet	gaugino
Higgs					
$\Phi_{u,d}$	$\begin{bmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{bmatrix}_{u,d}$	$\frac{1}{2}$	$\frac{1}{2}$	0	Higgs boson
Δ_L	$\begin{bmatrix} \frac{1}{\sqrt{2}}\Delta^+ & \Delta^{++} \\ \Delta^0 & -\frac{1}{\sqrt{2}}\Delta^+ \end{bmatrix}_L$	1	0	2	Higgs boson
Δ_R	$\begin{bmatrix} \frac{1}{\sqrt{2}}\Delta^+ & \Delta^{++} \\ \Delta^0 & -\frac{1}{\sqrt{2}}\Delta^+ \end{bmatrix}_R$	0	1	2	Higgs boson
δ_L	$\begin{bmatrix} \frac{1}{\sqrt{2}}\delta^- & \delta^0 \\ \delta^{--} & -\frac{1}{\sqrt{2}}\delta^- \end{bmatrix}_L$	1	0	−2	Higgs boson
δ_R	$\begin{bmatrix} \frac{1}{\sqrt{2}} & \delta^0 \\ \delta^{--} & -\frac{1}{\sqrt{2}}\delta^- \end{bmatrix}_R$	0	1	−2	Higgs boson
$\tilde{\Phi}_{u,d}$	$\begin{bmatrix} \tilde{\phi}_1^0 & \tilde{\phi}_1^+ \\ \tilde{\phi}_2^- & \tilde{\phi}_2^0 \end{bmatrix}_{u,d}$	$\frac{1}{2}$	$\frac{1}{2}$	0	Higgsino

Table I (*Continued*).

Higgs					
$\tilde{\Delta}_L$	$\begin{bmatrix} \frac{1}{\sqrt{2}}\tilde{\Delta}^{+} & \tilde{\Delta}^{++} \\ \tilde{\Delta}^{0} & -\frac{1}{\sqrt{2}}\tilde{\Delta}^{+} \end{bmatrix}_L$	1	0	2	Higgsino
$\tilde{\Delta}_R$	$\begin{bmatrix} \frac{1}{\sqrt{2}}\tilde{\Delta}^{+} & \tilde{\Delta}^{++} \\ \tilde{\Delta}^{0} & -\frac{1}{\sqrt{2}}\tilde{\Delta}^{+} \end{bmatrix}_R$	0	1	2	Higgsino
$\tilde{\delta}_L$	$\begin{bmatrix} \frac{1}{\sqrt{2}}\tilde{\delta}^{-} & \tilde{\delta}^{0} \\ \tilde{\delta}^{--} & -\frac{1}{\sqrt{2}}\tilde{\delta}^{-} \end{bmatrix}_L$	1	0	−2	Higgsino
$\tilde{\delta}_R$	$\begin{bmatrix} \frac{1}{\sqrt{2}}\tilde{\delta}^{-} & \tilde{\delta}^{0} \\ \tilde{\delta}^{--} & -\frac{1}{\sqrt{2}}\tilde{\delta}^{-} \end{bmatrix}_R$	0	1	−2	Higgsino

of breaking is brought about by $\langle\Phi\rangle\neq 0$ and (but this is not essential) $\langle\Delta_L\rangle\neq 0$.

As in the standard model, in order to ensure that $U(1)_{em}$ remains unbroken, only the neutral Higgs fields are allowed to have nonzero VEV's. These values are

$$\langle\Delta_L\rangle=\begin{bmatrix}0 & 0\\ v_L & 0\end{bmatrix},\quad \langle\Delta_R\rangle=\begin{bmatrix}0 & 0\\ v_R & 0\end{bmatrix},\quad \langle\Phi\rangle=\begin{bmatrix}\kappa & 0\\ 0 & \kappa' e^{i\alpha}\end{bmatrix}. \tag{2.6}$$

$\langle\Phi\rangle$ causes $\mathbf{W}_L$ and $\mathbf{W}_R$ to mix with a *CP*-violating phase $e^{i\alpha}$.

It should be noted that all LR models contain a bidoublet field Φ, whereas the additional Higgs fields can be members of doublets or triplets. The choice of a triplet representation is preferred because, as Mohapatra and Senjanovic[9] show, it has the ability to generate a large Majorana mass for v_R and, at the same time, a small one for v_L, thus providing a natural explanation for the smallness of the v_L mass. The doublet-Higgs[14] representations generate only Dirac masses and, consequently, achieve the objective of a small v_L mass in a more contrived way.

The VEV's of the Higgs field used in this paper are then taken as

$$\langle\Delta\rangle_R=\begin{bmatrix}0 & 0\\ v_R & 0\end{bmatrix},\quad \langle\Delta_L\rangle=\langle\delta_{L,R}\rangle\equiv 0,$$
$$\langle\Phi_u\rangle=\begin{bmatrix}\kappa_u & 0\\ 0 & 0\end{bmatrix},\quad \langle\Phi_d\rangle=\begin{bmatrix}0 & 0\\ 0 & \kappa_d\end{bmatrix}. \tag{2.7}$$

For this, the following assignments have been made:

$$v_L=0,\quad \kappa'=0 .$$

The first one is a stringent case of the phenomenologically required hierarchy[15] $v_R \gg \max(\kappa,\kappa') \gg v_L$; the second one is due to required cancellation of flavor-changing neutral currents.

The LR-symmetric model presented here offers several interesting possibilities for a further refinement of electroweak theory. It is particularly appealing for the following two reasons: It restores parity to the status of a conserved quantum number in electroweak theory—just as it is in the other fundamental interactions—and it introduces $B-L$ as a generator of gauge symmetry.

The full Lagrangian is then

$$L=L_{\text{gauge}}+L_{\text{matter}}+L_y-V+L_{\text{soft}} . \tag{2.8}$$

A. Gauge Lagrangian

The first part of the Lagrangian concerns itself with the gauge fields. It contains the kinetic and self-interaction terms for the vector fields and the Dirac Lagrangian of the gaugino fields. The covariant derivative D_μ is of the general form $\partial_\mu+igT_aG_\mu^a$, where T_a are the generators of the gauge group and G_μ is the gauge field:

$$\begin{aligned}\mathcal{L}_{\text{gauge}}=&-\tfrac{1}{4}\mathbf{W}^L_{\mu\nu}\cdot\mathbf{W}^{\mu\nu}_L+\tfrac{1}{2}\bar{\lambda}_L\bar{\sigma}_\mu D^L_\mu\lambda_L\\ &-\tfrac{1}{4}\mathbf{W}^R_{\mu\nu}\cdot\mathbf{W}^{\mu\nu}_R+\tfrac{1}{2}\bar{\lambda}_R\sigma_\mu D^R_\mu\lambda_R\\ &-\tfrac{1}{4}V_{\mu\nu}V^{\mu\nu}+\tfrac{1}{2}\bar{\lambda}_V\bar{\sigma}_\mu\partial_\mu\lambda_V .\end{aligned} \tag{2.9}$$

B. Matter Lagrangian

This piece contains the kinetic terms for the fermionic and bosonic matter fields (the Higgs fields are also included in this category), as well as the interactions of the gauge and matter multiplets:

$$\begin{aligned}
\mathcal{L}_{\text{matter}} = &+ Q_L^\dagger \bar{\sigma}_\mu \left[\partial_\mu - \frac{ig_L}{2}\tau\cdot\mathbf{W}_\mu^L - \frac{ig_V}{6}V_\mu \right] Q_L + Q_R^\dagger \bar{\sigma}_\mu \left[\partial_\mu - \frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R - \frac{ig_V}{6}V_\mu \right] Q_R \\
&+ L_L^\dagger \bar{\sigma}_\mu \left[\partial_\mu - \frac{ig_L}{2}\tau\cdot\mathbf{W}_\mu^L + \frac{ig_V}{2}V_\mu \right] L_L + L_R^\dagger \bar{\sigma}_\mu \left[\partial_\mu - \frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R + \frac{ig_V}{2}V_\mu \right] L_R \\
&+ \mathrm{Tr}\left[(\tau\cdot\tilde{\Delta}_L)^\dagger \bar{\sigma}_\mu \left[\partial_\mu - \frac{ig_L}{2}\tau\cdot\mathbf{W}_\mu^L - ig_V V_\mu \right] \tau\cdot\tilde{\Delta}_L \right] + \mathrm{Tr}\left[(\tau\cdot\tilde{\delta}_L)^\dagger \bar{\sigma}_\mu \left[\partial_\mu - \frac{ig_L}{2}\tau\cdot\mathbf{W}_\mu^L + ig_V V_\mu \right] \tau\cdot\tilde{\delta}_L \right] \\
&+ \mathrm{Tr}\left[(\tau\cdot\tilde{\Delta}_R)^\dagger \bar{\sigma}_\mu \left[\partial_\mu - \frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R - ig_v V_\mu \right] \tau\cdot\tilde{\Delta}_R \right] + \mathrm{Tr}\left[(\tau\cdot\tilde{\delta}_R)^\dagger \bar{\sigma}_\mu \left[\partial_\mu - \frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R + ig_V V_\mu \right] \tau\cdot\tilde{\delta}_R \right] \\
&+ \mathrm{Tr}\left[\bar{\tilde{\Phi}}_u \bar{\sigma}_\mu \left[\partial_\mu - \frac{ig_L}{2}\tau\cdot\mathbf{W}_\mu^L - \frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R \right] \tilde{\Phi}_u \right] + \mathrm{Tr}\left[\bar{\tilde{\Phi}}_d \bar{\sigma}_\mu \left[\partial_\mu - \frac{ig_L}{2}\tau\cdot\mathbf{W}_\mu^L - \frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R \right] \tilde{\Phi}_d \right] \\
&+ \left| \left[\partial_\mu - \frac{ig_L}{2}\tau\cdot\mathbf{W}_\mu^L - \frac{ig_V}{6}V_\mu \right] \tilde{Q}_L \right|^2 + \left| \left[\partial_\mu - \frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R - \frac{ig_V}{6}V_\mu \right] \tilde{Q}_R \right|^2 \\
&+ \left| \left[\partial_\mu - \frac{ig_L}{2}\tau\cdot\mathbf{W}_\mu^L + \frac{ig_V}{2}V_\mu \right] \tilde{L}_L \right|^2 + \left| \left[\partial_\mu - \frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R + \frac{ig_V}{2}V_\mu \right] \tilde{L}_R \right|^2 \\
&+ \mathrm{Tr}\left| \partial_\mu\Phi_u - \frac{ig_L}{2}\tau\cdot\mathbf{W}_\mu^L\Phi_u + \Phi_u\frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R \right|^2 + \mathrm{Tr}\left| \partial_\mu\Phi_d - \frac{ig_L}{2}\tau\cdot\mathbf{W}_\mu^L\Phi_d + \Phi_d\frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R \right|^2 \\
&+ \mathrm{Tr}\left| \left[\partial_\mu - \frac{ig_L}{2}\tau\cdot\mathbf{W}_\mu^L - ig_V V_\mu \right] \tau\cdot\Delta_L \right|^2 + \mathrm{Tr}\left| \left[\partial_\mu - \frac{ig_L}{2}\tau\cdot\mathbf{W}_\mu^L + ig_V V_\mu \right] \tau\cdot\delta_L \right|^2 \\
&+ \mathrm{Tr}\left| \left[\partial_\mu - \frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R - ig_V V_\mu \right] \tau\cdot\Delta_R \right|^2 + \mathrm{Tr}\left| \left[\partial_\mu - \frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R + ig_V V_\mu \right] \tau\cdot\delta_R \right|^2 \\
&+ i\tilde{Q}_L^\dagger \left[\frac{g_L}{\sqrt{2}}\tau\cdot\lambda_L + \frac{g_V}{3\sqrt{2}}\lambda_V \right] Q_L + \mathrm{H.c.} + i\tilde{Q}_R^\dagger \left[\frac{g_R}{\sqrt{2}}\tau\cdot\lambda_R + \frac{g_V}{3\sqrt{2}}\lambda_V \right] Q_R + \mathrm{H.c.} \\
&+ \frac{i}{\sqrt{2}}\tilde{L}_L^\dagger (g_L\tau\cdot\lambda_L - g_V\lambda_V) L_L + \mathrm{H.c.} + \frac{i}{\sqrt{2}}\tilde{L}_R^\dagger (g_R\tau\cdot\lambda_R - g_V\lambda_V) L_R + \mathrm{H.c.} \\
&+ i\sqrt{2}\,\mathrm{Tr}[(\tau\cdot\Delta_L)^\dagger (g_L\tau\cdot\lambda_L + 2g_V\lambda_V)\tau\cdot\tilde{\Delta}_L] + \mathrm{H.c.} + i\sqrt{2}\,\mathrm{Tr}[(\tau\cdot\delta_L)^\dagger (g_L\tau\cdot\lambda_L - 2g_V\lambda_V)\tau\cdot\tilde{\delta}_L] + \mathrm{H.c.} \\
&+ i\sqrt{2}\,\mathrm{Tr}[(\tau\cdot\Delta_R)^\dagger (g_R\tau\cdot\lambda_R + 2g_V\lambda_V)\tau\cdot\tilde{\Delta}_R] + \mathrm{H.c.} + i\sqrt{2}\,\mathrm{Tr}[(\tau\cdot\delta_R)^\dagger (g_R\tau\cdot\lambda_R - 2g_V\lambda_V)\tau\cdot\tilde{\delta}_R]\ \mathrm{H.c.} \\
&+ \frac{i}{\sqrt{2}}\mathrm{Tr}[\Phi_u^\dagger (g_L\tau\cdot\lambda_L + g_R\tau\cdot\lambda_R)\tilde{\Phi}_u] + \mathrm{H.c.} + \frac{i}{\sqrt{2}}\mathrm{Tr}[\Phi_d^\dagger (g_L\tau\cdot\lambda_L + g_R\tau\cdot\lambda_R)\tilde{\Phi}_d] + \mathrm{H.c.}
\end{aligned} \tag{2.10}$$

C. Yukawa Lagrangian

This piece involves the self-interactions of the matter multiplets; again, this includes the Higgs multiplets:

$$\begin{aligned}
\mathcal{L}_Y = \{ & h_u^L(L_L^\dagger\Phi_u L_R) + h_d^L(L_L^\dagger\Phi_d L_R) + h_u^Q(Q_L^\dagger\Phi_u Q_R) + h_d^Q(Q_L^\dagger\Phi_d Q_R) + h_u^L(\tilde{L}_L^\dagger\tilde{\Phi}_u L_R) + h_d^L(\tilde{L}_L^\dagger\tilde{\Phi}_d L_R) \\
&+ h_u^Q(\tilde{Q}_L^\dagger\tilde{\Phi}_u Q_R) + h_d^Q(\tilde{Q}_L^\dagger\tilde{\Phi}_d Q_R) + h_u^L(\tilde{L}_R^\dagger\tilde{\Phi}_u L_L) + h_d^L(\tilde{L}_R^\dagger\tilde{\Phi}_d L_L) + h_u^Q(\tilde{Q}_R^\dagger\tilde{\Phi}_u Q_L) + h_d^Q(\tilde{Q}_R^\dagger\tilde{\Phi}_d Q_L) \\
&+ \mathrm{Tr}[\mu_1(\tau_1\tilde{\Phi}_u\tau_1)^T\tilde{\Phi}_d] + \mathrm{Tr}[\mu_2(\tau\cdot\tilde{\Delta}_L)(\tau\cdot\tilde{\delta}_L)] + \mathrm{Tr}[\mu_3(\tau\cdot\tilde{\Delta}_R)(\tau\cdot\tilde{\delta}_R)] + h_{LR}(L_L^T\tau_1\tau\cdot\Delta_L L_L + L_R^T\tau_1\tau\cdot\Delta_R L_R) \\
&+ h_{LR}(\tilde{L}_L^T\tau_1\tau\cdot\tilde{\Delta}_L L_L + \tilde{L}_R^T\tau_1\tau\cdot\tilde{\Delta}_R L_R) \} + \mathrm{H.c.}
\end{aligned} \tag{2.11}$$

D. Scalar potential

$$V = |F|^2 + \tfrac{1}{2}|D|^2 + V_{\text{soft}}\ , \tag{2.12}$$

where

$$|F|^2=|h_u^Q\tilde{Q}_L\tilde{Q}_R+h_u^L\tilde{L}_L\tilde{L}_R|^2+|h_d^Q\tilde{Q}_L\tilde{Q}_R+h_d^L\tilde{L}_L\tilde{L}_R|^2+|h_u^Q\Phi_u\tilde{Q}_R+h_d^Q\Phi_d\tilde{Q}_R|^2+|h_u^Q\Phi_u\tilde{Q}_L+h_d^Q\Phi_d\tilde{Q}_L|^2$$
$$+|h_u^L\Phi_u\tilde{L}_R+h_d^L\Phi_d\tilde{L}_R+2h_{LR}\tau\cdot\Delta_L\tilde{L}_L|^2+|h_u^L\Phi_u\tilde{L}_L+h_d^L\Phi_d\tilde{L}_L+2h_{LR}\tau\cdot\Delta_R\tilde{L}_R|^2+\text{H.c.} \tag{2.13}$$

$$\tfrac{1}{2}|D|^2=\tfrac{1}{2}g_L\sum_L\left|\sum_A A^\dagger\tau_L A\right|^2+\tfrac{1}{2}g_R\sum_R\left|\sum_A A^\dagger\tau_R A\right|^2+\tfrac{1}{2}g_V\left|\sum_A A^\dagger\mathcal{V}A\right|^2, \tag{2.14}$$

where $A=\tilde{Q}_L, \tilde{Q}_R, \tilde{L}_L, \tilde{L}_R, \Phi_u, \Phi_d, \Delta_L, \Delta_R, \delta_L$, and δ_R, and τ_L, τ_R, and $\mathcal{V}$ are the generators of the gauge groups:

$$V_{\text{soft}}=m_s(\{h_u^Q\tilde{Q}_L^\dagger\Phi_u\tilde{Q}_R+h_d^Q\tilde{Q}_L^\dagger\Phi_d\tilde{Q}_R+h_u^L\tilde{L}_L^\dagger\Phi_u\tilde{L}_R+h_d^L\tilde{L}_L^\dagger\Phi_d\tilde{L}_R+h_{LR}(\tilde{L}_L^T\tau_1\tau\cdot\Delta_L\tilde{L}_L+\tilde{L}_R^T\tau_1\tau\cdot\Delta_R\tilde{L}_R)$$
$$+\text{Tr}[\mu_1(\tau_1\Phi_u\tau_1)^T\Phi_d]\}+\text{Tr}[\mu_2(\tau\cdot\Delta_L)(\tau\cdot\delta_L)]+\text{Tr}[\mu_3(\tau\cdot\Delta_R)(\tau\cdot\delta_R)]+\text{H.c.})$$
$$+m_{QL}^2\tilde{Q}_L^\dagger\tilde{Q}_L+m_{QR}^2\tilde{Q}_R^\dagger\tilde{Q}_R+m_{LL}^2\tilde{L}_L^\dagger\tilde{L}_L+m_{LR}^2\tilde{L}_R^\dagger\tilde{L}_R\ . \tag{2.15}$$

E. Soft-breaking Lagrangian

This is the term which gives Majorana mass to the gauginos:

$$L_{\text{soft}}=m_L(\lambda_L^a\lambda_L^a+\bar{\lambda}_L^a\bar{\lambda}_L^a)+m_R(\lambda_R^a\lambda_R^a+\bar{\lambda}_R^a\bar{\lambda}_R^a)+m_V(\lambda_V\lambda_V+\bar{\lambda}_V\bar{\lambda}_V)\ . \tag{2.16}$$

III. MASS EIGENSTATES

The choice of VEV's of the Higgs fields [Eq. (2.12)] makes it possible to analyze the generation of mass for the gauge bosons in two separate stages. In the first stage, $\langle\Delta_R\rangle$ generates masses for $W_R^\pm$, W_R^0, and V. The two neutral states mix, yielding the physical fields Z_R and B. In the next stage, taking place at a much lower-energy scale, $\Phi_{u,d}$, which couples to both left- and right-handed fields, mixes W_L and W_R. However, the amount of mixing is so small that, effectively, the right-handed fields can be considered to have decoupled from this part of the theory, and only $W_L^\pm$, W_L^0, and B acquire mass from this stage. Once again, the neutral fields mix and the familiar Z_L and A_μ are formed.

For the first stage of symmetry breaking, the relevant term to consider in the Lagrangian [Eq. (2.12)] is

$$+\text{Tr}\left|\left[-\frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R-ig_VV_\mu\right]\tau\cdot\Delta_R\right|^2. \tag{3.1}$$

Substituting the VEV $\langle\Delta_R\rangle$ for Δ_R, we obtain the physical fields

$$Z_R=\frac{g_RW_R^0-2g_VV}{(g_R^2+4g_V^2)^{1/2}}, \tag{3.2}$$

$$B=\frac{g_RV+2g_VW_R^0}{(g_R^2+4g_V^2)^{1/2}}, \tag{3.3}$$

and their masses are

$$M_{Z_R}=\frac{1}{\sqrt{2}}V_R(g_R^2+4g_V^2)^{1/2}, \tag{3.4}$$

$$M_B=0\ . \tag{3.5}$$

The new massless state B_μ is the gauge boson of the symmetry group $U(1)_Y$, which survives the breaking of $SU(2)_R\times U(1)_{B-L}$. $W_R^\pm$ and Z_R, being very massive, decouple from the low-energy theory, leaving only B_μ to go through to the next stage of symmetry breaking.

For the second stage of symmetry breaking, the terms to consider in Eq. (2.12) are

$$\text{Tr}\left|\partial_\mu\Phi_u-\frac{ig_L}{2}\tau\cdot\mathbf{W}_\mu^L\Phi_u+\Phi_u\frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R\right|^2$$
$$+\text{Tr}\left|\partial_\mu\Phi_d-\frac{ig_L}{2}\tau\cdot\mathbf{W}_\mu^L\Phi_d+\Phi_d\frac{ig_R}{2}\tau\cdot\mathbf{W}_\mu^R\right|^2. \tag{3.6}$$

Since $W_R^\pm$ and Z_R have effectively decoupled from this part of the theory, the charged bosons emerging from this stage are, to a good approximation, $W_L^\pm$, whose masses are simply calculated as

$$M_{W_L}\simeq\frac{1}{\sqrt{2}}g_L(\kappa_u^2+\kappa_d^2)^{1/2}\ . \tag{3.7}$$

As for the neutral bosons, W_R^0 now has to be written in terms of the fields B and Z_R:

$$W_R^0=\frac{g_RZ_R+2g_VB}{(g_R^2+4g_V^2)^{1/2}}\ . \tag{3.8}$$

Defining g', the gauge coupling constant of $U(1)_Y$, as

$$g'\equiv\frac{g_Rg_V}{(g_R^2+4g_V^2)^{1/2}}, \tag{3.9}$$

the neutral mass eigenstates are found to be

$$Z_L=\frac{g_LW_L^0-2g'B}{(g_L^2+4g'^2)^{1/2}}, \tag{3.10}$$

with

$$\text{mass}=[(\kappa_u^2+\kappa_d^2)(g_L^2+4g'^2)]^{1/2}, \tag{3.11}$$

and the massless photon A_μ given by

$$A_\mu=\frac{2g'W_L^0+g_LB}{(g_L^2+4g'^2)^{1/2}} . \tag{3.12}$$

In the following the mass eigenstates for supersymmetric partners are developed following the two-stage symmetry-breaking examples developed in the gauge-boson sector. The development owes much to the discussion of mixing matrices by Haber and Kane.[16]

The terms in Eqs. (2.10), (2.11), and (2.16) relevant to the mixing of gauginos and Higgsinos are

$$\begin{aligned}\mathcal{L}_{\text{gh}}=&i\sqrt{2}\,\mathrm{Tr}[(\tau\cdot\Delta_L)^\dagger(g_L\tau\cdot\lambda_L+2g_V\lambda_V)\tau\cdot\tilde{\Delta}_L]+\text{H.c.}+i\sqrt{2}\,\mathrm{Tr}[(\tau\cdot\Delta_R)^\dagger(g_R\tau\cdot\lambda_R+2g_v\lambda_v)\tau\cdot\tilde{\Delta}_R]+\text{H.c.}\\&+\frac{i}{\sqrt{2}}\mathrm{Tr}[\Phi_u^\dagger(g_L\tau\cdot\lambda_L+g_R\tau\cdot\lambda_R)\tilde{\Phi}_u]+\text{H.c.}+\frac{i}{\sqrt{2}}\mathrm{Tr}[\Phi_d^\dagger(g_L\tau\cdot\lambda_L+g_R\tau\cdot\lambda_R)\tilde{\Phi}_d]+\text{H.c.}\\&+\mathrm{Tr}[\mu_2(\tau\cdot\tilde{\Delta}_L)(\tau\cdot\tilde{\delta}_L)]+\mathrm{Tr}[\mu_3(\tau\cdot\tilde{\Delta}_R)(\tau\cdot\tilde{\delta}_R)]\\&+m_L(\lambda_L^a\lambda_L^a+\bar{\lambda}_L^a\bar{\lambda}_L^a)+m_R(\lambda_R^a\lambda_R^a+\bar{\lambda}_R^a\bar{\lambda}_R^a)+m_v(\lambda_v\lambda_v+\bar{\lambda}_v\bar{\lambda}_v)+\mathrm{Tr}[\mu_1(\tau_1\tilde{\phi}_u\tau_1)^T\tilde{\Phi}_d] .\end{aligned} \tag{3.13a}$$

Considering first the charged gauginos and Higgsinos (charginos), the first step is to substitute the VEV's of the Higgs fields into Eq. (3.13a) yielding

$$\begin{aligned}\mathcal{L}_{\text{c.m.}}=&[i\lambda_R^-(\sqrt{2}g_Rv_R\tilde{\Delta}_R^++g_R\kappa_d\tilde{\phi}_d^+)+i\lambda_L^-g_L\kappa_d\tilde{\phi}_d^++i\lambda_R^+g_R\kappa_u\tilde{\phi}_u^-+i\lambda_L^+g_L\kappa_u\tilde{\phi}_u^-\\&+m_L\lambda_L^+\lambda_L^-+m_R\lambda_R^+\lambda_R^-+\mu_1\tilde{\phi}_u^+\tilde{\phi}_d^-+\mu_1\tilde{\phi}_u^-\tilde{\phi}_d^+]+\text{H.c.} ,\end{aligned} \tag{3.13b}$$

where, for simplification, it is assumed that $\mu_2=\mu_3=0$.

Then the two stages of symmetry breaking are considered separately. First, charged fermions combine into four component Dirac spinors from the $\mathcal{L}_W$ part of the Lagrangian

$$\mathcal{L}_W=(i\sqrt{2}g_Rv_R)\lambda_R^-\tilde{\Delta}_R^++\text{H.c.} \tag{3.14}$$

At this stage supersymmetry is unbroken and the mass of $\tilde{W}_R^+$, $\sqrt{2}g_Rv_R$ is the same as that of W_R^+. The particles produced at this stage are very massive and decouple from the low-energy theory.

At the next step the remaining terms in the Lagrangian

$$\mathcal{L}_c=-\frac{1}{2}(\psi^+,\psi^-)\begin{bmatrix}0 & X^T\\ X & 0\end{bmatrix}\begin{bmatrix}\psi^+\\ \psi^-\end{bmatrix}+\text{H.c.} , \tag{3.15}$$

where

$$\begin{aligned}\psi^+&\equiv(-i\lambda_L^+,-i\lambda_R^+,\tilde{\phi}_u^+,\tilde{\phi}_d^+) ,\\ \psi^-&\equiv(-i\lambda_L^-,-i\lambda_R^-,\tilde{\phi}_u^-,\tilde{\phi}_d^-) ,\end{aligned} \tag{3.16}$$

and

$$X=\begin{bmatrix}m_L & 0 & 0 & g_L\kappa_d\\ 0 & m_R & 0 & g_R\kappa_d\\ g_L\kappa_u & g_R\kappa_u & 0 & -\mu_1\\ 0 & 0 & -\mu_1 & 0\end{bmatrix} . \tag{3.17}$$

The mass eigenstates are defined as

$$\chi_i^+=V_{ij}\psi_j^+,\quad \chi_i^-=U_{ij}\psi_j^-,\quad i,j=1,2,3,4 , \tag{3.18}$$

where V and U are unitary matrices chosen such that

$$U^*XV^{-1}=M_D , \tag{3.19}$$

where M_D is a diagonal matrix with non-negative entries. Positive square roots of the eigenvalues of $X^\dagger X$ will be the diagonal entries of M_D:

$$M_D^2=VX^\dagger XV^{-1}=U^*XX^\dagger(U^*)^{-1} . \tag{3.20}$$

Thus the diagonalizing matrices U^* and V are obtained by computing the eigenvectors corresponding to the eigenvalues of $X^\dagger X$ and $XX^\dagger$, respectively.

The Lagrangian for the neutral gaugino and Higgsino (neutralino) mass terms analogous to Eq. (3.13) is

$$\mathcal{L}_{\rm NM}=\Bigg[-i\lambda_R^0\sqrt{2}g_R v_R\tilde{\Delta}_R^0+i\lambda_V^0 2\sqrt{2}g_V v_R\tilde{\Delta}_R^0+i\lambda_R^0\frac{1}{\sqrt{2}}g_R\kappa_u\tilde{\phi}_{1u}^0-i\lambda_L^0\frac{1}{\sqrt{2}}g_L\kappa_u\tilde{\phi}_{1u}^0$$
$$-i\lambda_R^0\frac{1}{\sqrt{2}}g_R\kappa_d\tilde{\phi}_{2d}^0+i\lambda_L^0\frac{1}{\sqrt{2}}g_L\kappa_d\tilde{\phi}_{2d}^0+m_L\lambda_L^0\lambda_L^0+m_R\lambda_R^0\lambda_R^0+m_V\lambda_V^0\lambda_V^0+2\mu_1\tilde{\phi}_u^0\tilde{\phi}_d^0\Bigg]+\text{H.c.} \tag{3.21}$$

The mass eigenstates that "condense out" at the first stage of symmetry breaking are all those involving v_R, i.e.,

$$\mathcal{L}_{\rm NMH}=-\frac{1}{\sqrt{2}}v_R(\xi^0)^T Y\xi^0+\text{H.c.}\ , \tag{3.22}$$

where

$$(\xi^0)^T\equiv(-i\lambda_R^0,-i\lambda_v^0,\tilde{\Delta}_R^0)\ , \tag{3.23}$$

and

$$Y=\begin{bmatrix}0 & 0 & -g_R\\ 0 & 0 & 2g_V\\ -g_R & 2g_V & 0\end{bmatrix}. \tag{3.24}$$

Proceeding as before, define mass eigenstates by

$$\chi_i^0=N_{ij}\xi_j^0\quad(i,j=1,2,3)\ , \tag{3.25}$$

where N is a unitary matrix satisfying

$$N^*YN^{-1}=N_D\ , \tag{3.26}$$

where N_D is a diagonal matrix with non-negative entries. The eigenvalues of

$$Y^\dagger Y=\begin{bmatrix}g_R^2 & -2g_Rg_V & 0\\ -2g_Rg_V & 4g_V^2 & 0\\ 0 & 0 & g_R^2+4g_V^2\end{bmatrix} \tag{3.27}$$

are twice $(g_R^2+4g_V^2)$, and 0. The diagonal entries of N_D^2, and the diagonalizing matrix N is given by

$$N=\begin{bmatrix}g_R/\sqrt{2}(g_R^2+4g_V^2)^{1/2} & -2g_v/\sqrt{2}(g_R^2+4g_V^2)^{1/2} & -\frac{1}{\sqrt{2}}\\ g_R/\sqrt{2}(g_R^2+4g_V^2)^{1/2} & -2g_V/\sqrt{2}(g_R^2+4g_V^2)^{1/2} & +\frac{1}{\sqrt{2}}\\ 2g_V/(g_R^2+4g_V^2)^{1/2} & g_R/(g_R^2+4g_V^2)^{1/2} & 0\end{bmatrix}. \tag{3.28}$$

Using Eqs. (3.25) and (3.26), the physical neutralinos resulting from the first breaking are

$$\tilde{\chi}_{Z1}=\begin{bmatrix}\dfrac{-i(g_R\lambda_R^0-2g_V\lambda_V)}{\sqrt{2}(g_R^2+4g_V^2)^{1/2}}-\dfrac{\tilde{\Delta}_R^0}{\sqrt{2}}\\ \dfrac{+i(g_R\bar{\lambda}_R^0-2g_V\bar{\lambda}_V)}{\sqrt{2}(g_R^2+4g_V^2)^{1/2}}-\dfrac{\bar{\tilde{\Delta}}_R^0}{\sqrt{2}}\end{bmatrix}, \tag{3.29}$$

with mass $=(1/2)v_R(g_R^2+4g_V^2)^{1/2}$, (3.30)

$$\tilde{\chi}_{Z2}=\begin{bmatrix}\dfrac{-i(g_R\lambda_R^0-2g_V\lambda_V^0)}{\sqrt{2}(g_R^2+4g_V^2)^{1/2}}+\dfrac{\tilde{\Delta}_R^0}{\sqrt{2}}\\ \dfrac{+i(g_R\bar{\lambda}_R^0-2g_V\bar{\lambda}_V^0)}{\sqrt{2}(g_R^2+4g_V^2)^{1/2}}+\dfrac{\bar{\tilde{\Delta}}_R^0}{\sqrt{2}}\end{bmatrix}, \tag{3.31}$$

with mass $=(1/2)V_R(g_R^2+4g_V^2)^{1/2}$, and

$$\tilde{\chi}_B=\begin{bmatrix}\dfrac{-i(g_R\lambda_V^0+2g_V\lambda_R^0)}{(g_R^2+4g_V^2)^{1/2}}\\ \dfrac{+i(g_R\bar{\lambda}_V^0+2g_V\bar{\lambda}_R^0)}{(g_R^2+4g_V^2)^{1/2}}\end{bmatrix}, \tag{3.32}$$

with mass=0.

Thus the neutralino spectrum, at this mass scale, consists of two Majorana fermions, degenerate in mass, and a massless Majorana fermion. The two massive states can be written as a single Dirac spinor:

$$\tilde{\xi}^0=\begin{bmatrix}\tilde{\Delta}_R^0\\ \dfrac{i(g_R\bar{\lambda}_R^0-2g_V\bar{\lambda}_V^0)}{(g_R^2+4g_V^2)^{1/2}}\end{bmatrix}. \tag{3.33}$$

In the absence of supersymmetry-breaking terms, this is exactly as expected: $\tilde{\xi}^0$ is the superpartner of Z_R, and $\tilde{\chi}_B$ that of B_μ. To complete the symmetry that exists between ordinary particles and their superpartners, $\tilde{\xi}^0$ decouples from the low-energy theory, and the massless $\tilde{\chi}_B$ goes through to the next stage of symmetry breaking.

In view of the rearrangements that have taken place among the fields at the high-energy scale, the particles taking part in the low-energy interactions are no longer those of Eq. (3.21). Specifically, λ_R^0 and λ_V^0 have to be rewritten in terms of λ_z^0 and λ_B^0, which are defined by

$$\lambda_z^0 \equiv \frac{g_R \lambda_R^0 - 2g_V \lambda_V^0}{(g_R^2 + 4g_V^2)^{1/2}} , \tag{3.34}$$

$$\lambda_B^0 \equiv \frac{g_R \lambda_V^0 + 2g_V \lambda_R^0}{(g_R^2 + 4g_V^2)^{1/2}} , \tag{3.35}$$

giving

$$\lambda_R^0 = \frac{g_R \lambda_z^0 + 2g_V \lambda_B^0}{(g_R^2 + 4g_V^2)^{1/2}} , \tag{3.36}$$

$$\lambda_V^0 = \frac{g_R \lambda_B^0 - 2g_V \lambda_z^0}{(g_R^2 + 4g_V^2)^{1/2}} . \tag{3.37}$$

The mass Lagrangian of the light neutralinos, $\mathcal{L}_{\rm NML}$, is obtained by substituting for λ_R^0 and λ_V^0, and removing the contributions of the fields which have decoupled (i.e., $\tilde{\Delta}_R^0$ and λ_z^0). The result of this is

$$\begin{aligned}\mathcal{L}_{\rm NML} = \Bigg[& -i\lambda_L^0 \frac{1}{\sqrt{2}} g_L \kappa_u \tilde{\phi}^0_{1u} + \frac{i\lambda_B^0 \sqrt{2} g_V g_R \kappa_u \tilde{\phi}^0_{1u}}{(g_R^2 + 4g_V^2)^{1/2}} \\ & + i\lambda_L^0 \frac{1}{\sqrt{2}} g_L \kappa_d \tilde{\phi}^0_{2d} - \frac{i\lambda_B^0 \sqrt{2} g_V g_R \kappa_d \tilde{\phi}^0_{2d}}{(g_R^2 + 4g_V^2)^{1/2}} \\ & + m_L \lambda_L^0 \lambda_L^0 + \frac{m_R 4g_V^2 \lambda_B^0 \lambda_B^0}{g_R^2 + 4g_V^2} + \frac{m_V g_R^2 \lambda_B^0 \lambda_B^0}{g_R^2 + 4g_V^2} \\ & + 2\mu_1 \tilde{\phi}^0_u \tilde{\phi}^0_d \Bigg] + \text{H.c.} \end{aligned} \tag{3.38}$$

The identification of the mass eigenstates follows the now familiar procedure.

IV. $g-2$ OF THE MUON

The collection of graphs not present in the supersymmetric standard model, but occurring in the present model are displayed in Fig. 2. Consider the term calculated from the graphs of the form

$$\frac{ie}{2m_\mu} F(q^2) \bar{u} \sigma_{\alpha\beta} q^\beta u . \tag{4.1}$$

Then the muon anomaly is defined by

$$a_\mu \equiv (g-2)_\mu / 2 = F(0) . \tag{4.2}$$

The graph in Fig. 2 involving $\tilde{\gamma}$ was first considered by Fayet.[2] In addition, the graphs in Fig. 2(a) involving $\tilde{\nu}$ and Fig. 4(e) involving $\tilde{Z}$ are considered by Ellis, Hagelin, and Nanopoulos[17] and by Grifols and Mendez.[18] Additionally, the graph in Fig. 2(f) is considered by Barbieri and Maiani;[19] the graphs in Figs. 2(h) and 2(c) involving $\tilde{\nu}$ by Kosower, Krauss, and Sakai,[20] and finally Figs. 2(a)–2(d) involving $\tilde{N}$, and Fig. 2(b) by Frank and Kalman.[7] The new contributions in this paper are based on Figs. 2(g), 2(i), and 2(j) and some modifications to these based on Figs. 2(a)–2(c). Note that the graphs in Figs. 2(a)–2(d) involve a coupling of charginos to physical scalar neutrinos $\tilde{\nu}_{1,2}$. In the results given below, we use the usual convention that

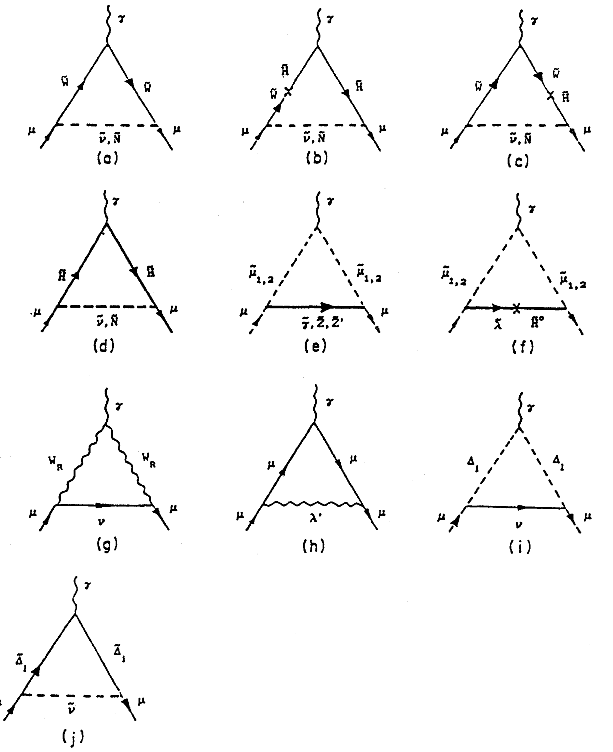


FIG. 2. Additional one-loop diagrams contributions to $(g-2)_\mu$.

$$\tilde{\nu}_L = \tilde{\nu} = \tilde{\nu}_1 \cos\theta_\nu - \tilde{\nu}_2 \sin\theta_\nu , \tag{4.3}$$

$$\tilde{\nu}_R = \tilde{N} = \tilde{\nu}_1 \sin\theta_\nu + \tilde{\nu}_2 \cos\theta_\nu . \tag{4.4}$$

The contribution to $(g-2)_\mu$ from Figs. 2(a)–2(d) is

$$\begin{aligned} a_\mu^{(\chi^\pm)} = & \frac{g_L^2}{96\pi^2} \sum_{i=1}^{4} |V_{i1}|^2 [\cos^2\theta_\nu F'(x_{1i}) + \sin^2\theta_\nu F'(x_{2i})] \tilde{W}_L \tilde{W}_L \\ & + \frac{g_R^2}{96\pi^2} \sum_{i=1}^{4} |V_{i2}|^2 [\sin^2\theta_\nu F'(x_{1i}) + \cos^2\theta_\nu F'(x_{2i})] \tilde{W}_R \tilde{W}_R \\ & + \frac{h_u^{L2}}{96\pi^2} \sum_{i=1}^{4} |V_{i3}|^2 [F'(x_{1i}) + F'(x_{2i})] + \frac{h_u^{L2}}{96\pi^2} \sum_{i=1}^{4} |V_{i4}|^2 [F'(x_{1i}) + F'(x_{2i})] \tilde{H}^u_L \tilde{H}^u_L + \tilde{H}^d_R \tilde{H}^d_R \end{aligned}$$

$$
\begin{aligned}
&-\frac{h_d^L g_L}{96\pi^2}\sum_{i=1}^{4}(\mathrm{Re}V_{i1}^*V_{i3})[\cos^2\theta_\nu F'(x_{1i})-\sin^2\theta_\nu F'(x_{2i})]\tilde{W}_L\tilde{H}_L^d\\
&-\frac{h_u^L g_L}{96\pi^2}\sum_{i=1}^{4}(\mathrm{Re}V_{i1}^*V_{i4})[\cos^2\theta_\nu F'(x_{1i})-\sin^2\theta_\nu F'(x_{2i})]\tilde{W}_L\tilde{H}_L^u\\
&-\frac{h_d^L g_R}{96\pi^2}\sum_{i=1}^{4}(\mathrm{Re}V_{i2}^*V_{i3})[\sin^2\theta_\nu F'(x_{1i})+\cos^2\theta_\nu F'(x_{2i})]\tilde{W}_R\tilde{H}_R^d\\
&+\frac{h_u^L h_d^L}{96\pi^2}\sum_{i=1}^{4}(\mathrm{Re}V_{i3}^*V_{i4})[F'(x_{1i})+F'(x_{2i})]\tilde{H}_L^u\tilde{H}_L^d\\
&-\frac{h_u^L g_R}{96\pi^2}\sum_{i=1}^{4}(\mathrm{Re}V_{i2}^*V_{i4})[\sin^2\theta_\nu F'(x_{1i})+\cos^2\theta_\nu F'(x_{2i})]\tilde{W}_R\tilde{H}_R^u\\
&-\frac{g_L g_R}{8\pi^2}\sum_{i=1}^{4}(\mathrm{Re}V_{i1}^*V_{i2})\cos\theta_\nu\sin\theta_\nu[F(x_{1i})+F(x_{2i})]\tilde{W}_L\tilde{W}_R\\
&+\frac{g_L h_d^L}{8\pi^2}\sum_{i=1}^{4}(\mathrm{Re}V_{i1}^*V_{i3})\cos\theta_\nu\sin\theta_\nu[F(x_{1i})+F(x_{2i})]\tilde{W}_L\tilde{H}_R^d\\
&+\frac{g_L h_u^L}{8\pi^2}\sum_{i=1}^{4}(\mathrm{Re}V_{i1}^*V_{i4})\cos\theta_\nu\sin\theta_\nu[F(x_{1i})+F(x_{2i})]\tilde{W}_L\tilde{H}_R^u\\
&+\frac{g_R h_d^L}{8\pi^2}\sum_{i=1}^{4}(\mathrm{Re}V_{i2}^*V_{i3})\cos\theta_\nu\sin\theta_\nu[F(x_{1i})-F(x_{2i})]\tilde{W}_R\tilde{H}_L^d\\
&+\frac{g_R h_u^L}{8\pi}\sum_{i=1}^{4}(\mathrm{Re}V_{i2}^*V_{i4})\cos\theta_\nu\sin\theta_\nu[F(x_{1i})-F(x_{2i})]\tilde{W}_R\tilde{H}_L^u\ .
\end{aligned}
\tag{4.5}
$$

From Fig. 2(g),

$$a_\mu^{(W_R)}=5\frac{g_R^2}{96\pi^2}\frac{m^2}{m_{W_R}^2}\ . \tag{4.6}$$

From Fig. 2(i),

$$a_\mu^{(\Delta)}=-\frac{h_{LR}^2}{96\pi^2}[\cos^2\theta_\nu G'(x_1)+\sin^2\theta_\nu G'(x_2)]\ . \tag{4.7}$$

From Fig. 2(j),

$$a_\mu^{(\tilde{\Delta})}=\frac{h_{LR}^2}{96\pi^2}[\cos^2\theta_\nu F'(x_{1i})+\sin^2\theta_\nu F'(x_{2i})]\ , \tag{4.8}$$

where

$$G'(x_k)=\frac{m^2}{m_{\Delta_L}^2}\left[\frac{2+5x_k-x_k^2}{2(1-x_k)^3}+\frac{3x_k}{(1-x_k)^4}\ln x_k\right],\quad x_k=\frac{m_{\tilde{\mu}_k}^2}{m_{\tilde{\Delta}_L}^2}\ , \tag{4.9}$$

$$F'(x_{km})=\frac{m^2}{m_{\tilde{\chi}_m}^2}\left[\frac{1-5x_{km}-2x_{km}^2}{(1-x_{km})^3}-\frac{6x_{km}^2}{(1-x_{km})^4}\ln x_{km}\right]\ , \tag{4.10}$$

$$F(x_{km})=\frac{m}{m_{\tilde{\chi}_m}}\left[\frac{1-3x_{km}}{(1-x_{km})^2}-\frac{2x_{km}^2}{(1-x_{km})^3}\ln x_{km}\right]\ , \tag{4.11}$$

$x_{km}=m_{\tilde{\nu}_k}^2/m_{\tilde{\chi}_m}^2$, and V is the unitary mixing matrix defined in Eqs. (3.18)–(3.20). Based upon supergravity,[21]

$$h_{u,d}^L\simeq g\frac{m}{m_W}\ , \tag{4.12}$$

with $g=g_L\simeq g_R$, and m everywhere refers to the muon mass. The contributions from Figs. 2(l) and 2(f)–2(h) are unchanged from Frank and Kalman,[7] but we repeat them here for completeness. In the following we use the usual convention

$$\tilde{\mu}_R = \tilde{\mu}_1 \cos\alpha_\mu - \tilde{\mu}_2 \sin\alpha_\mu \ , \tag{4.13}$$

$$\tilde{\mu}_L = \tilde{\mu}_2 \sin\alpha_\mu + \tilde{\mu}_2 \cos\alpha_\mu \ . \tag{4.14}$$

From Fig. 2(1),

$$a_\mu^{(\tilde{\gamma})} = -\frac{\alpha m^2}{12\pi}\left[\frac{1}{m_{\tilde{\mu}_1}^2} + \frac{1}{m_{\tilde{\mu}_2}^2}\right] . \tag{4.15}$$

$$a_\mu^{(\tilde{Z})} = -\frac{g^2}{24\pi^2\cos^2\theta_W}\{\cos^2\beta[\cos^2\alpha_\mu G'(x_{11}) + \sin^2\alpha_\mu G'(x_{21})] + \sin^2\beta[\cos^2\alpha_\mu G'(x_{12}) + \sin^2\alpha_\mu G'(x_{22})]\}\left[\frac{1}{4} - \sin^2\theta_W + 2\sin^4\theta_W\right] , \tag{4.16}$$

where

$$G'(x_{km}) = \frac{m^2}{m_{\tilde{\chi}_m}^2}\left[\frac{2+5x_{km}-x_{km}^2}{2(1-x_{km})^3} + \frac{3x_{km}}{(1-x_{km})^4}\right], \quad x_{km} = \frac{m_{\tilde{\mu}_k}^2}{m_{\tilde{\chi}_m^0}^2} , \tag{4.17}$$

$$a_\mu^{(\tilde{Z}')} = \frac{g_1^2+g_2^2}{48\pi^2}\left\{\left[\frac{g}{g_1}\right]^4 \tan^4\theta_W[\cos^2\alpha_\mu G'(x_1)] + \left[\left[\frac{g}{g_1}\right]^2 2\tan^2\theta_W - 1\right]^2 [\sin^2\alpha_\mu G'(x_1) + \cos^2\alpha_\mu G'(x_2)]\right\} , \tag{4.18}$$

where

$$G'(x_k) = \frac{m^2}{m_{\tilde{Z}}^2}\left[\frac{2+5x_k-x_k^2}{2(1-x_k)^3} + \frac{3x_k}{(1-x_k)^4}\ln x_k\right], \quad x_k = \frac{m_{\tilde{\mu}_k}^2}{m_{\tilde{Z}}^2} . \tag{4.19}$$

From Fig. 2(f),

$$a_\mu^{(\tilde{Z},\tilde{H})} = \frac{g h_d}{32\pi^2\cos\theta_W}\sin 2\beta\{(\cos^2\alpha_\mu \cos 2\theta_W + \sin^2\alpha_\mu 2\sin^2\theta_W)[G(x_{11}) + G(x_{12})] + (\sin^2\alpha_\mu \cos 2\theta_W + \cos^2\alpha_\mu 2\sin^2\theta_W)[G(x_{21}) + G(x_{22})]\} , \tag{4.20}$$

where

$$G(x_{km}) = \frac{m}{m_{\tilde{\chi}_m}}\left[\frac{1+x_{km}}{(1-x_{km})^2} + \frac{2x_{km}}{(1-x_{km})^3}\ln x_{km}\right], \quad x_{km} = \frac{m_{\tilde{\mu}_k}^2}{m_{\tilde{\chi}_m}^2} . \tag{4.21}$$

Based upon supergravity,[21]

$$h_d \simeq g\frac{m}{\text{mass of the most massive neutralino in the loop}} . \tag{4.22}$$

Finally, from Fig. 4(h),

$$a_\mu^{(Z')} = \frac{g_1^2+g_2^2}{48\pi^2}\frac{m^2}{m_{Z'}^2}\left[\left[\frac{g}{g_1}\right]^4 \tan^4\theta_W + \left[\frac{g}{g_1}\right]^2 \tan^2\theta_W - 1\right] . \tag{4.23}$$

Since the anomalous magnetic moments are so accurately measured, it is essential for any major change in particle theory that one must check that the addition of new particles will not adversely affect the present theoretical success. The present experimental value[22] from the last CERN $g-2$ experiment is

$$a_\mu(\text{expt}) = 1\,165\,922(9)\times 10^{-9} , \tag{4.24}$$

where the number in parentheses represents the error in the last significant figure. The Weinberg-Salam (WS) standard-model contribution is[23]

$$a_\mu(\text{WS}) = 1.95(1)\times 10^{-9} . \tag{4.25}$$

The most recent calculated total theoretical value of all standard-model contributions[24] is

TABLE II. Different scenarios of supersymmetric particle masses in GeV chosen to cover values of a_μ in the range $\simeq\pm10^{-8}$. The contribution of the $\tilde{Z}'$ is very small, and the effect of its variation is not shown.

$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$	$m_{\tilde{\nu}_1}$	$m_{\tilde{\nu}_2}$	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\mu}_1}$	$m_{\tilde{\mu}_2}$	$m_{\tilde{Z}'}$	a_μ
400	400	10	10	11	11	10	10	60	9.2×10^{-8}
35	200	10	10	11	11	10	10	60	9.0×10^{-8}
35	100	10	10	11	11	10	10	60	8.4×10^{-8}
40	40	10	10	11	11	10	10	60	6.2×10^{-8}
60	60	10	10	11	11	10	10	60	7.9×10^{-8}
60	60	30	30	11	11	10	10	60	8.0×10^{-8}
60	60	10	10	11	13	10	10	60	5.6×10^{-8}
60	60	10	10	13	13	12	12	60	5.2×10^{-8}
60	60	10	10	15	15	12	12	60	3.8×10^{-8}
60	60	10	10	18	18	17	17	60	2.1×10^{-8}
60	60	10	10	25	25	17	17	60	5.8×10^{-9}
200	200	201	201	202	202	203	203	60	1.5×10^{-9}
60	60	10	10	25	30	20	20	60	4.9×10^{-10}
50	60	10	10	25	30	20	20	60	-6.7×10^{-10}
30	60	10	10	30	30	20	20	60	-4.8×10^{-9}
30	60	10	10	40	40	20	20	60	-1.2×10^{-8}
70	70	30	30	50	50	40	40	60	-3.0×10^{-9}
20	20	12	12	11	11	10	10	60	-1.4×10^{-8}
16.5	16.5	16	16	10.001	10.001	10	10	60	-2.4×10^{-8}

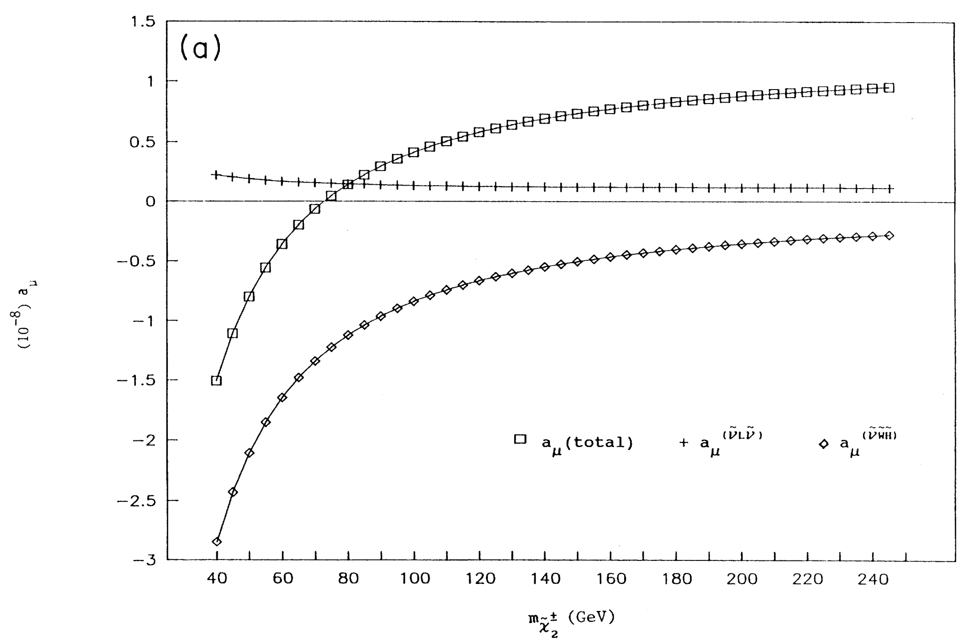


FIG. 3. (a) Variation of $a_\mu\equiv(g-2)_\mu/2$ as the mass of the heavier chargino $m_{\tilde{\chi}_2^\pm}$ is increased. The masses of the other supersymmetric particles are kept at their lower bound: $m_{\tilde{\chi}_1^\pm}=40$ GeV, $m_{\tilde{\chi}_1^0}=m_{\tilde{\chi}_2^0}=30$ GeV, $m_{\tilde{\nu}_1}=m_{\tilde{\nu}_2}=25$ GeV, $m_{\tilde{\mu}_1}=m_{\tilde{\mu}_2}=40$ GeV, and $m_{\tilde{z}}=60$ GeV. The components of a_μ contributing to the change in its value are also shown. (b) Same as (a) except that the masses of both $m_{\tilde{\chi}_1^\pm}$ and $m_{\tilde{\chi}_2^\pm}$ (set equal to each other) are increased. (c) The mass of the heavier neutralino, $m_{\tilde{\chi}_2^0}$ is increased while all other masses are kept at their lower bound. (d) Both the $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0}$ masses, set equal to each other, are increased. (e) The effect of increasing the mass of one of the s muons. Only two of the components of a_μ affected by this change are shown; the others contribute significantly less. (f) Same as (e), but with both s-muon masses, set equal to each other, increasing. (g) The effect of changing the mass of one of the s-neutrinos. (h) Same as (g), with both s-neutrino masses (equal to each other) increasing.

$$a_\mu(\text{theory}) = 116\,591\,920(191)\times 10^{-11}\ , \qquad (4.26)$$

where the error comes predominantly from the hadronic contribution. When the new muon $g-2$ experiment (E821) which is in progress at the Brookhaven National Laboratory (BNL) and associated experiments needed to improve the hadronic contribution to a_μ(theory) are completed, it will be possible to test the prediction of the standard model at the one-loop level. Any deviation from that prediction [Eq. (4.25)] would most likely be due to supersymmetry.[25]

In calculating values of the supersymmetric contribution to the anomalous magnetic moment, it is immediately seen that the values of θ_v and α_μ are unimportant. The maximum contribution for V_{ij} occurs when they are all

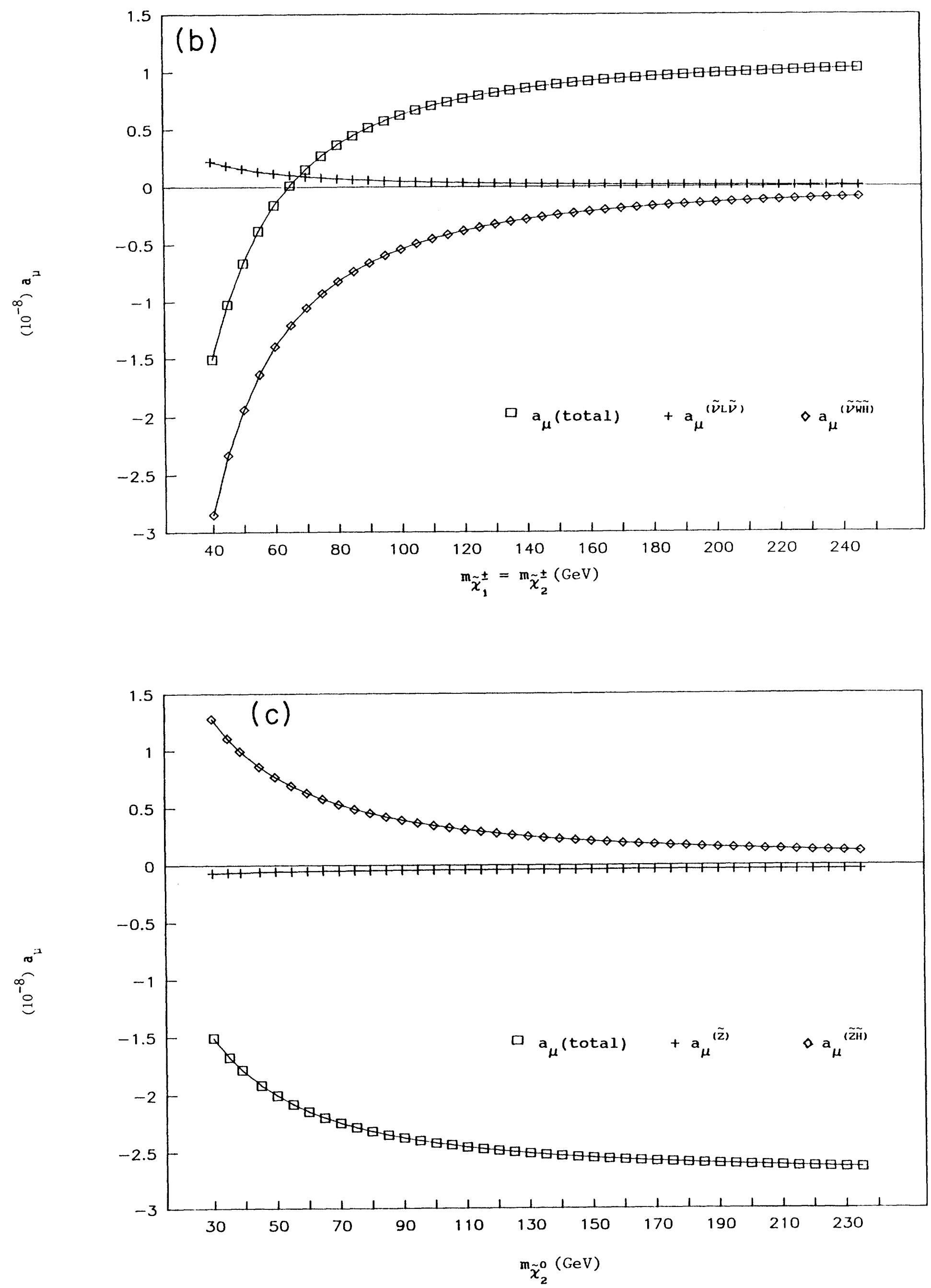


FIG. 3. (*Continued*).

equal ($|V_{ij}|^2=\frac{1}{2}$).

In the $SU(3)\times SU(2)_L\times U(1)_{I_3R}\times U(1)_{B-L}$ gauge group, V_{ij} has the simple form

$$\begin{bmatrix} \cos\varphi_\pm & \pm\sin\varphi_\pm \\ \mp\sin\varphi_\pm & \cos\varphi_\pm \end{bmatrix}$$

found in Frank and Kalman.[7] The value of β has little effect on $a_\mu^{(\tilde{Z})}$, but has a maximum effect on $a_\mu^{(\tilde{Z},\tilde{H})}$ (and thus on the total contribution from this model) for $\beta=45°$. Using the maximum values of β and V_{ij}, the effect of the masses of the supersymmetric particles on the anomalous magnetic moments was considered. The first question that was posed was the effect of the present

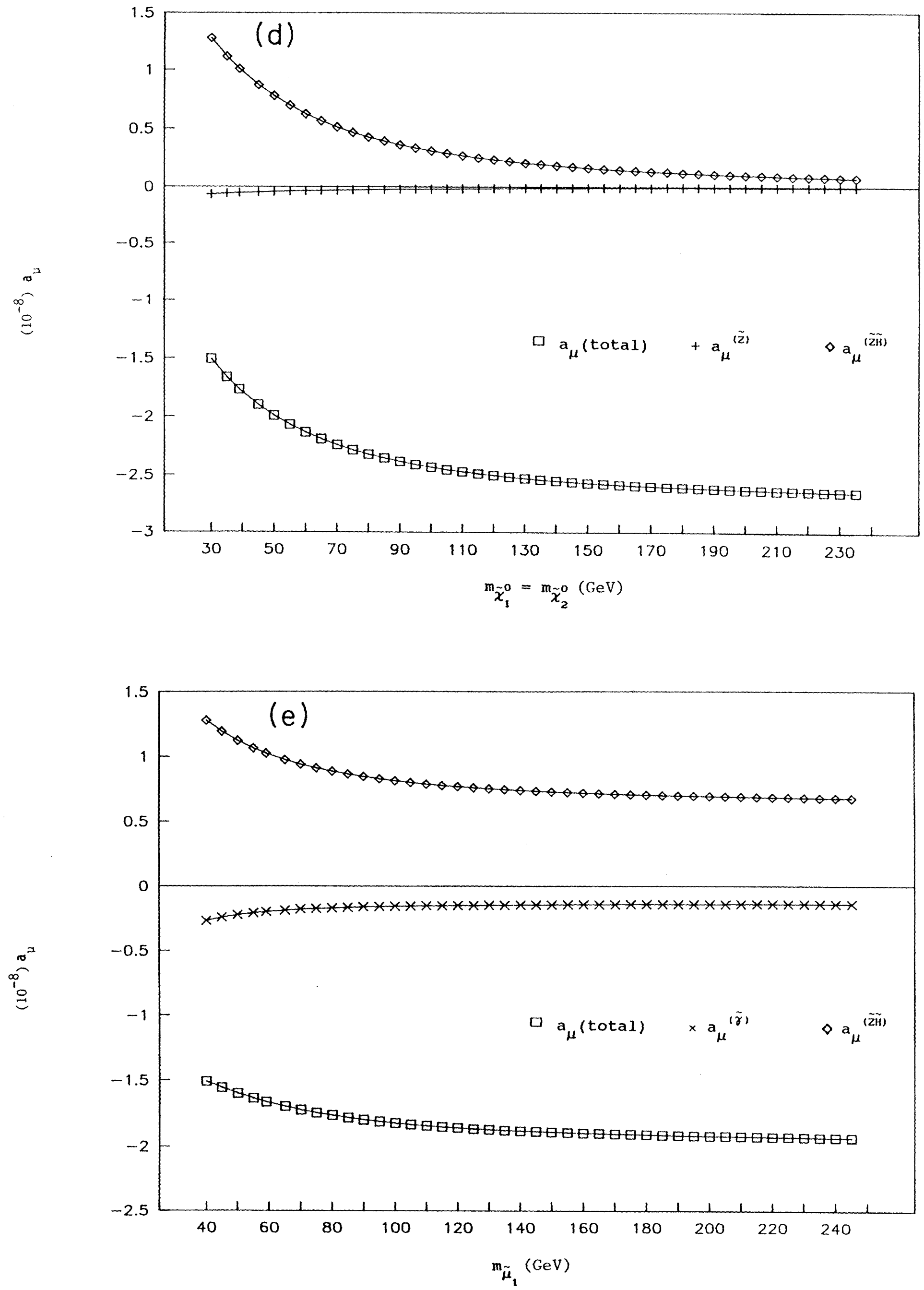
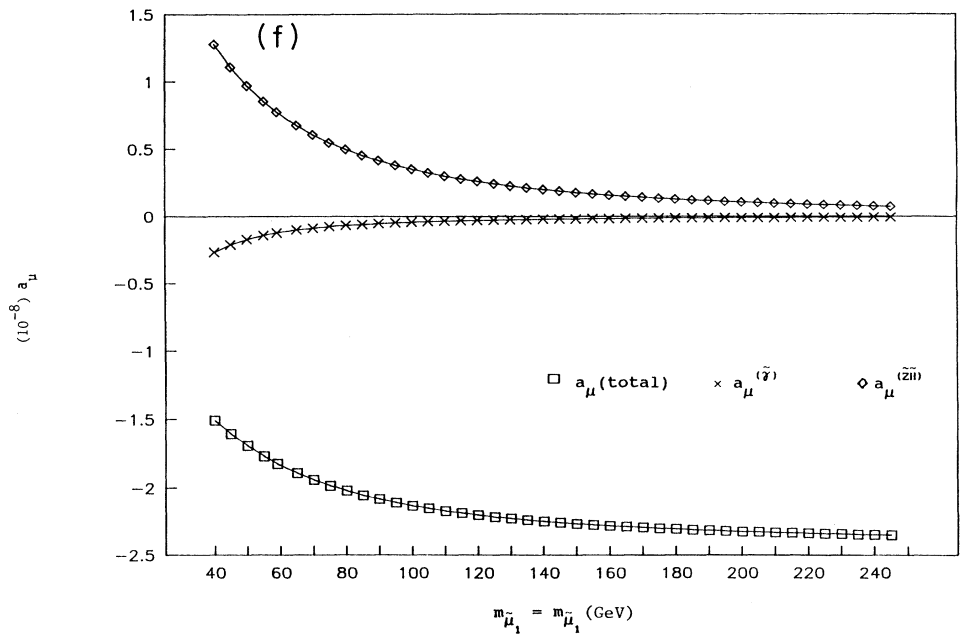


FIG. 3. (*Continued*).

experimental limits [Eq. (4.24)] on the masses of $m_{\tilde{\mu}}$ and $m_{\tilde{\chi}^{\pm}}$. The idea was to see whether independent confirmation of the present experimental limits on the masses of $\tilde{\mu}$ and $\tilde{\chi}^{\pm}$ found at CERN LEP,[26] KEK TRISTAN,[27] the CERN $p\bar{p}$ collider,[28] and the Collider Detectorat Fermilab[29] (CDF) could be obtained. This proved to be impossible as seen in Table II and Fig. 3. For sufficiently light $\tilde{\nu}$ and $\tilde{\chi}^{0}$, masses of $\tilde{\mu}$ and $\tilde{\chi}^{\pm}$ as light as 10 GeV can occur, consistent with present experimental limits on the anomalous magnetic moment or indeed of limits likely to be found by the BNL experiment. The values of $m_{\tilde{\mu}}$ and $m_{\tilde{\chi}^{\pm}}$ were then fixed to be bounded below by their present experimental limits of $\sim$40 GeV. The value of $m_{\tilde{\chi}_0}$ was taken to be bounded below by the experimental limit[30] of 30 GeV and $m_{\tilde{\nu}}$ by the experimental limit[31] of 25 GeV. (Based upon present experimental



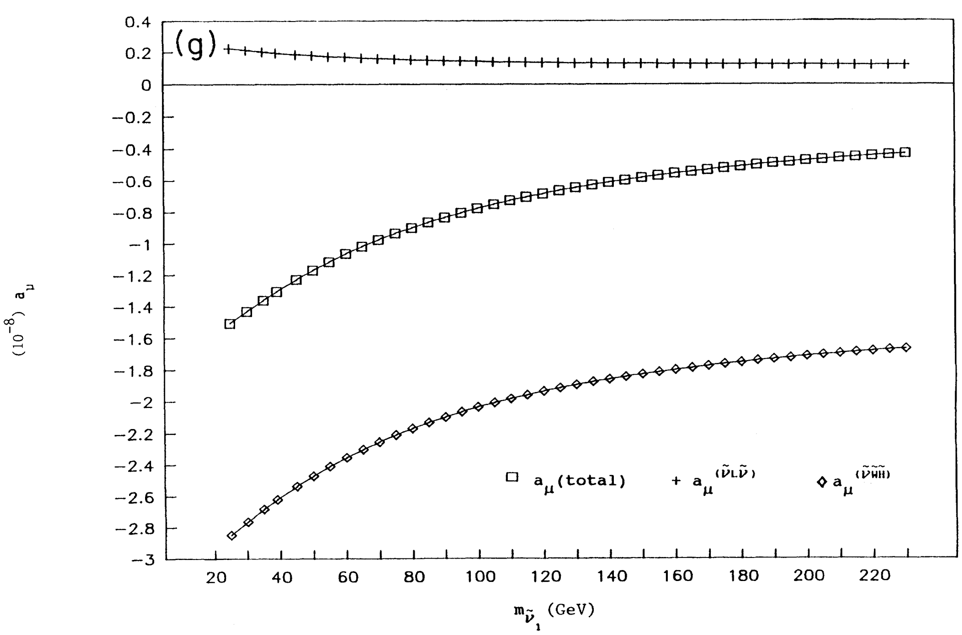


FIG. 3. (*Continued*).

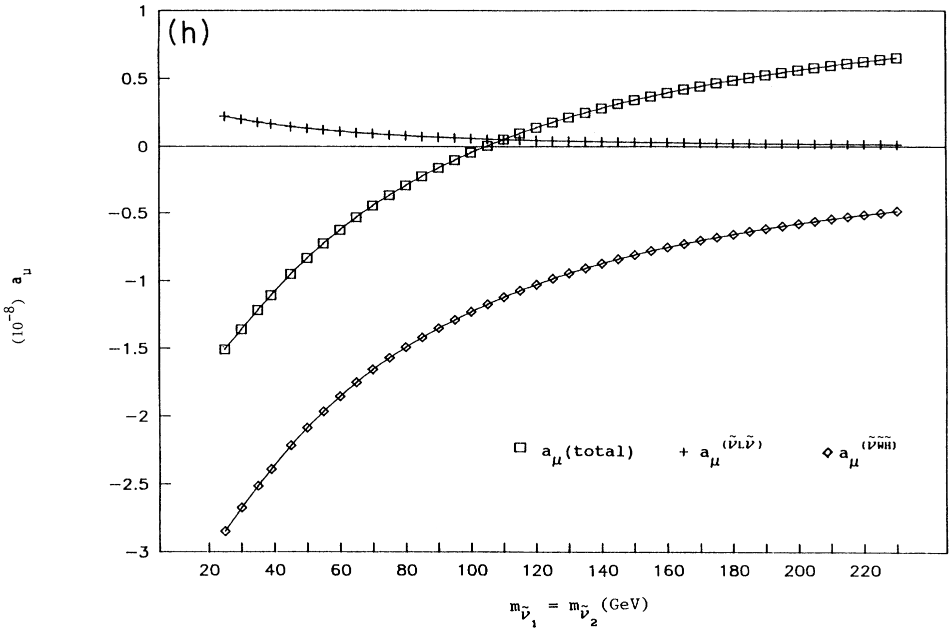


FIG. 3. (*Continued*).

limits[32] of M_{Z_R} and $m_{W_R^\pm}$, the contribution to the anomalous magnetic moment from Z_R and $W_R^\pm$ is expected to be $<10^{-9}$ and is ignored in this analysis. Contributions from $\tilde{\Delta}$ and Δ have also been ignored, although the value of their contributions is an enigma as values of $m_{\tilde{\Delta}}$ and m_Δ are unclear.)

The mass of each of the supersymmetric partners was then varied individually, giving rise to total contributions to the anomalous magnetic moment of the muon from the supersymmetric left-right model and the standard model shown in Fig. 3. These results imply that it is likely that deviations from the anticipated contributions to $(g-2)_\mu$ from the standard model due to supersymmetry will be measured in experiment E821 at BNL.

V. CONCLUSIONS AND PROSPECTS

Since there are many cogent reasons for extending the standard model to a fully left-right-symmetry model, it is imperative to consider a supersymmetric version of this extension to solve the gauge hierarchy problem. Application to the anomalous magnetic moment of the muon opens many experimental possibilities for the future. Also, as there are additional *CP*-violating phases in any supersymmetric model, further investigations into *B* physics and the electric dipole moment of the neutron are in progress.

ACKNOWLEDGMENTS

We would like to thank Dr. R. N. Mohapatra for checking the form of the Lagrangian used in this paper. One of us (C.S.K.) would like to also thank Dr. P. Nath for enlightening discussions about the affect of supersymmetry on this model during a 1-week stay at Northeastern University. The authors would also like to thank the Natural Sciences and Engineering Research Council of Canada for financial support through Grants Nos. A0358 and U0529. One of us (R. F.) would also like to thank Concordia University for direct financial support while working on this project.

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